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TITLE OF THESIS

The impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry

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DECLARATION

I declare that "The impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry at a Khayelitsha school'' is my own work; that this work has not been submitted before for any examinations or degree purposes in any university, and that all sources I have used have been indicated and acknowledged in complete references.

WILLIAM SHONHIWA   DATE: 30 NOVEMBER 2019

SIGNATURE:

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ABSTRACT

Euclidean geometry was recently re-introduced as a compulsory topic in the Mathematics Curriculum for learners in the Further Education and Training (FET) band in 2012. The diagnostic analysis reports on the National Senior Certificate (NSC) Mathematics Paper 2 examinations since 2014 has repeatedly expressed concern of the poor performance of leaners in proof and reasoning items linked to circle geometry. Various efforts have been made to examine the composition of the curriculum to find ways of motivating learners in the study of circle geometry and enhancing their performance but not much has been realized. The use of technology or cooperative learning approaches for the teaching of geometry is beneficial for pedagogical purposes, particularly for improving learners' performance in geometry. Hence, this study investigated the impact of using technology through cooperative learning on learners' performance on grade circle 11 geometry. It was thus an attempt to focus on blending these two teaching methods with an emphasis on the use of technology. The research took place at a Khayelitsha school and the scope of technology was limited to using a mathematical computer programme called Heymath.

This research was grounded on the cognitive level framework that is used by the Department of Basic Education (DBE) in the setting of National Senior examination mathematics papers, as well as the set of social constructivist views of mathematics teaching and learning. In the case of the latter, both social constructivism and cognitive constructivism views were considered and applied for the purposes of this study. Using a positivist paradigm, this convergent parallel mixed methods study employed a quasi-empirical design, where the control group consisted of a group 26 grade 11 learners who were comparable to the group of 27 grade learners that made up the experimental group.

Initially, data was collected from both the experimental and control groups via a geometry pre-test. Then the experimental group (E) was taught circle geometry using technology in the context of cooperative learning while the control group (C) was taught using conventional methods. Thereafter data was collected via a geometry post-test from both groups. Finally, the experimental group completed a questionnaire designed to ascertain the extent to which learners exhibit changes in motivation when answering grade 11 circle geometry questions when afforded the use of technology within a cooperative learning environment.

Data was initially collected through the pre-test and post-test in an endeavour to determine impact of teaching grade 11 circle geometry using technology within the context of cooperative learning, was analysed through using both quantitative and qualitative analyses techniques, namely the statistical t-test and qualitative content analysis. The statistical analysis showed that that using technology to teach in a context of cooperative learning improves learners' performance. The qualitative content analysis provided a deeper exposition of pertinent errors such as the non-use of a diagram in the development and presentation of a proof of a theorem, and the non-provision of a reason to justify a mathematical statement in a proof write up. Furthermore, the statistical analysis of leaners responses to the questionnaire, which elicited the views of learners who had been taught in groups using digital technology, showed that their levels of motivation was raised in geometry lessons.

Teaching and learning of geometry using technology supported by cooperative learning approaches has some significance. It is suggested that teachers, subject advisors, curriculum planners should take greater responsibility to incorporate the use of technology jointly with cooperative learning approaches in the facilitation of teaching and learning of circle geometry across our classrooms.

Keywords: Technology, Heymath, cooperative learning, impact, circle geometry, learners' performance

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ABBREVIATIONS

CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
EMDC	Education Management and Development Centre
GSP	Geometer's Sketchpad
NSC	National Senior Certificate
OBE	Outcomes-based Education
RNCS	Revised National Curriculum Statement
SAQA	South African Qualifications Authority
SASA	South African Schools Act
WCED	Western Cape Education Department
ZPD	Zone of Proximal Development
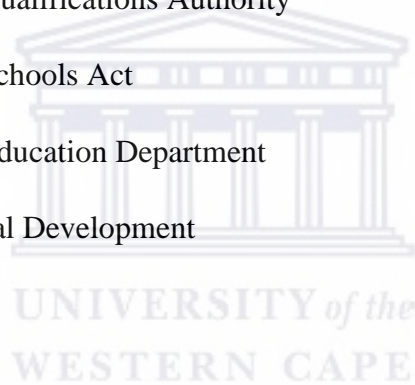
CHAPTER 1: INTRODUCTION OF THE STUDY

1.0   Introduction

The fourth industrial revolution has ignited a broad spectrum of ideas and approaches to enhance teaching and learning across our classrooms, and the South African government has pinned its hope of economic prosperity and job creation on it. The use and advancement of technology has become the slogan of new policies and at meetings. The education sector has refused to be left out in this race and has advocated for the integration of technology in teaching by "encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths" (Department of Basic Education, 2011:5)

This study explores the use technology within a context of cooperative learning in the teaching of grade 11 circle geometry. The focus of this study, thus, was to ascertain whether the benefits of using both approaches, technology and cooperative learning, simultaneously, could significantly help learners to improve their performance scores in circle geometry tests. The research was conducted at a Khayelitsha school in Cape Town. The performance of learners was measured using the results obtained from the pre-test and post- test written on circle geometry triangulated with learners' views expressed in a questionnaire.

Researchers, Koutsides (2001), Isik and Saygili (2015) have shown that cooperative learning improves results, and research teams have advocated for this teaching and learning strategy. Chianson's research for instance, forms part of this initiative (Chianson et al., 2011). Other studies have encouraged the use of technology in teaching, following the introduction of Geogebra. Haciomeroglu's work is illustrative of this (Haciomeroglu and Andreasen, 2013). Dynamic explorations of mathematical concepts are possible and gives multiple representations that are not possible to visualise without using technology. In their article, Haciomeroglu and Andreasen (2013:6) used Geogebra "which offers students tools that enable them to construct, manipulate and reason about mathematical objects or relationships.

This research was motivated by the perceived need for it, which is outlined in the section 1.2 of rationale and motivation. The literature part section assesses what other researchers have stated in their studies as it relates to this topic. This researcher compares and contrasts their findings to identify the knowledge gap subsequently taken up in this study. The research methodology and analysis of data are presented before the report is concluded.

This research was delimited to study the actual impact on learners' performance of a teaching and learning approach that relies on technology through cooperative learning.

1.1 Background and context

Circle geometry was a topic included in the South African post -democracy curriculum known as the National Curriculum Statement Grades 10-12 (2002) which was a result of a revision of Curriculum 2005 (C2005) due to implementation challenges (DBE,2011). The basic teaching approach for C2005 was referred to as Outcomes Based Education (OBE). In OBE a teacher was seen as a facilitator in the learning process and the learner a co-constructor of knowledge (DBE,2002). In this system the learner is said to have some control over the sequencing and pacing of content.

Due to the demands of OBE, the use of technology was advocated even though a number of schools had none to use in the classroom. In addition to the limited access to technology, the educators had not been trained or equipped to use it for teaching in either case. Thus one of the major reason for the failure of OBE as noted by Chisholm (2015), was the lack of training and development for teachers. This persistent problem could yet cripple the application of technology for e-learning in the Curriculum Assessment Policy Statement (CAPS).

OBE failed to improve the education system in South Africa for various reasons, as explained by Jansen (1998). The cited reasons included, but not limited to the fact that it was a borrowed curriculum and educators were not trained to implement it. In 2009, there was another revision of the National Curriculum Statement Grades 10-12 and the Revised National Curriculum Statement Grades R-9 to produce the current combined National Curriculum Statement Grades R-12 (DBE, 2011). This National Curriculum Statement Grades R-12 became functional in 2012 and the first group of learners matriculated in 2014. From 2009 to 2013, Euclidean geometry was a topic assessed in the optional Mathematics Paper 3. According to the foreword by the Minister of Education Angie Motshekga when she introduced CAPS (2011), the National Curriculum Statement Grade R-12 not only builds on the previous curriculum but also updates it. It is intended to provide clearer specifications of what is to be taught on a term-by-term basis.

