An Analysis of Mathematical Modelling Competencies of Grade 11 Learners in Solving Word Problems Involving Quadratic Equations

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I. ABSTRACT

This study analysed the modelling competencies of grade 11 learners and also explored the degree to which the learners' competency in setting up a mathematical model inhibits the development of an acceptable solution for word problems.

The research data comprised 30 learners drawn from a secondary school in the Western Cape Province, South Africa. Data was collected via a task-based activity response sheet containing five word problems linked to either one of the following concepts: rectangle, two-digit number, average speed and petrol price. Learners' responses were graded into four categories viz: correct, partially correct, incorrect and no response. Thereafter, the modelling competency framework was used to diagnose the modelling competencies of the sampled learners.

The findings of the study showed that most of the group of 30 learners were able to tackle the first 3 problems (rectangle, two digits and river problems). However, a small number of learners obtained correct solutions (2 for rectangle, 2 for 2 digit and 4 for river problem) compared to a large number who produced partially correct solutions (25 for rectangle, 23 for 2 digit and 24 for river problem). These set of learners demonstrated high competencies in mathematizing relevant quantities and relations by naming them, assigning variables and constructing relationships between variables. The finding revealed that learners who produced partially correct solutions for the first 3 problems experienced the following challenges: lack of knowledge to transform a quadratic equation into standard form; inability to use different methods to solve linear or quadratic equations; inability to multiply out brackets after substitution and also simplify algebraic fractions. Most (80%) of the learners who attempted problems 4 and 5 produced incorrect solutions because they lack essential mathematical modelling competencies right from understanding the problem to validating the solution (if any). Generally, the study has shown that learners struggled to solve more cognitively demanding problems. The learners' inability to construct a meaningful and relevant mathematical model prevented them from moving onto the next steps in the modelling cycle.

Key words: Mathematical model, modelling competencies, quadratic equations

II. DECLARATION STATEMENT

I, Memory Dizha, declare that the study entitled; *An Analysis of Mathematical Modelling Competencies of Grade 11 Learners in Solving Word Problems Involving Quadratic Equations* is the product of my own research. All sources used in the study have been indicated and fully acknowledged by means of complete references.

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Date: 28 February 2021

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` CHAPTER ONE: OVERVIEW OF THE STUDY

1.1 Introduction

Mathematical modelling provides opportunities for learners to use mathematics in solving real-life problems. It has the potential to develop skills and competencies that are essential for twenty-first century learning. In this study, an analysis of the mathematical modelling competencies of grade 11 learners with regards to solving word problems involving quadratic equations is pursued. The participants of this study were grade 11 mathematics learners. This chapter provides a background to the study, rationale for the study, problem statement, and the aims and objectives of the study. An overview of the research design and methodology, research questions, significance of the study, limitations of the study, and operational definitions are presented as well. Finally, a summary of the organization of this study is discussed.

1.2 Background to the study

The Further Education and Training (FET) Phase Mathematics Curriculum and Assessment Policy Statement (CAPS) asserts that: "mathematical modelling is an important focal point of the curriculum. Real-life problems should be incorporated in all sections whenever appropriate and examples used should be realistic and not abstract. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible." (DBE, 2011, p.13). This resonates with the understanding that mathematical modelling offers a platform to use mathematics to solve real world problems. Learners also develop mathematical competencies through mathematical modelling such as making assumptions, computing, interpreting solution and mathematical reasoning (Leong and Tan, 2020).

According to the National Council of Teachers of Mathematics (NCTM, 1980), problem-solving is the cornerstone of school Mathematics. Schoenfield (1992) avers that mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Mathematical word problems pose difficulties for many learners because of the complexity of the solution process they encounter. Children face difficulties in solving mathematical word problems because they often do not comprehend the wording of the problem (Van de Walle, 2004; Jitendra et al 2007). Hence, it is imperative for classroom teachers to motivate learners to move beyond the rules of learning mathematics and develop their capabilities on the usage of the language of mathematics to express things.

A strong grasp of mathematical language influenced the achievement in mathematics that was recorded. In the same vein, it was observed to influence problem-solving during the process of learning new mathematical concepts (Sajadi, Amiripour and Rostamy-Malkhalifeh, 2013). Significant studies have investigated the effect of language on learner achievement levels. The bulk of these studies have focused on the aspect of mathematical modelling of word problems (Ellerton and Clarkson 1996; Fasi 1999; Moschkovich 2002; Adams 2003; Latu 2004; Garegae 2007; Schleppegrell 2007; Sepeng 2013). These scholars have perceived that mathematical language is different from plain English used by learners in schools.

Considering the challenges involved in the teaching of mathematical modelling and applications, the natural tendency is to focus on difficulties encountered in bridging the real world to the world of mathematics. It is well documented that Mathematics performance in primary and secondary schools in South Africa is unsatisfactory (Taylor, 2011; Department of Basic Education, 2012) and that limitations on teachers mathematical content and pedagogical knowledge appear to contribute to learner under achievement in Mathematics (Department of Basic Education, 2007; National Planning Commission, 2011; Carnoy and Chisholm, 2008).

When learners are faced with a problem-solving task, it is imperative for the class teacher to monitor leaner progress and response. Also, the teachers should provide necessary scaffodled assistance so that learners are not left behind. According to Anghileri (2006), scaffolding means "*building on a child's own overt intention within a shared, functional learning environment (as when a parent assists a child), and strategic scaffolding adult deliberately teaches strategies which will enable the child to solve problems posed by a task (as related to lesson planning and the classroom*". Wood (1994) explained that in the process of scaffolding, the focus is on patterns of interaction that draw learners' attention to the critical aspects of word problems. The proces requirs that the teacher ask questions to turn the discussion back, thereby leaving the learners with the responsibility of resolving the tasks in the classroom.

Due to difficulties in the comprehension of the text and the identification of the "mathematical core" of the problem, primary school children frequently engage in a rather arbitrary and random operational combination of the numbers given in the text. In doing so, they fail to recognise the relationship between the given data and the real-world perspective. Failure in solving so-called "real-world problems" is obviously not related to a lack of practice.

In a quantitative study, Renkl and Stern (1994) analysed the data of 568 pupils from a total of 33 German primary classrooms. The scholars realised that the success rate in solving traditional word problems is not significantly improved by repeated practice. According to Winter (1984), real-world problem-solving involves the construction of a mathematical model with respect to the real-world situation, the finding (calculation) of the unknown, and the transfer of the mathematical result derived from the mathematical model to the real-world situation.

Research suggests that the greatest difficulty in this process is the identification of an appropriate mathematical model. This is the case because identifying an appropriate mathematical model requires contextual knowledge of the real-world situation as well as creativity (Winter, 1994). However, the last stage of this modelling process; which is the transfer of the (arithmetical) result in a real-world situation poses unforeseen problems for the school pupils. Word problems emphasize the precise definitions of terms, the formation of only those statements which specifically apply to the issues or objects under argument, and the application of careful thinking in problem-solving. These are all vital skills in any intellectually stimulating profession. They are also relevant in forming thoughtful decisions about political and educational issues, and in creating personal pronouncements.

The majority of previous publications on word problem-solving in mathematics focus on either the quantitative analysis of teaching and learning difficulties or the description of problem types. Some of these studies have concentrated on the level of difficulty of word problem-solving in mathematics, and/or their potential for the teaching and learning of mathematical modelling (Silver, 1995). On the other hand, the mathematical modelling process has not been given much attention in qualitative research on problem-solving in mathematics. Furthermore, Pehkonen (1991) stressed the need for the investigation of problem-solving strategies and modelling processes in mathematics classrooms. According to the scholar, such studies are needed in order to generate systematic knowledge, which is currently based on scientific investigations.

De Villiers (1993) distinguished three categories of modelling: direct modelling, analogous modelling, and explanatory modelling. These distinctions were useful since they could be used to categorize the style of teaching that takes place in most classrooms. Direct modelling can be associated with a traditional teaching approach since it involves immediate recognition and use of a known procedure by students. A limitation of direct model is that it does not capture the essence of the situation and predict outcomes. Direct models can be directly generated in the form of algebraic expressions tables of values or graphs. Analogous modelling entails recognizing a model from a previous situation and applying it to the current or new situation.

Explanatory modelling is more in line with the type of tasks proposed in this study. In this modelling category, students are required to build a model consisting of new ideas and structures. De Villiers (1993) notes that this category is a powerful teaching strategy. Therefore, it was paramount to investigate how the teachers implement this and how the learners are benefiting from it.

1.3 Problem Statement

The Department of Basic Education (DBE, 2011, p. 5) emphasizes that schools should produce learners who are able to identify and solve problems, and make informed decisions through critical and creative thinking. However, learners generally struggle to solve word problems on selected mathematical topics in their tests, examinations, assignments and daily classwork. Learners always regard each word problem as a new experience. As a result, they often fail to connect a given problem to a previous problem that has the same descriptions. Moreover, learners often find it difficult to sort out the important information in a word problem to the extent they cannot identify quantities that influence the situation including key variables. They failed to provide complete solution to complex classroom tasks assigned to them. Hence, this study seeks to track and analyse mathematical modelling competencies of grade 11 learners when they solve a set of word problems with the ultimate goal to enhance the learning and teaching of mathematics via problem-solving using a modelling approach. These word problems require the learners to work independently without the interference of the teacher when solving them. This way, the teacher will offer guidance to learners who are experiencing difficulties by helping them to arrive at the right solutions.

1.4 Rationale for the study

It is important to assess grade 11 learners modelling competencies as these skills are relevant to the 21st century learning skills. By knowing the extent and depth of learners' mathematical modelling competencies, teachers would be able to plan their mathematical lessons by including modelling tasks that help students develop modelling competencies. Hence, this study explored the range of modelling competencies demonstrated by grade 11 learners as they set about solving a set of graded word problems involving quadratic equations.

1.5 Aim and Objectives of this study

The study aimed at investigating the mathematical modelling competencies demonstrated by grade 11 learner when solving word probelms. The specific objectives of the study are as follows:

i. To investigate mathematical modelling competencies demonstrated by grade 11 learners when solving word problems involving quadratic equations.

ii. To explore the degree to which the learners in grade 11 learn a mathematical model that inhibits the development of an acceptable solution.

1.6 Research design and methodology

This section presents the methodological perspectives of the study. It offers explanation on the thesis outline and justification of the chosen research techniques. Qualitative research involves exploring, describing and explaining the data that is obtained. The major aim of the researcher in qualitative research was to appreciate and understand the situation under investigation primarily from the participants' and not the researchers' perspective (De Vos, 2005).

This study adopted observation, problem-solving and qualitative approaches in data collection. These approaches makes it possible to respond to the research problem with greater depth and insight within the context of the research questions (Henning , 2004; Creswell, 2003).

The data was collected using a worksheet containing a set of five-graded word problems that were premised on quadratic equations. The sample consisted of 30 grade 11 purposively-selected learners from a high school in the Western Cape. The selected group of learners was a convenient sample as the researcher was teaching these learners at the school. In addition, observations and focus group discussion were also used for data collection.

Data analysis was conducted using qualitative content analysis method. The learners' responses per problem were broadly classified into the following four types using a holistic rubric (see Table 4.1): no response, incorrect response, partially-correct response, and correct response. Also, learners' responses to each question were classified into each of the four types. Furthermore, second level analysis was performed on the respective type of responses using the modelling competency framework (see Table 4.2) by Blum and Kaaiser (1997) (c.f Maaß ,2004, p. 32). The results were presented in the form of graphs and tables.

According to Sutton and Austin (2015), the instrument of qualitative research is the human mind. Qualitative research captures the ability to explore, describe and explain the data that is obtained. The major aim of the researcher in qualitative research is to appreciate and understand the situation under investigation primarily from the participants' perspective rather than the researchers'.

Firstly, the researcher analysed the task written by participants. The researcher grouped the tasks according to the participants' modelling competencies: those who showed full modelling competencies and those who showed none. Data was analysed by grouping the mathematical modelling competencies namely: non-competent, meaning the participants attempted the problems but did not show any competence; partially competent, the participants demonstrated partial mathematical modelling competences; competent, means they could show all the mathematical modelling competencies. Most of the learners could not interpret the word problems and therefore could not even attempt to answer the questions. Simple tables and column graphs were used to exemplify the data with respect to the first level of analysis.

1.7 Research questions

1. What mathematical modelling competencies do grade 11 learners demonstrate when solving word problems involving quadratic equations?

2. What impact does learners' competency in mathematical modelling have on their ability to provide accurate solution to a problem?

1.8 Significance of the Study

This study aimed at ascertaining the modelling competencies grade 11 learners exhibited as they attempted to solve a set of word problems linked to quadratic equations. As with most 'word problems', the initial challenge was to set up the equation that the learners needed to solve, which required a 'translation' from words to mathematical expressions, and ultimately assimilation into an equation. It is envisaged that data generated from this study will help to provide an overview of the successes and challenges grade 11 learners experience as they traversed the set of problems in a double period. It is envisaged that this could serve as a blue print for the class teacher and other teachers in the system on how design task-based word problems and how to support and enhance meaningful learning in their own classrooms. Therefore, this study also yields information on how teachers and curriculum specialists could utilize modelling on a routine basis in the grade 11 mathematics classes. This particularly will assist the learners during the process of transitioning from word problems (algebra) to quadratic equations, and develop their competencies to validate solutions.

1.9 Limitations of this study

Various limitations were evident in the study despite its chronological and logical structure. The research was conducted in English. Given that the researchers and the participants' mother tongue is not English, it might have led to a deficiency in the interpretation of results and the questions. This was exacerbated by the fact that the learners in grade 11 complained about the problems associated with using English as a language of instruction.

Time factor was also a limitation since the researcher had to carry out the research during a double session period. The educator had to continue with everyday teaching because the study was conducted towards the end of the term. More so, the study was also limited to one secondary school in the Western Cape. As such the findings are limited to the peculiarities of the said school and cannot be generalised to all grade 11 learners.

1.10 Operational definitions

Mathematical modelling competencies: Competence is defined as an individual's readiness to act in response to the challenge(s) of a given situation (Blomhøj and Jensen, 2007). The most important characteristic of this definition is that it makes competence headed for action. Maab (2006) states that modelling competencies include the ability and skill to conduct modelling process in an adequate and goal-oriented manner; as well as the willingness to put these abilities and skills into practice.

Word Problems: Scholars such as Vernooy (1997, p.5) argue that "word problems are part of the mathematics curriculum because they illustrate the connection between mathematics and clear, critical thinking on any subject. Word problems emphasize the precise definitions of terms, the making of only those assertions which specifically apply to the issues or objects under discussion, and the application of careful reasoning in problem-solving. These skills are vital in any intellectually challenging profession, especially in the area of forming thoughtful judgments about political and educational issues, and making personal decisions.

Mathematical modelling: Mathematical modelling is very interesting and yet confusing when first introduced. It goes without saying that there is mathematics everywhere; yet, we find it difficult to apply it. Mathematical modelling is one of a number of reality-based learning activities. Krill, et al. (2002) defined mathematical modelling as the art of "translating problems from an application area into tractable mathematical formulations, whose theoretical and numerical analysis provides insights, answers and guidance which are useful for originating application".

Mathematical model: According to Sekerak (2010), mathematical modelling is a cognitive method where original object or situation is substituted by a model. By examining this model, we gain information that we would normally derive by examining the original object or situation.

1.11 Organization of the study

Chapter 1: An overview of the study: This chapter provides the introduction, background, research questions, rationale to the study, the problem statement and research questions. The significance of the study and an outline of the chapters comprising the study was discussed in this chapter. Research design and methodology was also mentioned in this chapter and in the chapter outline.

Chapter 2: Litearture review: The second chapter focuses on both local and international literature related to the study. A mathematical model is highlighted in this chapter in order to holistically understand how grade 11 learners develop mathematical modelling competencies with respect to quadratic equations. A model is also included to show how modelling competencies are developed. The effectiveness of understanding mathematics is also included as well as the theoretical framework.

Chapter 3: Methodology: This chapter focuses on the research design and the methodology for the study. It elaborates on the qualitative case study design; rationale for the choice of data collection tools, and data collection method. Ethical isues with respect to data collection was also mentioned in this chapter. Detailed explanations are given on the sample size, data analysis procedures and limitations to the study.

Chapter 4: Data analysis and findings: This chapter systematically presents analysis of the data collected from the task-based worksheets, focus group interviews and observations. The findings are summarised with recourse to how they address eacch research problem.

Chapter 5: Conclusions and recommendations: The final chapter provides a discussion of the findings in term of the research questions. The limitations of the study is presented, and is followed by conclusions and recommendations.

1.12 References

All the acknowledged sources of information are listed at the end of the research report according to the APA 6th edition referencing style.

1.13 Conclusion

In this chapter, the study background, problem statement, the rationale for the study, aim and objectives of the study, research methodology, and research questions were presented. In the same vein, discussions on the significance of the study, limitations of the study, operational definitions and organization of the five chapters of this thesis were presented as well.

In chapter 2, Literature review on mathematical modelling matters including problem solving and word problems are presented as well as the theoretical framework which directed the study design is presented.

CHAPTER TWO: LITERATURE REVIEW

1.14 Introduction

This study provides in-depth analysis of existing literature on mathematical modelling and competencies in solving quadratic equations. It reviews relevant literature on word problems in mathematics, the problems encountered while solving mathematical problems and the concept of mathematical modelling. It also addresses mathematical modelling competencies and the relevance of the social constructivism theory to the study.

1.15 Word Problems in Mathematics

Urquhart (2009) argues that word problems are part of the mathematics curriculum because they illustrate the connection between mathematics and clear critical thinking on any subject. Word problems emphasize the precise definitions of terms, the making of only those assertions which specifically apply to the issues or objects under discussion, and the application of careful reasoning in problem-solving.

Thomas, et al. (2015) assert that solutions to word problems in mathematics lie in appreciating the relationship between mathematics, language, and literacy. They argue that there is a need to conceptualise the difficulties faced by the identified students with regard to how they struggle to understand language problems versus mathematical problems. This is an indication that when one looks out for mathematical language as a discourse to aid learning, learners are able to inculcate a receptive and expressive mode that instructively deals with aspects such as solving word problems. The current study places emphasis on students' receptive and expressive difficulties, as well as language demands of mathematics they are faced with (Thomas et al., 2015).

Thomas, et al. (2015) go further to quip that there should be a universal design that deals with mathematical problems. While this study would offer a generic solution that fits all, it is rather presented in a top-bottom manner that does not adequately engage the problems before offering the solutions proposed by universal design. These may include the language used, the fact that it is a second language or its effect on the learning achievements in a society. English is one of the nine official languages in South Africa.

However, a significant number of studies have investigated the effect of language on learning achievement levels in the area of mathematical modelling of word problems (Abedi, 2006; Prediger et al., 2013, 2015; Sepeng, 2013; and Plath & Leiss, 2018). These scholars have alleged that mathematical language is different from plain English used by learners in schools. Learners achievements in mathematics is founded on a strong grasp of mathematical language, which influences problem-solving techniques during the process of learning new mathematical concepts such as quadratic equations.

Central to the need for a strong grasp of mathematical language is the need to evaluate challenges in solving mathematical problems that learners face. Thomas et al. (2015) study scope was limited to the challenges faced in learning mathematics amongst learners with disabilities. The study takes a conscious approach towards receptive language challenges. The first challenge is that the learner who has a problem with the processing of the problem in terms of attempting to listen, hold and evaluate also fails to arrive at logical conclusions. A major challenge for students who have receptive language difficulties is associating accurate meaning with words (Bley & Thornton, 1995). Similarly, Silver (2000) observed that middle grade students need to develop representational techniques for a profound understanding of, and fluency with linear equations. This may be evident in challenges such as the mathematical meanings of words, and the aspects of the mathematical language. In this vein, students may fail to understand particular words which offer instructions (Bryant, Bryant, and Hammill, 2000). However, Krawec et al. (2013) offered a different result using *Solve It!* instructional material. They realised that the *Solve It!* instructional material provides students with a research-validated, and problem-solving routine, that has proven results. It teaches students the processes and strategies required to represent mathematical word problems, and how to apply those processes and strategies when solving problems.

More so, the failure to relate to the receptive challenge of an underdeveloped mathematics language, or the failure to appreciate the basics of the mathematical instruction exacerbates the limited ability to solve word problems (Cawley, Parmar, Foley, Salmon, and Roy, 2001). This problem may be extended to word problems that involve semantic features of mathematics language (Allsopp, Kyger, and Lovin, 2007). For instance, learners may struggle to appreciate concepts with regard to the meaning of words and symbols. The limitation of these studies is the emphasis on learners who present mild disabilities. In addition, the studies focused on learners in less advanced grades (such as grade three) and thus findings may not be applicable to all learners, especially those in advanced grades (such as Grade 11). It follows that failure to understand mathematical terminology may add to the conflation of an aspect of a question for another.

The overriding aspect of receptive language problems is a failure to solve word problems generally (Bley & Thornton, 2001). This problem is largely due to lack of comprehension skills despite a strong command of mathematical language. Cawley et al (2001) state that such learners are not able to answer questions due to their reliance on cue words; which may be misunderstood in light of misconstrued perceptions. However, it is important to ensure that this concept is put to test so as to establish the possibility of a similar outcome in this empirical study. The key to such a solution lies squarely on a classroom scenario where the mathematics teachers intentionally select a mathematical word problem for learners to solve, so as to trigger the teaching and learning process. Intrinsically, the learners extract numbers and sketches through the translation of written language into quadratic or symbolic equation form, and application of learned algebraic manipulative procedures to solve one or more equations. Through modelling, the learners identify one number of reality-based learning activities (Maaβ, 2006).

In another study, Kroesbergem et al. (2004) compared three sets of conditions: traditional explicit instruction, constructivist instruction and a control group based on the regular curriculum in order to identify the benefits on students' fact automaticity and problem-solving. The study concluded that both explicit and constructivist instruction were effective compared to the regular curriculum in automaticity and problem-solving. The results support assumptions that students with learning difficulties, when compared to normal achieving students, can benefit more from instruction that utilizes the explicit teaching of mathematics strategies.

1.16 PROBLEM SOLVING IN MATHEMATICS

1.16.1 The concept of problem-solving.

The concept of problem-solving refers to the process from a given state to a goal state with no obvious method that offers solution (Van Merriënboer and Kirschner, 2017). This is an indication that the person solving the problem should be agile and be able to manoeuver multi-faceted challenges to arrive at a solution. Problem solving challenges can be mitigated by embracing the process other than executing memorised principles to solve mathematical word problems (Ganal and Guiab, 2014).

Other scholars like Stigler and Hiebert (2004) look at it as a resolution of any task which does not present any immediate method to the learner. This is an indication that the learner should have knowledge that he has previously acquired to enable him proffer logical solutions to the problem at hand (Krulik and Rudnick, 1980). It is on this basis that other writers relate this approach to the ability to solve a problem in different situations (Liljedahl, Santos-Trigo, Malaspina and Bruder, 2016). Based on these definitions, it appears that for one to solve a problem, he or she is required to have acquired some mathematical knowledge which can then be applied. Problem-solving is also known as a mathematical process that mitigates intellectual challenges hindering mathematical understanding and development of learners (Raoano, 2016).

1.16.2 Mathematical word problems in problem-solving

The art of solving word problems in mathematics forms a great aspect of the curriculum (Liljedahl, Santos-Trigo, Malaspina and Bruder, 2016). It is on this basis that various countries have set aside resources in the education sector to develop this skill (Jitendra, Sczesniak and Buchman, 2005). Solutions to arithmetic and algebraic word problems is a key factor of the mathematics curriculum. Therefore, it should be noted that mathematics word problems have attracted lots of attention due to the challenges they present to the learners (Tobias, 2005; Chapman, 2003; Chapman, 2004; Smith, Gerretson, Olkun and Joutsenlahti, 2010 and Kilic, 2017).

Word problems are referred to as any mathematical exercise whose contextual information is presented in a text format other than a mathematical notation (Boonen, Schoot, Wesel, Vries and Jolles, 2013; Boonende Koning, Jolles and van der Schoot, 2016). The question that arises is whether this definition describes mathematical word problems, or the exercises attempted by learners. Insights into what mathematical word problem entail can be gathered from earlier definitions by Cai and Lester (2010). Cai and Lester (2010) state that some word problems are not problematic enough for learners, and should only be considered as exercises for learners to perform. However, this definition does not cover the continued challenges that learners face when they attempt to solve such problems. In the same vein, the definitions do not ameliorate the situation. The neglect of such a crucial aspect of the literature calls for a study of such nature.

It is on this basis that other definitions equate solving word problems to a process which involves reading and understanding the problem, formulating a strategy, applying the strategy to produce a solution, and reflecting on the solution to ensure that it produces an appropriate result (Oktaviyanthi and Agus, 2018; Bodner, 1987). In contrast with the above, Boonen and Jolles (2015) and Raoano (2016) note that word problems combination is a subset that has to be solved on the basis of the information about two other sets.

The learner is expected to depict an appreciation of part-whole relationships and the knowledge that the whole set is equal to the sum of its parts. In the final analysis, the comparison of word problems presents a cardinality of one set which has to be computed through a comparison of information about relative sizes of the other set sizes; where one set is the comparison set and the other is the referent set. While the learner may look at the problem as challenging, it should be perceived as surmountable through the exercise of his or her discretion (McIntosh and Jarett, 2000, Johnson et al. 2009).

Taconis et al. (2001) classify mathematical word problems into four major forms. For instance, learners may easily recall an algorithm that offers a solution once it is applied. Provided that mathematical word problems are generic and well-structured, its goal is clear-cut to the learners who need to understand the problem, recall the algorithm, apply it to the problem, and reflect on the solution. More so, learners need to understand the problem, formulate or recall an algorithm, apply it to the problem, and reflect on the solution. The final form is an ill-structured problem whose problems and goals are unclear, not to mention missing information. Hence, the learners have to establish the goal, formulate an algorithm and apply it to the problem. However, other works have focused on learning with respect to mapping math expressions into formal languages (Roy et al. 2016). Ling et al. (2019), in their study, generated natural language rationales where the bindings between variables and the problem-solving approaches are mixed into a single generative model that attempts to solve the problem while explaining the approach taken.

1.16.3 Problem-solving strategies

There are various problem-solving strategies. This study adopts the Polya's problem-solving strategy. This problem solving strategy is based on a model coined by George Polya (1957), and represents one of the earliest problem-solving models. The model presents four main stages: understanding the problem; devising a plan which will lead to the solution; carrying out the plan and looking back or reflecting on the solution as a way of verifying it (Polya, 1957). There is a body of literature that suggest that Polya's model improves the performance of learners in dealing with problems that require the exercise of skills in offering solutions (Olaniyan, Omosewo and Nwankwo, 2015).

Understanding the problem is crucial to an individual's preparation to understanding what the problem entails, and relating the necessary mathematical concepts. This can be done through a consideration of the terminology and notation in the problem in light of its nature; what is required of the learner, what the terms mean and how they offer guidance to the solution sought (MSU, 2010). It is advised that the question is rephrased in one's own, with specific examples that relate to the identified requirements.

Devising a plan that will lead to the solution involves taking time to think with regard to where one starts in the process of dealing with the problem (MSU, 2010). In this regard, one considers how to tackle the question, how to depict the problem in diagrams, how to use variables for the known and unknown aspects, and how to be systematic in one's approach towards developing a pattern in the solution. Once this is done, the learner subconsciously keeps working at it, and tends to be relaxed until a solution is reached. This process is synonymous with a that of a modeller who simplifies the problem by arranging and structuring a model that aids him or her at arriving at the solution (Govender, 2018).

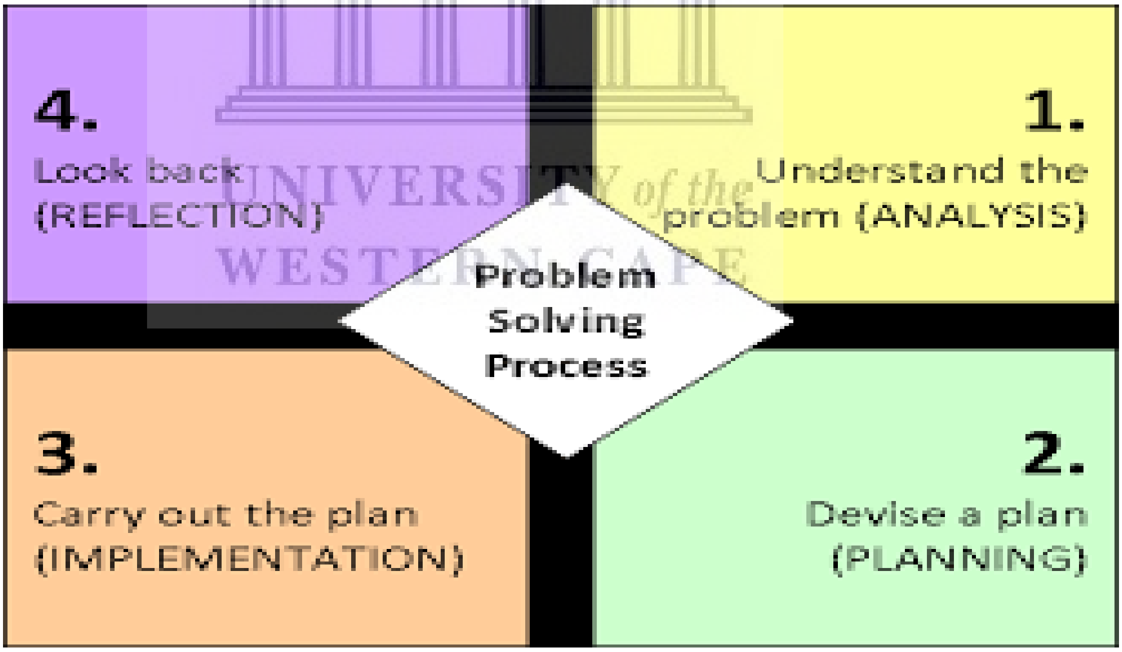


Figure 1.1: Problem-Solving Stages (Process) (Kimberley, 2018)

Carrying out the plan involves engagement of insights using the idea of the approach which the learner has identified (MSU, 2010). This may be done by writing it down, and trying it out until a logical solution is reached. In the event that the plan does not lead to a solution, it is advisable that one starts over with another approach. Solving the problem lies in trying until something works. The person is thereafter expected to look back or reflect on the solution that he has been able to devise.

This is crucial to ensuring that a verification of the correctness of the answer is done (MSU, 2010). The art of verification lies in looking at the solution in order to ascertain whether it works. Some of the questions that are instructive in the course of verification include: whether one answered the question, whether the result is reasonable, whether the conditions related to the problem have been fulfilled, and whether the computations are correct. This requires a lot of flexibility, especially where the initial steps do not lead to the correct answer (MSU, 2010).

1.17 MATHEMATICAL MODELLING

Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations. It is a method whose theoretical and numerical analysis provides insight, answers, and guidance useful for the driving application (Neumaier, 2003). Mathematical models go beyond the physical characteristics of a real-life situation to its structural features through mathematics (Amirgaliyeva, 2012). On this basis, a mathematical model is a representation of real mathematical problem that enable one to develop a better understanding of the problem, and arrive at the logical solution (Govender, 2018; Lesh and Finnwarld, 2010). As a result, the ability to have both creative and problem-solving attitudes is a crucial need that is central to the development of these competencies (Govender, 2017).

1.17.1 The Importance of Mathematical Modelling.

Various reasons validate the importance of mathematical modelling; which stimulates the learner's activities. According to Govender (2018), the engagement of mathematical modelling in real- life situations develops a students' thinking capacity. In an earlier study, Julie (2006) see mathematical literacy as subject that is specifically driven by the application of mathematics in life (Julie, 2006). This implies that anyone who studies mathematics is possible to possess and develop the ability to think critically and systematically in interpreting a situation through mathematical and mathematical modelling.

Blum (2015) avers that mathematical modelling is not a spectator sport, but done by the mathematics learners themselves; to stimulate the process of learning new mathematical concepts. Blum (2015) explains that the most important aspect of mathematical modelling is that learners learn to work independently. In the paper presented by Sanfratello and Dickman on mathematical modelling, the scholars stated that:

> "…Independent work means that the teacher is available, if necessary, and tries to let students work as independently as possible, but not to leave them alone in the desert. A key aspect, which sounds trivial but is in some sense the key to effective teaching, is always to seek a balance between students' independence, on the one hand, and teacher's guidance, on the other hand…" (Sanfratello and Dickman, 2014, p.58).

Smith and Pourchot (2013), state that models describe our beliefs about how the world functions. In addition, these scholars believe that in mathematical modelling, learners translate those beliefs into the language of mathematics, which is observed to have many advantages. Blum (2015) describes mathematical model as the principle of minimal support. Sanfratello and Dickman (2014, p.58) further state that:

> "…here is adaptive teacher intervention, i.e., an intervention which allows the individual to continue his/her work independently, which helps him/her to overcome a cognitive barrier but not more than that, and, in particular, does not prevent mistakes before a cognitive hurdle is even presented. Of course, whether an intervention is adaptive or not can only be judged afterward: Did the student overcome the hurdle? If so, it could have been adaptive or not and if not, it was certainly not adaptive. Let us turn next to another example in the context of the "Filling up" task".

A lot of students make mistakes by assuming the distance travelled is twenty kilometers, forgetting that the return journey needs to be accounted for. A simple but often successful intervention is to say, “imagine the situations concretely, imagine you are Mr. Stein and you drive to Luxembourg” (Sanfratello and Dickman, 2014, p.58). In this case, tutoring students to learn from their own mistakes is part of the cognitive development that mathematical modelling can facilitate.

Haylock and Manning (2014) acknowledge the need to consider mathematical modelling as a key process in teaching and learning mathematics. Wolfram (2013) states that mathematics is a very precise language which helps us to formulate ideas and identify underlying assumptions. It is also true that these processes are important given that mathematics is a concise language, with well-defined rules for manipulations (Lawson and Marion, 2008; Ferri, 2017; Mustaffa, 2017).

Mathematical modelling has been engaged by the Department of Basic Education (DBE) to enable teachers utilise opportunities, and explore the application of mathematics to real-life situations. This way, they can make mathematics more meaningful for learners that are intent on acquiring other important skills. In the meantime, applications involve the use of models in the modelling process. The Department of Higher education and Training (DHET) and the DBE underscore the importance of applications and modelling, due to the vital role it plays in the development and enhancement of mathematical competencies in the classroom (DBE, 2010; DHET, 2010).

1.17.2 Contextualising Mathematical Modelling.

Mathematical modelling is very interesting and yet confusing when first introduced. It is very interesting that we keep saying that there is mathematics everywhere and yet when we have to apply it, we find it difficult. Mathematical modelling is one of a number of reality-based learning activities (Stillman, Kaiser, Blum and Brown, 2013). Most emphasis of mathematical problem is in solving rather than finding an answer that must exist.

Sometimes we may not even be able to solve the problem entirely although we hope to move one step closer to finding the solution. Whereas other scholars believe that whenever the teacher uses an equation, draws a graph or complete a table of values to describe a situation, she/he has a mathematical model (Kühne, 2004).

During the process of modelling, learners are expected to form and manipulate algebraic expressions and formulae, simplify and combine algebraic fractions, rearrange harder formulae and generate further formulae from physical situations given to them by the teachers (Hall and Chamberlee, 2013). In the case of the current study, the Grade 11 learners were expected to find approximate solutions of quadratic equations, a pair of simultaneous equations, one linear and one quadratic depending on the requirement of the word problem they were exposed to at that particular time.

Hence, when one approaches the teaching of mathematics through mathematical modelling, they are really teaching mathematical problem-solving as mathematics is presented in action, instead of as a confusing set of formulae scribbled on the chalkboard. This places mathematics in some context and focus on why mathematics exists in the first place. Moreover, many challenging and exciting skills are used in developing models and these have often been ignored in traditional school mathematics (Abrams, 2001).

Foret (2012) argues that when you use physical contexts to plot and interpret the graphs of linear, quadratic, cubic, reciprocal, and exponential and logarithm functions, the sine and cosine functions and the modulus function are deemed important. Scholars such as Acosta-Tello, (2010) affirm that the learners recognise symmetry properties of functions whether even or odd and also if these functions are increasing, decreasing or stationary then solve problems using inverse and composite functions. In such a situation, the learners are required to work independently without the interference of the teacher when solving word problems. As such, the teachers only come in to guide the learners who are experiencing difficulties to help them arrive at the solution (Acosta-Tello, 2010).

Prominent researchers such as Piaget refer to this process of teaching as scaffolding the learners. Walshaw and Anthony (2008) use Anghileri's (2006, p.36) definition of scaffolding to mean:

> "Building on the child's own overt intention within a shared, functional learning environment (as when a parent assists a child), and strategic scaffolding adult deliberately teaches strategies which will enable the child to solve problems posed by a task (as related to lesson planning and the classroom".

Van Lier (2014) explains that in the process of scaffolding, the focus is on patterns of interaction that draw learner's attention to the critical aspects of word problems with the teacher asking questions to turn the discussion back, leaving responsibility for resolving the problem with the learners in the classroom. Hence, Julie (2013) holds the perception that mathematical modelling familiarizes the learners with the main aspects of teaching and learning mathematical knowledge.

1.17.3 Modelling and Applications

Mathematical modelling can be conceptualized as the art of translating problems from an application area into tractable mathematical formulations. It is a concept whose theoretical and numerical analysis provides insight, answers, and guidance which are useful for originating application (Krill, Glowa, Niles, Parson and Wiita, 2002).

For example, Yang (2013) believes that mathematical modelling is indispensable in many applications. According to the scholar, it gives precision and direction for proffering solution to problems, enables a thorough understanding of the system model, prepares the way for better design or control of a system, and allows the efficient use of modern computing machinery. This line of thought aligns with Hall et al. (2012) who state that the important step from theoretical-oriented mathematical training to the application-oriented typology is that it makes the learner fit for mastering the challenges of modern technological culture in a mathematics classroom.

According to Lakatos (2015), a mathematical problem is considered to be either a pure problem; if the problem situation in question is embedded entirely within 'the mathematical universe or if the problem situation addresses some other disciplines or real-world situations. Other Scholars like Kaur and Dindyal, (2010) argue that using mathematics to solve real-world problems is often called application-oriented mathematics. By implication, real-world situation which can be undertaken by means of mathematics is called an application of mathematics. For that reason, the notion of "applying" is used for any kind of linking of the real world and mathematical problem-solving. Therefore, scholars such as Kelly and Baek (2014) are adamant that the application of mathematics cannot be dissociated from the use of models and the modelling process during the teaching and learning of new mathematical concepts in class.

1.17.4 The Modelling Processes

The process of modelling and application is clearly illustrated in Figure 2.2, indicated as the modelling cycle (Blomhøj and Jensen, 2007).

Earlier on, Blomhøj, (2009, pp.5-6) states that:

> "… The main idea of the educational perspective is to integrate models and modelling in the teaching of mathematics both as a means for learning mathematics, and as an important competency in its own right. Accordingly, classical didactical questions about educational goals and related justifications for teaching mathematical modelling at various levels and branches of the educational system, ways to organise mathematical modelling activities in different types of mathematics curricula, problems related to the implementation of modelling in school culture and teaching practices, and problems related to assessing the students' modelling activities are all been addressed under this research perspective..."

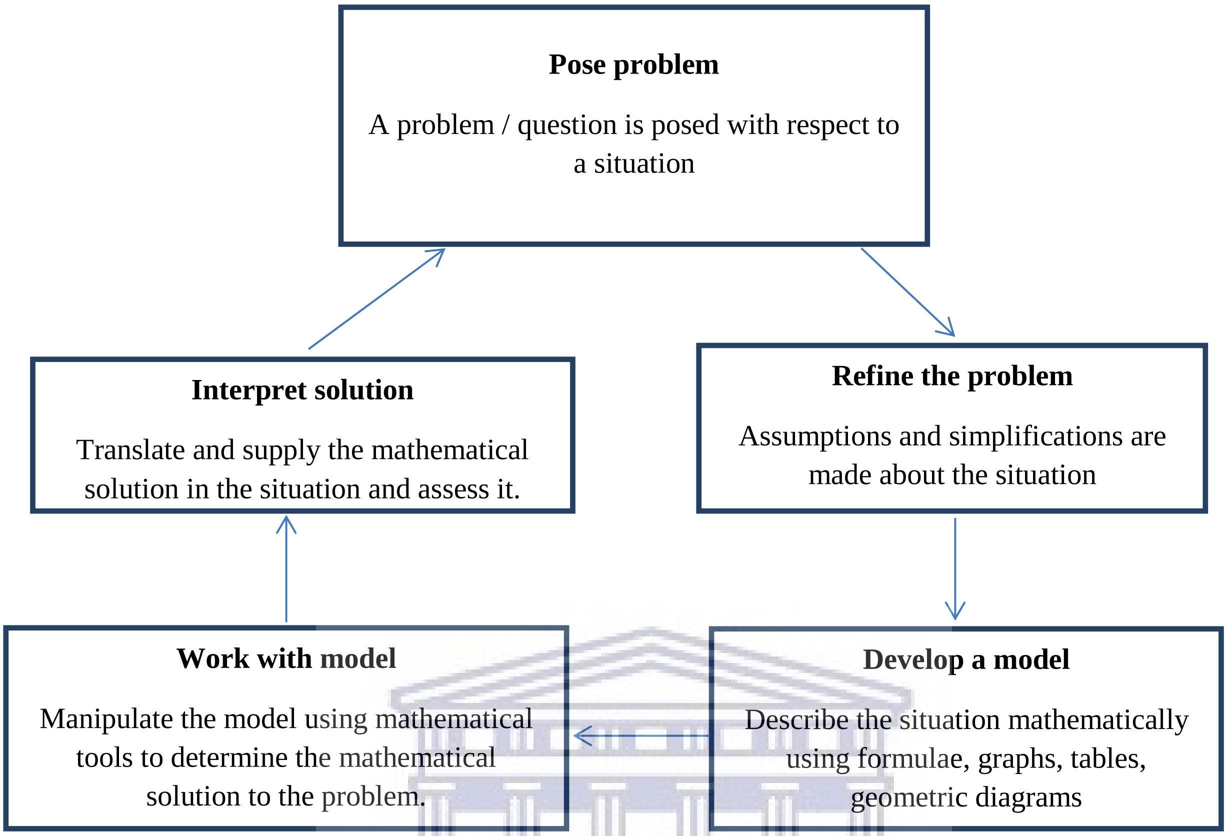


Figure 2.2: Modelling Cycle (Adapted and modified from Blomhøj and Jensen, 2007, p. 48)

On the other hand, Tyler (2013) argued that the problem-solving experiences that children typically meet in schools are no longer adequate for learners and teachers in today's teaching and learning situations. Nevertheless, mathematical problem-solving involves more than working out how to go from a given situation to an end situation. However, Lesh and Lamon (2013) aver that the most challenging aspect of problems encountered by many mathematics teachers in high schools today involves developing useful ways of thinking mathematically about relevant relationships, patterns, and regularities of the mathematical concepts. Bahmaei (2011, p.1) also stated that:

> "…in typical elementary schools worldwide, the teaching of early arithmetic is predominantly focused on computational proficiency. Even word problems that putatively link mathematics and aspects of the real world are often no more than thinly disguised exercises in

> the four basic operations and with the increased importance of mathematics in our ever-changing global market, there are greater demands for workers who possess more flexible, creative, and future-oriented mathematical and technological capabilities, Powerful mathematical processes such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organizing data, provide a foundation for the development of these capabilities, Also of increasing importance is the ability to work collaboratively on multidimensional projects, in which planning, monitoring, and communicating results are essential to success…"

Kaput and Roschelle (2013) agree with Greer et al. (2007) who suggest a new perspective or change in review of the essential pedagogical process when teaching mathematical concepts such as measurements. The introduction of mathematical modelling and applications into elementary schools have positive implications for solving problems associated with the teaching and learning of mathematical concepts.

1.17.5 Effectiveness of Understanding Mathematics

Dixon and Brown (2012) state that mathematical concepts and skills should be structured around the problems which need to be solved. This problem-solving approach seeks to discourage the mastery of, and application of skills while encouraging the resolution of problems. This research does not support the position that presents the ability of the student to solve problems as a function of his or her ability to understand the mode in which the problem is presented. Therefore, the question arises with regard to the failure to solve a problem where it is not presented in a mathematical manner.

Other scholars suggest that students should use a reflective inquiry to facilitate the understanding of problem-solving in the curriculum (Ganal and Guiab, 2014). This position does not cover how the presentation of a problem in English facilitate the understanding of problems in Mathematics. With regard to solving problems that are not presented in a mathematical way, collection of data from the field is instructive because they will either validate this position or inform methods for reform. This is because developing efficient ways of learning is crucial to learner's educational development in South Africa.

Protheroe (2007) are of the opinion that students should be encouraged to work cooperatively with others. Similarly, Sumarna and Herman's (2017) used of group problem-solving to stimulate students to apply their mathematical thinking skills. This position is provided against the backdrop that mathematical concepts such as speed and time depend on the appreciation of the problem and proposing a solution that fits the student's ability to work with their peers. However, this approach does not consider the issues peculiar to mathematics class in settings such as the South African where English is just one of the official languages.

In addition, Sumarna and Herman (2017) advocate for student interaction as a way of challenging each other's strategic thinking. This option does acknowledge the fact that student need to be self-reliant in the process of solving problems. It other words, it suggests that all mathematical problems have to be solved individually. Subject to empirical research, there is a probability that this approach does not support the development of individual capacities. It is on this basis that specific mathematical problems, such as the concepts of speed, distance and time are addressed in the research in order to ascertain how students' understanding of these concepts would differ from an individual's perspective. We hope that this approach will offer nuanced ways of exploring, extending and evaluating their progress from an individual perspective.

Ashley (2016) engages three critical components that are instructive to effective mathematical modelling from a teacher's perspective: teaching for conceptual understanding, developing children's procedural literacy and promoting strategic competence through meaningful problem-solving investigations. The authors do not create a distinction in the presentation of a problem in a mathematical way and in the English language.

In another study, Protheroe (2007, p. 52) makes an important comment about students' developmental challenges in learning mathematics when he stated that they are 'forming conclusions about their mathematical abilities, interest, and motivation which will in turn influence how they approach mathematics in later years.'

This is an indication that the way they comprehend solutions to problems that are presented in both Mathematical and English modes should be geared toward understanding the problem. Regardless of the mode through which mathematical problems are presented, students should be able to think hypothetically, and depict reasoning in logical terms. Protheroe's (2007) submission that students should be able to reason in both concrete and abstract terms do not clearly indicate how this can be done in solving a mathematical problem, and it is on this basis that a logical solution is encouraged.

1.18 MATHEMATICAL MODELLING COMPETENCIES

While modelling a real-world problem, we move between reality and mathematics. Competence is defined as someone's insightful readiness to act in response to the challenges of a given situation (Blomhøj and Jensen, 2007). The most important characteristic of this definition is that it makes competence headed for action. However, Julie (2006, p.63) is convinced that the National Curriculum Statement (Grades 10 - 12) on Mathematical Literacy only favors competence to open criticism to mathematical models and applications, yet "Mathematical Literacy is embedded in mathematical modelling and applications" (Julie, 2006, p. 63). He adds that the intended critical competence can only be realized in the mathematical applications and modelling component of the Mathematical Literacy curriculum in South Africa.

Therefore, Julie (2012) believes that mathematical modelling intimates the learners with the main aspect of teaching and learning mathematical knowledge. Laurillard (2013) on the other hand suggested that mathematical modelling brings out the main aspect of teaching which is grasping mathematical knowledge for a purpose. Julie (2013) argues that the general classification of mathematical knowledge is firstly conventions, representations, and notations. Secondly, it is the concepts and definitions; that is equations.

1.18.1 Approaches to Mathematical Modelling Competencies

Artigue and Blomhøj (2013) agree with Barbosa (2006) who averred that there are different theoretical frameworks and research agendas concerning mathematical modelling which originated from different mathematical modelling perspectives. Julie (2002) posited that mathematical modelling as content emphasises the development of competencies to model real-world situation. Moreover, the mathematical model as a vehicle is understood as a way to teach mathematical concepts to learners. Schoenfeld (2014) argues that models are used to describe our beliefs about how the world functions. In this case, the learners in grade 11 will use mathematical modelling to translate their beliefs into the language of mathematics to solve word problems.

During the process of modelling, the learners are expected to form and manipulate algebraic expressions and formulae, simplify and combine algebraic fractions, rearrange harder formulae and generate further formulae from physical situations given to them by the teachers (Kaput et al., 2013). In this study, the Grade 11 learners will be expected to find approximate solutions of quadratic equations, a pair of simultaneous equations, one linear equation, and one quadratic equation depending on the requirement of the word problem they will be exposed to.

Vernooy (1997, p.5) argues that "Word problems are part of the mathematics curriculum because they illustrate the connection between mathematics and real world. Word problems emphasize the precise definitions of terms, the making of only those assertions which specifically apply to the issues or objects under discussion, and the application of careful reasoning in problem-solving. These are all vital skills in an intellectually challenging profession that are necessary in forming thoughtful judgments about political and educational issues, and in making informed decisions. It is on this basis that the principles of mathematical modelling are used to develop the analytical skills of students in addition to solve word problems.

According to Schoenfeld (2014), mathematical competency is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. The following cognitive modelling competencies were considered in the main study and they include understanding, simplifying, mathematising, working mathematically, interpreting, validating, presenting and arguing (Frejd, 2013).

A mathematical modelling competency is the ability to identify the relevant questions, variables, relations or assumptions in a real-world situation, to translate this into mathematics and interpret and validate the solution. Competence is the sum of available abilities and skills and the willingness of a learner to solve a problem and to act responsibly concerning the solution (Weinert, 2001; Clements, and Sarama, 2004; Henning and Keune, 2007). Modelling competencies encompass all the abilities and skills a learner uses in solving a modelling problem.

In addition, competencies have a sphere of exertion; that is, a domain within which competency can be brought to maturity (Vidhi, 2014). This does not mean that competency is contextually tied to the use of specific methods for solving a given task. If this was the case, the attempt to define general competencies would have no meaning. Competencies are only contextual in the sense that they are framed by the historical, social, or psychological circumstances of the "given situation" mentioned in the definition of competence (NRC, 2002).

Mathematical modelling competency is in accordance with the general definition of competence. Competence is defined as the insightful readiness to go through the whole mathematical modelling process in a given situation (Kjeldsen and Blomhøj, 2013). The competencies are a declaration of assumptions, building a model based on assumptions, testing and validating of the model, possible improvement of the model, and summarizing what the various authors are saying about mathematical modelling.

Eli et al. (2013) carried out a series of studies in mathematics and the results showed that schemata exhibited by teachers who have a well-developed pedagogical content knowledge prepared them for their job. Hogan et al. (2003) compared expert and novice teachers with the aim of providing insight into the understanding of mathematics teachers. In their work, they acknowledged the importance of extensive content knowledge and explained that there are differences between novice and expert teachers in terms of teacher-focus or student-focus during the presentation of lessons in class.

Furthermore, Hogan et al. (2003) argued that further research is needed to determine pedagogical strategies for novice teachers to obtain mathematical representational skills of expert teachers. Research comparing the questioning patterns of novice and experienced mathematics teachers has not been done. Such studies will contribute to a more specialized understanding of a teacher" (McAninch, 2015, p. 20).

Frejd, (2013) argue that mathematical modelling competence means being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context. In his explanations, he stated that it coheres with the understanding of the concept of competence. Blomhøj and Jensen (2003) continue to give several aspects of the definition that must be mentioned in order to fully capture the essence of mathematical competency. In addition, the scholars posited that:

> "…Firstly, competence is headed for action. We use 'action' in a broad sense, as the term 'readiness to act' in our definition of competence could imply a positive decision to refrain from performing a physical act, or indirectly being guided by one's awareness of certain features in a given situation. But no competence follows from being immensely insightful if this insight cannot be activated in this broad interpretation of the word action. Secondly, all competencies have a sphere of exertion, i.e. a domain within which the competence can be brought to maturity. This does not mean that competence is contextually tied to the use of a specific method for solving a given task…" (Blomhøj and Jensen, 2003, p.126).

Boud, Keogh and Walker (2013) also argue that competence is someone's insightful readiness to act in response to the challenges of a given situation in a learning process. As such, the most significant characteristic of the definition given by these scholars is that it makes competence headed for action. According to Killen (2015, p. 419), the definition of competence embodies the idea of a person, who adequately demonstrates knowledge, skills and abilities. Eraut (1998) defined competence as an ability to perform a task and the roles required for the expected standard. Killen (2015) proposed that assessing competencies differ from content-driven assessment in that it requires an assessor to have an explicit description of the performance

However, it has been argued that this will only become a mathematical competence only when the challenges mentioned in the definition of competence becomes a mathematical challenge (Edward and Mecer, 2007). Interestingly, presenting both mathematical competence and challenge becomes a straightforward and uninteresting extension of the general definition of competence. Blum et al. (2006) are adamant that you can only make the analysis interesting and binding for the development of mathematics concepts such as measurement when there is a need to be more specific, to analyse and discuss the elements that constitute a mathematical challenge.

This way, one will create a connection which will result in a focus of definitions of mathematical competency as someone's insightful readiness to act in response to certain kinds of mathematical challenges posed by a given situation (Blomhøj and Jensen, 2003). Subsequently, the literature on mathematical competence can be improved upon by researching other mathematical competencies. Such scholarly works harbor potential to be innovative because student could be given priority, and come up with more suggestions and examples of such elements.

1.18.2 Understanding Mathematical Modelling Competencies

Mathematical competencies derive from an understanding of how models are used. It finds expression in situations where the modelling process plays a key role in the translation between the real world and mathematics in both directions (Blum and Feri, 2009). A proper understanding of mathematical modelling competencies requires that an individual engages a problem which requires such competencies to be applied. Dede and Bukova-Güzel (2018, p.33), while citing Schleicher (2012) stated that 21st -century students are expected to understand mathematical concepts; translate new problems into mathematical ones; make them amenable to mathematical treatment; and identify relevant mathematical knowledge.

To this end, an understanding requires that the individual is able to interact with concepts such as real word problems; real models; mathematical models; mathematical solutions; interpreted solutions; simplifying, mathematising, working with mathematics, as well as interpreting and validating. At the center of this interaction should be the ability to distinguish between all ten concepts.

These ten concepts are largely grouped into two. The first group is referred to as the reality, while the second group is referred to as a mathematics group (Maaß, 2006). The reality consists of the real world problem that has to be understood by the person attempting to solve it. Any attempt to understand and solve it requires construction and simplification of the problem into a real model (Blum & Ferri, 2006). The application of the mathematics model is in the use of mathematical concepts like equations that have to be solved (Maaß, 2006). The solution that is arrived at when the mathematical model is put into operation is referred to as the mathematical solution.

The foregoing solution is then interpreted by the problem solver, which entails a crossover from mathematics to the real group (Maaß, 2006). The essence of interpretation is to convey a solution to the problem in a simplified manner. The final step is a validation of the answer by testing the solution to ensure that it solves the real problem without offering different solutions (Maaß, 2006).

The outcome of such interaction leads to the engagement of the competencies that require an individual to do the following:

A. Understand the real problem and to set up a model based on reality.
B. Set up a mathematical model from reality.
C. Solve mathematical questions within this mathematical model.
D. Interpret mathematical results in a real situation.
E. Validate the solution.

It is expected that engagement with these competencies should be intertwined with the problem-solving strategies by Polya (1957) which requires that one uses the four strategies of understanding the problem; devising a plan; carrying out the plan and reflecting on the solution (Polya, 1957).

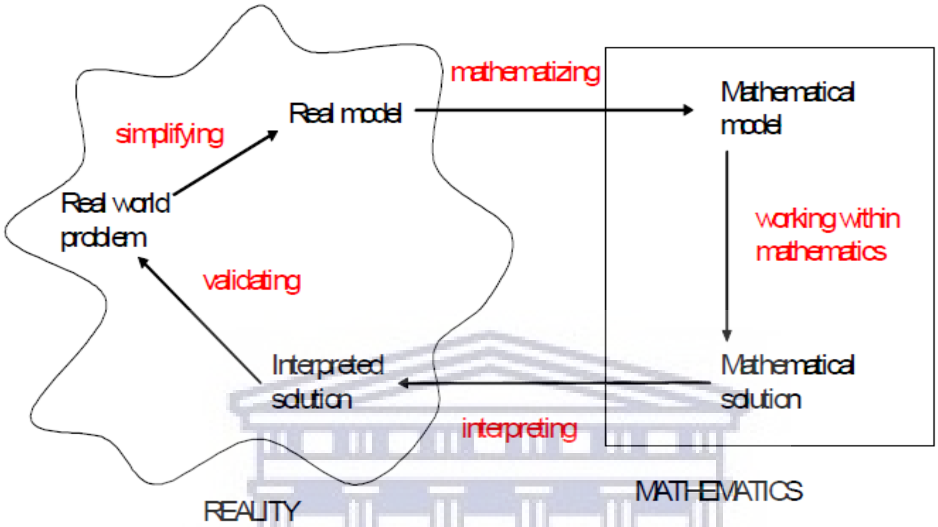


Figure 3.3: Modelling Process (Blum, 1996, p.18)

A look at the above proceeding explanation shows that the solution to this problem is simplified by developing a real model. Furthermore, the mathematic model is engaged with mathematical solutions that lead to interpreted solution. This solution is then validated by testing it against the real problem. It is hoped that the validation confirms the interpreted solution as the correct solution to the real-world problem.

1.18.3 Studies Focusing on Modelling in School Mathematics

There are various studies that have been used to solve word problems through mathematical modelling using equations and other mathematical methods. These studies that have basically been carried out in primary, secondary and other institutions, and are instructive in as far as they add relevance to the use of modelling in mathematics.

Julie (2020) used quantitative survey design to examine the mathematical modelling competency displayed by a cohort of South African learners which comprised of 471 grade 10 and 200 grade 12 learners in the school mathematics programme in South Africa. The study showed that there were significant differences for the overall modelling competencies. In addition, the mathematical modelling competencies were found to be low for both groups. Significant differences between the groups were established in the four competencies but not all in favour of the grade 12 group.

De Almeida (2018) stated that when given a mathematical problem, it is best to regard modelling as a way to deal with mathematical problems, and to strengthen the use of mathematical modelling activities in the classroom. Furthermore, from de Almeida's (2018, p.19) viewpoint, the introduction of classes should take the following modelling activities into account.

The guiding research question is whether students can co-operate to develop confidence and competence in both written and verbal communication. The authors employ an evaluation methodology that uses given scrutiny and criteria to evaluate the program against particular criteria set at the beginning of the study. It was established that while the students found it interesting to use mathematics to get solutions to real-life problems, the greatest challenge was identifying the correct variables due to the problem of selecting irrelevant factors, thereby leading to wrong choices.

While this project engaged mathematical competency, it was just part of a larger aspect of components that the study seeks to establish. Hence, the questions on modelling were not given adequate attention. This study seeks to depart from that position and offer better scrutiny.

1.19 THEORETICAL FRAMEWORK

A theoretical framework is defined as

> "…The foundation from which all knowledge is constructed (metaphorically and literally) for a research study. It serves as the structure and support for the rationale for the study, the problem statement, the purpose, the significance, and the research questions. The theoretical framework provides a grounding base, or an anchor, for the literature review, and most importantly, the methods and analysis…" (Osanloo and Grant, 2014, p.12)

In the same vein, researchers such as Maxwell (2013) argue that a theoretical framework provides a well-supported rationale to the study conducted. It helps the reader to clearly understand the researcher's perspective.

In keeping with the aforementioned nuances, the theoretical framework of this study embraces social constructivism.

1.19.1 Constructivism

The researcher adopted a constructivist theory. The theory stipulates that knowledge is constructed through what scholars such as Piaget believes humans make meaning of the world based on their experiences and their ideas. The current study interrogates how learners interact with their experiences and ideas when modelling quadratic equations from mathematical word problems.

The researcher believes that constructivism provides a theoretical framework for the study because it uses a modelling approach as the learning technique during the process of data collection. Through this theoretical framework, the researcher may be able to better understand how the learners model quadratic equations from mathematical word problems. Constructivist thinking has been very influential in learning generally, although these influences are rarely made explicit to problem-based learning. This section of the theoretical framework will discuss constructivism as a theory and how it has influenced learning mathematical concepts. Also, the implications of constructivism in the modelling approach will be discussed with recourse to how learners' model quadratic equations from a mathematical word problem.

1.19.2 What is Constructivism?

Phillips (1995) states that the term constructivism is often used to describe different theories by researchers globally. Richardson (2003) defines constructivism as a theory that sees human knowledge as constructed by individuals within social communities, disciplines and groups.

It is believed that the varied representations of constructivism emerge from responses to two major issues, which initially arose from the basis of constructivism. Adopting constructivist beliefs implies a rejection of the perception that knowledge is "discovered" by humans, and is passively acquired. Nevertheless, scholars such as Phillips (1997), Prawat and Floden (1994) and Von Glasersfeld, (1984) believe that this is an epistemological position which presents the first point of divergence amongst constructivists. The central debate about the extent of commitment to the notion is that knowledge is either made or discovered. This implies that learners will generate new knowledge when modelling quadratic equations from mathematical word problems or discover knowledge from the same mathematical word problems.

Furthermore, other scholars align with earlier argument that the area of contention centers around the focus of attention on the individual learner, and cognitive processes associated with the construction of an individual's own knowledge (Phillips, 1997; Prawat and Floden, 1994; Von Glasersfeld, 1984). For example, as the learners solve a mathematical problem individually, they construct their own individual knowledge using mathematical concepts given to them to solve. By so doing, the different forms of constructivism can be represented as adhering to different positions on these issues, therefore distilling a single description of constructivism as a learning theory is a difficult and contentious task to solve.

The argument of Von Glasersfeld (1984) was particularly influential in the science and mathematics; where his version of constructivism gained the roots from Piaget's radical constructivism. Based on the previous categorizations by Phillips (1997), the radical constructivism would be incorporated under psychological constructivism.

1.19.3 Types of Constructivism

It was noted that Phillip (1997) attempted to divide constructivism into two different groups: social constructivism and psychological constructivism. This provides a summarised categorisation in a complex and diverse part of the theory. The major arguments of the two categories is based on the assumption that meaning or knowledge is constructed by individuals in society. However, social constructivism focuses attention on the economic, social and political arena within which the knowledge has been constructed. In the same way, psychological constructivism is more concerned with the creation of meaning at an individual level and how meaning develops as formal knowledge within groups of people (Creswell, 2013).

On the other hand, Wertsch (1985) and Brophy (2002) who believed that social constructivists derived predominantly from Vygotsky (1994) and have made substantial contributions to these two major constructivist approaches and that are linked to variations in the goals of teaching and learning mathematical concepts. However, scholars such as Vadeboncoeur (1997) were adamant that Piaget's constructivism is aligned with an emphasis on education for individual cognitive development while Vygotsky's (1994) constructivism is aligned with an emphasis on education for social transformation.

1.19.4 Rationale for Social Constructivism for this study

Learners in a mathematics class engage in similar processes of cognitive development as they solve mathematical problem, tasks and construct mathematical knowledge. The questioning categories and associated theoretical framework transfer appropriately to the field of mathematics especially in High Schools teaching and learning.

The learning of cognitive development of learners in the field of mathematics is helpful to think in terms of cognitive representations as well as external representations of mathematical concepts such as quadratic equations. As learners use modelling skills, they construct mathematical meanings which undergo some kind of interpretive activity for them. Cobb et al. (1992) outline three features for the representational view of the human mind. First, the goal of instruction is to help learners construct mental representations that correctly or accurately mirror mathematical relationships that are located outside the mind in instructional representations.

In addition, the method for achieving this goal is to develop transparent instructional representations that make it possible for learners to construct correct internal representations. More so, external instructional materials presented to learners are the primary basis from which they build mathematical knowledge. Therefore, it is noted that when considering this representational view, there is an obvious piece missing: guidance from the teacher in helping students make a connection between materials and internal construction of conceptual understandings.