A close examination of the CAPS document reveals that most aspects of the old curriculum are still there as it is not a new curriculum but a repackaged one. However, greater emphases on roles of the teacher and the learner, for example the learner is viewed as an active participant

in the learning process and is expected to engage collaboratively in projects, investigations and learn to apply content learnt in assessments.

Within CAPS, circle geometry is a topic introduced to every mathematics learner. It requires a great deal of application as learners work with riders (geometrical riders). The assessments must meet the four cognitive demands of knowledge, routine procedure, complex procedure and problem solving (CAPS document). This nexus of requirements introduces technology as an important and necessary teaching aid if the demands of CAPS, which include application and problem solving, are to be fulfilled in mathematics.

Circle theorems were tested only in the optional Mathematics Paper 3 in the National Senior Certificate (NSC) examinations from the year 2008 to 2013 as part of the National Curriculum Statement (NCS-Grades 10-12). They were re-introduced as a compulsory examination topic in 2014 with the introduction of CAPS and abolishment of Mathematics Paper 3. This may mean that Euclidean geometry was seen to be a topic important for every learner in the new curriculum.

'Circle theorems' is a difficult topic for most learners for whom it has meant poor results in mathematics, as the different NSC diagnostic reports of 2014-2018 have shown (DBE, 2014, 2015, 2016, 2017, 2018). Despite the fact that these diagnostic reports recommend possible interventions in relation to this topic, it has been shown that the "integration of concepts required in geometry still proved to be a challenge to many candidates" (DBE, 2015:162). From the researcher's experience as a grade 11 mathematics teacher, it has been observed that Euclidean geometry in general, and circle geometry in particular, which requires visualisation and integration of separate theorems poses a big challenge to learners.

According to DBE (2015), the following statistics show that the performance of learners in geometry is dismal. The average percentage performance in Euclidean geometry per question, for questions 8 to 11 was 56%, 28%, 38% and 29% respectively. In 2016 the average performance was 57%, 44% and 36% for questions 8 to 10 respectively on Euclidean geometry DBE (2016). In 2017, average percentages for questions 8 to 11on Euclidean geometry were equally pathetic with averages of 46%, 34%, 34% and 41% per question respectively (DBE, 2017). This is an indication that there is a problem in this section of the work. Further research is definitely required if the quality of mathematics results is to improve. According to Bhagat and Chang (2015), subjects like geometry and calculus are too abstract for the majority of

learners as they cannot figure out the mathematics behind, leading to poor scores in assessments. This has led to a decline in the love for mathematics.

There are several comments that the researcher was able to obtain from the NSC diagnostic reports (DBE 2014, 2015, 2016, 2017 & 2018) which include, but are not limited to the common mistakes listed here:

- drawing unnecessary geometric constructions to answer a question
- failure to give correct acceptable reasons as indicated in examination guidelines
- making wrong assumptions in the process of solving geometrical questions, leading to mathematical breakdown, and
- failure to show construction lines in proofs constituting a breakdown.

According to the DBE (2015:175), "candidates showed little understanding of the difference between a theorem and its converse". This deficiency indicates that in general, learners struggle to link concepts and they lack essential knowledge, which should have been established in the lower grades. It is thus difficult for them to connect ideas. This predicament might be addressed through exploration using technology to concretise the concepts. The idea of resorting to technology is in line with what de Villiers, Hanna and the International Program Committee (2008:332) meant when they said, "the dragging function opens up new routes to theoretical knowledge within a concrete environment that is meaningful to students."

The necessity for a research such as this is borne out in the recommendations given in several diagnostic reports for the years 2014 to 2018. These recommendations are concerned on how best to improve the results in geometry. The recommendations include items on the list that follows.

- Exploratory methods must be used for learners to understand the theorems and relate them to diagrams.
- Learners must be taught to deconstruct a complex diagram to identify theorems. It is suggested that learners should look carefully for clues in given drawings to help answer the questions.
- Teachers are advised to use diagrams to explain theorems.
- Teachers should use an exploratory method for teaching Euclidean geometry.

- The use of examination guidelines is advised for learners to get used to acceptable reasons in geometry examinations. Examination terminology should also be used in class as learners engage with their work.

- It is further suggested that more time should be given to teaching this topic as learners need much time to practise the application of the concepts involved (DBE, 2014, 2015, 2016, 2017, 2018)

Learners in Japan, as elsewhere in the world, including South Africa, struggle with learning deductive proofs and hence struggle with geometry (Miyazaki, Fujita, & Jones, 2017). Teachers must therefore find other approaches to help learners improve in their performance when tackling this challenging topic.

In the context of this study it was felt that the researcher should pursue the recommendation to use an exploratory method with the aid of technology in the context of cooperative learning.

1.2   Rationale and Motivation

Having been a mathematics educator for over fifteen years, the researcher encountered challenges in trying to convey information to learners, as topics like Euclidean geometry appear too difficult for some. Several learners fail to connect the theorems deductively in working with riders. Majority of learners do not understand what they are required to do when asked to prove in geometry. The years between 2008 and 2013 were very good years for the researcher in terms of matric results because topics like Euclidean geometry were optional in the curriculum. However, results began deteriorating from 2014 onwards with the introduction of Euclidean geometry which by that time had a weighting of 33% in the final examination of the CAPS curriculum (Department of Basic Education, DoE, 2011).

Prior to this the researcher had observed that learners also had problems with grade 9 geometry; for most, reasoning deductively was not easy at that level. Grade 10 geometry was equally difficult for the researcher's learners. For instance, they found it hard to prove that a certain quadrilateral is a kite. This personal experience resonated with the observations of Fuys, Geddes and Tischler (1988) who referred to the van Hieles' model of geometric thinking to cultivate higher order thinking in high school geometry students. Alex and Mammen (2016) also found that the level of cognition in high school geometry was lacking in students as they had not acquired the requisite skills. Geometry concepts are difficult for the majority of students (Brannon, Liengme, & Liengme, 2018). Furthermore, in my own grade 11 classes, learners struggle to prove the theorems and often they just provide numerical answers without

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providing supporting reasons. In the light of the aforementioned problems, the researcher felt it was necessary to investigate the impact of using technology through cooperative learning in teaching grade 11 circle theorems

The low percentage performance in geometry questions, ranging from 28% to 56% recorded in the 2015 diagnostic report was significantly disturbing to the researcher. The averages per question for the year 2017 in Euclidean geometry ranged between 34% and 46% (DBE, 2017). Hence, an approach using technology through cooperative learning was selected following a thorough literature survey. This literature review confirmed the point that technology improves learning capacity. Furthermore, it was noteworthy that findings of enhanced interaction in lessons from the literature also encouraged the use of cooperative learning amongst learners (Shadaan & Kwan Eu, 2013).

It was thus logical for the researcher to opt for an approach aligned to using technology, but because of a shortage of resources at the school where the research was to be conducted, it became convenient to share the available technological devices thereby simultaneously introducing some aspects of cooperative learning. This is significant as the researcher felt that blending the two methods would be likely to improve the students' performance in the topic. In this way the research project evolved.

Isik and Saygili (2015) and Herceg and Herceg (2010) provided the impetus to investigate the potential in harnessing both methods as they had advocated for the use of cooperative learning and technology respectively. The motivation was to find out whether learners would be likely to improve with this dual approach. The researcher developed the idea that technology and cooperative learning complement each other, and if combined may improve performance since separately the methods had proved to be effective as indicated by Shadaan and Kwan Eu (2013) and Chianson et al., (2011). It was assumed that the results of this research would contribute towards improving learners' performance in geometry.