Constructivists believe that social interaction plays an important role in learners' mathematical learning process. However, the teachers might then consider the various ways that learners actively interpret the mathematical word problems. Teachers can go about this by paying attention to how they convert them into quadratic equations with recourse to the social context of the classroom. According to scholars such as Benson (2013), there is a need for learners to develop constructive competence through social interaction with their teachers and classmates. He recommends negotiation of mathematical meaning between teachers and learners through discussion of solved tasks, learning by contrasting with negative instances, and discussion of underdetermined tasks in the classroom environment. The educators in mathematics classrooms serve a similar navigator's role as in science classrooms. They perform this role by helping learners navigate between everyday understandings and the world of mathematics as they transition to the concepts, symbols, and practices of the mathematics community.

In the current study, this theoretical argument becomes important because ultimately it has an impact on teachers and teaching as well as learners and learning because they have to model the word problems into quadratic equations and solve them. For this reason, one needs to understand that constructivism is a theory that describes learning. It is not a method of teaching. It is important to note that constructivism does not suggest how an individual learner should learn, but offers an account of how learners construct knowledge in the process of modelling mathematical problems. Constructivists automatically provide a prescription for principles of teaching and learning new ideas, especially when modelling mathematical problem.

Tobin and Tippins (1993) posit that the researchers endeavour to use constructivism more as a reference to analyse learning through new ways of solving mathematical problems by grade 11 learners.

1.19.5 Mathematical Modelling Competency and Sub-Competencies Framework

For the purposes of this study, mathematical modelling competency is defined as one's ability to traverse the respective phases constituting the modelling process when solving a given problem (as indicated in Figure 2) (Blum, 1996). Essentially, the five broad mathematical competencies can be tabulated as follows:

A. Competency to understand the real problem, and to set up a model based on reality.
B. Competency to set up a mathematical model from reality.
C. Competency to solve mathematical questions within this mathematical model.
D. Competency to interpret mathematical results in a real situation.
E. Competency to validate the solution.

Specific sub-competencies (sub-skills and abilities) exist for each modelling competency (A-E). Blum and Kaaiser (1997, p.9) as cited in Maaß (2006, pp. 116-117) provides the following detailed list of sub-competencies that permeates each phase (or competency) of the modelling process:

A. Sub-competencies to understand the real problem, and to set up a model based on reality.

- To make assumptions for the problem and simplify the situation.
- To recognise quantities that influences the situation, name them and identify the key variables.
- To construct relationships between variables.
- To look for available information and to differentiate between relevant and irrelevant information.

B. Sub-competencies to set up a mathematical model from reality:

- To mathematize relevant quantities and their relations.
- To simplify relevant quantities and their relations if necessary, and reduce their number and complexity.
- To choose appropriate mathematical notations, and to represent situations graphically.

C. Sub-competencies to solve mathematical questions within this mathematical model:

- To use heuristic strategies such as division of the problem into part problems, establishing relations to the similar or analogue problem, viewing the problem in a different form, varying the quantities or the available data, etc.
- To use mathematical knowledge to solve the problem.

D. Sub-competencies to interpret mathematical results in a real situation:

- To interpret mathematical results in extra-mathematical contexts.
- To generalize solutions that was developed for a special situation.
- To view solutions to a problem by using appropriate mathematical language and/or communicate the solutions.

E. Sub-competencies to validate the solution:

- To critically check and reflect on found solutions.

- Some parts of the model or crosscheck the modelling process if solutions do not fit the situation.
- To reflect on other ways of solving the problem or explore the possibility of developing solutions differently.
- To generally question the model.

1.20 SUMMARY OF CHAPTER 2

This chapter has presented a literature review and a theoretical framework to guide the study. In this regard, it has reviewed the literature on word problems in mathematics, the problems encountered in solving mathematical problems and the concept of mathematical modelling. It has also addressed mathematical modelling competencies and showed the relevance of the social constructivism theory to the study. The next chapter will present and discuss the methodology and theoretical framework which guided the data collection.

CHAPTER THREE: RESEARCH METHODOLOGY

1.21 Introduction

This chapter outlines the methodology that guided data collection, analysis and interpretation. A research methodology is a logical plan that enables the researcher to provide answers to the research questions assosciated with a research problem (Yin, 2009). Therefore, this chapter provides pertainent details concerning the research methodology employed in this study. It starts by indicating the qualitive research approach adopted for this study. Other aspects of the methodology includes the research design, data collection methods and data analysis strategies and research ethics.

1.22 Qualitative Research

According to Yilmaz (2013), there are two major ways of doing research in social sciences: quantitative and qualitative research methodologies. This study employed the qualitative research method which involves exploration, with the aim of gaining an understanding of reasons, opinion, and motivations for a given research theme. Qualitative research approach focuses on non-numerical data and the research information is gathered from the phenomenon being explored, examined or investigated. The researcher carrying out a qualitative research seeks to provide an understanding of the phenomenon by analysing participants' responses, experiences, and views.

According to Sutton and Austin (2015), the instrument of qualitative research is the human mind. Qualitative researches involves the use of interviews, focus group discussions, observations and a review of existing documentation. The major aim of the researcher in qualitative research is to appreciate and understand the situation under investigation primarily from the participants' viewpoint rather than the researcher's. Due to the focus on the participant as the source of data, the researcher becomes the primary instrument for data collection and analysis in qualitative research (Hancock & Algozzine, 2006).

In qualitative researches, the researcher is the main instrument of research and his or her interpretation of the data leads to the creation of information or knowledge. This is an indication that the researcher has not drowned the voices of the participant offering the information or data to be captured. Qualitative research captures the ability to explore, describe and explain the data that is obtained. This can be done through case studies. The use of case study embraces a topic under study through intensive analysis and description of a single unit or system bounded by space and time. Topics often examined in case studies include individuals, events and groups.

Through case studies, researchers hope to gain an in-depth understanding of situations and the meaning of those involved. According to Merriam (2001) insights gleaned from case studies can directly influence policy, procedures and future researches. Although case studies are discussed extensively in the literature and employed frequently in practice, little has been written regarding the specific steps one may use to successfully plan, conduct, and share the results of a case study (Hancock and Algozzine, 2006).

When qualitative research is used, trends in thoughts and opinions are uncovered from the sample. The sample size is typically small, and respondents are selected to fulfil a given quota to extensively explore and investigate the subject matter. The qualitative research approach is relevant to this study because grade 11 learners were interviewed and assessed in order to ascertain their competency in mathematical modelling.

1.22.1Research Design

A research design is defined by McMillian and Schumacher (2010) as a plan for selecting subjects and data collection procedures to answer the research questions. Given that this study adopts a qualitative approach, a case study design was adopted. A case study is defined as a single instance of a bounded system, for example, a child, a clique, a class, a school, or a community. A Case study involves profound and thorough interrogation and examination of an authentic phenomenon in a natural context (Cresswell, 2015). It provides a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply by presenting them with theories or principles. Indeed, a case study can enable readers to understand how ideas and abstract principles can fit together. Case studies can penetrate situations in ways that are not always susceptible to numerical analysis (Cohen, Manion, and Morrison, 2000; Gerring, 2006). Case studies are single-subject designs or single-case research designs.

A case research design is most often used in applied fields of psychology, education, and human behaviour in which the subject serves as her own control, rather than using another group. In this case, the study was situated in the mathematics education and a grade 11 mathematics class formed a single case study for data collection. In educational setting, the use of case study approach as a research tool has gained widespread prominence. In this research, the case is the worksheet with word problems which learners were expected to solve at a high school in Western Cape township. Observations were done by the researcher and learners were interviewed. This is the reason why the researcher opted to use the case study for the current study in order to gain insight of what is happening in that mathematics class.

1.22.1.1 Strengths and weaknesses of the case study approach

This study adopted case study design with a greater reliance on qualitative data. According to Cresswell (2015), a case study provides room for the interrogation of a phenomenon in a natural context. The information generated from case studies leads to rich and in-depth insights into the cases in question (Yin, 2009). According to Bassey (2004) the term "case study" is ill-defined because it is not seen as an experiment or a survey but as an investigation of one, or a few cases in naturally occurring situations. In this research, the case study is on the mathematical modelling competencies of grade 11 learners with respect to algebraic expressions

Good educational research through a case study leads to trustworthy outcomes (Bassey, 2004). A descriptive case study was chosen for this study because it provides the leverage to describe the unobstructed phenomena emerging from the collected data, including a description of the data collected (McMillan and Schumacher, 2012). In this research, the case was the worksheet containing the set of word problems that learners were expected to solve. Yin (2009) stated that in a descriptive case study, the researcher must determine the analytical component of the study before the study sets off. The component of analysis in this study was the Grade 11 learners from school X in Western Cape.

1.22.2 Sampling Procedure

Sampling refers to the extraction of a representative population of data or respondents from a general population for the purpose of obtaining or eliciting information (Babbie & Mouton, 2004). There are various sampling techniques which may be adapted to select samples from a population. These methods include purposive sampling, quota sampling, and snowball sampling.

Purposive sampling finds expression when the participants are grouped in preselected criteria relevant to a particular question. According to Given (2008), adopting purposive sampling signifies that the researcher sees sampling as a series of strategic choices about whom, where and how the study can be done. Examples of purposive sampling used in qualitative research include stakeholder sampling, extreme or deviant case sampling, typical case sampling, paradigmatic case sampling, maximum variation sampling, criterion sampling and theory-guided sampling.

The study focused on 30 learners of grade 11 from school X in Cape Town. The researcher chose this school because this is where the researcher works and it also spared the researcher the cost of traveling. It also ensured that the school system is spared the incovenience that comes with requating additional time to conclude the research. Stangor (2011, p.110) defines a representative sample as one that is approximately the same as the population in every important respect. The researcher selected the grade 11 class because the researcher teaches mathematics to that class and had the intention of using that grade for the study. The class generally struggled when working on word problems so the researcher identified this as trend that should be investigated.

1.22.3 Data Collection

The problem under study used both in-depth probing and focus group to holistically investigate the development of mathematical modelling to enhance the competencies of grade 11 learners with regards to quadradic questions from mathematical word problems. Data gathered from a single source via a single gathering method would be insufficient and would not lead to valid conclusions. Hence, to deal with this problem, it was imperative to adopt different primary and secondary data collection methods such as observations, worksheet and interviews. The approach devised was validated by McMillian and Schumacher (2010) who state that 'strategies for collecting data such as worksheets, observations and interviews, makes it possible for researchers to gather data needed for a particular study.'

Creswell, (2003) refers to content analysis as various procedures involved in analysing and interpreting data generated from the examination of documents and records relevant to a particular study. The researcher used a task-based activity worksheet containing 5 graded word problems associated with the grade 11 topic quadratic equations. The learners worked on the task activity given by the researcher. In this case, learners' work were kept and analysed in a bid to obtain more information.

Observation was another technique used to collect the data. Observations were used as a frequent source of information. According to Hancock and Algozzine (2006), observations of a setting by case study researcher may provide more objective information related to the research topic unlike interviews, which rely on people's potentially biased perceptions and recollections of events. It is important to note that it requires a great deal of skill and persistence. In an observation, the researcher creates her version of what is "there" and creates a situation to observe.

In Hancock and Algozzine (2006) opinion, four factors are required for conducting a successful observation. The most important factor is for the researcher to identify the phenomenon to be observed in order to shed light on possible answers to the research questions. Secondly, the researcher created an observation guide – a list of features to be addressed during a particular observation (see appendix 9). Using the observation technique and the observation schedule contained in Appendix 9, the researcher as a participatory observer tracked learners initial impressions and interpretations of the activities, their mathematical modelling moves, successes, and challenges including salient errors and mistakes.

Interviews are a common form of data collection in qualitative research. Interviewing individuals or groups allow the researcher to attain rich, personalized information (Hancock and Algozzine, 2006). Interviews may be structured, semi-structured or unstructured. Semi-structured interviews are particularly well-suited for case study research. Researchers using semi-structured interviews pose predetermined questions; and ask follow-up questions designed to probe interviewees for additional information.

In this manner, semi-structured interviews encourage interviewees to express their experiential knowledge of the world from their own lived experiences rather than the perspective of the researcher. Within this scope, the researcher used semi-strcuctured, focus group interviews to triangulate some of the data collected from the task-based worksheet and related observation. Semi-structured focus group interviews were used to probe learners on specific aspects of their written responses to problems as well as noted observations.

Five focus groups interviews were planned to facilitate the collection of supplementary data from the 30 learners, whose perspectives would be difficult to obtain on a one-to-one basis. Each focus group was composed of 6 learners and held in their grade 11 classroom. In addition, it was easier for the students to express themselves freely at school because of the presence of their peers. The learners discussed the questions that were on the task activity sheet which was given by the researcher. The questions that the researcher asked the learners were based on the interview schedule that had been prepared prior to the interviews. These questions were chosen because the learners had to be tested on their level of competency in interpreting and solving mathematical questions within this mathematical model and in a real situation. This was followed by the competencies to validate the solution and finally, the competencies to understand the real problem and to set up a model based on reality. The discussion allowed learners to think and probe each other without much assistance from the researcher. Table 3.1

provides an overview of the data collection plan in relation to each research question.

Table 3.1: Data Collection Plan

Research question	Data collection method	Instrument
1. What mathematical modelling competencies grade 11 learners demonstrate when solving word problems involving quadratic equations?	1. Task based activity 2. Observations 3.Dialogue and discussions in groups	1.Task based activity: mathematical modelling of word problems in algebraic expressions 2. Observation schedule task-based activity 3. Audio recording during interview 4. Focused group interview schedule
2. To what degree does learners' competency in setting up a mathematical model inhibits development of an acceptable solution?	1. Task based activity 2. Dialogue and discussions in groups 3.Observation	1.Task based activity: mathematical modelling of word problems in algebraic expressions 2. Observation schedule 3. Audio recording during task-based activity 4. Focused group interview schedule

1.23 DATA ANALYSIS

1.23.1 Qualitative Data Analysis

Qualitative content analysis was used by the researcher to analyse and make sense of learners responses to the set of problems presented in the task-based worksheet.
To facilitate first level analysis of learners' responses in solving the word problems involving quadratic equations, the holistic rubric furnished in Table 3.2, was used to classify the learners' responses per problem into four broad response options: No response, Incorrect Response, Partially Correct Response.

Table 3.2: Holistic Rubric Used to Classify Learner's Responses

Type of Response	**Description of response**
No response	No evidence of any written attempt to solve problem
Incorrect Response	No relevant response or response has major errors Omits significant parts or all the questions
Partially Correct Response	Shows some understanding of the problem and/or mathematical concepts Contains and incomplete response and/or carries out mathematical responses with errors Explanation muddled
Correct Response	Contains a complete response The mathematics responses are carried out correctly and completely Satisfies the requirements of the immediate problem

After learners' responses for each question were classified into each of the types as indicted in Table 3.2, second level analysis was performed on the respective type of responses problem-wise using the modelling competency framework by Blum and Kaaiser (1997), cited in Maaß (2006, pp. 116-117). It requires that an educator test the competence of the learner in ten different aspects. This is shown in the table below (Table 3.3).

Table 3.3: Competencies and Sub Competencies - Modelling Process

A. Competencies to understand the real problem and to set up a model based on reality. Competency: • To make assumptions for the problem and simplify the situation.	B. Competencies to set up a mathematical model from reality. Competency: • To mathematize relevant quantities and their relations. • To simplify relevant quantities and their relations if necessary

• To recognise quantities that influences the situation, to name them and identify the key variables. • To construct relationships between variables. • To look for available information and to differentiate between relevant and irrelevant information.	and to reduce their number and complexity. • To choose appropriate mathematical notations and to represent situations graphically.
C. Competencies to solve mathematical questions within this mathematical model. Competency: • To use heuristic strategies such as division of the problem into part problems, establishing relations to a similar or analogue problem, viewing the problem in a different form, varying the quantities or the available data, etc. • To use mathematical knowledge to solve the problem.	D. Competencies to interpret Mathematical results in a real situation. Competency: • To interpret mathematical results in extra-mathematical contexts. • To generalize solutions that were developed for a special situation. • To view solutions to a problem by using appropriate mathematical language and/or communicate the solutions.
E. Competencies to validate the solution. Competency: • To critically check and reflect on found solutions. • To gain review on some parts of the model or again go through the modelling process if solutions do not fit the situation. • To reflect on other ways of solving the problem or if solutions can be developed differently. • To generally question the model.	

Source: Blum and Kaaiser, (1997) as cited in Maaß (2006, pp. 116-117).

An overview of the analysis process of the audio recordings was based on a model from Smith et al. (2010).

Step 1 - Transcription

Verbatim transcription of the content of each interview and video- recording done by the researcher. (See appendix 11)

Step 2 – Reading and re-reading

Immersion in the data and active engagement with the search for richer detailed sections.

Step 3 – Initial noting

Detailed examination of data to note the relevance to the research question.

Step 4 – Developing emergent themes

Reducing the volume of data to only using the mathematical concept of number. Mapping the use of a number to relate to a number of skills identified in the literature review. (See Appendixes 5, 6, 7)

Step 5 – Searching for connections across emergent themes

Searching for the connection of how these number skills fit to the demands of the CAPS document to relate to the research questions. This was manually done and it culminated into the themes that emerged from the study.

The data from audio-recorded and semi-strutured focus group interviews were transcribed (See Appendix 11). The thoughts and insights derived from the data were also analysed, coded and recorded into a memo file attached to the participants' files for interpretation. In addition, to approve the results or outcomes of the first analysis, they were re-analysed by considering biographical details of the participants such as age and gender. These were analysed by coding significant sentences and paragraphs relating to themes. This process aimed at summarising the information obtained from participants, and analysing it horizontally. This is because each interview and subject in the proposed study was considered as having independent coherence and structure of meanings.

1.24 Validity and Reliability

The concepts of validity and reliability are multi-faceted, meaning there are many different types of validity and reliability. Hence, there were several ways in which they could be addressed. The effects of the threats to validity are attenuated by attention to validity and reliability throughout a piece of research (Cohen et al. 2000). Validity is an important key to effective research. If a piece of research is invalid, then it is worthless (Cohen et al. 2000). Reliability according to Eisner Golafshani, (2003) is a concept used for testing or evaluating qualitative research; the idea is most often used in all kinds of research. Testing as a way of eliciting information and the most important test of any qualitative study is its quality.

1.24.1 Validity and Reliability in Observations and Interviews

According to Cohen et al. (2000), there are several threats to validity and reliability. The researcher avoided becoming too attached to the group so that emotions do not get in the way of judgment. The researcher endeavoured to give equal attention to all the groups during the observation and interviews. Cohen et al. (2000) revealed that the most practical way of achieving greater validity is to minimize the amount of bias as much as possible. The sources of bias could be from the interviewer's mannerism, the respondents' mannerism, or perhaps from the substantive content of the questions. In this research, the sources of bias included:

- The attitudes, opinions, and expectations of the interviewer.
- A tendency for the interviewer to see the respondent in her own image.
- A tendency for the interviewer to seek answers that support her preconceived notions.
- Misperceptions on the part of the interviewer about what the respondent is saying.
- Misunderstandings on the part of the respondent about what is being asked.

In each of these instances, the researcher tried by all means to explain the questions in simple language to avoid ambiguity.

A trustworthy relationship was established with the Grade 11 learners and this made them comfortable when answering the research questions during the interview.

In the methodology of the research, the issue of reflexivity has been a huge challenge due to the fact that the researcher is a teacher in the same school. The researcher however dealt with this by dissociating the study from her bias or interests.

1.25 Research Ethical Considerations

The study adhered to the ethical considerations such as permission, confidentiality, informed consent and participant safety. The proposed research was conducted after the University of the Western Cape's School of Science and Mathematics' Education and Senate approved this proposal. The research was conducted in alignment with ethical research standards and the legal ethical requirements of the University of the Western Cape. Written permission to conduct interviews with the research participants was sought prior to the meetings. Participation in the research study was voluntary.

Research participants were informed that they had the right to withdraw from the study at any point. Where the prospective participant asked for clearance to be obtained from his/her superior before the interview, the researcher obliged and permissions were obtained from the superior of the prospective participant. Where necessary, the researcher endeavoured to explain the objectives of research and interview to participants. The researcher adhered to legal and ethical requirements for all researches involving respondents. Interviewees must give permission to be interviewed and should not be deceived or coerced (Hancock and Algozzine, 2006).

The researcher also endeavored to assure and live by his promise to treat all information provided by participants as sensitive and confidential. In this regard, the researcher ensured that research participants remained anonymous.

1.25.1.1 Permission

Before proceeding to the field for data collection, the researcher addressed a letter to the school Principal from the University of the Western Cape's School of Science and Mathematics' Education. The purpose of the letter was to give permission to the researcher to conduct the proposed study in the institution. The researcher also drafted the letter of introduction to all participants selected for the study. All participants were fully informed that the lessons were observed and audio recorded during the interview session. The letters explained in detail the aims and objectives of the study, including the role of the researcher.

1.25.1.2 Confidentiality

Once permission was granted, the participants were assured of their confidentiality. The participants were assured that when compiling the report for the study, their names would not be mentioned. Instead, pseudonyms will be used. Participants were assured that the information provided was safe with the researcher and would be used for the purpose of the study only. They were told that participation in this study was voluntary and they had a right to withdraw at any time even though they had signed the consent form. The informed consent forms organized by the researcher was signed by all the participants to confirm their participation in the study. The letter was used to inform all the participants that their names were not to be mentioned and recorded audio files as well as written field notes will be destroyed after the submission of the final report.

1.25.1.3 Informed Consent Forms

Partcipants were asked to sign the consent that grants permission to carry out the proposed study. The parents signed the informed consent forms on behalf of the learners.

1.26 SUMMARY OF CHAPTER 3

This chapter has presented the methodology adopted for the study and explained the research processes employed throughout the study. The methodology of the research is important because it is the blueprint that helps the researcher in providing answers to the research questions. This chapter has offered a detailed elucidation of the research design. Sampling techniques, data collection methods, data analysis processes and a statement of ethics that guided the conduct of the research have been discussed as well.

CHAPTER FOUR: DATA ANALYSIS, INTERPRETATION AND PRESENTATION OF FINDINGS

1.27 Introduction

The previous chapter presented the research methodology adopted for the study and the research processes employed throughout the study. In particular, the methodology of the research drew emphasis on systematic ways of providing answers to the research questions. This chapter presents the data analysis and interpretation.

1.28 Data Analysis and Interpretation

To facilitate first level analysis of learners' responses in solving the word problems involving quadratic equations, the holistic rubric outlined in Table 3.2 was used to classify the learners' responses per problem broadly into four types: No response, Incorrect Response, Partially Correct Response.

After learners' responses for each question were classified into each of the types as indicted in Table 3.2, second level analysis was performed on the respective type of responses problem-wise using the modelling competency framework by Blum and Kaaiser (1997), cited in Maaß (2006, p.117). The modelling competency framework requires that an educator test the competence of the learner in ten different aspects, which are shown in the table below (Table 3.3).

The analysis of the observational data and semi-structured focused group data helped to some extent to better understand and interpret the findings of this study.

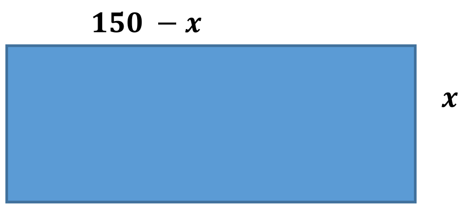
1.29 ANALYSIS OF MODELLING COMPETENCIES DEMONSTRATED BY LEARNERS IN RESPONSE TO PROBLEM 1

Problem 1 read as follows:

A piece of wire of length 300mm is bent to form a rectangle with area 3125 square mm. Determine the dimensions of the rectangle.

An expected solution for Problem 1 is as follows:

This is a representation of the rectangle in 2 dimensions



Step 1:

The perimeter of the rectangle:

$p = 2(l + b)$

$300 = 2(l + b)$

$150 = l + b,$

If we let breadth = x *mm (expressing the dimension of a side of the rectangle in terms of a variable), then length =* $(150 - x)$*mm (expressing the dimension of a side of the rectangle in terms of variable)*

Step 2:

Area of the rectangle:

$A = l \times b$ *(Substitution of the breadth in the formula for area)*

So, $3\,125 = x(150 - x)$ *(Formulation of the quadratic equation)*

$150x - x^2 = 3\,125$

$x^2 - 150x + 3\,125 = 0$ *(Quadratic equation)*

$(x - 125)(x - 25) = 0$ *(Solving the quadratic equation by factorisation)*

$x = 125$ *or* $x = 25$ *(Roots of the equation)*

- *Learners need to check whether the answer makes sense*
- *The learners test their answer using the formula*
- *Interpretation of the roots of the equation*

$$\boldsymbol{Length = 125mm\ and\ breadth = 25mm}$$

a) The learners could show some reasoning through the formulas they selected to solve the problem.

The implication of the above is that the learners understood the concept of graphical representation of word problems. It also means that they have good knowledge of formulas necessary for solving graphical mathematical problems. This finding is supported by Verschaffel et al (2020)observation that graphical representations may particularly help in the initial phases of the solution process of a word problem wherein the situational and mathematical model are constructed, by providing an additional relevant source of information.

1.29.1 Learners Performance in Problem 1

Problem 1 focused on the representation of a rectangle as a 2-dimensional shape, the concepts of area and perimeter with respect to rectangle, and the calculations of the measure of dimensions (length and breadth) of a rectangle. The holistic rubric as described in Table 3.2 was used to classify each of the 30 learners' responses to problem 1 into one of the following categories: correct; partially correct; incorrect; or no response. Figure 4.1 provides an overview of learners' performance with respect to Problem 1.

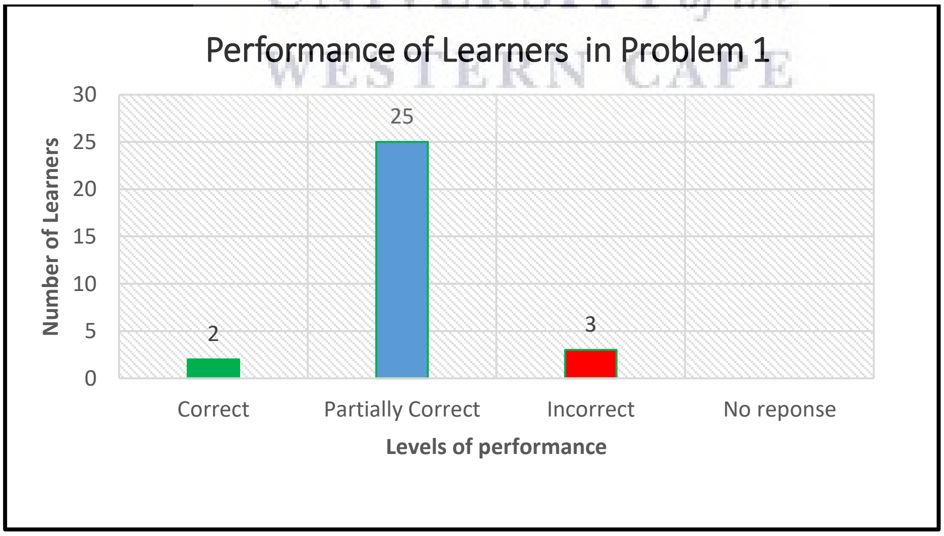


Figure 4.1: Levels of Performance of Learners in Problem No. 1

The anlaysis presented in the next subsections (4.3.2 to 4.3.4) exemplifies the mathematical modelling competencies demonstrated by learners who produced correct, partially correct, and incorrect responses in relation to Problem 1.

1.29.2 Exemplification of mathematical modelling competencies by learners who obtained the correct response to Question 1

As per Figure 4.1, only two learners produced a correct solution for Problem 1. These two learners were L9 and L15, and their responses are shown in Figures 4.2 and 4.3 respectively.

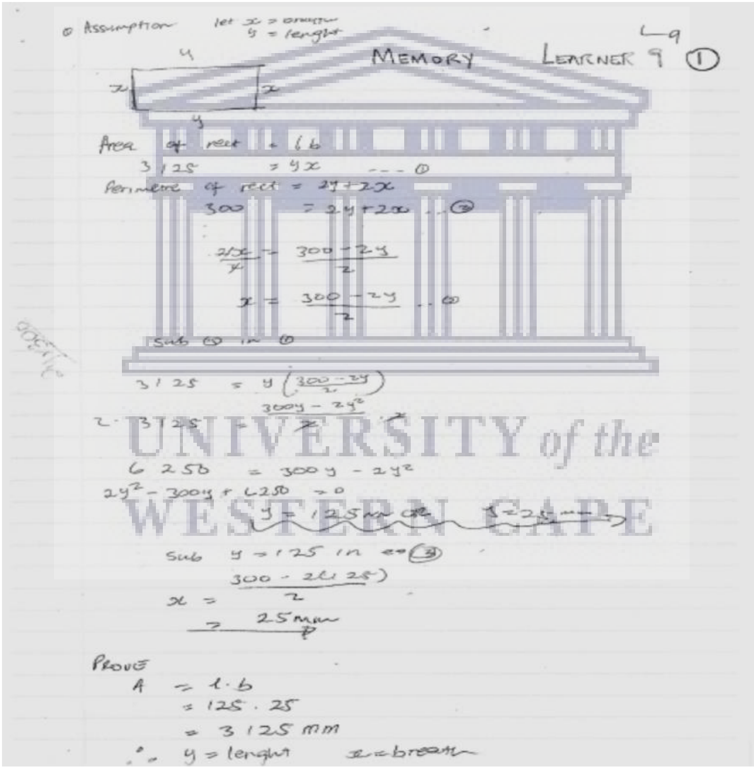


Figure 5.2: Learner 9's Correct Response to Problem No. 1

Learners L9 and L15 seemed to have read the text precisely and imagined the situation clearly. They demonstrated this competence by drawing a diagram to represent the bent wire; a two-dimensional shape looking like a rectangle. To simplify the situation, L9 assumed the breadth to be x units and the length to be y units, whilst L15 assumed the breadth to be y units and the length to be x units with his diagram. This way, L15 was able to show that the breadth is longer than the length in relative terms. In the process, they recognized quantities that could impact on the situation, named them and assigned variables accordingly. The opposite sides were labelled x and y respectively, which indicates that L9 and L15 were aware that each pair of opposite sides of a rectangle are equal. In doing so, both learners ignited process of mathematizing that is characterized by the translation of the problem into a mathematical universe of mathematical structures and equations to yield a mathematical model that can be operated upon (Brady, 2018; Niss, 2015). The two learners were able to build relationships between the variables and given information, thus demonstrating reasonable understanding of the problem. For example, L9 and L15 realized equation 1 as follows:

$$\text{area of rectangle} = l \times b$$

$$3125 = xy \text{ .... (1)}$$

Both learners, L9 and L15 continued to mathematize the perimeter quantity to obtain equation 2 as follows:

$$\text{perimeter of rectangle} = 2y + 2x$$

$$300 = 2y + 2x \text{ .... (2)}$$

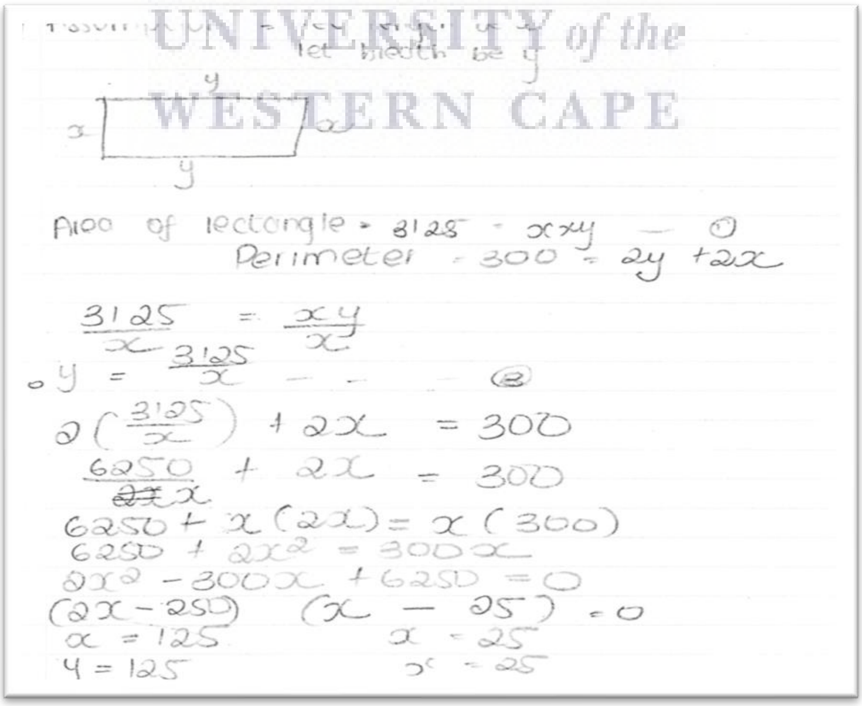


Figure 6.3: Learner 15's Correct Response to Problem No. 1

By constructing equations (1) and (2), it is quite evident that the diagram helps them to understand the problems. Also, through the process of mathematizing relevant quantities, they were able to build a system of linear equations that best represents the situation

As evident in both learners' solutions, they were able to recognize that they need to find the values for x and y by solving their derived system of linear equation simultaneously. Learner L9 and L15 seemed to be conversant with the procedure of solving a system of linear equations simultaneously. Hence, they proceeded to solve x and$y$. However, L9 should have simplified equation (2) to read $x = 15 - y$ instead of leaving it as $x = \frac{300-2y}{2}$ as this could prevent unnecessary working with fractions in solving a system of linear equations. Although L9 solved $2y^2 - 300y + 6\,250 = 0$ correctly to obtain 2 values for y, namely $y = 125$ *or* $y = 25$, the learner could have first simplified $2y^2 - 300y + 6\,250 = 0$ first to read $y^2 - 150y + 3\,125 = 0$ to make it easy to factorize $y^2 - 150y + 3\,125$. Irrespective, L9 did not provide a reason for selecting $y = 125$ and leaving out $y = 25$. However, it is plausible to assume that L9 took cognizance that y represents the length of the rectangle as indicated in his/her diagram in Figure 4.2. Hence, the decision to opt for $y = 125$.

Subsequently, L9 substituted $y = 125$ in (2) to obtain $x = 25$. This leaner (L9) decided to show that the values $y = 125$ and $x = 25$ satisfied the conditions of the given problem as indicated in the last 4 lines of the solution shown in Figure 4.2. It is quite evident from the steps that L9 has interpreted the length to be 125mm and the breadth to be 25mm. Although the learner L9 substituted the values $y = 125$ and $x = 25$ into formula for the area of a rectangle, $A = lx\,b$, to show that conditions of the given problem were satisfied the learner L9 made a small slip in the units for area by writing = 3125 mm instead of writing = 3125 mm^2 . Despite this, it seems that learner L9 was satisfied that answers work and hence finally concluded that $y = length\ and\ x = breath,$ which in effect means that the length of the rectangle is 125mm and the breadth is 25mm.

Reflecting on learner L15 solving of the system of linear equations, the leaner procedurally moved through the smoothly obtain values $x = 125$ and $x = 25$ as shown in the penultimate line of the solution shown in Figure 4.3. In the very last line, the leaner L15 has written $y = 125$ and $x = 25$ (this $x = 25$ should have read $y = 25$). Although learner L15 has not shown how he/she arrived at these final solutions, it is could be probable that after obtaining the values $x = 125$ and $x = 25$, the learner decided to take $x = 25$ to be the breadth of the rectangle, and then substituted it in equation (3), namely $y = \frac{3125}{x}$, to obtain $y = 125$. If L15 had adopted a similar strategy, he or she could have arrived at $x = 25$ which in fact should have read $y = 25$. Despite this error in judgment, L15 made no attempt to crosscheck the answer like how L9 did.

With regard to the written answer, participant L9 and L15 were able to experience the mathematical process to the extent that they could write down the final results but with only L9 able to validate the solution (i.e. check if the answer satisfies the given conditions)

It was observed that in the course of answering the question, L9 and L15 used a calculator and both displayed confidence in answering the question. When L9 was interviewed about his attempt at the problem, he admitted that the problem was easy as evidenced in the following interview extract:

> *L9: In my opinion, the task was not that difficult, if you understood the logic behind what you were doing. It would have been difficult if I did not understand the problem.*

L9's response resonates with the emphasis placed by Polya (1954) on the importance of understanding a problem which impacts on establishing what is given and what is expected to be found or shown, before building a mathematical model to solve the problem using relevant mathematical knowldge.

1.29.3 Exemplification of mathematical modelling competencies by learners who provided a partially correct response to Problem 1

As shown in Figure 4.1, twenty five learners presented a partially correct response to Problem 1. In most instances, they were able to draw a two-dimensional diagram to represent a rectangle under consideration, and assign the variable x to represent the breadth and the variable y to represent the length. Knowing that each pair of opposite sides of rectangle are equal, they appropriately assigned the variable x to each of the opposite sides representing the breadth, and the variable y to each of the opposite sides representing the length as illustrated by L10’s response in Figure 4.4.

Using the formula for the area of a rectangle, $A = lx\ b$, the most learners like L10 were able to construct the equation $3125 = xy$. The learners also used the formula for a perimeter of a rectangle to construct the equation $300 = 2y + 2x$. To a large extent, 11 of the 25 learners were able to mathematize relevant quantities and construct relationships between the variables. This culminated in a pair of linear equations, such as $3125 = xy$ and $x = 150 - y$. However, like in the case of L10, these 10 (out of 25) learners could not solve this system of equations using the simultaneous strategy to generate a quadratic equation in terms of either $x\ or\ y$, which could help to solve for x and y. Reflectively, this means that they were not conversant with the heuristics of solving a system of equations as illustrated by L10’s last 3 moves in Figure 4.4. Hence, they did not find the possible values of x and y, which represent the dimensions of the rectangle. Also, they did not move onto the interpretation stage of the modelling process.

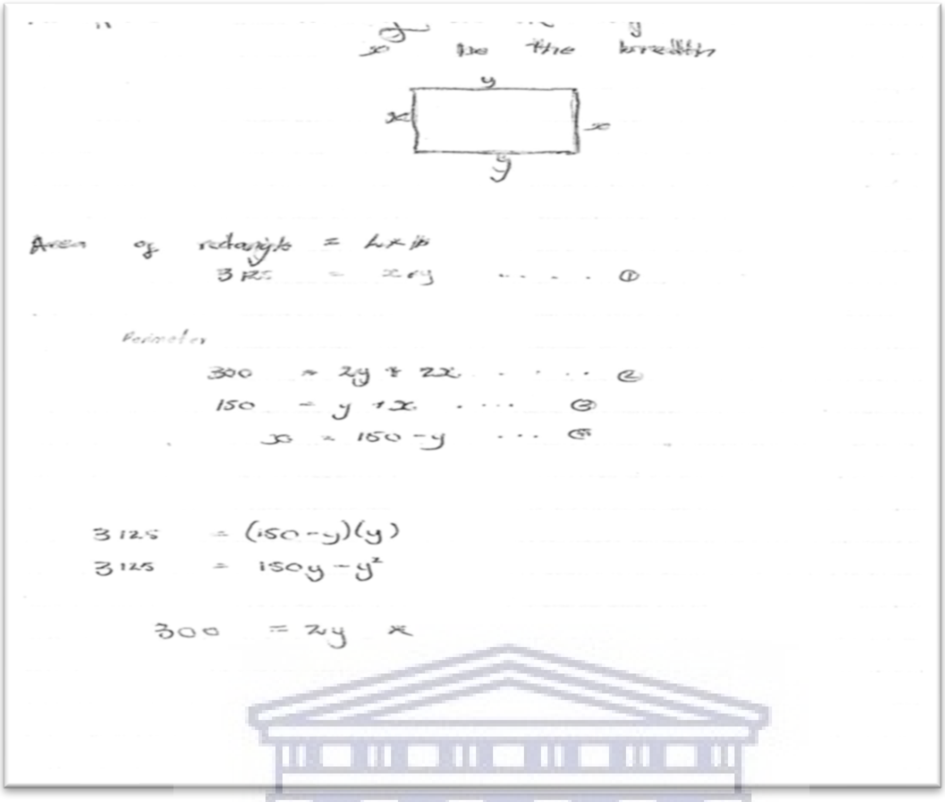


Figure 7.4: Learner 10's Response to Problem No. 1

Although 5 out of these 25 learners did not draw a diagram in their written submission, it seems that they read the text precisely and imagined the situation clearly. For example, as illustrated in Figure 4.5, learner L16 identified the quantities related to the situation, and assigned variables as follows: Let x be the breadth and let y be the length. Using these variables assigned to the dimensions of the rectangle, the 5 learners like L16 used the formula for the area of a rectangle, $A = lx\, b$ to build the equation $3125 = x \times y$, as illustrated in steps 3 and 4 in Figure 4.5. Using the facts that opposite sides of a rectangle are equal, these 5 learners managed to express the perimeter of the rectangle in terms of x and y as follows: Perimeter $= 2y + 2x$.

Using the given information that the length of wire used to form the rectangle shape, these 5 learners like the earlier group of 11 learners formulated the following equation: 300= $2y + 2x$ (see line 6 in Figure 4.5)

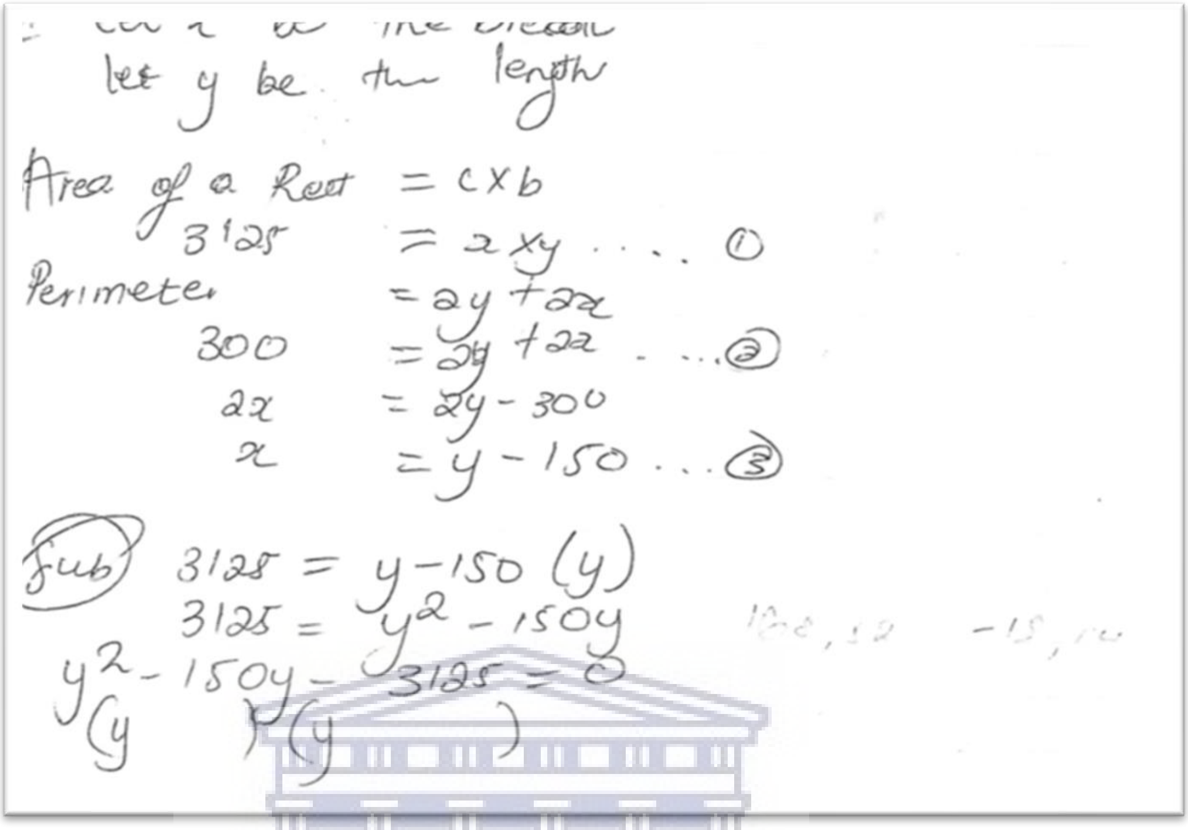


Figure 8.5: Learner 16's response to Problem No. 1

Learner L16 seems to know that in order to find x and y, he must solve the following system of linear equations simultaneously: $3125 = x \times y$ ………... (1)

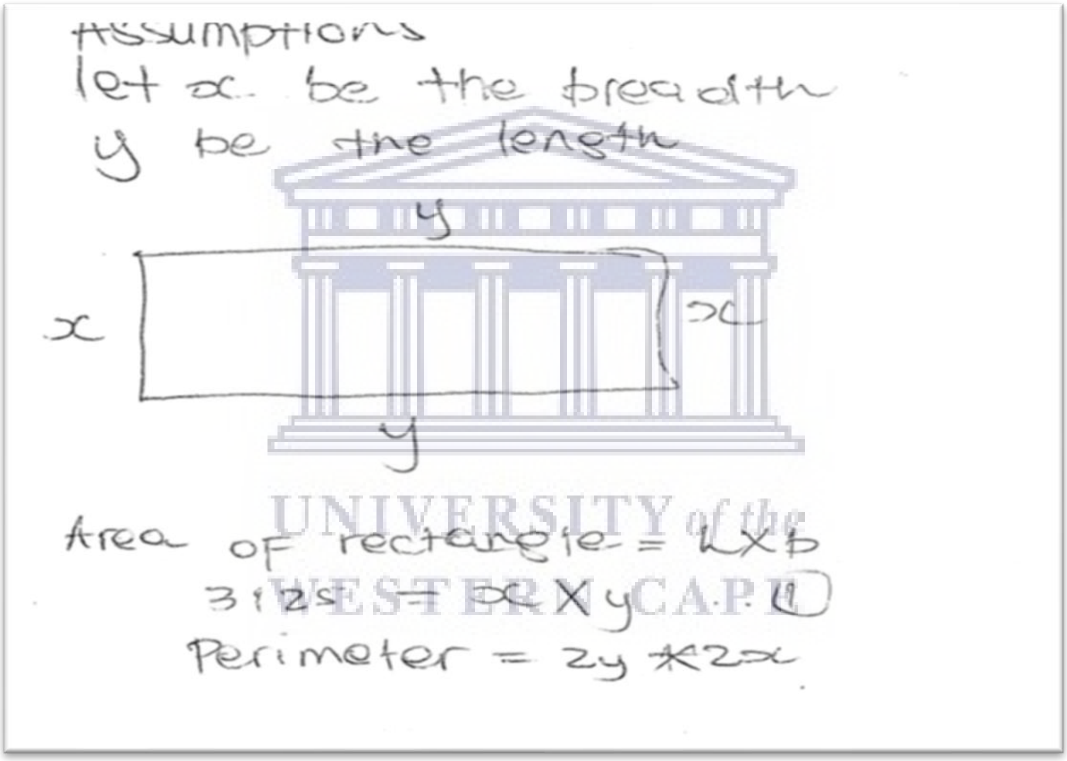
and $x = y - 150$ ……… (3)

L16 proceeds by substituting $y - 150$ in place of x in (1). This approach turned out to be accurate as the learner was able to obtain the quadratic equation $y^2 - 150y + 3\,125 = 0$. However, L16 fails to factorize $y^2 - 150y + 3\,125 = 0$, fails to solve for y in the process. Failure to solve for y inhibited L16 from proceeding further to find corresponding y values. Although the learner did not take a long time develop the quadratic equation $y^2 - 150y + 3\,125 = 0$, it seems that his struggles with factorizing $y^2 - 150y + 3\,125$ was his major obstacle. However, when interviewed, L16 stated that:

'I had difficulty in answering the questions. The requirement of understanding English made answering the question difficult' (L16).

The remaining 4 out 25 learners, managed to assign variables to the main quantities like the length and breadth. For example, as illustrated in Figure 9, learner L13 drew a diagram to represent a rectangle and assigned the variable y to both pairs of opposite sides and the variable x to both pairs of opposite sides, thereby demonstrating that he knows that the opposite sides of a rectangle are equal.

Figure 9.6: Learner 13's Response Problem No. 1



Using the formula for the area of a rectangle, $A = lx\, b$, L13 proceed to build the equation $3125 = x \times y$, as illustrated in Figure 4.6. Furthermore, knowing that the perimeter of a rectangle is given by 2 × length + 2 × breadth, L13 correctly expressed the perimeter of the rectangle in terms of x and y as follows: perimeter $= 2y + 2x$. However, after mathematizing the relevant quantities and the relationships to obtain the mathematical model as indicated in the last 2 steps of L16's solution (in Figure 4.6), the learner did not proceed any further.

In effect, L13 failed to use mathematical knowledge to move further to solve the problem, or validate the answers. Although L13 barely used his calculator, when interviewed he stated:

> *'It was difficult as it had a lot of statements and it was hard to understand. (L13).*

.

1.29.4 Exemplification of mathematical modelling competencies by learners who provided an incorrect response to Problem 1

As indicated in Figure 4.1, three learners' (namely L1, L16; L18) responses were incorrect and incomplete. For example, learner L1 produced the response as illustrated in Figure 4.7.

This learner did not draw a diagram on his page but could have developed a visual representation in his mind. Mere looking at the equation (2), $300 = 2y + 2x,$ it is possible to conjecture that the learner assigned variable x to represent the breadth. Furthermore, it appeared that the learner used the knowledge that opposite sides of a rectangle are equal to marked the opposite sides, thereby representing the breadth of the rectangle each equal to x.

Similarly, L1 could have marked the opposite sides representing the lengths of the rectangle by using the variable y. Thus, we can say that L1 was able to recognize quantities that influence the situation and assign them variables to help build relationships between what is given to enable to the determination of a reasonable solution. However, it seems from the equation $3125 = x + y$ written in line 1, leaner L1 was not able to recall that the area of a rectangle is given by the formula Area = length × breadth and not Area = length + breadth. The latter misconception (or error) has resulted in the gross error of forming the equation $3125 = x + y$ instead of the equation $3125 = x \times y$.

It seems that L1 had some inclination and solved for x and y simultaneously. Hence, he or she proceeded simplifying $300 = 2y + 2x$ to $150 = y + x$ and call it equation (3). However, the subsequent steps seem to demonstrate that L1 was not very conversant with the heuristic strategy of solving a system of algebraic equations simultaneously to the extent the learner, while performing the operation $(3) - (1)$ obtained an expression instead of an equation, and abruptly stopped by writing $-2975 - x$.

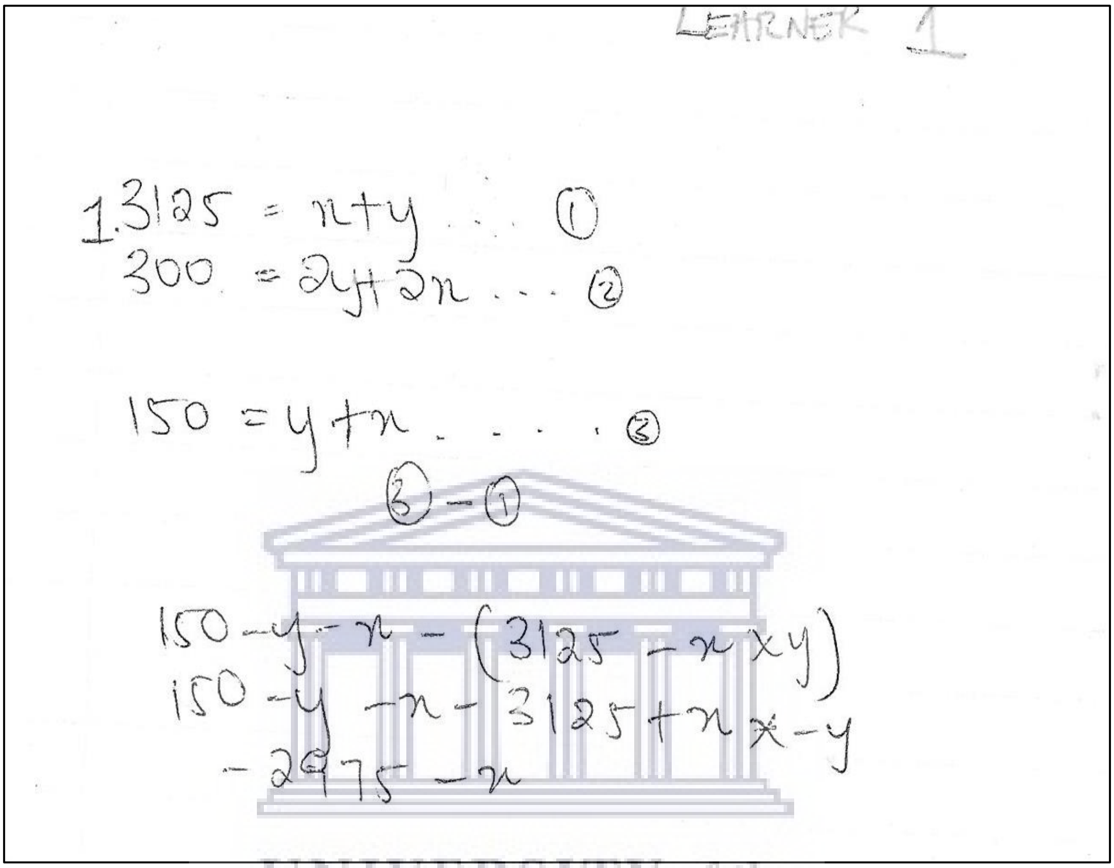


Figure 10.7: Learner 1's Response to Problem No. 1 is Incorrect

As illustrated in 4.8, learner L18 while realizing that he/she must work with rectangle immediately assumed the breadth to be x units and length to y units. This is assigning variables to relevant quantities such as length and breadth. This is appropriately reflected in the diagram drawn by the learner as illustrated in Figure 4.8. The learner did not assign variables to each of the remaining sides of the rectangle. Furthermore, no further calculations were done to show complete understanding or completion of the question. It was observed that the learner spent quite a short time on the question. He was visibly stuck with attempting to answer the question. When he was asked how the question was, he stated:

'I did not know how to calculate. I only knew the mathematical assumptions and the diagram' (L18).

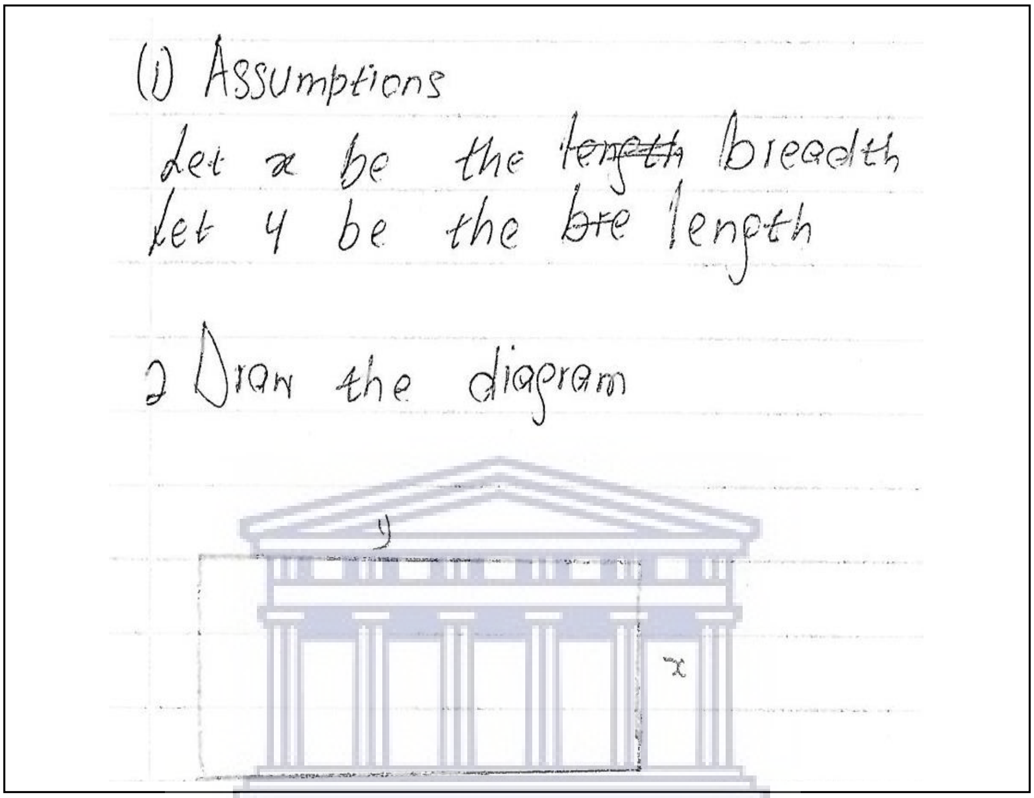


Figure 11.8: Learner 18 did not make any calculations

1.29.5 Summary of Findings with Respect to Problem 1

i. All 30 learners attempted problem 1 but majority (83,3%) of them developed a partially correct solution (see Figure 4.1, which shows 25 out of 30 learners)

ii. An extremely small number (2 out of 30 learners) developed a correct solution for problem 1, and an extremely small number (3 out of 30 learners) developed an incorrect solution for problem 1.

iii. Nearly all learners, who attempted this problem focused on area and perimeter concepts within the context of a rectangle. They also drew a diagram of rectangle to help understand the problem.

iv. Many learners (like L9 shown in Figure 4.2) stipulated assumptions such as: Let the breadth be x; and let length be y.

v. All learners, even those who did not draw diagram of a rectangle, ascribed the variables x and y to the breadth and length of a rectangle respectively, and those who produced a diagram correctly represented each dimension on their diagram of the rectangle using variables x and y. In principle, learners were able to recognise quantities that influence the situation, name them and identify the key variables;

vi. All learners who produced correct or partially correct solutions were able to recall a key property of a rectangle; opposite sides are equal and appropriately invoke the property and apply it to the situational diagram of a rectangle. However, only 1 out of 3 learners (namely L1) who produced incorrect incomplete solution was able to do the same. All of this demonstrates that majority of the learners (28 learners) were able to begin the process of constructing relationships between variables;

vii. Using the concept of the area within the context of a rectangle, all learners who produced correct or partially correct solutions were able to succinctly recall that the area of a rectangle is expressed by the formula: $Area = length \times breadth$, and use it to build the relationship: $3125 = x \times y$. In essence, they were able to mathematize relevant quantities and their relations to help lead to the development of a plausible mathematical model.

viii. Using the concept of the perimeter within the context of a rectangle, all learners who produced correct or partially correct solution including 1 learner (L1) who produced incorrect incomplete solution were able to succinctly recall that the perimeter of a rectangle is expressed by the formula: $Perimeter = 2 \times length + 2 \times breadth$, and use it to build the relationship: $300 = 2x + 2y$.

ix. All learners who produced correct or partially correct solutions were able to set up a correct system of linear equations to help solve the problem:

$$3125 = x \times y \quad \ldots\ldots\ldots (1)$$

$$300 = 2x + 2y \ldots\ldots\ldots (2)$$

This in essence, means that majority of the learners were able to mathematize relevant quantities and build a mathematical model.

x. The learner L1, who produced incorrect or incomplete solution was able to set up a system of linear equations which was partially correct as follows:

$3125 = x + y$ ……… (1)

$300 = 2x + 2y$ ……… (2)

The error was noted in equation (1), which should have read $3125 = x \times y$.

xi. In order solve the mathematical model, learners had to be conversant with the heuristics of solving a system of linear equations simultaneously.

a) The two learners, L9 and L15, who obtained correct solutions, invoked and applied the procedures to solve a system of linear equations appropriately to obtain correct values for $\boldsymbol{x}$ and y.

b) Majority of the 25 learners who produced partially correct solutions managed to simplify $300 = 2x + 2y$ (equation 2) to get $= 15 - y$. They managed to move onto the next critical step of substituting $x = 15 - y$ into $3125 = x \times y$ (equation 1) but in the process committed mistakes and demonstrated shortcomings such as:

- Leaving out the brackets (see Figure 4.5 for L16 response); which resulted in obtaining a quadratic equation that was different from the expected one.
- Failure to factorize a quadratic expression into linear factors (see Figure 4.5 for L16's response)
- Inability to realize that a quadratic equation should be expressed in the form $ax^2 + bx + c = 0$, where after $ax^2 + bx + c$ can be factorized and expressed as a product of 2 linear factors. This shortcoming is illustrated in L10's response in Figure 4.4.

c) A small number of learners (4 out of 25), who obtained a partially correct solution, did not show any attempts to solve their assimilated system of linear equations (see for example L13 response in Figure 4.6)

d) From the small group of learners who produced incorrect/incomplete solutions, only 1 learner, namely L1 (see Figure 4.7) did not seem to know how to proceed to solve the system of simultaneous linear equations.

xii. From all the learners who attempted to solve this problem, there was just one learner (L9 – see Figure 54.2) that related the results emanating from solving the model to problem situation. This learner firstly appropriated the correct unit of measurement for each variable, namely mm; and interpreted $x = 25mm$ to be the breadth and

$y = 125mm$ to be the length of the rectangle. These interpretations were consistent with the learner's assumptions that were articulated at the initial stages of solving the problem, namely: let the let breadth be x; and let length be y.

xiii. Only L9, who produced a completely correct solution made the necessary effort to check if their mathematical answer makes sense in terms of the original situation. As shown in Figure 5, this learner substituted the value of 25 for breadth (b) and 125 for length (l) in the formula for area of rectangle, $A = lb$, to see if s/he gets 3125. Despite the slip in the square units (the writing of mm instead of mm^2), the learner on the basis of getting 3125, which was the given area of the rectangle concluded that y (which is 125 mm) is the length and that x (which is 25 mm) is the breadth.

1.30 Analysis of Modelling Competencies Demonstrated by Learners in Response to Problem 2

Problem 2 read as follows:

The sum of the digits of two- digit number is 13 and the product of the digits is 36. Determine which two numbers fit this description.

An expected solution for question 2

The learners were expected to create an assumption:

Let the digits be:

x and $13 - x$

Sum: $x + (13 - x) = 13$………………………… (1)

Product: $x(13 - x) = 36$………………………… (2)

$x^2 - 13x - 36 = 0$……………………………….. *Quadratic equation*

$(x - 9)(x - 4) = 0$……………………………….. *Factorisation*

$x = 9$ *or*

$x = 4$.

The digits are 4 and 9…………………… *solutions*

The number could be 49 or 94 ………. *Conclusion*

1.30.1 Learners performance in Problem 2

Problem 2 focused on formulating equations and solving them. In this question, data was presented and analysed. This question represented information in a very simple format and it was very short in the phrasing. The students were expected to understand the concepts of sum and product. It included the need to identify the key variable, which required the student to construct simultaneous equations. Hence, one had to use mathematical knowledge to solve the problem.

Learners were expected to reorganize the problem into smaller parts to help solve it. They had to do this by showing an understanding of the mathematical language and solving the simultaneous equations. The response of each learner in the group of 30 learners was marked and classified into 4 categories: correct; partially correct; incorrect; and no response. Figure 4.9 provides an overview of learners' performance with respect to Problem 2. There were 2 learners who answered question 2 correctly, 23 learners answered the question partially since they made mistakes in formulating and factorising the quadratic equation, and 5 learners who could not solve question 2 as presented in the sections 4.4.2 to 4.2.4 respectively.