This approach is also in keeping with the National Curriculum Statement Grades R-12 which advocates an active and critical approach to learning (DoE, 2011). Also indirectly supportive of this initiative is the research of Shadaan and Kwan Eu (2013) who suggest that because of the massive general availability of technological devices, educationists have begun to see that there is a need to incorporate technology into education (Shadaan & Kwan Eu, 2013).

According to Roberts (2012), using technology has many merits some of which include providing students with opportunities to gain a deeper understanding or a broader

comprehension of the subject while promoting their active engagement. This perspective is shared by White (2012), who found that the use of technology encourages discovery learning. Such findings are pertinent to this study since in geometry it is vital to students' grasp of abstract concepts that they are able to construct and visualise shapes, and technology makes this both actual and accurate. Indeed technology has brought about a paradigm shift in the learning of mathematics as it permits multiple representations of mathematical concepts thus allowing the learner to interact with the actual mathematics in the learning process (Bhagat & Chang, 2015).

With the use of computer-based software it becomes easier to "play around" with variables and in this way to end up learning the mathematics behind a concept. For instance, someone playing with the Geometer's Sketchpad will be enabled to visualise the circle theorems and this visualisation will reinforce permanent learning. In the classroom, interactive whiteboards and other teaching software bring joy and motivation to learners whilst the traditional chalk and talk method leads to rote learning and boredom. According to Pijls, Dekker, and Van Hout-Wolters (2003:211), "it is possible that integration of textbook-like exercises and investigation tasks in the context of computer simulations encouraged students to try to build up a concept with their own constructions, by reflection on their experiences and perceptual level". Authors Pijls *et al*. (2003:211) further note that "computer simulations offer the opportunities to explore the subject on a concrete level".

It was such critical reflection, and research studies of this nature that further propelled this researcher to carry out the study to ascertain whether technology actually makes a significant difference in the learning of grade 11 geometry.

Constructivism was selected as the main philosophy for this research and it will thus be discussed at greater length in Section 2.6.1. In brief, Constructivism refers to the construction of knowledge. Constructivism does not give direct instructions to learners but sets up learner-centred environments. These foster the focussed collaboration of learners as they work at the learning tasks.

According to Squires and Preece (1999, cited in de Villiers 1990), constructivists are against approaches that help learners avoid errors in their learning as they believe that learners learn best from their mistakes. This approach promotes problem solving, teamwork in finding solutions, research and communication skills as learners communicate and discover the

meaning of concepts. The foregoing aspect concerning collaboration motivated the researcher to use technology though cooperative learning.

1.3   Problem statement

The teaching and learning of circle geometry have been under review on different platforms, which includes the different diagnostic reports used in this study (DBE, 2015, 2016 and 2017). These reports highlight mistakes made by learners in examinations on the topic of geometry and suggest ways in which teachers might reduce the challenges. For instance, the NSC diagnostic report (2015) described several misconceptions by learners which included drawing irrelevant lines to solve riders and presented possible approaches that may be followed by teachers, as explained in Section 1.1 of this research. These misconceptions are similar to the ones highlighted by Prescott, Mitchelmore and White (2002) which found that learners face challenges in geometry and that many struggle to connect facts in circle geometry. (Luneta, 2015) also observed that geometry as a topic is difficult for teachers to teach and equally difficult for learners to learn because of the large amount of geometrical knowledge required to deal with topic.

According to Delgado, Wardlow, Mcknight and Malley (2015:398), "the research on the effects of technology in the classroom is increasing rapidly, but there seems to be much debate on whether or not technology has been making a significant impact on student achievement". A more positive view is that technology plays a major role in the simulation of real situations and in closing the gaps in the problems learners have in geometry (Shadaan & Eu, 2013). Another perspective, affirming the value of technology in teaching is that it has levelled the playing field for learners around the world (Adams, 2013). To these Chianson's insight adds a point of particular interest, that is, that cooperative learning enables the clarification of concepts as learners give each other responsibilities and share information (Chianson *et al*., 2011).

Despite the many positive attributes concerning the use of technology or cooperative learning in the classroom, the challenge to improve performance in geometry questions persists, as was observed in the researcher's experience as a mathematics educator and established through the literature survey.

Efforts to address the problem of poor results have not stopped with the Department of Education encouraging the use of discovery methods in teaching, both in the Revised National Curriculum Statement Grades R-9 and the National Curriculum Statement Grades 10-12 (DoE, 2002) and in its revised versions. These include CAPS where the curriculum overtly aims to

use science and technology effectively (DoE, 2011). However, Jansen has been sceptical about OBE, pointing out that it had the use of technology in teaching as the main priority and this did not succeed for various reasons (Jansen, 1999). From this critique the researcher was persuaded to believe that the use of technology or cooperative learning does not bring the required results if used separately in the teaching of geometry and hence the averages remain low. There was therefore a need to research the impact of using technology through cooperative learning to improve learners' understanding of grade 11 circle geometry.

This challenge was taken up at Khumbulani (pseudonym) High School, in Khayelitsha Township, Western Cape. The next section outlines how this was done as it forms the aim of this study.

1.4   Purpose of the study

The purpose of this study was to investigate the impact on learners' performance of using technology through cooperative learning. Grade 11 geometry was the area of focus. The idea was to teach a group of learners to learn more about the theorems of circles and their application using Heymath, a computer software programme. Learners were put into groups to engender a sense of collaboration as they worked with each other through the geometry problems. As already stated, it was hoped that learners' use of the computer programme within a context of team learning, would have a positive outcome.

1.5   Research objectives

The objectives of this research were:
- to investigate the impact on learners' performance of using technology in a cooperative learning context in answering grade 11 circle geometry questions; and
- to observe and describe changes in learners' motivation in participation when answering grade 11 circle geometry questions using technology in a cooperative learning context.

1.6   Research questions

In order to pursue the objectives of this research it was important to set guiding questions, the answers to which would lead to a conclusion at the end of the study. Hence the questions that follow were devised as a scaffold for the research:

- How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 circle geometry?

- To what extent, if at all, do learners exhibit changes in motivation to participate in answering grade 11 circle geometry questions when using technology within a cooperative learning context?

1.7   Significance of the study

Having foregrounded the research objectives and research questions in preceding sections it is of paramount importance that the significance of this study be explained. The findings from this study may possibly contribute in the following ways.

1. The findings may provide evidence to policy makers of the need to incorporate technology through cooperative learning as the basis for good practice;

2. They may guide educators towards choosing teaching methods that best help learners to improve performance and ultimately help increase the number of learners interested in taking circle geometry as a topic in mathematics. This point addresses Bankov's (2013) observation that there is a decline in theoretical geometry instruction in a number of countries because of the difficulties the topic poses to a number of learners.

3. The findings may uncover practicable insights that contribute towards learners' improvement in this topic. The untenable alternative is that a repeat may occur of what was done in 2008 when geometry was removed from the Outcomes based Education (OBE) curriculum following a review of the content covered in geometry in our South African curriculum.

4. Also, the findings may serve to assess the impact of using technology in the context of cooperative learning on grade 11 geometry. This study is consistent with Vygotsky's and Piaget's (Vygotsky (1978) theories of constructivism as it aimed to see how interaction achieved through using technology and cooperative learning helped learners improve in knowledge construction and participation in circle geometry. According to Vygotsky (1978) the role of social interaction in the learning process may be strengthened as learners acquire competence using technology.

5. The findings may help curriculum advisors to appreciate the significance of the intervention given to learners in the experimental group; it may enlighten them about methods by which to plan the teaching of a geometry programme. This also implies

better planning in terms of resources needed in schools and in developing pace-setters for the delivery of the topic.

6. The findings may foreground the common problems encountered by learners as they answer questions on the topic and raise the level of motivation to participate in the learning of geometry.