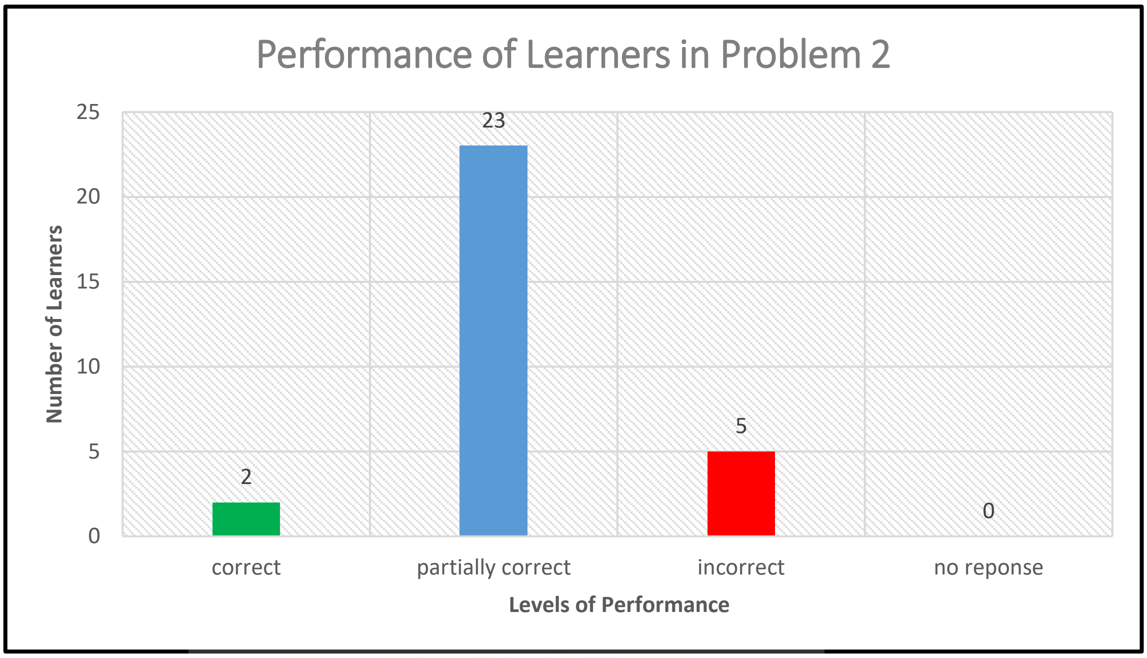


Figure 12.9: Levels of Performance of Learners in Problem No. 2

1.30.2 Exemplification of mathematical modelling competencies by learners who obtained the correct response to problem 2

Two learners (L9 and L28) as shown in the graph in Figure 4.9, solved the problem although they did not explicitly write down the final result, namely the 2 numbers that satisfy the given conditions imposed in the problem. However, the 2 numbers can be inferred from the learner's workings as shown in Figure 4.10 and Figure 4.11 respectively. Although the learners did not explicitly define the 2-digit number in the form xy or yx, it is plausible to conjecture from the relationships expressed between x and y in the form

$x + y = 13$ and $x.y = 36$

that the learner was working with a 2-digit number in the form xy or yx. The building of the two linear equations:

$$x + y = 13 \text{ and}$$
$$x.y = 36$$

as a system of linear equations suggests that the learner was able to make connections between mathematics and the given information governing the problem. These moves characterize some sense of understanding of the problem, and the corresponding ability of the learner to choose appropriate mathematical notations to represent the situation through a system of related linear equations (which we call a mathematical model).

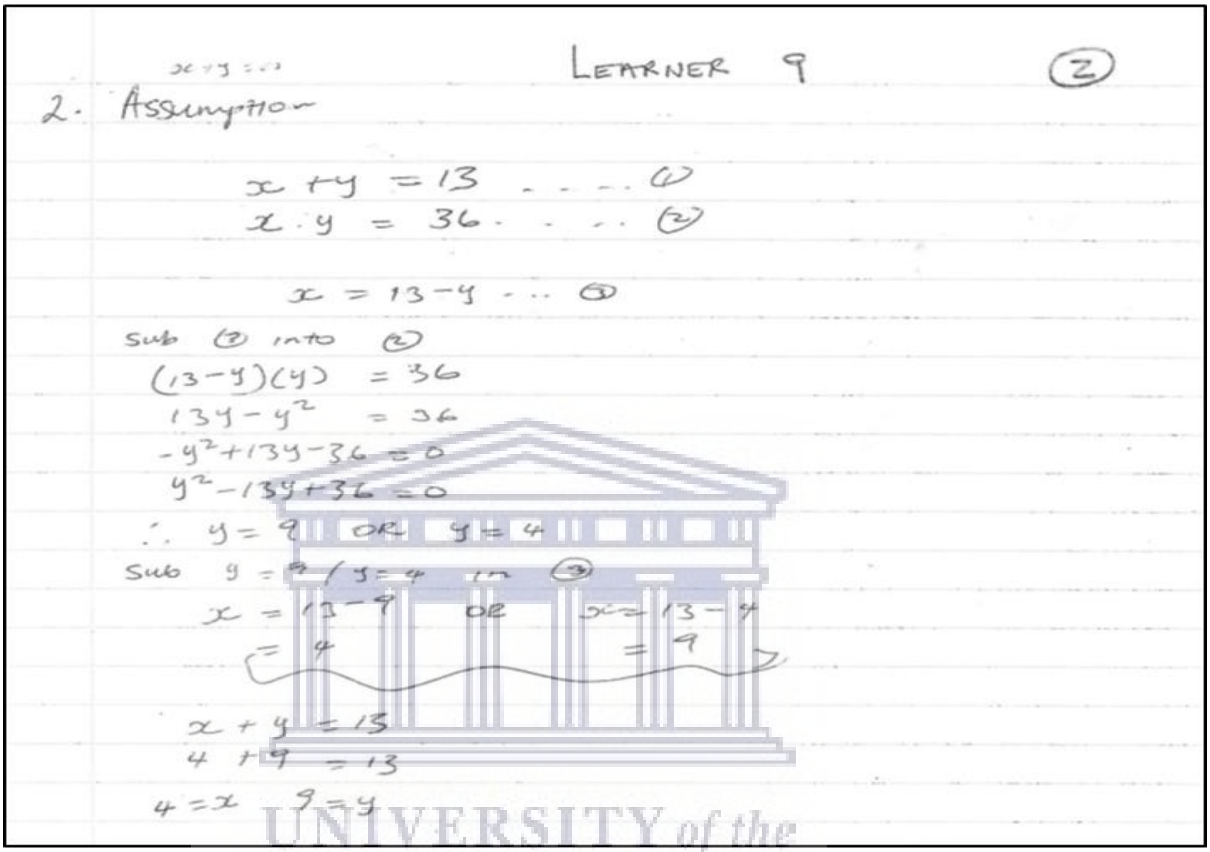


Figure 13.10: Learner 9's Correct Response to Problem No. 2

As illustrated in L9's written solution in Figure 4.10, it is evident that the learner is conversant with the heuristic strategy of solving a system of linear equations. Through correctly arriving at the quadratic equation:

$$y^2 - 13y + 36 = 0,$$

the learner correctly solves for y to get $y = 9 \; or \; y = 4$. Furthermore, through substitution in (3), $x = 13 - y$, the learner obtains $x = 4 \; when \; y = 9$ and $x = 9 \; when \; y = 4$. However, from the learner's last 3 steps, it seems that the learner only considered the case $x = 4 \; when \; y = 9$ and not the case $x = 9 \; when \; y = 4$, when testing whether the values of x and y satisfy one of the conditions, namely that sum of the digits of a 2-digit number is 13.

In addition to learner not checking that the values of x and y satisfied the second condition which requires that the product of the digits be 36, the learner did not write down the final result; the 2 two-digit numbers. The latter omission could be attributed to the learner not fully understanding what is expected to be found. It was observed that in the course of answering the question, L9 was composed and concentrated and did not display any uneasiness. When interviewed, he stated that:

Researcher: How did you manage to answer question whereas other learners could not?

Learner L9: 'I understood the logic behind the question.

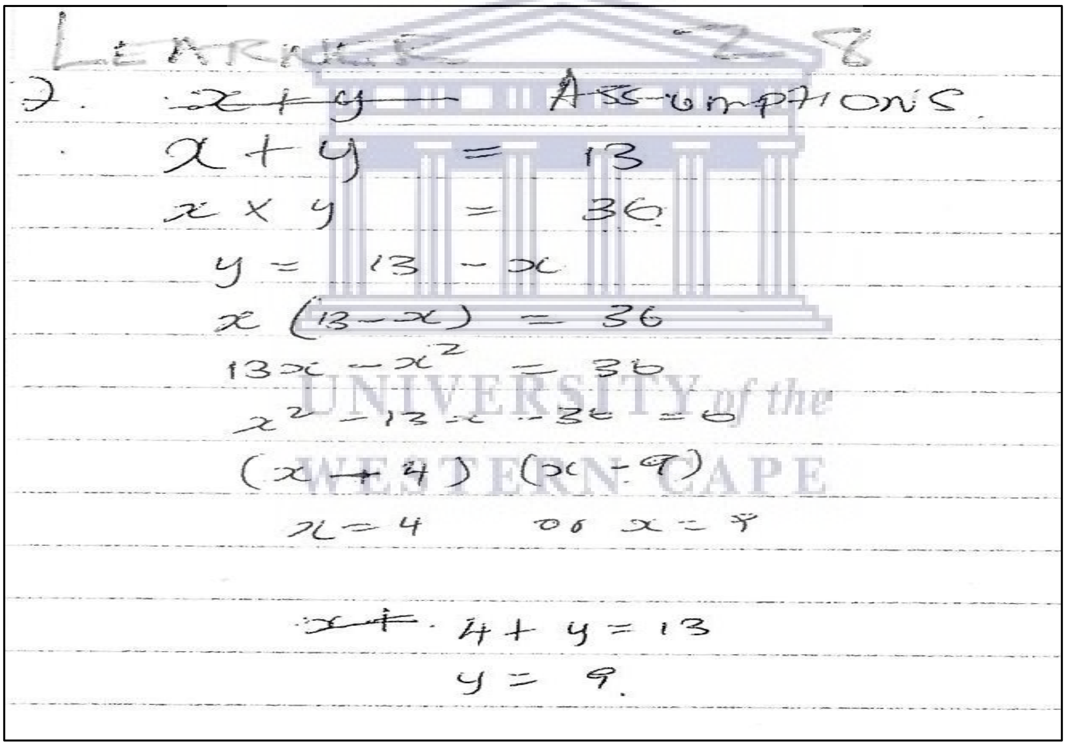


Figure 14.11: Learner 28's Correct Response to Problem No. 2

To a good extent, learner L28 like learner L9 in the previous case was able to successfully move through most of the modelling processes. The learner formulates the quadratic equation, $\boldsymbol{x^2 - 13x - 6 = 0}$, and then factorised the trinomial on the left-hand side equation correctly but omitting to set equal to zero. Despite this gross mistake, the learner moved on to obtain the roots of the quadratic equation $\boldsymbol{x^2 -}$

$\boldsymbol{13x - 6 = 0}$ to read $\boldsymbol{x = 4 \; or \; x = 9}$. Furthermore, the learner only chose to work with $\boldsymbol{x = 4}$, and then substituted in the equation $\boldsymbol{x + y = 13}$ to obtain $\boldsymbol{y = 9}$. The reason for choosing to work with $\boldsymbol{x = 9}$ is not provided by the learner.

Although $\boldsymbol{x = 4 \;\; and \; y = 9}$ is one of the ordered pairs that could be used to assemble the two-digit number, namely 49, the learner did not explicitly assemble the two-digit number as 49. Moreover, in not considering $\boldsymbol{x = 9}$, which could substitute in $\boldsymbol{x + y = 13}$ to obtain $\boldsymbol{y = 4}$, the learner failed to generate a second possible solution, namely 94. The latter moves seem to suggest that the learner did not fully understand the problem and discern what exactly must be found. In addition, the learner did not crosscheck his/her solution. However, the researcher observed that learner L28 was well composed. He concentrated on answering this particular question and this was evident in the interview where he stated that:

> Researcher: Did you find any difficulty in solving question 2?
>
> *Learner 28: 'I did not have any difficulty answering the question. I went ahead and tried my best' (L28).*

1.30.3 Exemplification of mathematical modelling competencies by learners who provided a partially correct response to Problem 2

As illustrated in Figure 4.9, 23 learners partially provided correct answers. For example, Figure 4.12 illustrates a partially correct solution produced by one such learner, namely L14. Learner L14 explicitly assigned variables to the two digits, namely x and y. Much like learners L9 and L28 (as shown in Figures 4.10 and 4.13 respectively), learner L14 was able to mathematise the relationships between what was given and create an algebraic model as follows:

$x + y = 13$ and

$x \times y = 36.$

The learner (L14) started to solve for x and y using the simultaneous equations but stopped abruptly after building the required quadratic equation,

$y^2 - 13y - 6 = 0$, which was indeed correct. From this stage, L14 could not proceed with the modelling competence of solving for *x* and *y*. This meant that L14 could not factorise the quadratic equation formulated. The learner L14 had mathematical modelling competences of mathematising the word problem and assuming that the unknown values were *x* and *y*, but as shown Figure 4.12, this question remained incomplete with no effort made by the learner to see if the answer made sense or not.

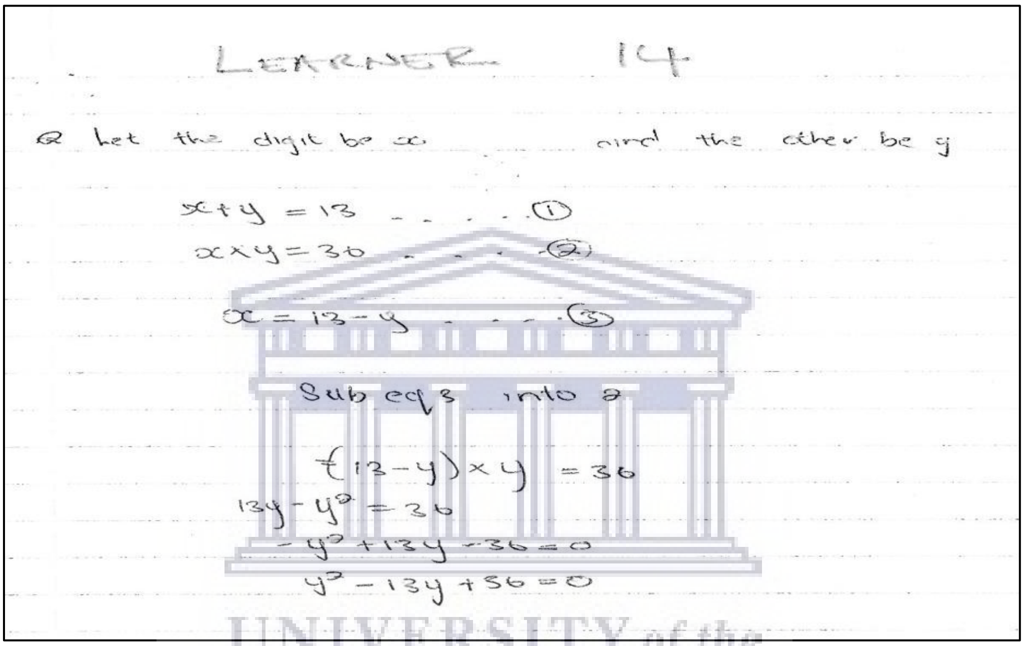


Figure 15.12: Learner 14 Partially Correct Response to Problem No. 2

1.30.4 Exemplification of mathematical modelling competencies by learners who provided an incorrect response to Problem 2

Learner L29 as shown in Figure 4.13 and learner L22 as shown in Figure 4.14 represent a sample of learners who provided incorrect responses to Question 2. L29 and L22 did not explain the assignment of variables $\boldsymbol{x}$ **and** $\boldsymbol{y}$. However, as both L29 and L22 constructed the required set of equations (1) and (2) in terms of the variables $\boldsymbol{x}$ **and** $\boldsymbol{y}$, it is highly plausible that they considered $\boldsymbol{x}$ **and** $\boldsymbol{y}$ to be representing the digits of the two digit number. The shortcoming here is that both learners were silent about whether the 2-digit number is in the form $\boldsymbol{xy}$ or $\boldsymbol{yx}$. Gauging from (3), $\boldsymbol{x = 13 - y}$, it seems that both learners had the hunch that equations (1) and (2) could be used to solve for $\boldsymbol{x}$ and $\boldsymbol{y}$ simultaneously.

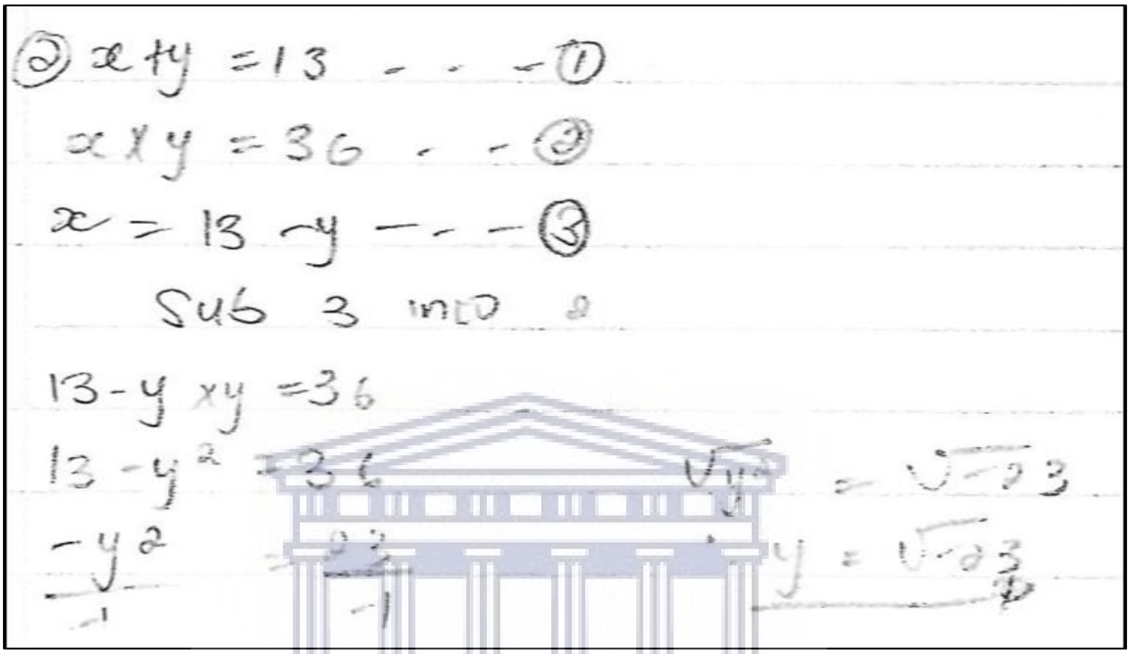


Figure 16.13: Learner 29 with Incorrect Response to Problem No. 2

However, both learners seemed to have lacked the necessary skills set to proceed systematically to solve for $\boldsymbol{x}$ and $\boldsymbol{y}$ simultaneously. In the case of learner 29, this is evident by the lack of the insertion of $\boldsymbol{13 - y}$ in brackets when substituting for $\boldsymbol{x}$ in (2). In particular, learner L29 wrote $\boldsymbol{13 - y \times y = 36}$ instead of $\boldsymbol{(13 - y) \times y = 36}$. This error seemed to have caused the learner L29 to obtain $\boldsymbol{13 - y^2 = 36}$ instead of $\boldsymbol{-y^2 = 36}$ **through simplification**. Working with $\boldsymbol{13 - y^2 = 36}$, the learner L29 proceeded to solve $\boldsymbol{y}$ and obtain = $\boldsymbol{\sqrt{-23}}$.

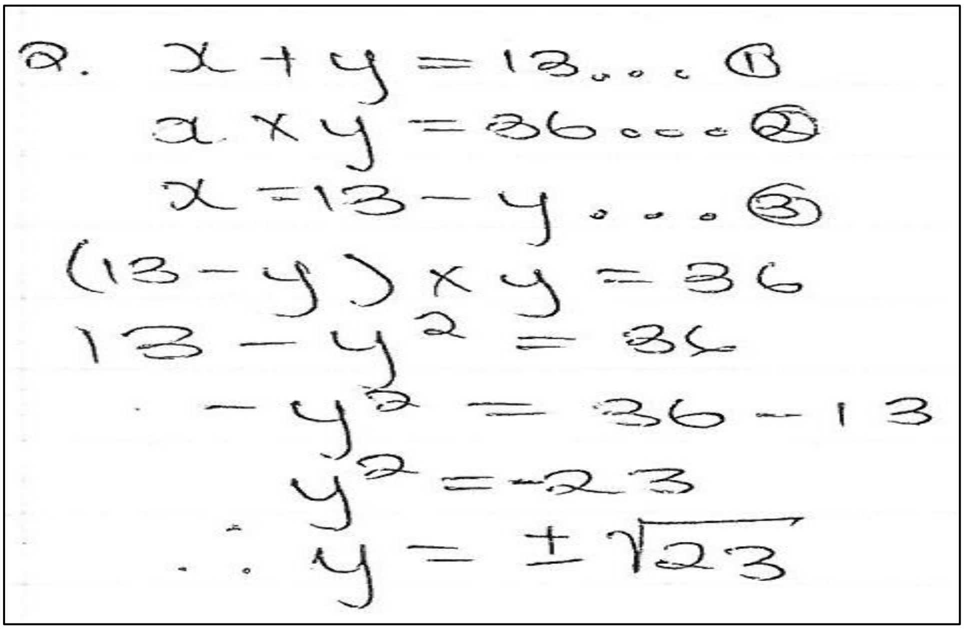


Figure 17.14: Learner 22 with Incorrect Response to Problem No. 2

On the other hand, learner L22 used brackets when substituting for $\boldsymbol{x}$ in (2) by writing

$$(\mathbf{13}-\boldsymbol{y}) \times \boldsymbol{y} = \mathbf{36}\ .$$

However, L22 failed to apply the distributive property correctly when simplifying $(\mathbf{13}-\boldsymbol{y}) \times \boldsymbol{y}$. This caused L22 to obtain $\mathbf{13}-\boldsymbol{y}^{\mathbf{2}}=\mathbf{36}$ instead of $\mathbf{13}\boldsymbol{y}-\boldsymbol{y}^{\mathbf{2}}=\mathbf{36}$. However, L22 proceeded with carried accuracy to obtain $\boldsymbol{y} = \pm\sqrt{-\mathbf{23}}$. Regrettably, both learners did not interrogate the value $\boldsymbol{y}$ to realize that when working strictly in the real number system, it is not possible to find the square root of a negative number. In the case of L22, the learner did realize that the digit of a number cannot be negative. However, when the learner L22 was interviewed, he stated that:

The question was hard and … I do not know how I arrived at it'

As shown in Figure 4.15, the set of equations built by learner L17 seem to suggest that the learner assumed both digits to be strictly the same. On this basis, the learner correctly wrote down $x + x = 13$ to give $2x = 13$. This showed the learner understood that two unknowns were although he or she could not differentiate x and x to be x and y and $x \times x = 36$ to give $x^2 = 36$. There were no further moves by learner L17. Hence, the leaner's response remains incomplete.

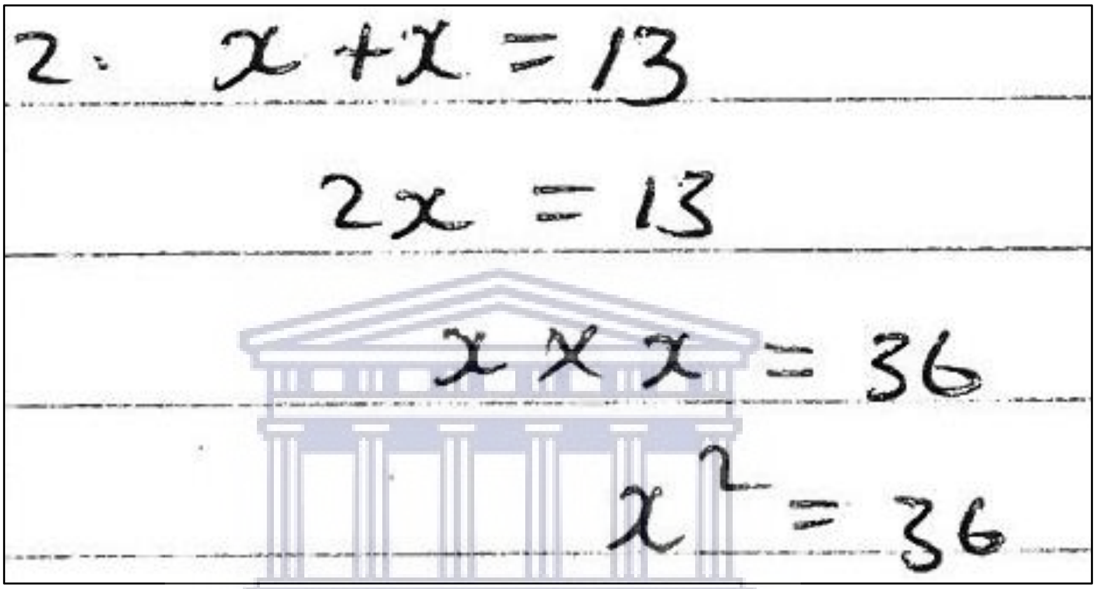


Figure 18.15: Learner 17 With the Partially Incorrect Response Problem No. 2

1.30.5 Summary of findings with respect to Problem 2

(i) Although all 30 learners attempted Problem 2, majority of the learners (23 out of 30, which is about 77%) produced a partially correct solution, whilst an extremely small number (2 out of 30 learners) developed a correct solution and an extremely small number (5 out of 30 learners) developed an incorrect solution.

(ii) It is evident from the response of learners that nearly most (29 out of 30) of the learners were able to recognize quantities that influence the situation and routinely assign them variables without necessarily defining them upfront. These 29 learners were able to construct relationships between variables even though 23 solutions were partially correct and 5 were not completely correct. The mathematization of relevant quantities and variables was appropriately enacted by majority of learners (29 out of 30) to produce a correct system of linear equations. This showed that nearly all of the learners were able to build a correct mathematical model that was essential to enable learners solve the 2-digit problem.

(iii) The two learners, who produced correct solutions, were able to apply the heuristic strategy to solve the linear equations, while L9 was able to produce both solutions. Whereas L28 only produced 1 possible solution only. However, none of them really showed evidence of interpreting or validating their solutions.

(iv) Most learners who produced a partially correct solution proceeded correctly up to the stage of generating the quadratic equation, $y^2 - 13y - 6 = 0$, and then stopped (like L14). In addition, they did not make any attempt to see if their answers made sense or not.

(v) Four out of the 5 learners, who produced an incorrect solution slipped up after ignoring the use of the brackets when substituting $x = 13 - y$ into $x \times y = 13$, hence resulting in $13 - y^2 = 36$. This error resulted in them proceeding to obtain solutions like $y = \sqrt{-23}$ which is incorrect. These learners did not make any attempt to see if their solutions made sense. Also, they did not demonstrate evidence of interpretation and validation of their solutions.

1.31 Analysis of Learners Responses Task-Based Question 3.

Problem 3 read as follows:

Vula is a river guide on the Gariep River. Robert, a member of the group is injured. Vula paddles Robert to the nearest pickup point, 12 km away. Vula paddles back to his group. If the total paddling time for the trip (there and back) is five hours and the river flows at a constant speed of 1km/h, calculate the average speed that Vula paddles.

Expected Solution for Problem 3

	DISTANCE	SPEED	TIME

With current	12	$x + 1$	$\frac{12}{x+1}$
Against current	12	$x - 1$	$\frac{12}{x-1}$

$$\frac{12}{x+1} + \frac{12}{x-1} = 5$$

$$12(x-1) + 12(x+1) = 5(x+1)(x-1)$$

$$12(x-1) + 12(x+1) = 5x^2 - 5$$

$$12x - 12 + 12x + 12 = 5x^2 - 5$$

$$5x^2 - 24x - 5 = 0$$

$$(5x+1)(x-5) = 0$$

With regard to the verification, the learners had to show that speed cannot be negative.

$$\boldsymbol{x \neq -\frac{1}{5}}\ or\ x = 5\ [\boldsymbol{his\ speed\ is\ positive}]$$

Vula's average padding speed is 5 km/h.

1.31.1Learners performance in Problem 3

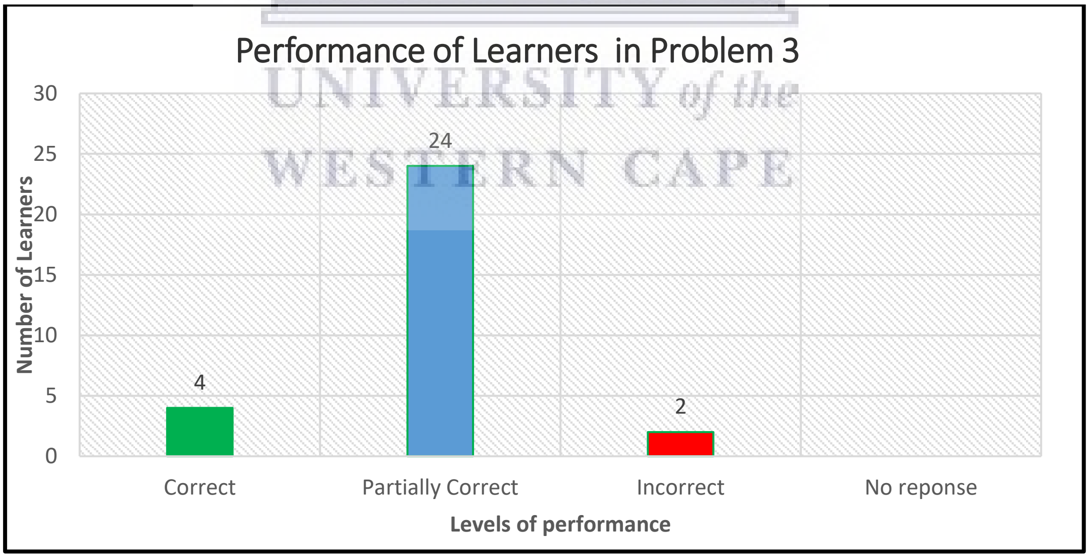


Figure 19.16: Levels of Performance of Learners in Problem No. 3

Figure 4.16 shows that out of the 30 learners, 4 learners had correct responses although they could not scale the final step of modelling which is validating the answer, while

24 learners managed to partially solve the problem 3. In the latter group of 24 learners, some could make the table, deduce the equations and formulate the quadratic equations but failed to solve them. More so, some of the learners could only make the table but failed to formulate correct quadratic equation which made reaching the solutions cumbersome. There were 2 learners with wrong responses, and this could suggest that they did not understand the word problem or they did not have previous knowledge of dealing with problems that involve speed, time and distance.

1.31.2 Exemplification of Mathematical Modelling Competencies by Learners Who Obtained the Correct Response to Problem 3

Four learners (L2; L4; L9 and L17) solved the problem correctly. Figure 4.17 and Figure 4.18 illustrates the detailed response of two of such learners (L2 and L17). Although both learners constructed a table to help understand the problem, they should have explicitly stated what the variable 'x' represents' so that the reader could follow their table with a greater sense of clarity. Nevertheless, from careful analysis of the information presented in the table, it seems that both learners (by co-incidence) assumed that the speed at which Vula paddles upstream and downstream is constant and considered it to be $x\ km/h$ even though this may not necessarily be the case.

However, this suggests that both learners read the text with a deep sense of understanding and imagined the situation clearly. In doing so, they recognized quantities like distance, speed and time that impact on the situation, and built relationships between them in terms of the variable x. It takes deep thinking to realize that the resultant speed of the paddling boat downstream is equal to the speed at which Vula was paddling plus the speed of the river. Also, the resultant speed of the paddling boat upstream is equal to the constant speed at which Vula was paddling upwards minus the downward speed of the river.

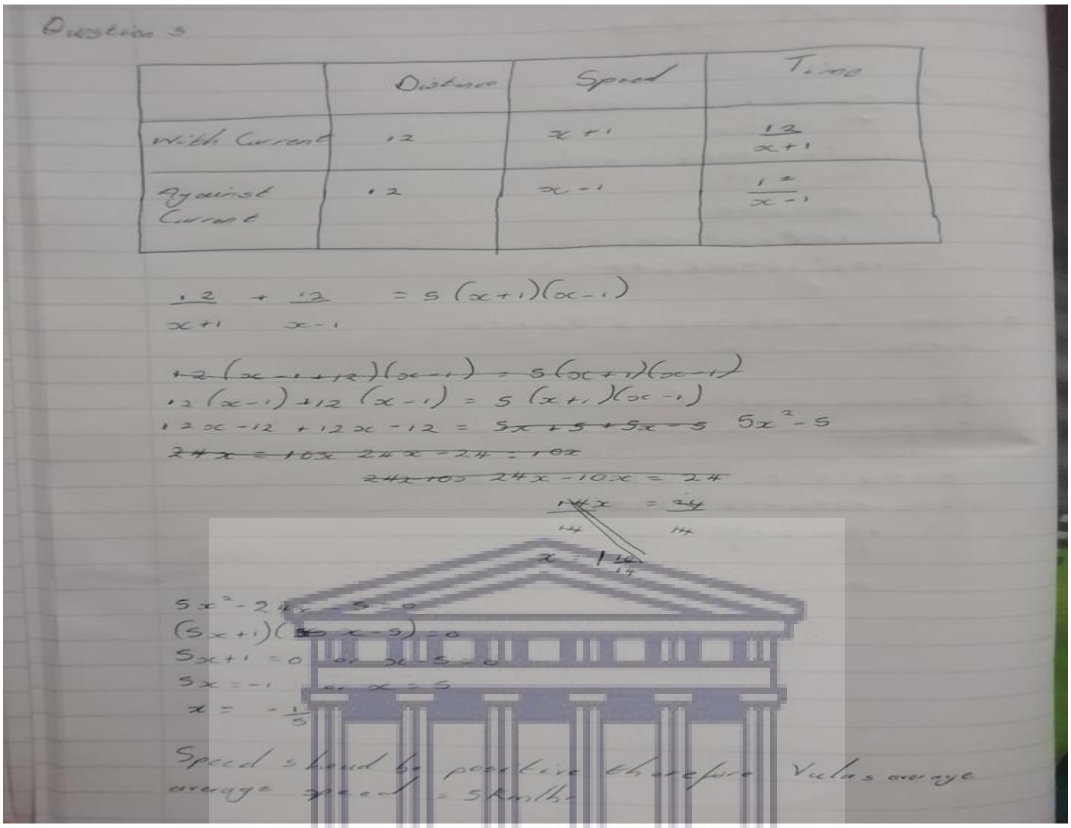


Figure 20.17: **Learner 2 with Correct Response to Problem No. 3**

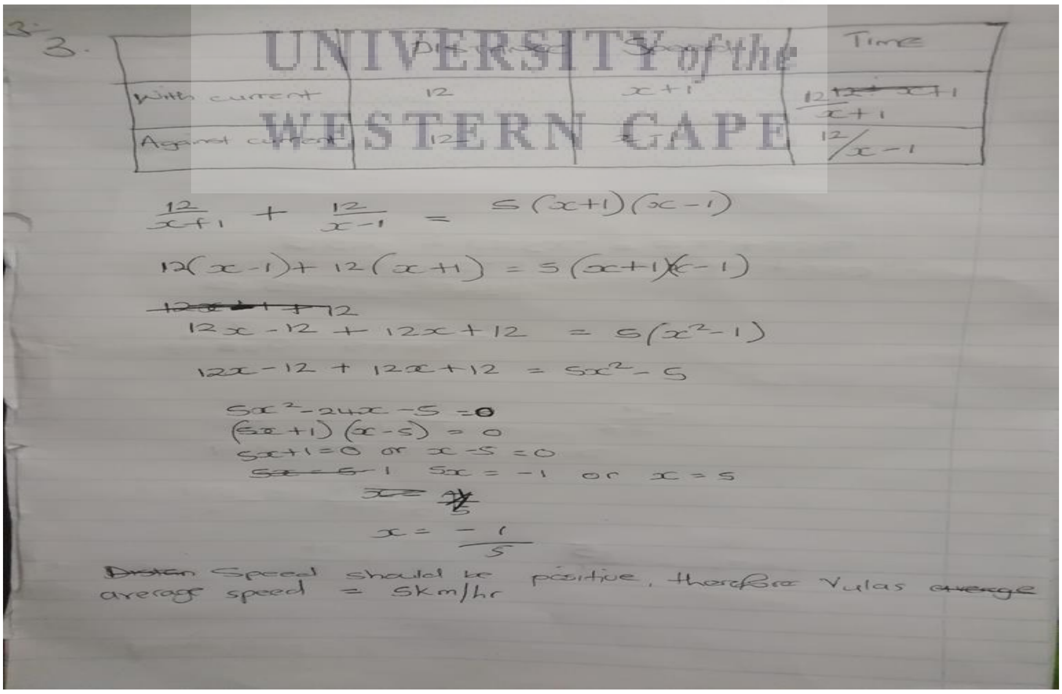


Figure 21.18: Learner 17 with Correct Response to Problem No. 3

While reflecting on the data assimilated in their respective tables, it is evident that both learners established what is given, what is expected to be found, and have mathematized relevant quantities and relationships to reach the following mathematical model expressed in terms of time:

$$12/(x+1) + 12/(x-1) = 5\,(x-1)\,(x+1).$$

Careful reflection on this model shows that it is the case that both learners did not substitute correctly into the formula, $time = distance/speed$ or did not realize that the total time for the forward and return trip is just 5 hours. However, it seems that in the next step, both learners simplified their algebraic equation to:

$$12(x-1) + 12(x+1) = 5(x-1)(x+1)\ ,$$

this signals that they do have some challenges in simplifying algebraic equations. Nevertheless, this latter simplification is equivalent to the algebraic model that they ought to have initially formulated to represent the situation, namely:

$$\frac{12}{x-1} + \frac{12}{x+1} = 5.$$

Despite this anomaly, both learners elegantly solved for x first through simplifying the situation to obtain the quadratic equation $5x^2 - 24x - 5 = 0$, and then through factorization of the left-hand side obtaining $(5x+1)(x-5) = 0$, and finally, through setting each factor equal to zero thereby obtaining $x = -\frac{1}{5}$ and $x = 5$.

Subsequently, both learners seemed to reflect on their answers to see if it makes sense or whether they are reasonable. On their understanding and knowledge that speed is a positive quantity, they discarded

$$x = -\frac{1}{5} \text{ and accepted } x = 5,$$

and hence concluded that Vula's speed is $5\ km/h$. Even though both learners wrote down their final answer, none of them took the initiative to check if their final answer works or satisfies the conditions of the given problem.

1.31.3 Exemplification of mathematical modelling competencies by learners who provided partially correct responses to Problem 3

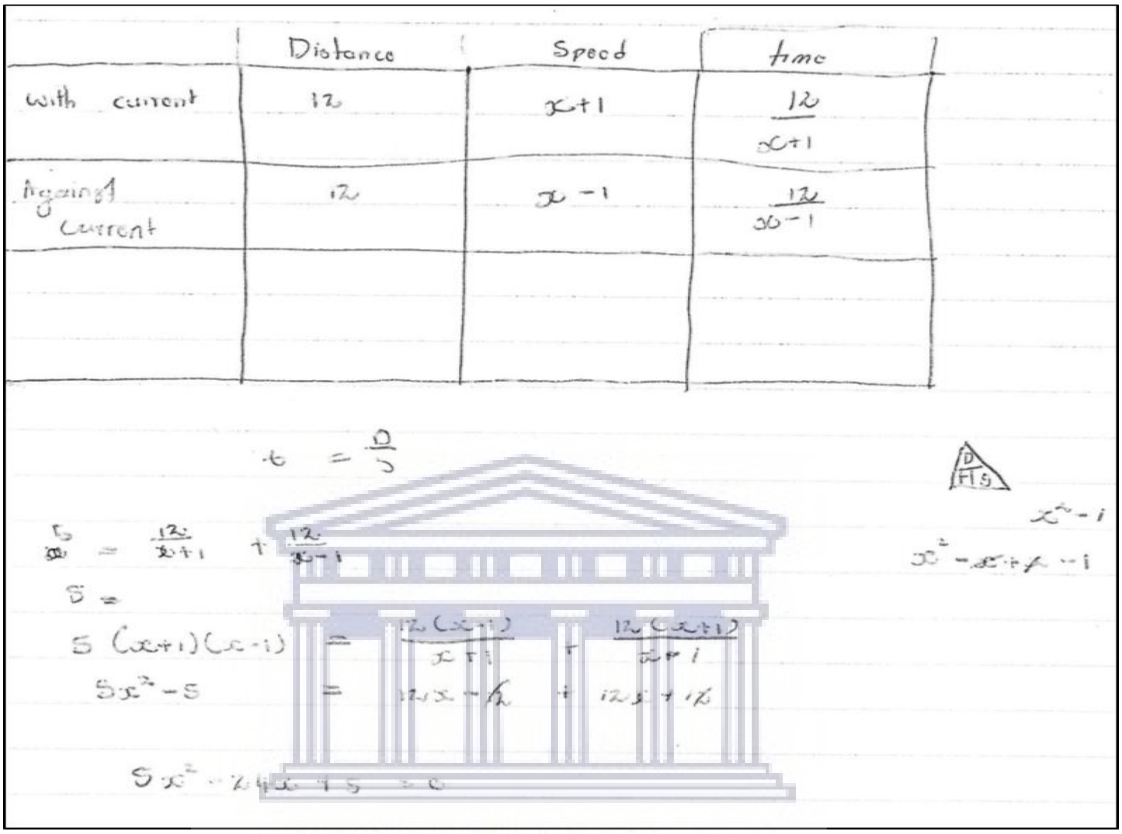


Figure 22.19: **Learner 6 with the Partial Correct Response Problem No. 3**

Leaner 6 in Figure 4.19 was able to assign the variable x to the average speed Vula travels just like the two learners who could not obtain the correct answer. Like learners A and B, learner L6 used a table to help understand the given information in the problem and build meaningful relationships in terms of x between distance, speed and time for with current and against current. Differentiating the case when travelling and against current shows a deep sense of understanding of the situation. This helped learner L6 to build a critical equation, which is the gateway to solving the problem, namely

$$5 = \frac{12}{x-1} + \frac{12}{x+1} \ .$$

This learner used the appropriate mathematical procedures and skills to simplify this fractional algebraic equation to the form

$$5x^2 - 5 = 12x - 12 + 12x + 4 \text{ but}$$

made slip in simplifying it to a quadratic equation in terms of x, namely

$$5x^2 - 24x + 5 = 0\text{, instead of}$$

$$5x^2 - 24x - 5 = 0\ .$$

Nevertheless L6's quadratic equation is incorrect, the learner did not solve for x, and neither made any attempt to look back at the last step to see if it befitted the solution to the problem. Hence, this learner did not complete solving the problem and did not exhaust all stages of the modelling process.

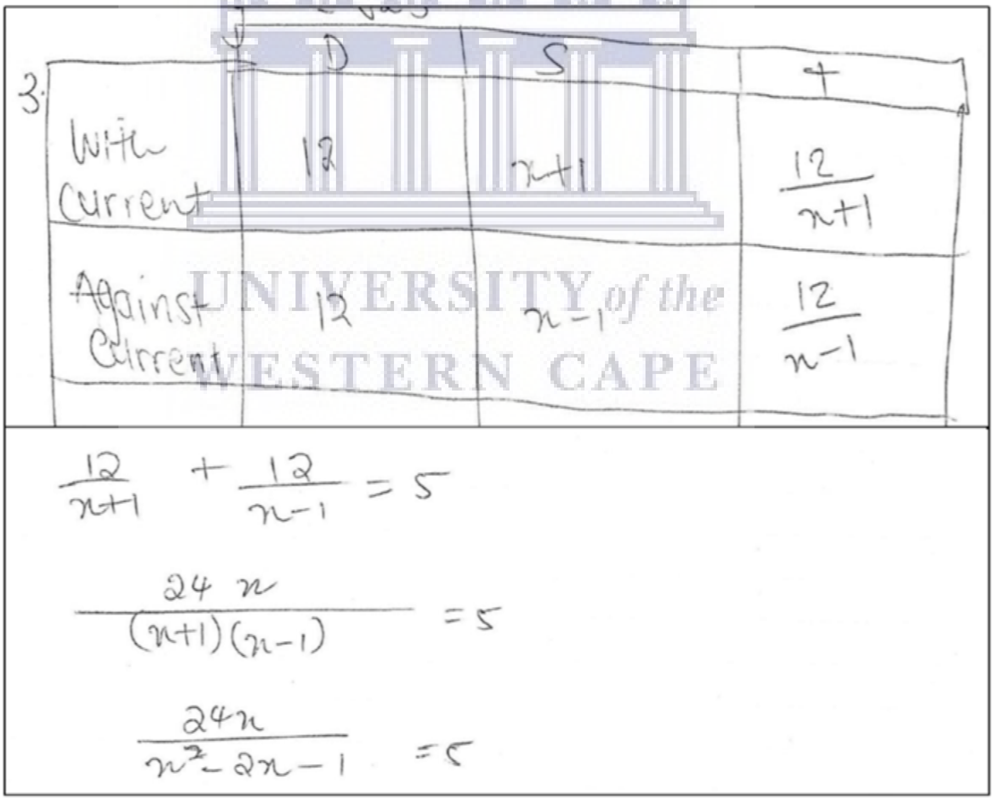


Figure 23.20: Learner 1 with Partially Correct Response Problem No. 3

As illustrated by learner L1 response in Figure 4.20 , the table helped most of the learners in this group to build a critical equation, which is the gateway to solving the problem, namely

$$\frac{12}{x-1}+\frac{12}{x+1}=5.$$

Despite this significant milestone in building a model, most learners in this group as shown in the last 2 steps for the case of L1 in Figure 4.20, could not solve for x in the established equation involving algebraic fractions. They abruptly stopped form proceeding any further. This in essence demonstrates that these learners were not able to use knowledge of solving algebraic equations to solve the model, and did not make any attempt to see if the answers satisfy the conditions of the given problem. When interviewed, learner L1 said:

> "It *was hard for me to find the correct answer. I did not know how to move from my last step. Solving fractions is difficult for me* "

This answer was an indication that the learner got stuck in the course of solving the problem. This learner lacks the competence of solving the fractions and formulating the quadratic equation. This also makes it difficult for the researcher to know whether the learner can solve the quadratic equation since the learner could not formulate the quadratic equation.

A few learners like learner L12 (as shown in the last step in Figure 4.21) were unable to simplify the algebraic equation containing algebraic denominators to a quadratic equation of the form

$$ax^2+bx+c=0.$$

In particular, L12 failed to remove the brackets and simplify the mathematical expression so as to formulate the quadratic equation. During the interview, the learner said:

> *'I was so confused by the question. I did not know what to do next' (L12).*

This confirmed the struggles learners experienced with the pre-requisite mathematical knowledge (**solving fractions, removing brackets**) that was necessary for solving the problem. This seemed to have hindered learners like L12 and others in this group from moving forward with solving the problem.

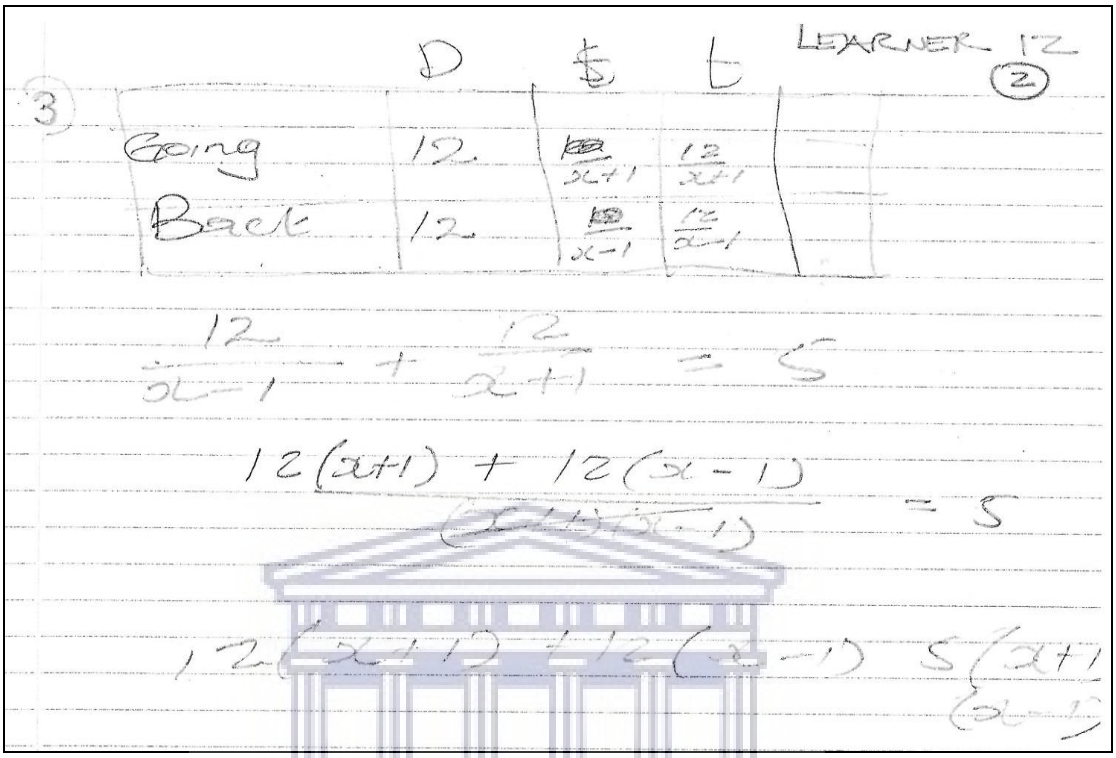


Figure 24.21: **Learner 12 with Partially Correct Response to Problem No. 3**

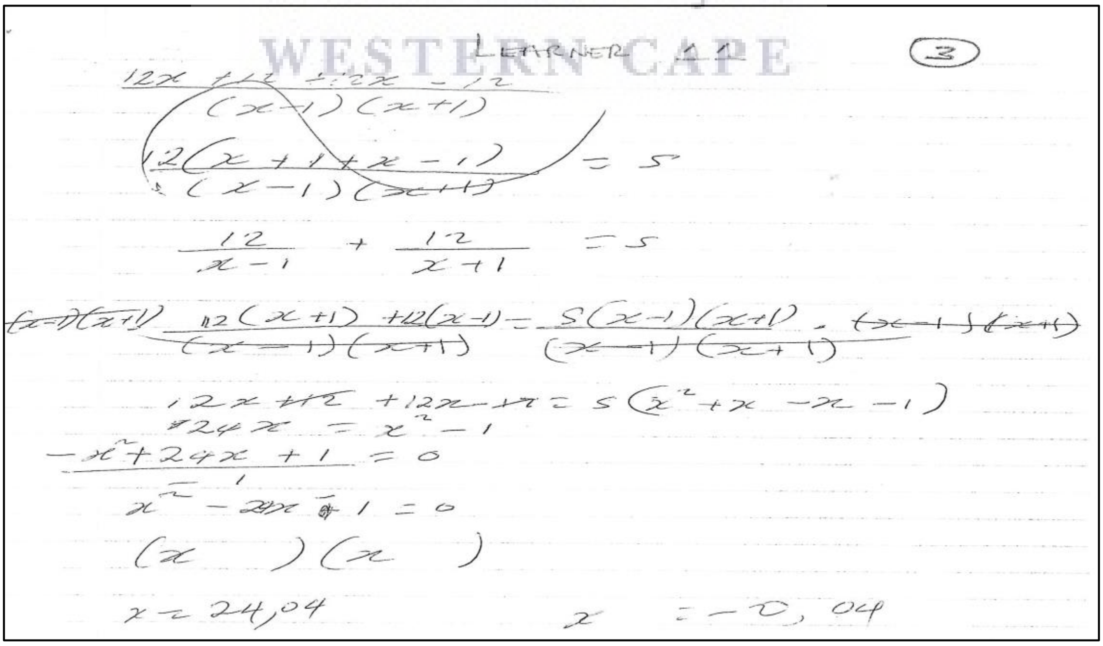


Figure 25.22 Leaner 11 with Partially Correct Response Problem No. 3

Although leaner L11 in Figure 4.22 did not show the use of a table as L1 and L12 did, he/she managed to construct the key equation $\frac{\mathbf{12}}{\boldsymbol{x}-\mathbf{1}}+\frac{\mathbf{12}}{\boldsymbol{x}+\mathbf{1}}=\mathbf{5}$, that could help solve the problem as indicated in Figure 24. The learner managed to multiply both sides of the equation by the LCD

$(x+1)(x-1)$, to obtain

$$12(x-1)+12(x+1)=5(x^2+x-x-1).$$

However, the learner's simplification of the equation in the immediate next step was incorrect. It seems that on the RHS, the learner simplified $(x^2+x-x-1)$ correctly to (x^2-1) but forgot to multiply it by 5. While applying consistent accuracy method, the learner then proceeded to obtain the quadratic equation $x^2-24x-1=0$, for which the LHS could not be factorized into linear factors. The learner could have used the calculator to solve for x in the quadratic equation $x^2-24x-1=0$. Although the learner obtained 2 values for x, the learner made no attempt to write down the final solution and/or crosscheck the answers.

1.31.4 Exemplification of mathematical modelling competencies by learners who provided incorrect responses to problem 3

It was observed that learners L2 and L19 struggled to answer question 3 as shown in Figures 4.23 and 4.24. They seem to have resigned to fate and were not willing to try. When they were interviewed, they stated that:

> *'I did not know what to do at first; I just answered the question to finish it' (L19).*

'The question confused me, when I tried to create an equation' (L2)

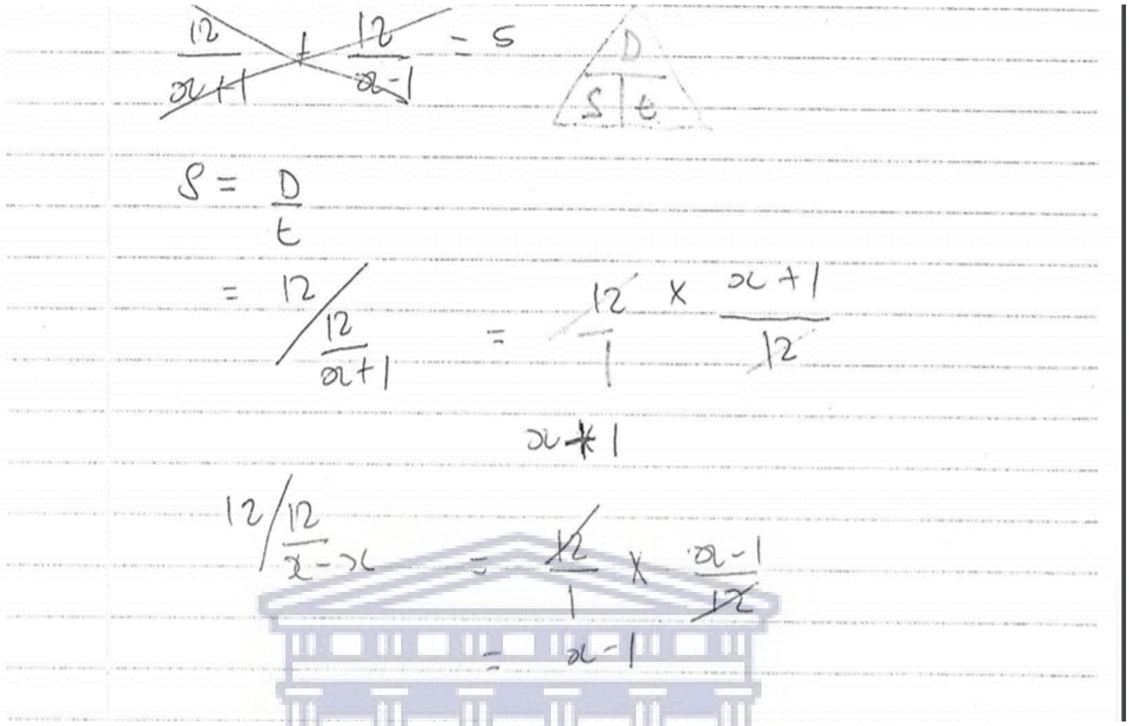


Figure 26.23: **Learner 2 with Incorrect Response to Problem No. 1**

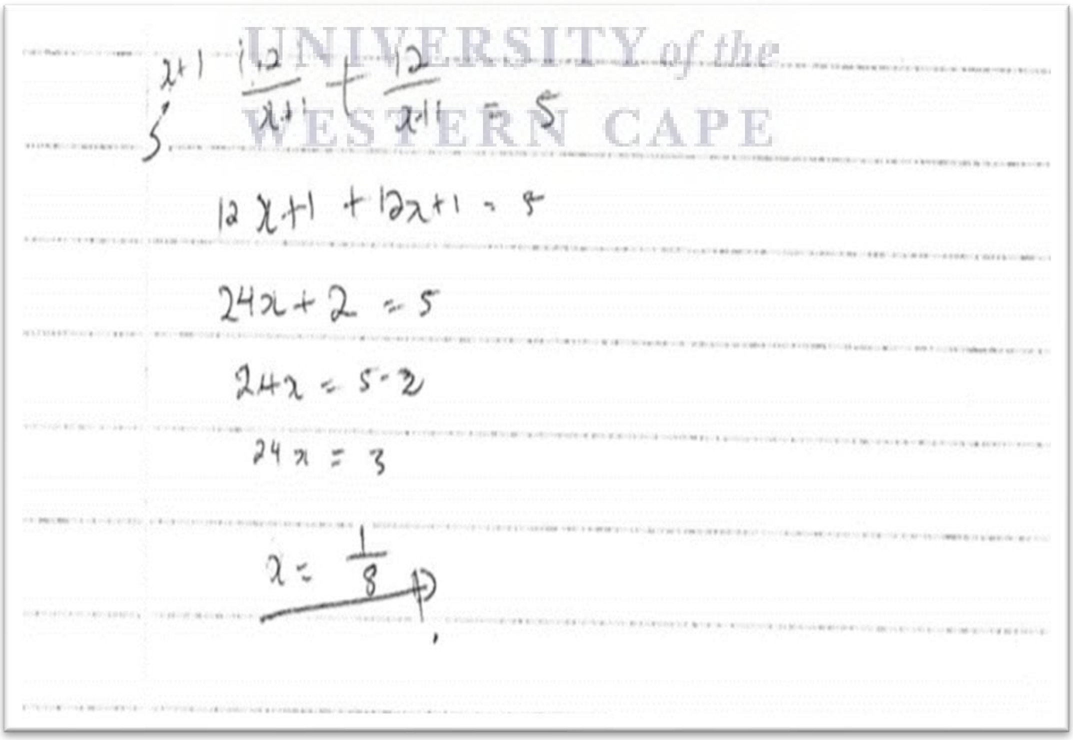


Figure 27.24: Learner 19 with Incorrect Response to Problem No. 3

These responses reflected what the learners wrote on their answer scripts. They appeared to have been demoralised by their inability to answer the questions. This showed that L19 and L2 failed to show the mathematical modelling competences that would start from formulating the table, quadratic equations and solving the quadratic equation. Learner 2 was far worse. The learner could not demonstrate that he or she has any idea of what was asked. L19 showed the fractions but could not solve them. When interviewed, the learners ascribed difficulties in solving the problem and cited their ability to visualize the situation as key challenges. For example, L9 stated:

> *'If you are struggling to understand the question, it becomes difficult to understand the question. Therefore, it was hard to visualise kilometres and hours' (L9)*

Equivalenty, L8 stated:

> *'about 50 per cent to 75 per cent of the tasks required us to visualise to understand the story behind the work' (L8).*

1.31.5 Summary of Findings with Respect to Problem 3

(i) Four out of 30 learners (13.3%) provided correct responses. In doing so, learners (L2, L4, L9 and L17) were able to recognise quantities that influence the situation such as distance, speed and time. Also, they established key variables were used to construct relevant relationships with the aid of a table. All of this suggests that they understood the problem to a good extent. In effect, they were able to mathematize relevant quantities and relationships but initially failed to correctly construct mathematical models to represent the situation. From the table, they wrote down $\frac{12}{x+1} + \frac{12}{x-1} = 5(x+1)(x-1)$ instead of $\frac{12}{x+1} + \frac{12}{x-1} = 5$. This could be attributed to incorrect substitution in the formula in terms of distance and speed.

However, the next simplification step seemed to characterise a correct model. Whether it was by coincidence or by deliberate correction of one's own mistake is unknown since the researcher did not have the opportunity to probe the learners. Both students were able to solve the quadratic equation successfully. They were able to reach the solution on the basis of their understanding that speed is a positive quantity. Hence, they discarded $x = -\frac{1}{5}$ and accepted x=5. Hence, it appeared that both students have used their mathematical knowledge appropriately to solve the quadratic equation. They also managed to interpret the roots within the context of the given problem. However, they did not show any evidence of validating their solution or checking the answer to establish its correctness.

(ii) In the group of 24 learners, nearly all of the learners were able to recognise quantities like distance, speed and time that influence the situation. With the aid of a table, the learners were able to meaningfully assign variables which they used to show understanding in building key algebraic terms for relationships. Whilst majority of these 24 learners were able to mathematise relevant quantities and establish their relations and set up the mathematical model: $\frac{12}{x+1} + \frac{12}{x-1} = 5$, there were some that failed to do so. The few learners who failed to form a correct algebraic equation were not able to make any headway towards the correct solution. Even if they did, they struggled with applying the procedural steps to generate roots.

(iii) It was evident that most learners in this group of 24 learners, who were able to construct a correct mathematical model for the situation, were not proficient in solving algebraic equations which contained algebraic fractions. Hence, they stopped abruptly after a few incorrect steps (see L1, L12) or proceeded by applying incorrect heuristics to generate roots that were not within an acceptable range for the problem (see L11). In the case of L11, those learners who generated erroneous values for x did not make any attempt to confirm whether the answers are accurate.

(iv) There were 2 learners (L2 and L19) with wrong responses. L2 seemed to have recalled the formula for speed which was correctly written down. Like L2, L19 made use of a variable x, with no clear assignment for what quantity it represents. Whilst the model L2 built seemed to demonstrate non-comprehension and lack of understanding of the problem, the model built by L19 $\frac{12}{x+1} + \frac{12}{x+1} = 5,$ does demonstrate some reasonable understanding of the problem even though the 2^{nd} term on the left-hand side of equation has an incorrect algebraic expression $\frac{12}{x+1}$ as divisor. However, L19 showed lack of ability to simplifying the algebraic fraction in the second step. This deficiency was illustrated by the simple action of multiplying throughout by the lowest common denominator. This demonstrates a lack of mathematical knowledge to help solve the projected model. Despite the attempts of both L2 and L19, none of them demonstrated any attempt at crosschecking or validating their solution.

1.32 Analysis of Modelling Competencies Demonstrated by Learners in Response to Problem 4

Question 4 read as follows:

A motorcyclist travels from A to B at 40 km/h and from B to A at 60 km/h. If the distance between
A and B is x km, determine the average speed at which the motorcyclist travels from A to B
and back to A. (The average speed is not 50 km/h).

Expected Solution for Question 4

Total distance is 2x km

Time from A to B is $\frac{x}{40}$ *hours*

Time from B to A is $\frac{x}{60}$ *hours*

Total time is $\frac{x}{40} + \frac{x}{60}$

$Average\ speed = \frac{total\ distance}{total\ time}$

$$s = \frac{2x}{\frac{x}{40}+\frac{x}{60}} = \frac{2x}{\frac{3x+2x}{120}} = 2x\ x\ \frac{120}{5x} = 48\ km/h$$

Average speed to A to B and back: 48km/h

1.32.1Learners performance in Problem 3

As illustrated in Figure 4.25, all thirty learners failed to answer question 4 with 24 thereby producing incorrect answers. More so, learner 6 did not attempt the question.