1.8.   Definitions of terms used

This section provides detailed explanations of the meaning particular terms used in this study.

a) Technology

Technology refers to the use of computers and associated software programmes like Heymath in teaching and learning.

b) Cooperative learning

Slavin (2010) defines cooperative learning as a conducive learning environment provided by educators for learners to learn new content by helping each other in small groups. Cooperative learning is built on the assumption that group members have a responsibility to learn and making sure that every other member of the group understands the concepts by collaboration and discussions.

c) Heymath

Heymath is a computer software programme used for learning mathematics through videos, examples, practice questions, drawing and simulation tools. The Heymath Software Programme can be installed on a computer or be accessed online. The online version was used in this study because the school had been licensed for this version only.

d) Geometer's Sketchpad (GSP)

The Geometer's Sketchpad is a common computer software programme used for learning mathematics. Like Heymath it has drawing tools and simulation properties. GSP was not used in this study though it is used extensively in the literature referred to in this study.

e) Euclidean Geometry

Euclidean geometry is the geometry used for exploring properties of shapes including sides and angles without Cartesian coordinates.

f) Curriculum 2005(C2005)

Curriculum 2005 (C2005) was designed in 1997 to remove the curricula divisions of the apartheid government in South Africa (DoE, 2011). The teaching approach accompanying this curriculum was known as OBE.

1.9. Scope and delimitation of the study

Technology is a broad term used to define a number of devices including computers with associated programmes. In this study, the use of technology was limited to tablets and Heymath software. The online version of Heymath was used. However, this presented many challenges due to unstable internet availability, thus making it difficult to execute the teaching plan as intended. As indicated earlier, the online version was chosen as Khumbulani High School had only an online version user licence. The study was limited to theorems that formed part of the CAPS curriculum.

Regarding the second prong of this investigation, cooperative learning was applied in a limited way: in doing tasks, only the group approach was used for cooperative learning. This was because of the short time prescribed by the CAPS curriculum for the teaching of the topic. Only three weeks are allocated to this topic; and using cooperative learning in its fullness would have required more time.

The next section gives an overview of how the study is organised.

1.10. Thesis outline

This thesis consists of five chapters outlined in the following way:

Chapter 1: Overview of the study

This chapter introduce study and reports on the background of the study, statement of the problem, purpose of study, research question, and significance of the study, scope and delimitations of the study, and the operational definitions.

Chapter 2: Literature Review and Theoretical framework

This chapter focussed on the literature that was reviewed in this study, which mainly covered cooperative learning, technology and its application in mathematical contexts, motivation, proof and reasoning in mathematics, and van Hiele Theory. The underlying theoretical

framework guiding this study, cognitive and social constructivism are discussed. This is supplemented by the four levels of cognitive demand as prescribed in CAPS, that guide the setting of National Senior Certificate examination question papers.

Chapter 3: Research Methodology

This chapter presents and explains the research design and methodology used in this study. It begins by focusing on the research paradigm that resonates with the purpose of this study, then describes and discusses the research approach, research approach, sample and sampling strategy, research setting, data collection methods and procedures, data analysis strategies, issues of trustworthiness and as well as ethical considerations

Chapter 4: Data Analysis, Findings and Discussion

The findings that were gathered from the analysis of the data are presented in this chapter, as well as a detail discussion of findings in relation to the literature reviewed and theoretical framework is presented. The discussion is done for each research question.

Chapter 5: Conclusions and Recommendations

This chapter provides summary of the findings that assist to answer the research question, discussion of implications of findings to different stakeholders and recommendations, and ultimately with the conclusion of the study.
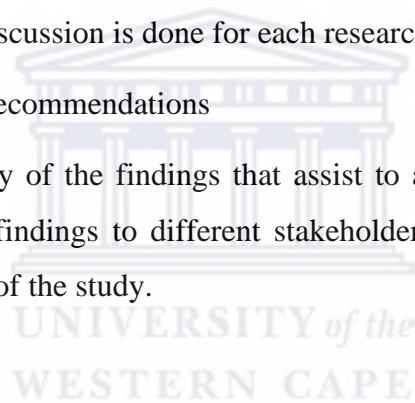
CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.0   Introduction

In a review of literature relevant to this study, Chapter two explores at some depth the concept of cooperative learning and the use of technology in mathematics education. This chapter also introduces constructivism as the theoretical framework to pursue the enquiry foregrounded in this study. Furthermore, proofs and reasoning are discussed as aspects critical to the investigation. The van Hiele levels of geometrical thinking and cognitive demand levels are also explored as they are important for the researcher in carrying out the study.

Hence, in this chapter the concept of cooperative learning is defined, related research is explored, and different aspects of cooperative learning are brought to light. Authors who have produced common findings are highlighted and their findings explained. Also in this chapter is literature concerning the use of technology in mathematics education. Related definitions and findings from other studies in the area of technology are explained and reflected upon here to show how the literature review guided the process of identifying existing gaps, so allowing the objectives of this study to emerge. The body of reading, thus, enabled me first to determine what other researchers have had to say on the topic and thereby to take the critical step of laying the groundwork for this study.

 The literature on proofs was very important for this study as the topic of grade 11 circle theorems is mainly about discovering and justifying conjectures, proving theorems and applying them to solve geometry riders. It was thus of paramount importance to establish an understanding of proofs in general. The gap identified was then elaborated upon.

Cognitive and social constructivism are discussed as they underpin the design and enactment of the activities for this study. The van Hiele theory of geometric thinking, and the cognitive levels as explained in the CAPS document conclude the literature review.

Some studies like Shaldaan and Leong (2013) have revealed that technology and cooperative learning stimulate motivation and enable a better understanding of geometry, even if used separately. While much has been written on circle geometry already Chianson, Kurumeh and Obida (2011); Slavin (2010); Almeqdadi (2000); Shaldaan and Leong (2013); and Choi-Koh (1999) have researched teaching using cooperative learning and technology in general. Chianson et al. (2011) has examined how cooperative learning and collaboration improve retention of concepts learnt. These scholars have concluded that using technology and

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cooperative learning produces positive results with respect to retention and test scores in geometry.

2.1.1 Cooperative learning

According to Slavin (2008), cooperative learning simply means learners work together in small groups where they help each other to learn. In cooperative learning every learner contributes to the group task. The activity maybe done at once or learners might give each other responsibilities and then later sit together to compile an assignment. Chen (1999) defines cooperative learning as a learning method where students study together to achieve common goals. According to Slavin (2010), cooperative learning simply means students are required to work together to accomplish an academic task. Thus, each member of the team is responsible for their own learning as well as maximising the learning of the entire group (Slavin, 2011). Cooperative learning is regarded as a successful tool as it enhances the construction of new ideas, and allows for high level reasoning, social competence, and cognitive and affective perspective-taking (Mayo, 2010). Cooperative learning changes the roles within a classroom. It takes the sole responsibility of teaching and learning away from the teacher and allows it to be shared amongst groups of students. Mckeachie (1999) says cooperative learning encourages communication and elaboration of concepts learnt while enhancing collaboration.

According to Lam and Li (2013), cooperative learning generates five essential elements. The first is an element of positive interdependence as every member has a contribution to make for the good of the group. A second crucial element is individual accountability where all learners are accountable for providing their contribution to the group's success. Marks are given by assessing individual contributions to the task. A third important element in cooperative learning is face-to-face interaction, which helps all members of the group to understand the task as members present to the group their own feedback on the question. In so doing, learners understand the concepts better. The fourth is the element of collaboration itself, through which learners build trust in each other and develop skills for conflict management. The fifth element concerns group processes as members of a group set goals for the group. It is important to be aware that these elements have to be properly incorporated into teaching for proper learning to take place (Lam & Li, 2013).

Cooperative learning employs many different strategies, which traditionally did not include technology. However, these strategies can be adapted to include technological tools and

applications to make learning relevant in an increasingly technologically orientated world. Some of these strategies discussed.