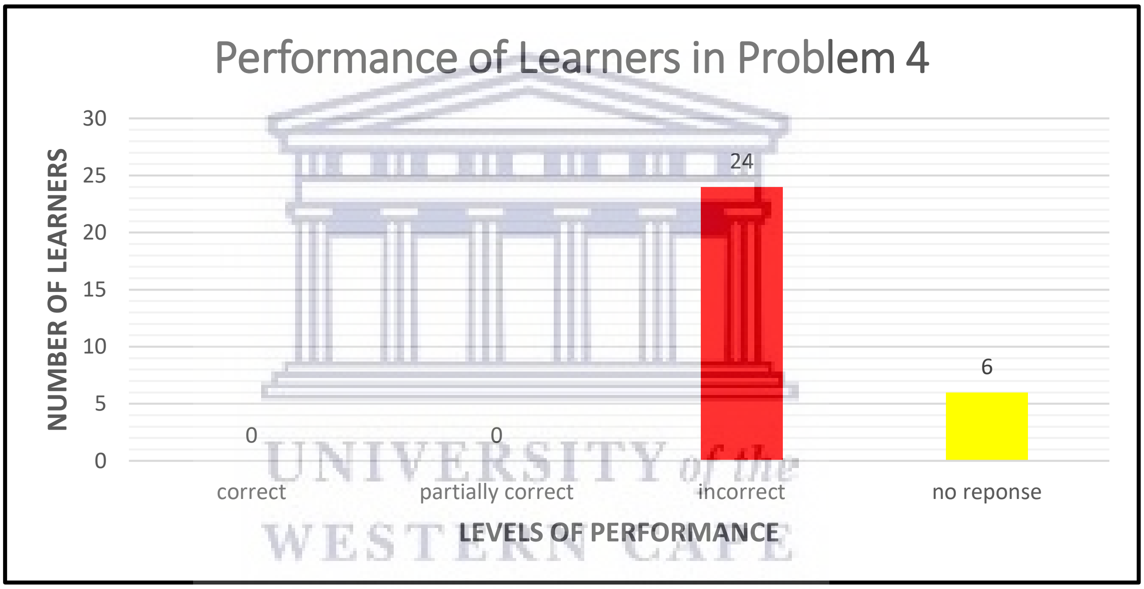


Figure 28.25: Levels of Performance of Learners in Problem No. 4

1.32.2Exemplification of Mathematical Modelling Competencies by Learners Who Provided an Incorrect Response to Problem 4

None of the 30 learners managed to provide a correct or partially correct solution. However, majority of the 24 learners who attempted the problem seemed not to have read the text precisely or imagine the situation clearly. None of them made use of a diagram or table to help understand the problem. Although, most recalled the formula for speed as $speed = \frac{distance}{time}$, they were not able to build relationships between what was given and the anticipated goals.

As evident in the responses of learners L13 and L15 (as shown in Figures 4.26 and 4.27), they were unable to mathematize the relevant quantities and their relations. For example, they were unable to show understanding that the forward time from A to B could be expressed in terms the distance $x\ km$ as $\frac{x}{40}$ hours and the return time from B to A could be $\frac{x}{60}$ hours. In essence, both learners, like others in this group did not recall or realise that the total time for the journey (forward and back) can be expressed in terms of x as follows: $\frac{x}{40} + \frac{x}{60}$. Although learner L13 seems to have inserted the total distance ($2x\ km$) in the numerator of the formulae $= \frac{distance}{time}$, the total time was represented by 40 instead of $\frac{x}{40} + \frac{x}{60}$ in the denominator..

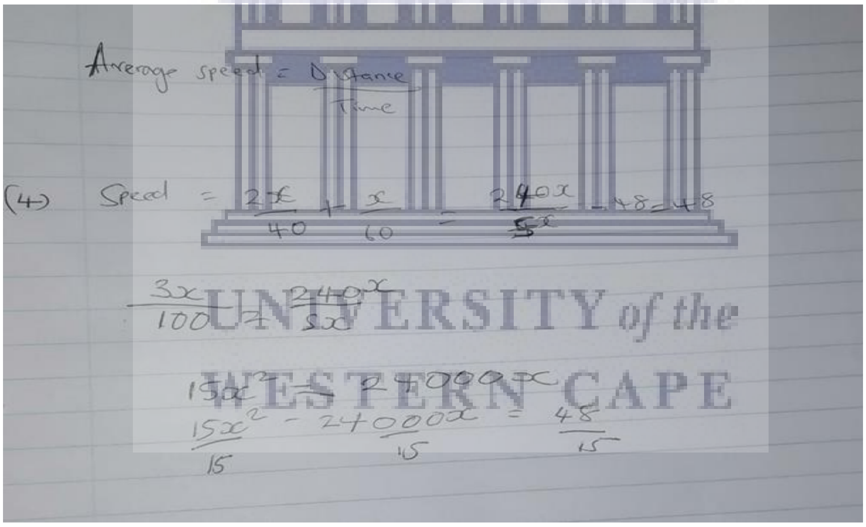


Figure 29.26: Learner 13 with Incorrect Response Problem No. 4

Also, learner L15 was unable to express the speed for the entire journey correctly in terms of x, and the expressions became muddled up. However, it is unclear why both learners set their expressions equal to 48.

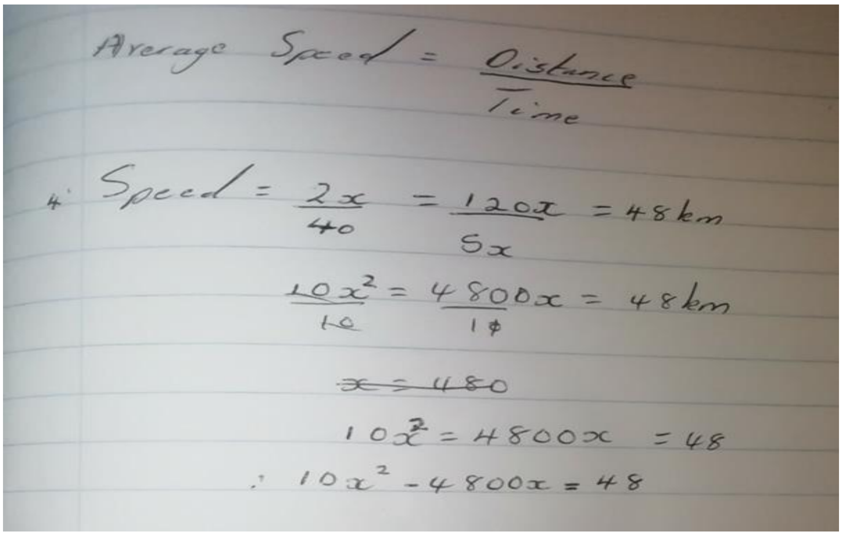


Figure 30.27: Learner 15 with Incorrect Response to Problem No.4

This is a complete mathematical breakdown and signals lack of conceptual understanding of the situation when making connections between mathematics and context. In addition, their calculations are muddled up, incorrect and incomplete. In a nutshell, the learners who attempted the problem demonstrated lack of understanding of problem. They also demonstrated incompetence as they could not build and solve problems using appropriate mathematical models; speed, distance and time relations.

With regard to question 4, most of the learners felt question four 4 is the most difficult. It was observed that the learner's attempt to answer the question was futile. L13 stated that:

> '*No. 4 and 5 required us to make a table. As we understood the question, we began to see what was happening but it took us time.'(L13)*

This response contradicts his answer as the learner did not show any attempt at engaging the competencies. This showed that the learner did not even attempt the question. Meanwhile, learner L15 did not make any assumptions. It was observed that he was visibly tired of doing the work. The researcher observed this as the learners were working. L15 stated that:

> *'Number 4 was difficult due to distance; I did not understand the whole table. I was tired of doing all four questions.' (L15).*

This response pointed to the presence of fatigue on the part of the learner by the time he started to attempt the fourth question. The same problem of fatigue was evident with learner L18 who attempted to offer the algebraic expressions in the course of answering question 4. The point of departure was the failure to offer solutions or validate the answers. The learner stated that he could have done this in two sessions so as to avoid fatigue. He averred that:

> *'To me, it was no, 4 which was difficult. I never understood the early stages of speed and time' (L18).*

In this vein, while mathematical concepts and skills should be structured around the problems to be solved (Dixon and Brown, 2012); this literature does not consider the issue of fatigue as a detrimental feature. Hence, it is flawed to the extent that it does not consider the fact that a learner may be affected by the number of questions that he or she is required to answer. From a theoretical perspective, Phillip (1997) perceived constructivism to include a psychological aspect. The scholar notes that there is a need to look beyond the social construction of knowledge. This should be extended to the psychological or cognitive aspect that embraces the need for the learner not to be emotionally drained or tired.

1.32.3 Summary of Findings with Respect to Problem 4

(i) Twenty-four of the 30 learners wrote incorrect responses, while the remaining four wrote incorrect responses.

(ii) Although most of those who produced incorrect responses made use of x as variable, it was not clear as to what quantity it was assigned to notwithstanding the fact that most of the learners seemed to have invoked the speed distance formula, $S = \frac{d}{t}$. Despite this, there has been a general failure for these learners to mathematize relevant quantities and their relationships and hence build the expected mathematical model that could help solve this speed, distance, time problem (as illustrated in the examples cited).

This could be attributed to many reasons. However, one of salient reasons seems to be the lack of conceptual insight that the total time for the journey (forward and back) can be expressed in terms of x as follows: $\frac{x}{40}+\frac{x}{60}$. In some cases, the algebraic expressions put forth by learners like L15 was muddled up, thereby confirming lack of conceptual understanding of the situation and hence inability to build a plausible mathematical model to solve the problem. In most cases, learners demonstrated neglect for mathematical conventions and procedures when formulating equations and solving them (see L13 for example). In principle, the learners' mathematical knowledge and capabilities to solve this higher order problem was really poor.

Regrettably, there was no evidence of any attempt to interpret (or make sense) or validate their final steps with respect to what was required of them in terms of proffering solution to the given problem.

1.33 Analysis of modelling competencies demonstrated by learners in response to Problem 5

Question 5 read as follows:

Thabiso has a budget of R 525 per month for petrol. The present price of petrol is x cents per liter. If the price rises by 50 cents per liter, she can buy five liters less petrol for R 525. Calculate the present price of petrol.

Expected Solution for question No. 5

At the present petrol price of x cents per litre, Thabiso can buy $\frac{52\,500}{x}$ *litres.*

At the increased price of $(x+\ 50)$ *cents per litre she can buy* $\frac{52\,500}{x+50}$ $litres$.

$$\frac{52\,500}{x}-\frac{52\,500}{x+50}=5$$

$$52\,500(x+50)-52\,500(x)=5\,[\,x(x+50)]$$

$$52\,500x\ +\ 2\,625\,000\ -\ 52\,500x\ =\ 5x^2\ +250x$$

$5x^2+250x-\ 2\,625\,000=0$

$x^2+50x-525\,000=0$

$(x+750)(x-700)=0$

$x \neq -750 \; or \; x = 700$

The present petrol price is 700 cents per litre (i.e. R7 per litre). If learner cannot find the factors, they must use the quadratic formula.

1.33.1 Learners performance in Problem 5

The holistic rubric as described in Table 3.2 was used to classify each of the 30 learners' responses to Problem 5 into one of the following categories
: correct; partially correct; incorrect; or no response. Figure 4.28 provides an overview of learners' performance with respect to Problem 5. Although 26 learners attempted the problem, their responses were all incorrect for a range of reasons. Four learners (4) learners provided no response.

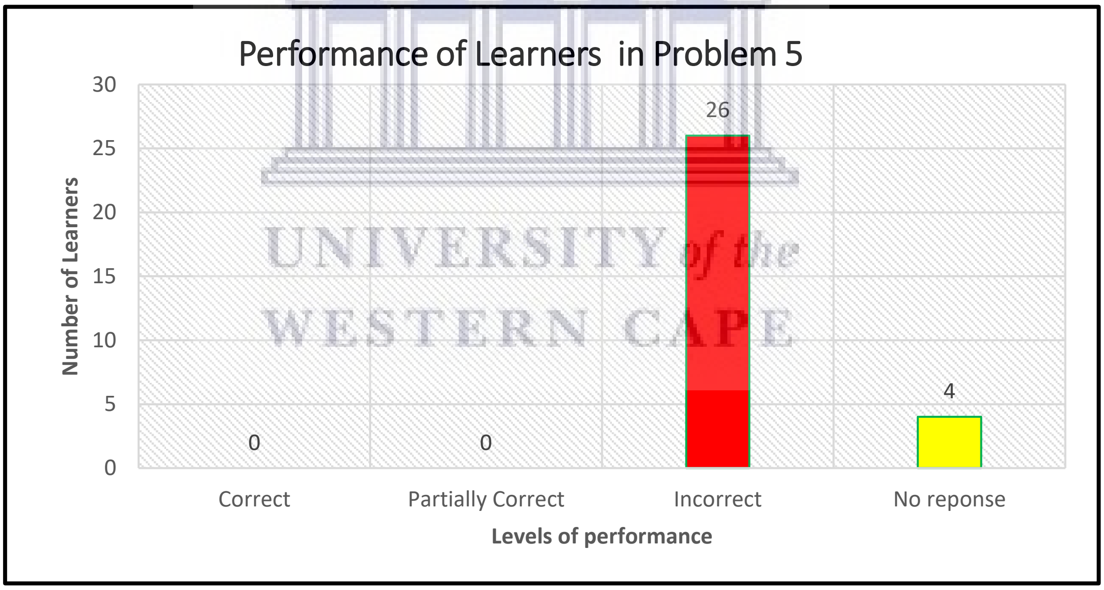


Figure 31.28: Levels of Performance of Learners in Problem No. 5

In Problem 5, learners were expected to demonstrate how to calculate the present price of petrol based on the change in price from the old price to the new price, using the algebraic expression formulated from the word problem. This would be done through mathematical modelling.

This problem required the learners to write down all important information into a very easy to read format. This required noting down all the relevant information to the effect that the learners understood the question. The learners were expected to write the price at present and after increment. In addition, the learners had to show that they were able to translate the phrases into expressions with variables. It follows that the variables were expected to be deduced from the available information. The learners were expected to create an assumption (as shown below in the expected solution) that fits into the formulation of an equation. They were expected to choose the variable x to represent the speed. After solving the equation, they were expected to validate the answer. In addition, they were expected to give the answer in a way that relates to the problem that was asked, using appropriate units.

Much like question 4, four learners did not respond and 26 learners attempted the question but produced incorrect responses as shown in Figure 4.28.

1.33.2 Exemplification of mathematical modelling competencies by learners who provided an incorrect response to Problem 5

As indicated in Figure 4.28, none of the learners managed to solve problem 5. Some learners developed some expressions but lacked conceptual understanding of the problem, to the extent they muddled up the expressions, and formed incorrect algebraic equations to represent the situation. In effect, they failed to mathematize relevant quantities appropriately and their relations. For example, learner L14 (as shown in Figure 4.29) seemed to form an expression that could represent the difference between the litres of petrol when cost was at $x\ cents$ and when the cost was increased from $x\ cents$ by 50 cents. The minus end in the first expression is correct as it represents the number of litres of petrol that can be bought for R525 at x cents per litres. However, the number of litres that can be bought for R525 when the cost was increased from $x\ cents$ by 50 cents was incorrectly represented in the subtrahend of the second expression as $\frac{52\ 500}{x-50}$ instead of $\frac{52\ 500}{x+50}$. Nevertheless, learner L14 did not express a relationship between these quantities; which represent the number of liters of petrol purchased at x cents per litre and $(x + 50)\ cents$ per litre.

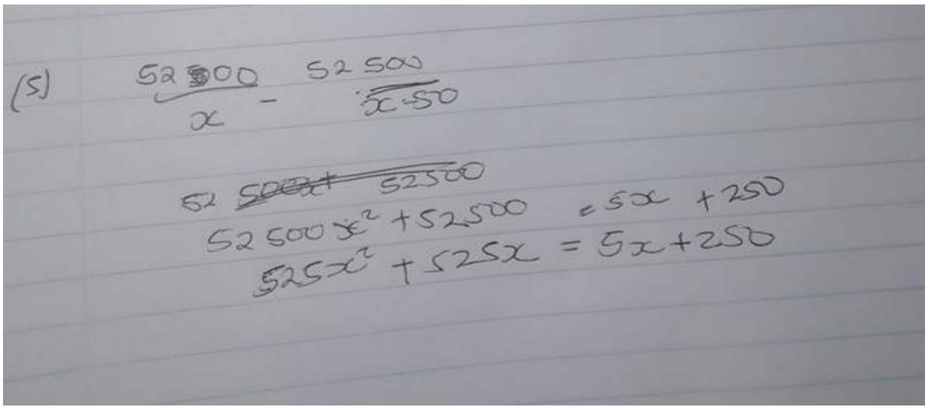


Figure 32.29: Learners L4's Response to Problem No. 5

The second statement represents a complete breakdown as it is not clear how the learner introduced an equation even though it is incorrect. This breakdown is further exemplified in the third line by the leaner's division of the LHS by 100 and not the RHS. All of this clearly indicates that the learner (L14) was not able to make connections between the mathematics and the information presented in the problem.

In the case of learner L21, the expression on the LHS of the presented equation is in the wrong order since the minuend represent the number of litres that can be bought for R525 when the cost increased from $x\,cents$ by 50 cents (i.e. when the cost was $(x + 50)$ cents), and the subtrahend represent the number of litres that can be bought for R525 when the cost was $x\,cents$. Notwithstanding this conceptual default, the learner equated the LHS to $5x + 50$ on the RHS of the equation instead of just 5. This implies that the learner did not comprehend the text precisely, and did not realize that the difference between the number of litres that can be bought at x cents and $(x + 50)$ cents respectively should be actually 5 litres and not $(5x\ + 50)$ litres. In effect, the learner was unable to build appropriate relations between what was given in the problem in terms of the variable x.

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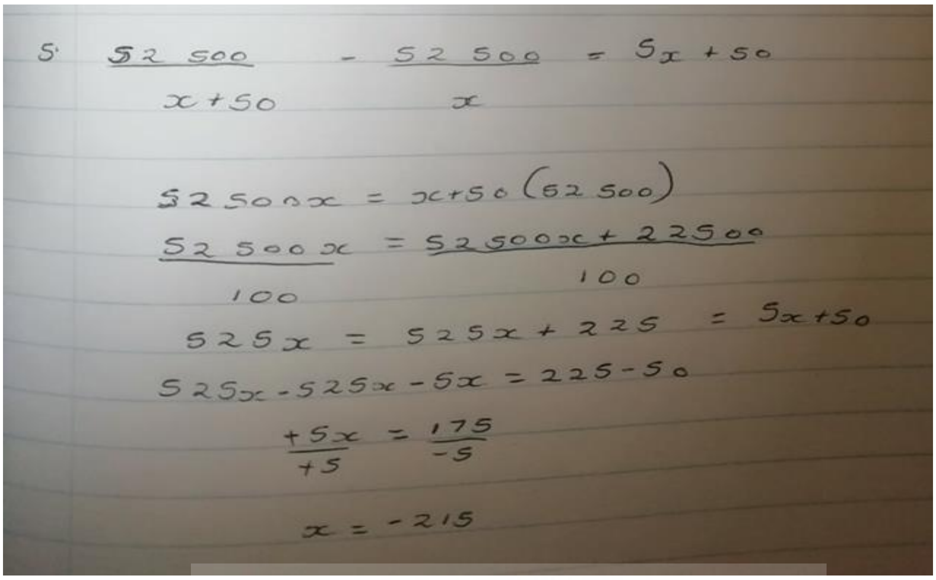

Figure 33.30: Learner 21 Response to Problem No. 5

Although learner L21 (see Figure 4.30) formulated an equation to represent the situation, it was incorrect. This implies that learner L21 was unable to build a meaningful model to help solve the problem. Furthermore, the equation written down in the second line is an incorrect simplification of the equation written in the first line. This is an umpteenth mistake which culminated in obtaining a negative value of -215 for x. This clearly demonstrates that the learners did not have the necessary mathematical knowledge and competencies to solve an algebraic equation. More importantly, the learner did not make any attempt to validate the answer. This way, he or she would have realised that the price of the petrol cannot be a negative monetary quantity

It was observed that learner L16 stated that the normal class questions were presented in mathematics (meaning as numbers), while the questions were presented as word problems in English Language and this constituted a barrier. He stated:

> *'In our everyday maths lessons, we are usually given short explanations. For example, if you want to prove something like a reason based on a triangle, you will be given all the necessary information and sometimes with formula but these questions do not even guide on which formula to use'* (L16).

Learner L17 also state that the approach that was used in asking the question was different from what they were used to in everyday mathematics class right from the primary school level. He stated that:

> *'Our daily classes give the equation and some explanation. This question required us to find the equation first and assumptions which was very difficult' (L17)*

It is evident from the assertions of learners' L16 and L17 that they are not conversant with solving word problems on their everyday mathematics classroom or homework activities. By implication, they appear to allege that their lessons are not learner-centred. Hence, when word problems are a 'bug bear' to learners, it is often compounded by language barrier.

1.33.3 Summary of Findings with Respect to Problem 5

(i) A sizeable number of learners (18 out of 26 who attempted the problem) could recognize quantities that influence the situation, and assign them key variables (like L14 and L21).

(ii) Twelve of the 18 learners (as identified in (i) above) were able to construct relationships between variables to a very limited extent but not holistically. They were not able to mathematize relevant quantities and their relationships completely and hence failed to set up a mathematical model.

(iii) The 12 learners (as identified in (i) and (ii) above) who generated an algebraic equation; albeit an incorrect one did not even have the necessary mathematical knowledge and competencies to solve their own algebraic equations (see for example L21)

(iv) Even though some learners proceeded to work with the incorrect equation, and produce a value for the variable x (which was no way near the expected answer), none of them made any attempt to interpret their solution or validate it. (see for example L21)

CHAPTER FIVE: DISCUSSION AND CONCLUSION

1.34 Introduction

This chapter discusses the findings of this study in relation to each of stipulated research questions. It offers a discussion of the limitations to the study and provides the conclusion and recommendations.

1.35 Discussion of Findings in relation Research Question 1

> ***Research Question 1:*** *What are mathematical modelling competencies grade 11 learners demonstrate when solving word problems involving quadratic equations?*

The modelling competency framework by Blum and Kaaiser (1997) as cited in Maaß (2006) (as presented in Table 4.1 of Section 4.2) guided the discussion of findings on the modelling competencies of grade 11 learners.

1.35.1 Understanding the Problem

(i) Problem 1: The rectangle problem

Understanding a problem is the cornerstone of mathematical modelling. Without the ability to read and understand a problem, the usefulness and power of mathematical knowledge, skills and procedures are limited (Olaniyan, Omosewo and Nwankwo, 2015). Across the range of the five problems that the 30 learners were required to solve, understanding of the problem varied depending on the content and cognitive levels that the problem posed required. In the case of Problem 1, 29 of 30 learners were able to sketch a diagram to represent the situation and simplify it. Drawing a diagram or other type of visual representation is often considered a good starting point for solving all kinds of word problems. It is an intermediate step between language-as-text and the symbolic language of mathematics (Kaput, et al. 2013). In these instances, at least 27 of these 29 learners assigned variables that represent the dimensions of the rectangle (like x to represent the breadth and y to represent the length). Notwithstanding, the learners expressed the length in terms of x in some instances, where x represented the breath of the rectangle.

All of these show that with respect to the rectangle problem, majority of the learners were able to read the text precisely and imagine the situation clearly as observed by Govender (2018). To a good extent, they were able to establish what is given and what is expected to be found, namely the breadth and length of the rectangle. By inserting x on the opposite short sides and y on the opposite long sides of the rectangle, majority of the learners were able to demonstrate that they recognize the properties of a rectangle and apply it apply to the situation.

Furthermore, they were able to successfully build relationships between variables to established that $3125 = xy$ is reached by using the concept of area of a rectangle, and by establishing that $300 = 2y + 2x$ (or $150 = y + x$) by using the concept of perimeter of a rectangle. This finding resonates with Hall and Chamberlee (2013) veiw that for successful modelling, learners should be capable of building algebraic expressions which represent specific situations of a problem.

(ii) Problem 2: The Two-Digit Problem

Nearly all of the learners (29 out of 30) including those that produced partially correct and even incorrect solutions were able to recognize quantities that influence the situation and routinely assign them variables without necessarily defining them upfront. These 29 learners showed necessary competence in building relationships between variables like sum of the 2 digits $(x + y = 13)$ and product of the 2 digits $(x.y = 36)$. These competences provided the basis for developing a mathematical model (Schoenfeld, 2014). These key achievements signal that the learners demonstrated sufficient understanding of the problem. According to Van De Walle (1998), understanding a problem is paramount because it provides the learner with the leverage to do something productive in his or her attempt to solve a given problem.

(iii) Problem 3: The River Problem

Four out of 30 learners who provided correct solutions as well as majority of the 24 learners who produced partially correct solutions were able to recognise quantities like distance, speed and time that influence the situation. They also established key variables which they used to construct relevant relationships with the aid of a table.

All of this demonstrates that majority of the learners did understand the problem to sufficient extent.

(iv) Problem 4: The Motorcyclist Problem

None of the learners produced a correct or partially correct solution to the motor cyclist problem. The 24 learners who made some attempt at the problem did make use of x as variable without clarifying what quantity it represents even though they evidently recalled the formula for speed in terms of distance and time ($S = \frac{d}{t}$). Most learners were unable to build expressions in term of x to represent the time for the forward trip from A to B, return trip from B to A, and total time for both return and forward trips. These struggles suggest that learners did not fully understand the problem and lacked conceptual understanding of how to configure the total time for the trip. It can be further conjectured that given the fact that these learners were second language speakers of English, it is highly probable that language barrier could have posed a problem in their quest to develop a correct understanding of the problem (Raoano, 2016).

(v) Problem 5: Petrol Price Problem

A reasonable number of learners (18 out of 30) could recognize quantities that influence the situation, and assign them key variables (like L14 and L21). However, only 12 of these 18 learners made some attempt at building relationships between variables in isolated and incorrect ways, to the extent that they failed to mathematize relevant quantities and their relationships. This in essence demonstrate that learners struggled to comprehend and understand the problem, and like in the case of Problem 4 could be attributed to language being a barrier as the learners were learning in their second language (Lenchner, 1983). Furthermore, Polya (1945) recommends that we should develop learners' capabilities to read problems intensively to gain deep insight into its key elements. Having a deep insight of a problem could help enable a learner to make necessary connections with relevant concepts and ideas which could enable him/her to have a breakthrough with the problem at hand (Polya, 1945). It is also vitally important that key words and phrases in problems that may pose a challenge to learners are identified and necessary scaffolds are put in place to enable learners to move forward with problem.

1.35.2 Building a Mathematical Model

(i) Problem 1: The rectangle problem

As discussed in (a), majority (28 out 30) of learners were able to express the relationships between the sides of the rectangle in terms of the respective dimensions, namely length and breadth. So, in effect they were able to mathematize relevant quantities and build a mathematical model that in most instances constituted a system of linear equations as follows (see for example L9, L10, and L16):

$3125 = xy$ ………………. (1)

$300 = 2y + 2x$ ………… (2)

The above system simultaneous system of equations is a compact way of modelling the situation, which not only provides an exposition of learners' insights into problem but can help to solve the problem (Niss, 2010)

(ii) Problem 2: The Two-Digit Problem

As evidenced in Section 4.2.2, majority of learners (29 out of 30) were able to express the sum of the 2 digits as $x + y = 13$ and the product of the 2 digits as $x.y = 36$, and use these 2 equations to assimilate a system of linear equations to solve the problem as follows:

$x + y = 13$ ….. (1)

$xy = 36$ ….. (2)

Hence, in alignment with MaaB (2006), it is prudent assert that the learners demonstrated sufficient competence to mathematize relevant quantities generate a system of linear equations as a mathematical model, which could be solved through applying the method of solving a system of linear equations find the values of the unit digit and tens digit for the 2- digit number.

(iii) Problem 3: The River Problem

Notwithstanding some blemishes in a few cases, majority of the learners demonstrated sufficient competence to mathematize relevant quantities and relationships and assimilate pivotal constructed algebraic expressions to build the mathematical model to represent the situation correctly. The model was generally presented as follows: $\frac{12}{x+1} + \frac{12}{x-1} = 5$. As was found in Mousoulides (2007) study, it was found that most of the learners were able to invoke the formula for speed in terms of distance and time and develop interrelationships and with a high degree of fluency advance a meaningful relationship between core quantities that could make it possible to solve the problem in this instance.

(iv) Problem 4: The Motorcyclist Problem

The learners' inability to build critical algebraic expressions that represent given information or pertinent information coupled with lack of conceptual understanding of the problem could have caused them not to build a relevant and correct mathematical to solve the motorcyclist problem. In principal all the learners in this group were not able mathematize relevant quantities and their relations possibly due the lack of their representational fluency as was articulated in the study by Mousoulides (2007).

(v) Problem 5: Petrol Price Problem

Only twelve of the 30 learners made some attempt build relationships between variables in isolated and incorrect ways to an extent that they failed to mathematize relevant quantities and their relationships completely. This in essence demonstrate that learners were not capable to set up a mathematical model that represent the given situation adequately and appropriately, and transform the mathematical ideas into mathematical entities that could help to piece together a feasible model (Govender, 2018).

1.35.3 Solving the mathematical Model

(i) Problem 1: The rectangle problem

As evident in the presentation of data and findings, only 2 out of 30 students managed the complex procedure of working with 2 linear equations using the simultaneous strategy to arrive at a correct quadratic equation and then solve it to generate 2 roots correctly (see L9 and L15). One of these learners (L9) chose the smaller y value to represent the breadth, and correspondingly substituted in $300 = 2y + 2x$ to obtain $x = 25$. The second learner L15, who obtained 2 correct x values, did not clearly show how he/she arrived at $y = 125$ and $x = 25$ (it seems that instead of writing $y = 25$ the learner may by slip wrote down $x = 25$)

Doer and English (2006) found in their study students expressed difficulties with the mathematical content in topics associated with the mathematical problems posed. For example, they did not know what a graph is or how to calculate salient features of graphs including solving quadratic equations and algebraic equations. Similarly, majority of the 25 learners (i.e. 21), who produced a partially correct solution, failed to solve their mathematical model (which was correct) for one of the following reasons:

- Leaving out the brackets (see Figure 8 for L16 response) which resulted in obtaining a quadratic equation that was different from the expected one.
- Failure to factorize a quadratic expression into linear factors (see Figure 8 for L16 response)
- Inability to realize that a quadratic equation should be expressed in the form $ax^2 + bx + c = 0$, where after $ax^2 + bx + c$ can be factorized and expressed as a product of 2 linear factors. This shortcoming is illustrated for example in L10 response in Figure 7)

A small number of learners (4 out of 25), who obtained a partially correct solution, did not show any attempts to solve their assimilated system of linear equations (see for example L13 in Figure 9)

(ii) Problem 2: The Two-Digit Problem

A very small percentage (6.7%) of learners (namely L9 andL28) were able to apply the procedure of solving a system of simultaneous linear equations through generating a quadratic equation, and then solving it to yield a set of roots which represented $\boldsymbol{x}$ -values (or $\boldsymbol{y}$-value). These x-values where then substituted to yield the corresponding y values (or On the contrary, most learners who produced a partially correct solution proceeded correctly up to the stage of generating the quadratic equation, $\boldsymbol{y^2 - 13y - 6 = 0}$, and then stopped. This inability to move forward could possibly be attributed to them not knowing how to solve a quadratic equation. On the other hand the five learners who produced incorrect answers made gross mathematical mistakes as they tried to move on from the point of forming a mathematical model to an extent that they were not able to generate the quadratic equation $\boldsymbol{y^2 - 13y - 6 = 0}$. In the limited cases some of these 5 learners did produce a quadratic equation which was different from the expected $\boldsymbol{y^2 - 13y - 6 = 0}$, they made serious errors to an extent they arrived at undefined solutions. For example, L29 worked with $\boldsymbol{13 - y^2 = 36}$ (which was incorrect) and proceeded to solve for $\boldsymbol{y}$ and obtain y = $\boldsymbol{\sqrt{-23}}$. In summary it seems that most of the students. Taking all of the aforementioned discussion into account, it seems that generally majority of learners failed to solve the system of simultaneous equations primarily because they did not seem to be aware of what to do after generating a quadratic equation, notwithstanding that few who produced an incorrect quadratic equation, made serious mathematical errors as they proceeded further as was evident in the study by Gallegos (2008).