The jigsaw method is where learners are grouped into teams known as the home groups. Each of the members of a team are then re-assigned to another team that is tasked with mastering the understanding and application of a chunk of learning material. These reconstituted teams then return to their home groups once they have mastered the assigned work. The home group now consists of a team of experts which share their respective knowledge about different components of the work and support the individuals in the group to master all the work (Aronson, Blaney, Stephan, Sikes, & Snapp, 1978). This method can be adapted for use in technology-infused learning contexts by using applications such as: Skype which allows users to communicate from different locations. Other applications that could be used are *WhatsApp, and Google Duo.* Google docs could also be used which allows users to share a document with other group members. Each group member is allowed to edit and comment on the document in real time. This result in a document which is authored by the entire group. (Johnson, & Johnson, 2014).

Group investigation- Students are placed in groups and the workload is divided amongst each group member where they decide how each section will be researched and summarized (Sharan & Sharan, 1989). This can be convenient when using a Gannt chart (Kagan, 2019). Gannt chart is a software app which monitors progress of project work with virtual graphic bars, displaying the milestones of projects, the resources used as well as the dependencies.

Student teams achievement divisions consists of a group of four students who work together to master a lesson given to them by the teacher. Students then take individualized quizzes which are compared to their previous scores. The entire group's scores are added to see if the students were able to meet or surpass their previous performance (Slavin, 1995). The teacher provides learners with a reward if the previous performance was surpassed. This can be achieved by creating an online platform for learners to answer questions individually, Google forms have bridged the gap by allowing complete anonymity whilst individuals still receiving their respective results. It comes in the form of a survey, which can be customized to meet the educational aims of the teacher or lecturer. These customisations are a collection of timed sections, limited options and unlimited attempts (Johnson & Johnson, 2014).

Teams Games tournaments: This is where members compete against other teams in academic games in an attempt to earn points for their team. There are various online games and software

that contain multiple topics and variable levels that provide a challenge for learners (Slavin, 1995). The games are designed to suit every strength or difficulty students and teachers alike might encounter. Examples of such games are Shepperd software, PBS kids Games and Mr Naussbam (Schaaf, 2014). All these platforms are free to use and only require a stable internet connection and computer.

Academic controversy: Students are placed into groups holding opposing views where they engage in an academic form of debate and try to reach an agreement on the topics (Johnson, Johnson & Smith, 1991). This can be achieved by texting using WhatsApp or social media sites like Facebook and Twitter, Prepd which is a debate application, as well as video conferencing applications like skype where a discussion can be held and users are able to comment or video call one another to discuss a problem they need to solve. The advantage of this method is that discussion is able to continue at any time without having to be present at in one physical space, therefore, in cyberspace. These debates can also be completed by completing a google survey or poll, a free service offered by google which can be monitored in real time and gives immediate feedback (Johnson & Johnson, 2014).

2.1.2 Studies on cooperative learning

Different studies have investigated the effectiveness of using cooperative learning strategies with regard to learners' performances. Gambari, Shittu, Daramola and James (2016) examined the effects of video-based instruction strategies on geometry progress among senior secondary school learners in Nigeria. The instructions strategies included: The cooperative Instruction Strategy, Competitive Instruction Strategy, Individualized Instructional Strategy, and Conventional Teaching Method respectively. The study employed a pre-test, post-test, as well as an experimental and control group design. In this study by Gambari et al. (2016) there were three experimental groups, each employing one of the three instruction strategies along with using the video package. The control group used the video instructional package in a conventional manner. The intervention instrument that they used was the video instructional package, which was a user-friendly application that lasted 6 hours for six weeks. The package covered nine geometry lessons. They also made use of the Geometry Achievement test in the pre-and-post tests. The findings suggest that students using the video instructional package and the cooperative instruction strategy performed better than the other two experimental groups as well as the control group. This, therefore, indicates that cooperative learning enhances

greater improvement in mathematics performance, even with the use of technological resources such as mathematics instructional packages.

A South African study conducted by Naidoo and Kopung (2016) with 75 pre-service mathematics teachers explored the influence of WhatsApp (an instant messaging service) on mathematics learning. Participants were divided into an experimental and a control group with both groups doing a pre-test and post-test using the Mathematical Proficiency Questionnaire (MPQ) which consisted of various mathematical problems based on Algebra. The experimental group made use of WhatsApp for mathematics learning while the control group did not. The findings revealed that using WhatsApp was instrumental in improving mathematics performance in the MPQ maybe because the use of WhatsApp encouraged cooperative learning as students were able to advise others on how to solve mathematics problems (Naidoo & Kopung, 2016).

Another South African study by Mammali (2015) looked at the factors that enhanced learners' performance in geometry in Limpopo. One of the factors was the effect of group work on mathematics performance. The study revealed that 59.9% (242) of the learners strongly agreed that working in groups could improve learners' performance in Geometry; 29.4% (119) agreed, 6.4% (26) learners disagreed while 4.4% (18) strongly disagreed. The study also focussed on the use of educational media in enhancing the performance of learners' in mathematics. The findings suggest that 19.1% strongly agreed while 33% agreed (altogether 52%) that technology was effective in enhancing learner performance on mathematics. (Mammali, 2015). This indicates that educational media could be beneficial in enhancing learner's performance

Cooperative learning process fosters collaboration and elaboration on the task. This was illustrated when Chianson et al., (2011) conducted research on the effects of cooperative learning on students' retention in circle geometry in secondary schools, in Benue State, Nigeria. Chianson's objective was to ascertain the extent to which students in a cooperative learning and teaching environment retain new information, when compared with students taught using conventional methods the chalk-and-talk. Learners were assigned to the experimental and control groups using the "hat and draw method". This ensured randomness in assigning participants to groups. The distribution of participants was 174 and 184, to the cooperative learning and conventional learning groups respectively. The pre-test, known as Pre-GAT (Geometry Achievement Test) was first administered before geometry was taught, and then after five weeks of teaching geometry the Post-GAT was given. A month after, the Post-GAT,

another test was conducted to assess the retention of concepts learnt in the geometry which had been taught. This was called the RET-GAT (Retention Geometry Test). This last test was conducted after a month because it was assumed by the researcher that a month is long enough for learners to have forgotten the GAT.

The study confirmed that students, who were taught using cooperative learning were better able to retain information on what had been taught in circle geometry. It was also found that these learners were motivated and had the zeal to work tirelessly in a mathematics class. The researchers' recommendation was that the cooperative learning strategy should be used to teach mathematics (Chianson et al., 2011) .

In his analysis, Chianson et al. (2011) argues that learners struggle to remember what was taught because of the teaching method used because cooperative learning had proved to provide better retention. Udeinya and Okabiah (1991) blame the conventional teaching methods and conclude that the latter reduced motivation levels amongst learners. Harbor-Peters (2001) also highlights teaching method as a source of the problem affecting retention of concepts and performance in examination.

More broadly scholars observe that there is increased awareness amongst educationists resulting in their disillusionment with the conventional methods, and their advocacy of different methods and techniques for effective teaching. Slavin (1990) proposes that the cooperative learning method should be used as it improves both performance and retention in learners.

In a similar vein, Slavin's (2010) article on the behavioural and humanistic aspects of cooperative learning reinforces this point. From the behaviourists' perspective, cooperative learning is viewed as a group contingency where learners are rewarded for the group's performance. The humanistic view in particular, values the understanding that comes from interactions within the group. In combination, humanistic and behavioural views bring about effective cooperative learning.

According to Slavin (2010), to learn geometry, students must pay full attention and this attention is a result of motivation, which comes from different sources. Some motivation comes from home and some from the way teachers present their lessons. Schools have long tried to use different forms of extrinsic motivation including a grading system, which encourages competition. However, this grading system demotivates the weaker students who end up thinking that schooling is not for them (Coleman, 1961).