(iii) Problem 3: The River Problem

Four out of 30 learners ((L2, L4, L9 and L17), who provided correct responses, did possess the required heuristic knowledge on how solve their mathematical model, which required them to simplify an algebraic equation containing fractions to a quadratic equation of the form $ax^2 + bx + c = 0$, and then factorize the quadratic trinomial on the left hand side of the quadratic equation (which is in standard form) to produce to possible roots. As evidenced in the responses of L2 and L17, they were quite fluent in solving their mathematical model. On the other hand, most of 24 learners who produced partially correct solutions but constructed a correct and relevant mathematical model as a representation of the situation, lacked the capabilities of moving correctly through the salient of steps to solve the quadratic equation to an extent they stopped after performing a few steps incorrectly (like L1. L12) or moved on to produce roots were undefined (i.e. meaningless). Reflectively a very small number of learners (4 out of 30) demonstrated sufficient competency in solving their mathematical model correctly as was also found in the study by Ferreira and Jacobini (2008).

(iv) Problem 4: The Motorcyclist Problem

As none of the learners go to the stage of building a model that represents the situation correctly or partially correctly and hence did not engage in the solving of a mathematical model, it is indeed difficult to make any positive assertion. However, reflecting on a few candidates who in a muddled way established a compromising model (which was really inappropriate) and then proceeded to solve with an array errors permeating each step, it is plausible to say that these few learners demonstrated no ability to solve their model even though the model was irrelevant as was found by Govender (2018).

(v) Problem 5: Petrol Price Problem

Only 12 learners proceeded to generate an algebraic equation containing fractions even though they were seriously compromised and flawed with gross errors. Even worse, all 12 of them lacked the ability and procedural knowledge to solve their own incorrect algebraic equation containing fractions. As advanced by Oberholzer (1992), it is evident that in the case of a complex problem the learner's own mathematical skills were insufficient to tackle this problem to full extent.

1.35.4 Interpreting the mathematical results

As part of the modelling process, mathematical results should be interpreted in terms of the original problem and context (Maab, 2006). Ferreira and Jacobini (2008) affirmed that in the modelling tasks that students were required to solve in their study, presented difficulty during results interpretation irrespective whether they invoked applied mathematical relevant and mathematical procedures correctly or incorrectly or partially. The same kind of scenario prevailed across most of the problems as indicated in the following discussion for Problems 1-5.

(I) Problem 1: The rectangle problem

Only one of the learners, L9, who produce a correct solution, demonstrated necessary competence to interpret the mathematical result within the context of the problem to the extent of signalling clearly that the length is 125mm and the breadth is 25mm. This shows appreciation of the use of units as well. None of the 25 students who produced a partially correct solution showed deliberate evidence of interpreting their mathematical results.

(ii) Problem 2: The Two-Digit Problem

None of the learners even those who obtained a correct or partially correct solution signalled any attempt to interpret their mathematical result in relation to the given problem.

(iii) Problem 3: The River Problem

None of the learners including the four students who solved the algebraic equation containing fractions correctly, showed any evidence of interpreting their solution.

(iv) Problem 4: The Motorcyclist Problem

Those few learners who made some attempt to show some calculations did not make any attempt to interpret their solution if produced.

(v) Problem 5: Petrol Price Problem

As discussed in Section 4.4.5, only twenty four of the 30 learners attempted this problem, and none obtained a correct solution. Like for problem 4, none of these learners made any attempt to interpret their solution.

The implication of the findings in problems 1 to 5 above, is that majority of the learners exhibited low level of mathematical competencies. This finding is corroborated by the finding of Julie (2020) whose study highlighted low level of mathematical modelling competencies among learners in grade 10 and 12 in school mathematics programme in South Africa.

In addition, the inability of the learners to solve problems 4 and 5 could be ascribed to lack of competency of making assumption as put forward by Haines et al.,(2001, p. 373) "The difficulties students face with mastering mathematical modelling competencies are summarised as ' . . . Identifying broad assumptions influencing a simple model . . . posing clarifying questions with a mathematical formulation relating to assumptions [and] effective comparison of the model outcomes with the real world problem".

Another reason why these learners could not solve problems 4 and 5 ccan be attributed to the fact that they were not provided with models and formular sheets to haid their solving modelling questions contrary to what they are used to. In South Africa it is the norm to provide learners with models relating to mathematical words problems as observed by Julie (2015, pp.477-478)

1.35.5 Validating the solution

As a core step of the modelling process, leaners are expected to test the quality of their model and process (MaaB, 2006). This includes learners reflecting on whether their mathematical answer makes sense in terms of the given problem (Govender, 2018). In a study by Doer and English (2006), students manifested uncertainty in validating their solutions for reasons such as lack of clarity of the problem and insufficient mathematical knowledge, skills and procedures. Regrettably, in this study there was only one learner (L9) limited to only the rectangle problem hat appropriately and correctly validated his/her solution within the context of problem. This means that in all remaining attempts across all five problems, the learners' did not make any attempt to validate their solutions (correct /incorrect). This could be attributed to reasons as advanced by Doer and English (2006).

(i) Problem 1: The rectangle problem

L9, who was one of the learners who obtained the correct solution was the only learner who reflected on his solution to the extent that he verified by substituting the value of the length 125cm and the value of the breadth 25cm into the area formula, $A = lxb$ to show that $A = 25mm\ x\ 125mm = 3125mm$ (noting the slip in the square units: mm^2). Despite the minor slip, this learner demonstrated sufficient competence to check and validate his solution.

(ii) Problem 2: The Two-Digit Problem

None of the learners even those who obtained a correct or partially correct solution displayed any evidence of validating their solutions (or responses). This implies that the learners lack the skills of validating their solution as emphasised by Maaß, 2006, pp. 116–117).

(iii) Problem 3: The River Problem

None of the learners including the four students who solved the algebraic equation containing fractions correctly, showed any evidence of interpreting their solution.

(iv) Problem 4: The Motorcyclist Problem

As discussed in Section 4.4.4, only twenty four of the 30 learners attempted this problem, and none obtained a correct solution. None of these learners made any attempt to validate their solution.

(v) Problem 5: Petrol Price Problem

As discussed in Section 4.4.4, only twenty four of the 30 learners attempted this problem, and none obtained a correct solution. Like for problem 4, none of these learners made any attempt to validate their solution.

1.36 Discussion of Findings Research Question 2

Research Question 2: *To what degree does learners' competency in setting up a mathematical model inhibits the development of an acceptable solution?*

(i) Problem 1: The Rectangle Problem

Ninety present of the learners (27 out of 30) were able to construct the system of linear equations, which served as the mathematical model to represent the rectangle problem. However, only two students (L9 and L15) were able to use their model to generate a relevant set of roots (i.e. x-values), which was used to ascertain the corresponding y-values. Al large number (21 out of 25) who produced partially correct solutions managed to use their model to an extent where they simplified $300 = 2x + 2y$ (equation 2) to get $= 15 - y$, which was then into $3125 = x \times y$ (equation 1). However, further simplification of the situation was compromised by:

- poor use of brackets (see Figure 8 for L16 response),
- Failure to factorize a quadratic expression into linear factors (see Figure 8 for L16 response)
- Inability to realize that a quadratic equation should be expressed in the form $ax^2 + bx + c = 0$, where after $ax^2 + bx + c$ can be factorized and expressed as a product of 2 linear factors. This shortcoming is illustrated for example in L10 response in Figure 7)

A small number of learners (4 out of 25) who obtained a partially correct solution did not show any attempt to solve their assimilated system of linear equations (see for example L13 in Figure 9).

In the case of the rectangle problem, poor use of brackets, lack of mathematical knowledge on how to write a quadratic equation in standard form and/or the ability to factorize a quadratic expression into linear factors hindered 83,33% of the learners' from using their correctly established model to find solutions to the problem. Only 10% of the learners (3 out of 30) were hindered by their use of an incorrect mathematical model in their quest to find a solution to the problem. The learners' error in judgment was exacerbated by poor mathematical knowledge.

(ii) Problem 2: The Two-Digit Problem

As illustrated in Sections 4.2.2 and 5.2.1, an extremely large percentage of the learners were able to construct a mathematical model comprising of the following system of 2 linear equations:

$x + y = 13$ ….. (1)

$xy = 36$ ….. (2)

Hence, learners' competency in setting up a mathematical model did not hinder them from solving the problem but instead their lack of competency in solving the system of linear equations was the hindrance.

(iii) Problem 3: The River Problem

Majority (80%) of the learners including 4 of those who produced correct solutions and 20 of those who produced partially correct solutions were able to mathematize relevant quantities, their relations and also set up the mathematical model, of $\frac{12}{x+1} + \frac{12}{x-1} = 5$. To a large extent, learners' competency in setting up a mathematical model did not hinder them from solving the problem. Their lack of competency in solving an algebraic equation containing algebraic fractions was the hindrance. In 20% of the cases (6 out of 20 learners), failure to set up an appropriate mathematical model seemed to have hindered them from solving the problem.

(iv) Problem 4: The Motorcyclist Problem

In the motorcyclist problem, only 80% of the learners (24 out of 30) attempted the problem. However, none of these 24 learners were able to build a mathematical model that appropriately represented the problem. Hence, it is plausible to conjecture that in 80% of the cases, learners lacked competency in setting up a mathematical model could have inhibited the development of a plausible solution.

(v) Problem 5: Petrol Price Problem

Although 26 learners attempted the problem, none of them could set up a mathematical model that appropriately represented the problem. Hence, it is plausible to conjecture that in 87% of the cases, learners' lack competency in setting up a mathematical could have been the inhibiting factor in the development of a reasonable solution to the problem.

1.37 Acknowledgement of the limitations of the study

The limitations of the study revolved around the use of English. Since the mother tongue of the researcher and participants is not the English language, this study holds the perception that linguistic barrier was a factor that inhibited learners' competency with regard to solving the questions accurately. This was exacerbated by the fact that the learners in grade 11 complained about the problem of using English as the language of instruction.

Because the research was conducted towards the end of the term, the data collection process was carried out during a double-lesson period as the educator had to continue with everyday teaching. This situation took its toll on the data collection, participant observation and focused group interviews. The scope of the study was also limited to one secondary school in the Western Cape. Hence, the findings of this study cannot be representative of all grade 11 learners in South African schools. It can only be applicable to grade 11 learners of Western Cape school.

1.38 CONCLUSION AND RECOMMENDATIONS

The number of learners that obtained correct solutions was limited to two for the rectangle and two-digit problems. In the case of the river problem which was pivoted around average speed, four students obtained correct solutions. None of the students obtained correct solutions in the case of the motorcyclist problem and petrol price problem. The number of learners that obtained partially correct solutions was limited to 25 for the rectangle problem, 23 for the two-digit problems, 24 for the river problem, and none for each of the motorcyclist and petrol price problems. It was evident that as the problems increased in cognitive demand (like in the case of the motorcyclist and petrol price problems), none of the learners who attempted them got it either correct or partially correct. The low levels of performance exhibited by the learners seem to correspond with the levels of mathematical competency.

The finding of this study has shown that learners who obtained the correct solutions in the case of the first 3 problems demonstrated sufficient understanding of the problem. This was the case because they could recognize quantities that influence a situation, name them, assign variables, and construct relationships between variables. The few learners were competent to mathematize relevant quantities and relations to an extent that they constructed relevant mathematical models in the case of problems 1-3. In addition, they managed to recall and invoke the necessary mathematical knowledge and heuristics to solve their respective mathematical models. It was only in the case of the rectangle problem that one learner demonstrated sufficient evidence of interpreting and validating his solution, whereas in all other cases for all of problems 1-3 this was not the case.

Regrettably, none of learners who generated correct mathematical solutions for problems 1-3, showed any real understanding of problems 4 and 5. They could not effectively develop a feasible mathematical model to solve problems 4-5. Hence, they failed to crosscheck and validate the derived solutions.

Furthermore, the findings showed that majority of those learners who produced partially correct solutions for problems 1-3 were able to demonstrate understanding of problems 1-3. They were able to mathematize relevant quantities and relations and build appropriate mathematical models. However, solving each of the mathematical models posed a challenge to most of these learners as they appeared to be less proficient in solving a system of linear equations. They were deemed to be less proficient because they ignored the use of brackets, were unable to simplify an algebraic equation containing fractions.

Moreover, they were unable to write a quadratic equation into standard form. Inability to factorize a quadratic equation into linear factors, or not knowing the next steps after deriving a quadratic equation or the system of linear equations itself were demonstrated as well. None of these learners showed attempts to interpret and validate their solutions even though they were incorrect or incomplete. All of learners, who produced partially correct solutions for problems 1-3 produced incorrect solutions for problems 4 and 5 whilst a few did not attempt the questions. This indicates that the students did not understand problems 4 and 5, and this could have caused them not to generate a meaningful mathematical model or proceed further.

For those few learners who produced incorrect solutions for problems 1-3, only one of them was able to generate a system of linear equations (in the case of the rectangle problem) that was partially correct. This can be attributed to lack of knowledge on how to represent the product of 2 digits in the construction of a linear equation (see L1). Despite this anomaly, the learner appeared to be incapable of applying the heuristic procedure to solve a system of linear equations. In addition, none of these learners produced correct solutions for problems 4 and 5 if they attempted it. Also, they seemed not to have crosschecked the solutions reached; and this negligence was observed in problems 1-3. On a general note, it appears that the learners were not conversant with how to tackle and solve word problems pivoted on quadratic equations.

While reflecting on the extent to which learners' competency in setting up a mathematical model inhibits the development of an appropriate solution, findings show that in case of the river problem, 20% (6 out of 30) of the learners failed to set up an appropriate mathematical model and thus could have hindered learners from moving on successfully to the next steps of solving the problem. However, the situation worsened drastically for problems 4 and 5. Although only 80% (24 out of 30) and 87% (26 out of 30) of the learners attempted problems 4 and 5 respectively, none of them could build a mathematical model that appropriately represented the problem. Hence, it is plausible to conjecture that in 80% of the cases, learners' lack competency in setting up a mathematical model and, this could have inhibited the development of a reasonable solution.

To address the lack of mathematical modelling competencies demonstrated by learners, it is imperative that mathematics teachers provide sufficient opportunities for learners to engage with word problems on a continuous basis via classwork and homework exercises, group tutorials and assignments. It is essential that prior knowledge and relevant mathematical heuristic procedures (like solving a system of linear equations, solving a quadratic equations or simplifying algebraic equations containing fractions) are regularly revised so that it does not restrict learners from building and solving mathematical models. Mathematics teachers should inculcate in their learners' habits of interpreting their solution(s) and validating their solution at every opportunity possible when learners are engaged with mathematical tasks. With more mathematical modelling lessons, teachers can also improve their modelling teaching approach whilst learners can improve their reading, understanding and thinking skills.

Studies on exploring the mathematical modelling competencies should be in-depth so that we can help students improve their competencies. Furthermore, the results of this study could enable mathematics subject specialists at district, provincial and national levels to be more aware of learners' challenges and accomplishment in solving word problems via a modelling approach. Thus, professional growth activities such as seminars, workshops and symposiums can be organised for teachers accordingly.

Resource tools and textbooks used by educators and learners ought to enhance learners' mathematical modelling competency for solving word problems. When developing resource materials and textbooks, developers should take into cognisance the problems Grade 11 learners experience at each stage of the modelling process as they try to solve word problems. This will reinforce learner confidence and success in solving problems via the mathematical modelling approach.

In conclusion, the curriculum of Mathematics in South Africa should incorporate real life problems that are realistic and not contrived into all sections. Such problems according to DBE (2011, p.8) should include issues relating to "health, social, economic, cultural, scientific, political and environmental issues whenever possible".

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APPENDICES

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

Appendix 1: Background Information Sheet

Dear Sir/Madam

My name is **Memory Dizha,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting a research on the implementation of an intentional teaching model in developing mathematical modelling competencies of grade 11 learners using quadratic equations. The target group will be Grade 11 mathematics learners at Manzomthombo High School in Mfuleni.

Research Title: An analysis of mathematical modelling competencies of grade 11 learners in solving word problems involving quadratic equations

The research study is guided by the following research questions:

a. What are mathematical modelling competencies grade 11 learners demonstrate when solving word problems involving quadratic equations?
b. To what degree does learners' competency in setting up a mathematical model inhibits the development of an acceptable solution?

The research participants will comprise 30 Grade 11 Mathematics Learners from a secondary school in Mfuleni township in Cape Town, Western Cape. Data collection will be in the form of a problem-solving questionnaire, observations of learners doing the word problems, and focused group interviews with grade 11 learners.

Participation in this study is voluntary. Participants have the right to withdraw from the research at any stage of the research process without giving any explanations should they feel uncomfortable with my research. Participants are guaranteed utmost confidentiality regarding all information collected from them Pseudonyms will be used to protect their identity.

The researcher will make all the research information and correspondence available to each participant (learner) and their parents in their relevant home language.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof. Rajendran Govender,** whose contact details are provided below.

Yours sincerely,
Researcher: Ms Memory Dizha
Contact number: 0631409103
Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender
Tel: 021-9592248
Email: rgovender@uwc.ac.za

Signature of the researcher: ……………………… Date: …………………

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Appendix 2: Permission Letter

THE Western Cape EDUCATION DEPARTMENT

The Research Director

Western Cape Education Department

P/Bag X9114

Cape Town

Dear Dr. Wyngaardt

Re: Permission to conduct research at Manzomthombo High School

My name is **Memory Dizha,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on **An analysis of mathematical modelling competencies of grade 11 learners in solving word problems involving quadratic equations**. The target group will be Grade 11 mathematics learners at Manzomthombo High School in Mfuleni.

I would like to request your permission to sit in a grade 11 Mathematics classes at Manzomthombo High School and observe how the learners engage with a set of word problems and try to solve them. I would also like to observe the learners written activities and also conduct focus group interviews with the learners regarding their experiences in doing the word problems.

The research will not disrupt class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes other than to understand how the grade 3 learners solve particular word problems with a focus on the type of problem-solving strategy they use to do so.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof. Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

Signature of the researcher: ……………………………… Date: …………………

FACULTY OF EDUCATION

UNIVERSITY OF THE WESTERN CAPE

Private Bag X17, Bellville, 7535, South Africa

Appendix 3: Permission Letter

The Principal

Manzomthombo High School
Nkohla Street
Mfuleni
Western Cape
7580
South Africa

Dear Sir

Re: Permission to conduct research in your school

My name is **Memory Dizha,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on **An analysis of mathematical modelling competencies of grade 11 learners in solving word problems involving quadratic equations**. The target group will be Grade 11 mathematics learners at Manzomthombo High School in Mfuleni.

I would like to request your permission to sit in a grade 11 Mathematics classes at Manzomthombo High School and observe how the learners engage with a set of word problems and try to solve them. I would also like to observe the learners written activities and also conduct focus group interviews with the learners regarding their experiences in doing the word problems.

The research will not disrupt class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes other than to understand how the grade 3 learners solve particular word problems with a focus on the type of problem-solving strategy they use to do so.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof. Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Ms Memory Dizha

Contact number:0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

Signature of the researcher: ……………… Date: …………………

FACULTY OF EDUCATION

UNIVERSITY OF THE WESTERN CAPE

Private Bag X17, Bellville, 7535, South Africa

Appendix 4: Permission Letter

THE HEAD OF DEPARTMENT

Manzomthombo High School
Nkohla Street
Mfuleni
Western Cape
7580
South Africa

Dear Sir

Re: Permission to conduct research in your school

My name is **Memory Dizha,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on **An analysis of mathematical modelling competencies of grade 11 learners in solving word problems involving quadratic equations**. The target group will be Grade 11 mathematics learners at Manzomthombo High School in Mfuleni.

I would like to request your permission to sit in a grade 11 Mathematics classes at Manzomthombo High School and observe how the learners engage with a set of word problems and try to solve them. I would also like to observe the learners written activities and also conduct focus group interviews with the learners regarding their experiences in doing the word problems.

The research will not disrupt class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes other than to understand how the grade 3 learners solve particular word problems with a focus on the type of problem-solving strategy they use to do so.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof. Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

Signature of the researcher: ………………… Date: …………………

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UNIVERSITY OF THE WESTERN CAPE

Private Bag X17, Bellville, 7535, South Africa

APPENDIX 5: PERMISSION LETTER

THE PARENTS

Dear Parent/Guardian

Re: Permission for your child's participation in research

My name is **Memory Dizha,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on **An analysis of mathematical modelling competencies of grade 11 learners in solving word problems involving quadratic equations.** The target group will be Grade 11 mathematics learners at Manzomthombo High School in Mfuleni.

I would like to request your permission to sit in a grade 11 Mathematics classes at Manzomthombo High School and observe how the learners engage with a set of word problems and try to solve them. I would also like to observe the learners written activities and also conduct focus group interviews with the learners regarding their experiences in doing the word problems.

The research will not disrupt class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes other than to understand how the grade 3 learners solve particular word problems with a focus on the type of problem-solving strategy they use to do so.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Dr. Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Ms Memory Dizha	Supervisor: Prof Rajendran Govender
Contact number: 0631409103	Tel: 021-9592248
Email: 3080775@myuwc.ac.za	Email: rgovender@uwc.ac.za
Signature of the researcher: ……………	Date: …………………

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Private Bag X17, Bellville, 7535, South Africa

Appendix 6: Parent's Informed Consent Form

I …………………………………………………………. agree/disagree to be part of the study and I am aware that my participation son/daughter's participation in this study is voluntary. If for any reason, I wish to stop my son/daughter from being part of this study, I may do so without having to give an explanation. I understand the intent and purpose of this study.

I am aware the data will be used for a Master's thesis and research paper. I have the right to review, comment on, and/or withdraw information prior to the paper's submission. The data gathered in this study are confidential and anonymous with respect to my son/daughter's identity unless I specify or indicate otherwise. In the case of classroom observations, written work, and interviews, I have been promised that my son/daughter's identity and that of the school will be protected, and that my son/daughter's duties will not be disrupted by the researcher.

I have read and understood the above information. I give my consent for my son/daughter to participate in the study.

Parent's Signature   Date

Researcher's Signature   Date

Contact Details:

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

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Private Bag X17, Bellville, 7535, South Africa

APPENDIX 7: Assent letter from learners

I ………………………………………………………………... Agree/disagree to be part of the study and I am aware that my participation in this study is voluntary. If for any reason, I wish to stop from being part of this study, I may do so without having to give an explanation. I understand the intent and purpose of this study.

I am aware the data will be used for a Master's thesis and research paper. I have the right to review, comment on, and/or withdraw information prior to the paper's submission. The data gathered in this study are confidential and anonymous with respect to my identity unless I specify or indicate otherwise. In the case of classroom observations, written work, and interviews, I have been promised that my identity and that of the school will be protected and that my duties will not be disrupted by the researcher.

I have read and understood the above information. I give my consent for me to participate in
the study.

Learner's Signature Date

Researcher's Signature Date

Contact Details:

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

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APPENDIX 8: Task-based activity for learners

1. A piece of wire of length 300 millimeters is bent to form a rectangle with area 3 125 square millimeters. Calculate the dimension of the rectangle.
2. The sum of the digits of two- digit number is 13 and the product of the digits is 36. Determine which two numbers fit this description.
3. Vula is a river guide on the Gariep River. Robert, a member of the group, is injured. Vula paddles Robert to the nearest pickup point, 12 kilometers away. Vula paddles back to his group. If the total paddling time for the return trip (there and back) is five hours and the river flows at a constant speed of 1 km/h, calculate the average speed that Vula paddles.
4. Motorcyclist travel from A to B at 40km/h and from B to A at 60km/h. if the distance between A and B is x km, determine the average speed at which the motorcyclist travels from A to B and back to A. (the average speed is not 50km/h)
5. Thabiso has a budget of R525 per month for petrol. The present price of petrol is x cents per liter. If the price rises by 50 cents per liter, she can buy five liters less petrol for R525. Calculate the present price of petrol.

Learner's Signature ____________ Date ____________

____________ ____________

Researcher's Signature Date

Contact Details:

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

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APPENDIX 9: Observation Schedule

1. Can learners make use of distance and speed formulas?
2. Do learners understand or show understanding of terms like "product''?
3. Can learners scan the problem to identify the keywords?
4. Can learners decide which information is relevant or irrelevant?
5. Are the learners able to determine the area of shapes in terms of the variable?
6. Are the learners able to create an equation (model) that can help solve the problem?
7. Can learners reorganize the problem into smaller parts to start solving the problem?
8. How do learners use the given model (equation) to solve the problem?
9. Can learners think of what mathematics must be used, to solve a problem?
10. Can learners check whether the answer makes sense?
11. Can learners give the answer in a way that relates to the problem asked, for example, using appropriate units?
12. Can learners draw a diagram and note down important information?

Learner's Signature ____________ Date ____________

____________________ ____________

Researcher's Signature Date

Contact Details:

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

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Appendix 10: Interview Questions

Question 1: How did you find the task? Was it easy or difficult?

Question 2: Can you describe the problem, which one was difficult and why do you think it was the most difficult?

Question 3: Which question did you find easy and why did you find it easy?

Question 4: How did language affect you in answering the main question?

Question 5: Looking at the problems that you do in class, and the ones for the research, what was the difference?

Question 6: Did you visualise any of the problems you did before identifying the keywords, for example, drawing the diagrams?

Question 7: In respect to the easy questions, what steps did you follow?

Question 8: For example, in questions, which numerical data was useful and which one was not?

Question 9: Do you think you need word problems in mathematics?

Learner's Signature Date:

Researcher's Signature Date:

Contact Details:

Researcher: Ms Memory Dizha

Contact number: 0631409103

Email: 3080775@myuwc.ac.za

Supervisor: Prof Rajendran Govender

Tel: 021-9592248

Email: rgovender@uwc.ac.za

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Appendix 11: Transcriptions

Question 1: How did you find the task? Was it easy or difficult?

L1: In my opinion, the task was not that difficult if you understood the logic behind what you were doing, it was difficult if the learners did not understand the nature of the student.

L2: It was a bit easy but I struggle with such questions, I do not understand the distances.

L3: It was normal, but the trick was the problem with English. The question was easy but the way it was asked was hard to understand.

L4: It was easy…

L5: It was difficult with a lot of statements and hard to understand.

L6: It was difficult, I was coming from the holidays and could not comprehend

Question 2: Can you describe the problem, which one was difficult and why you think it was the most difficult?

L1: No. 4 and 5 which required us to make a table. As we understood the question, we began to see what was happening but it took time.

L2: No 4 was difficult due to distance. I do not understand the whole table.

L3: To me, it was no, 4 because I have never understood the early stages of speed and time.

L4: It was difficult because I did not understand that the wire covered the whole table.

L5: The English made it hard. I did not understand.

Question 3: Which question did you find easy and why did you find it easy?

L1: No. 1 because as it was explained clearly and we saw the nature of the question and how to apply the mathematisation behind it.

L2: No 2 was easy because I knew in my mind that 1 x, and 5+ a = 13. So I knew the answer.

L3: It was no. 2 because it referred to a two-digit number. I knew it was not less than 10. So when I combined 4 and 9 to give me 49 and 94, and adding = 13 and multiply = 36.

Question 4: How do think language affected you in answering the main question?

L1: English is an additional language and not a mother tongue. If it had been simplified, it would have been easy.

L2. English is an additional language, we are taught in isiXhosa, speak in isiXhosa, so it makes it difficult to understand it.

L3: I did not understand some words and my teachers do not use English and we do not understand English

L4: In our maths questions, we are given numbers and not words. So it was difficult.

Question 5: Looking at the problems that you do in class, and this one for the research, what was the difference?

L1: In our everyday maths lessons, we are usually given short explanations. For example, if you want to prove something like a reason based on a triangle, instead of writing a triangle, you draw a triangle. English elongated the triangle.

L2: Our Daily classes give the equation and no explanation. This question required us to find the equation first.

L3: In class, we are given simple or determine. These questions in paragraphs were not easy to read and stay focused.

L4: In our maths classes, classwork sometimes has numbers with no words in the question. So when we were doing this research work, we were answering questions in words and not numbers.

Question 6: Did you visualise any of the problems you did before identifying the keywords, for example, drawing the diagrams?

L1: 50% to 75%v required us to visualise to understand the story behind the work.

L2: If you are struggling to understand the question, it becomes difficult to understand the question. Therefore, it was hard to visualise kilometers and hours.

L3: Yes, I did visualise and drew some diagrams but they were not mathematically correct. I was able to understand question 4 like going from A to B. But for most of the questions I could not draw the diagrams and could not understand what was required.

Question 7: In respect to the easy questions, what steps did you follow?

L1: I read first, worked out carefully to avoid mistakes and then it was easy for me to solve.

L2: No 2, I followed the steps. I made the assumptions. Did the steps after figuring out the answer? I knew the answer but I did follow the steps afterwards.

L3: The way I attempted the easy questions, some of them were due to the afternoon programmes in numerical. I knew the answers and followed the mathematical modelling assumptions.

L4: I did not follow any steps I was just calculating. I already knew the answers as a two-digit number. I got 9 and I knew the second number as 4. Because when you add 4 to 9, you get 13.

L5: I followed some of the instructions and not others because I calculated first. I looked at the instructions, I only followed to draw a diagram and interpreted the diagram.

Question 8: For example, in questions, which numerical data was useful and which one was not?

L1: The numerical data include area and the length so this made it hard to know the equations. So what was taught by the teachers made it easy to answer.

L2: The numerical data was a plus for me, the alphabet like letters made it hard. Numerical data helped us.

L3: All the data that was given was useful and it made it easy for us to understand the question and calculate everything.

L4: Some data that was given in question 4, like the speed that Vula was running, it did not help me in using it to calculate it.

Question 9: Do you think you need word problems in mathematics?

L1: Yes. We do need them because they make you think. It is more difficult dealing with word problems than numbers. Maths is easier when dealing with numbers. The word problems should be a few because they will make us fail. What is important is understanding what is being asked.

L2: I think we do need the word problems because, in Mathematics, Word problems are not a usual encounter as we do not meet them repeatedly. It may be a huge asset if the teacher gets to practice the word problems with us. It's just that we do not see what the teachers do. The word problems show the reality of what is happening.

L3: I really feel we should be given word problems; we struggle in other subjects like geography. Just like you train a muscle and it grows, one should train a brain and it will grow.

L4: We need them but not in maths in a test or marks, I don't think that is good because it will take a lot of time to first translate English to Xhosa and then maths… it is a problem. You end up not understanding what is asked. You have to understand it in English, translate to Xhosa and when you are solving it in maths, you end up translating what is wrong or not writing at all.

L5: We last used the word problems in grade 9. So it is a long time. When you pass it, you feel happy? It helps our minds to get used to word problems other than numerical problems

L6: The word problems help to challenge us and make us ready for the next encounter. They have taught me not to underestimate anything. Consider two questions one with geometry and another with mathematical word problems, most people would choose the mathematical questions and think that they are geniuses, it is only when you look at the question that you discover that it is challenging.

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Private Bag X17, Bellville, 7535, South Africa

Appendix 12: Ethics Clearance letter from UWC

07 November 2016

Ms. M Dizha

Faculty of Education

Ethics Reference Number: HS/16/8/3

Project Title: AN ANALYSIS OF MATHEMATICAL MODELLING COMPETENCIES OF GRADE 11 LEARNERS IN SOLVING WORD PROBLEMS INVOLVING QUADRATIC EQUATIONS

.

Approval Period: 04 November 2016 – 04 November 2017

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above-mentioned research project. Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval. Please remember to submit a progress report in good time for annual renewal. The Committee must be informed of any serious adverse event and/or termination of the study.

Ms. Patricia Josias

Research Ethics Committee Officer

University of the Western Cape

PROVISIONAL REC NUMBER - 130416-049

Appendix 13

HORUS *Consulting*

Proofreading & Copyediting Services

Harrison Oghenerukevwe EJABENA
B.A Hons (English Language), M.A (English Language)

Language and technical proofreading | Bibliographic citation

DECLARATION OF PROOFREADING

House G4, Senior Staff Quarters, **+234 9067550911**
Obafemi Awolowo University **+234 8079767923**
Ile-Ife, Osun state
Nigeria
harrison.ejabena@gmail.com

This MEd thesis by **Memory Dizha**, titled 'An Analysis of Mathematical Modelling Competencies of Grade 11 Learners in Solving Word Problems involving Quadratic Equations has been proofread for grammatical and expression errors. The references have been checked for conformance with the APA Style of bibliographic citation. All references have been checked against the text, and all in-text citation have been checked against the reference list. The candidate has been advised to make the recommended changes.

Harrison Oghenerukevwe EJABENA

25 April, 2021