Furthermore, with this concern Slavin (2010) writes about group contingencies and cooperative learning as approaches to classroom motivation with few negative aspects compared to those brought about by competition. In group contingency, he argues, learners will be rewarded on the basis of the behaviour of its individual members. The group is rewarded if it collectively completes the task, and in order to receive their reward members sanction one another for poor behaviour.

Thus, Slavin (2010) concurs with Chianson et al. (2011) that cooperative learning brings about motivation and accelerates learner performance. Both aver that peer interaction and collaboration is central to cooperative learning. They see value in the elaborated explanations resulting from cooperative learning. Slavin (2010) explains that group rewards are significantly better than individual rewards. Further, he notes that cooperative learning becomes more effective if there are group rewards because members become accountable to the group.

Vedder (1985) points out that, studies that involve group task(s) methods but does not give group rewards have been equally disappointing in terms of achievement outcomes. He is of the view that cooperative learning must have group rewards to be effective and suggests that further research should be done on whether quality and quantity of group interactions are affected by reward structures.

Slavin (2008) also cites work relating to the John Hopkins' research on cooperative learning. He reminds the reader that there is a John Hopkins model on cooperative learning called Student Teams-Achievement Divisions (STAD) and Teams-Games-Tournaments (TGT). In this model, teams of 4 to 5 students are rewarded for the achievement of all its individual members. Studies conducted in the late 1970s at John Hopkins University showed that cooperative learning models are successful if they recognise group goals and the accountability of individual members to group goals (Slavin, 2008). By organising groups in this format, it becomes the responsibility of every member to teach other members so that the group can be recognised as a whole. It is, thus, the learning achieved by each group member that will gain scores for the group. Towards this end, members assess each other to check if learning has occurred, and this behaviour positively enhances learning.

As the research into cooperative learning expanded at John Hopkins University, they extended the programme by enrolling elementary and secondary schools to form part of the project. The educators in these schools were taught how to use cooperative learning by putting them in

groups where they would do group assignments as a way of teaching them how to use cooperative learning.

The university staff were disappointed to see that teachers who were taught how to use cooperative learning were not using it when they went back to their school. This was because educators were simply comfortable with using their textbooks and had become reliant on them. Some programs like Team Accelerated Instruction (TAI) were used to supplement teacher efforts on cooperative learning as this TAI was a program developed to integrate content and process. It was found that TAI improved student achievement and teachers were willing to work with such a programme to promote cooperative learning (Slavin, 2008).

Elaborating on this, Slavin (2008) comments on the positives brought about by cooperative learning and further argues that educational policies should be based on well researched ideas. Slavin (2008) proposes cooperative learning as an example of a well-researched teaching method and explains how it has been incorporated into the United States curriculum. For (Slavin, 2008), evidence based reform is possible in education. This paper differs from the foregoing ones in that it is mainly concerned with the question of whether government's policies will be determined by well researched evidence. (Slavin, 2008) seems to be in agreement with Chianson et al. (2011) as both state that cooperative learning improves learner achievement when it is based on group goals and accountability. (Slavin, 2008) and Slavin (2010) are in agreement when they say groups must be rewarded according to the individual learning of all its members instead of a single group product.

In the same vein, Adams (2013) researched the impact of cooperative learning on the achievement of students in literacy. His observation is that cooperative learning offers educators a teaching strategy, which is learner-centred. Adams (2013) is also in agreement with Slavin (2010) and Chianson et al. (2011) on the point that cooperative learning improves achievement and collaboration amongst learners. They also agree on the levels of motivation brought about by cooperative learning as learners interact. Adams (2013) concurs with Herrmann (2013) in his assessment that cooperative learning increases in-class participation. However, Herrmann (2013) holds the view that this increased participation does not necesarily imply an increase in cognitive activity.

Another study by Slavin et al. (2013) examined the evaluation of Power Teaching mathematics in primary schools, focusing on grades 4-5. Power teaching mathematics is a framework in which students are assigned to mixed ability teams who work together to solve mathematics

concepts. An important feature of Power Teaching Maths is that it makes use of multimedia, animation, and video sketches to reinforce learning objectives. A member of the team is called at random to answer questions on an interactive whiteboard. Slavin's et al. (2013) study consisted of a sample of 58 teachers in the experimental group and 60 in the control group at 21 experimental and 21 control schools. The experimental group teachers used Power Teaching Maths (PTM) whereas the control group made use of whatever methods and materials they usually used. Both experimental and control groups made use of a pre-test and post-test. The results indicated significant differences (effect size = -0.26, p<.001) favouring the control group. The overall impact of treatment was essentially zero, showing no difference between experimental and control groups. Both groups gained in maths achievement, but to the same degree. Analyses for pupils in the low, middle, and high achievement levels at pre-test show that all gained to about the same degree in the post-test regardless of whether they were in experimental or control schools.

Adams (2013) mentioned some disadvantages of cooperative learning indirectly when he suggests that educators have to plan the lesson properly so as to be able to manage learners in the classroom. It must be noted that some learners will not always cooperate with other members making it difficult to manage the group. Learners not contributing to the group will benefit from the efforts of others (Isik & Saygili, 2015). The differences in personalities may lead to conflicts in the class. Classrooms of diverse ethnic upbringing that are common today may pose a challenge to teachers in implementing cooperative learning (Du Plessis, 2016).

The considered literature on cooperative learning demonstrated that the use of cooperative learning has many positive effects on teaching and learning within the classroom. It enhances social, skills, fosters positive interdependence and increases confidence. As discussed, cooperative learning has its roots from the social constructivism theory (Vygotsky, 1978) which places emphasis on social interaction in the learning process. Furthermore, the studies reviewed, have indicated that the use of technology has had mostly positive effects on mathematics performance and motivation.

The next section deals with literature on technology, which is the main component of this research.

2.2.0   Technology

Delgado, Wardlow, Mcknight and Malley (2015:405) define educational technology broadly as "both hardware and software that support educational goals". They propose that such support implies the inclusion of technology in teaching as we move into the digital era, and that teachers and learners find best ways of incorporating technology into teaching and learning (Delgado et al., 2015). They point out that technology in the classroom is accessible in various ways. The list that follows outlines this range:

- **Own device ("Bring your own device"):** This is when learners bring their own instruments to school for learning.

- **Blended learning**: This is when teachers use digital devices or 'flipped classroom' for their teaching. According to Delgado et al. (2015), 'flipped classroom' is based on four pillars supporting a: flexible environment (F); learning culture (L); intentional content (I); with a professional educator (P). Flexibility here refers to loosening rules to allow learners to have control over their learning by creating an environment suitable to group and individual learning.

- **Online learning**: This means learners are able to acquire content and courses that are not available in their schools through the internet.

- **Investment in education technology**: This involves the use of instruments such as desktops, tablets, laptops, iPads and other devices.

Heymath online version offers a variety of animated lessons and interactive tools. There are so many math lab activities and games that can engage learners and drive them towards deeper learning. There are adaptive lessons tailored towards personalised learning (Heymath, n.d.)

According to Delgado et al. (2015), a number of studies have been carried out to measure the effectiveness of technology in the classroom. One such study, conducted by researchers Shadaan and Kwan Eu (2013) concluded that technology improves performance especially in mathematics. However, in some subjects like computer assisted instruction (CAI) in grade 1 and grade 4, it was found that the impact was negligible. Nonetheless, Cheung and Slavin (2011) are still of the view that technology has positive results and they attribute such inconsistencies in effectiveness to poor sampling techniques and poor methods of data collection and analysis.

2.2.1 Application of technology

Almeqdadi's (2000) researched on the use of GSP on Jordanian students' understanding of geometry revealed that the use of technology was steadily on the increase in schools. In this research, Almeqdadi (2000) had a sample of 52 students with 26 students as the experimental group and the other half as the control group. The experimental group was taught using GSP and the control group was taught using conventional methods. The results of the study showed that the experimental group performed significantly better than the control group. This conclusion was however deemed generalising of the results found in the survey. The researcher suggested that more research should be done on using technology in education, notably in GSP.

According to Giamatti (1995), GSP has many advantages which include allowing learners to learn geometrical theorems as they play around with diagrams to make any discovery that can be generalised to produce a theorem. Giamatti (1995) goes on to say that the power of GSP, together with the power of proofs will end up giving a complete illustration of the involved theorem and some aspects of doing mathematics. This is because learners are able to explore the behaviour of a particular geometric figure as they play around with the diagram in the programme and thus to have an opportunity to better understand the theorem.

Fabian, Topping and Barron (2018), investigated the use of mobile technologies among 52 grade 6 and 7 participants for mathematics, specifically the effect they had on students' attitudes and their achievements. The participants were divided into two groups, an experimental group that made use of the mobile learning activities and a control group that followed the normal curriculum. The findings revealed that the use of mobile technologies elicited positive responses on both students' performance and their perception of mobile technologies.

Miller and Robertson (2011) conducted a study on the effects of computer games amongst Scottish elementary school children aged 10 and11. A pre-and-post-test design was used with experimental and control groups across 32 schools. Children in the experimental group used a brain training computer game for 20 minutes each day. Children in the control group continued their normal routine. The results revealed that the experimental group had a 50% greater improvement in accuracy and a double improvement in speed when compared to the control group. Furthermore, within the experimental group the level of improvement among children who were considered as less abled, was higher in terms of accuracy than those who were considered more abled.

Adding voice to this finding, Growman (1996)) argues that learners who used Geometer's Sketchpad (GSP) wanted to buy their own copies of the software and had positive responses in testing geometry conjectures. The study also concluded that GSP gives excellent capabilities for improving teaching and learning as it gives learners a good opportunity to explore simulation which brings them much closer to real life situations.

However, these conclusions left the researcher wondering why results in geometry remain as poor as indicated in the diagnostic reports if programmes like GSP have such potential to improve performance in assessments.

Almeqdadi (2000) has made a number of recommendations for further studies on the topic, such as the point that studies should involve female participants; his study was for only male grade 9 students using GSP. Almeqdadi (2000) also suggests that further studies should investigate the use of GSP with samples constituting both males and females of other grades to see if the effectiveness of GSP can be generalised. He also suggested that since GSP had proved to have a significant benefit in the teaching of geometry, computers and their associated programmes should also be investigated to see if their use improves the performance of learners. Narrowing the research of GSP to its individual features and capabilities was another recommendation.

With a similar enquiry Shadaan et al. (2013) analysed the effectiveness of using Geogebra in the learning of circles. Shadaan et al. (2013) and Almeqdadi (2000) investigated the use of technological software to enhance the learning of circle geometry. Geogebra, like GSP, gives simulations, which are very close to real situations learners may figure out. Shadaan et al. (2013) also worked with grade 9 learners in the research while employing a quasi-experimental study technique. They had a sample that was divided into the control group and the experimental group. The experimental group was taught using Geogebra and the control group was taught using a conventional method. Both researchers came to the same conclusion that technology enhances the comprehension of circle geometry. For drawing conclusions they both used pre-tests and post-tests in the research (Shadaan & Kwan Eu, n.d.). In addition, they relied on a survey using a questionnaire to confirm that learners who used Geogebra were more motivated to learn geometry Shadaan et al. (2013).

Another study, which contributes to the discussion, is that of Haciomeroglu and Andreasen (2013). They also used dynamic geometry software and could confirm that students' mathematical understanding was enhanced through their use of dynamic geometry software.

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Haciomeroglu and Andreasen (2013), Shadaan et al. (2013), and Almeqdadi (2000) all agree on the benefits brought about by using technology in the classroom. Technology is seen to bring a rich learning environment which promotes interaction, sharpens critical thinking and enables understanding in the process of working with concepts. Haciomeroglu and Andreasen (2013) are also in agreement with Herceg and Herceg (2010) who had conducted research on using Geogebra to reduce the working process of numerical integration. In this instance, they found that learners who used computer-based learning software like Geogebra performed much better than those who did not. The computer-based program was especially helpful to learners who struggle with mathematical calculations.

In an additional study conducted at a Malaysian primary school, Tieng and Eu (2013) investigated the effect of using GSP on pupils according to the van Hiele levels of geometric thinking. The experimental group consisted of 16 learners and the control group comprised of 15. The experimental group was taught year three angles using GSP and the control group was taught using the chalk board. The research concluded that there was no correlation between students' literacy in using technology and their van Hiele level of geometric thinking, although the results of pre-tests were significantly different from those of post-tests for each of the two groups. The results of the post-tests of the two groups were not significantly different after using the t-test following two weeks of treatment. This was attributed to the short duration of experiment. However, learners in the experimental group were more confident in the newly acquired concepts and were able to discuss issues. This is consistent with what Meng and Idris (2012) found when they said primary school children benefit a lot from well-planned geometry activities that are taught using GSP with guidance from the teacher.

By comparison, Meng and Sam (2013) conducted similar research aimed at improving year 4 learners' geometrical thinking. They relied on stage based instructions, using GSP based on van Hiele theory. A group of 26 learners was investigated on their performance in working with polygons. The authors carried out the research using just one group instead of either an experimental or a control group. The group was given a pre-test and post-test, and the post-test was administered after the GSP intervention.

The results showed that the computer based programme helped learners to move from level 0 (pre-recognition) at pre-test to level 2 (analysis) after the post-test. It was concluded that GSP had significantly improved results from pre-test to post-test. This is in line with what was observed by Tieng and Eu (2013), that the van Hiele level of geometricel thinking improved

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for both experimental and control groups. In the latter study, the Wilcoxon test was used by Meng and Sam (2013) to arrive at their conclusion whereas Tieng and Eu (2013) used the t-test for theirs. These two research teams differed also in that the Tieng and Eu (2013) had an experimental and control group.

Funkhouser (2002) also found that students who were taught geometry using technology did much better than those who were taught using traditional instruction. They also observed that technology helps learners to be more flexible in their thinking as it allows visualization and exploration of mathematical concepts. Also Choi-Koh (1999) found that the use of GSP helped learners to move quickly from one stage of van Hiele to another. The simulation environment in GSP "provides a means for integrating problem-solving investigations with regular mathematics instructions"(Choi-Koh, 1999:310). Bhagat and Chang (2015) corroborate this by suggesting that teachers should therefore be encouraged to use technology in concepts where learners will benefit from visualising objects as in geometry.

Similarly, Eu's (2002) research into the impact of GSP on students' achievement in functions concluded that the use of GSP impoved students' motivation and subsequently their achievements in graph assessments. This research was carried out at a Malaysian school with grade 12s and the quasi-experimental method was used. Eu's study confirmed what was observed by (Rutten, Van Joolingen, & Van Der Veen, 2012) that computer simulations enhance learning. Rutten et al.'s (2012) enquiry had investigated how best to use simulations to improve learning. They also observed that the teacher's role should not be ignored in the procees of learning, but that technology had replaced only the traditional teacher centred approach with a learner-centred one where the learner is is an active participant in the learning process.

In a slight deviation from the research foci in the foregoing section, Bankov's (2013) study on the way geometry is being taught in Bulgaria does not look at the impact of a particular method. Rather it highlights the struggles encountered by learners in the learning of geometry. The study emphasises the importance of geometry and the need for it to be taught in schools. According to Bankov (2013), geometry forms the centre of theoretical and applied mathematics. Euclidean geometry represents the first time that formal proofs are encountered by students.

However, Bankov (2013) does acknowledge that the teaching and learning of this important section of mathematics is very difficult. He is aware that education systems around the world

are always reassessing how best to help the learner and the educator in the classroom. He points out that instead of listing the geometry objectives, the Bulgarian curriculum lists the step by step approach to teaching the topic. The findings of this research are consistent with what other researchers like Funkhouser (2002) have found on the topic that learners are performing badly. Educators were also held to account as lacking in the necessary geometry knowledge, hence making the teaching of the topic very difficult.

Unlike the research studies discussed up to this point where the interest has been mainly in the learner, Abdullah, Surif, Ibrahim, Ali and Hamzah (2014) conducted a study on technology which targeted the teacher .They observed that teachers struggle to integrate technology into their teaching. They found that teachers have limited time for lesson preparation and hence find it difficult to incorporate GSP into their lessons. A number of teachers also showed ignorance on how to use GSP. Abdullah et al. (2014) then developed a module known as MyGSP to help teachers with material for teaching using GSP and videos. Teachers were shown how best to use the GSP program. This research showed that MyGSP made it easy for teachers to learn GSP software. Their findings concurred with what Rahim (2002) reported – that most Canadian teachers found it easy to to teach geometry and other topics using GSP.

Despite many studies indicating a positive relationship between technology and mathematics achievement; other studies do not support this relationship. For example, Carr (2012) conducted a quantitative, quasi-experimental study was to examine the effects of iPad use as a 1-to-1 (1:1) computing device on 5th-grade students' mathematics achievement in two rural Virginia elementary schools. The experimental group made use of iPads and computers during mathematics lessons while the control group did not. A pre-test was administered before the use of the iPad and a post test was administered after using the iPad to check for the effects of the iPad on achievement. The findings revealed that the change from the pre-test to the post test was not significantly different between the two groups. Therefore, it was not meaningful enough to significantly influence students' mathematics achievement.

There are a number of problems associated with the use of technology in the classroom. Baba (2014) has it that some educators on their side are not competent enough to use technogy to deliver content for their subject and therefore teacher training on using technology should be a priority for the government. Secondly, some teachers feel that using technology in some subjects like languages is time consuming. The differences in socio-economic status of the learners make it difficult for teachers to give assignments requiring the use of technology as

some learners do not have access to these resources at home. Lastly, using technology in the classroom takes away the focus and concentration of some learners as they will be thinking of computer games and social media.

2.3 How can learner's motivation for doing mathematical tasks be stimulated using cooperative learning and technology?

A study by Plass, O'Keefe, Homer, Hayward, Stein and Perlin (2013) evaluated the Impact of Individual, Competitive, and Collaborative Mathematics Game Play on learning, performance, and motivation. The participants were 58 middle school students from grades 6-8. They made use of the programme *Factor Reactor* which is used to practice and automate arithmetic skills. From a theoretical framework perspective, the results demonstrated that although only the competitive mode of play increased within-game learning, both competitive and collaborative modes of play increased interest and enjoyment. Further investigation concluded that collaborative play was associated with greater enjoyment, situational interest, and intention of reengagement than individual play. Practically, the research provides support for the potential of educational games as effective learning environments that provide incentives for students. The figures showed its highest increase rate for interest at 5.82% when tasks were completed collaboratively compared to 5.67% when done competitively. It also suggested that in comparison to the individual mode of play the competitive and collaborative mode resulted in strongest mastery goal orientation which is highly associated with motivation and learning (Plass et al., 2013). In summary, this study emphasizes competitive modes of play over collaborative for games aimed at developing arithmetic skill fluency as well as increasing situational interest.

Zengin and Tatar (2017) conducted a mixed methods study to evaluate the effectiveness in using the cooperative learning model supported with Dynamic Maths Software (DMS). The participants consisted of 61 high school students. There was a control group as well as two experimental groups, one each for sequences and quadratic function. The cooperative learning model was used for a period of three weeks in both the experimental and the control groups. The experimental groups were also supported by the DMS. The data was collected using Sequences Knowledge Test (SKT), the Quadratic Functions Knowledge Test (QFKT) This was administered to the students both pre-and- post-tests, and an open-ended questionnaire was used after the implementation of the model to examine its feasibility. DMS was included in the experimental group's learning while the while the control group was taught sequences in a

traditional manner. The pre-test results indicated no significant difference between the experimental groups and the control group. The post- tests for both the experimental groups (the sequence and quadratic functions) in comparison with the control group (sequence) found that the use of the cooperative model supported with the DMS was more effective than the use of the cooperative learning model without the use of DMS in the control group for both achievement in quadratic functions as well as sequencing. The qualitative data indicated that students from the experimental groups reported an increase in motivation and interest (Zengin & Tatar,2017). Moreover, these students mentioned that it had increased their retention and aided their understanding and conceptual learning (Zengin & Tatar,2017).

A study conducted by Taleb, Ahamadi and Muvasi (2015) looked at the effects of mobile learning on the ability to learn maths. The study was conducted with 329 secondary school teachers of Mathematics from 19 districts of Tehran using a descriptive-field method during the 2012-2013 academic years. These researchers made use of a Likert-type questionnaire to identify the teachers' viewpoint of the effect of M-learning (mobile learning) in different aspects of Mathematics learning (Taleb et al., 2015). Twenty six questions measured the effect of different functional capabilities of mobile technology on increased motivation in learning Mathematics. Thirty seven questions measured the effect of different aspects of mobile learning on the diversity of training methods in learning Mathematics. Thirty one questions measured the effect of different functional capabilities of mobile learning on students' participation in learning Mathematics (Taleb et al., 2015). The main hypothesis highlighted how different capabilities of mobile technology (mobile phone, laptop, and tablet) have a positive effect on increased motivation in learning Mathematics (Taleb et al., 2015). The results showed that 95% of students agreed that mobile technology use in learning mathematics increased their motivation (Taleb et al., 2015).

A study conducted by Yorganci (2018) studied the views of graduate students on the use of GeoGebra (a mathematics software program) in the teaching of maths learning. The sample consisted of 7 graduate students at the University of Turkey enrolled in the maths education programme. A case study method was employed in this study. The data was collected using semis structured questionnaires and observations. The study found that GeoGebra was an effective learning tool which was associated with positive factors in mathematics learning. These factors were: "visualization, ease of use, motivation, rich content, competence, conceptual learning and algebraic thinking, frequently emphasized by students in the competence of learning environment created by GeoGebra" (Yorganci, 2018:72). Although

this study did not make use of a cooperative approach, the use of Geiger can be adapted to make use of a cooperative approach.

Bester and Brand (2013) conducted a South African study with grade 8 learners on the effect of technology on learner attention and achievement. Researchers made use of a control and an experimental group. During the mathematics lesson the experimental group was exposed to a video called Mobius strip while the control group was given the same information without the use of technology (Brand & Bester, 2013). The results indicated a high positive relationship between motivation and concentration in the experimental group as opposed to the control group who did not make use of technology (r = 0.64; p < 0.01). Therefore, indicating that high attention, concentration motivation will lead to higher levels of achievement (Brand & Bester, 2013).

2.4 Axiomatic system

The axiomatic system is made up of axioms, definitions, theorems, converses and corollaries, all of which are essential to solve geometry riders through calculations giving reasons or through constructing proofs by developing a connected set of arguments supported by mathematical statements backed by appropriate reasons.

Axioms

Axioms are mathematical facts that are assumed to be true in their own right and need not be proved. These are used to prove theorems (Govender, 2002). For example, if one straight line meets another, the sum of the angles so formed is 180°.

Theorems

A theorem is a true mathematical statement that comes as a result of a chain of mathematical definitions or axioms put together deductively. The following is an example of a theorem in Euclidean geometry.

The opposite angles of a cyclic quadrilateral are supplementary.

That is $\hat{A} + \hat{C} = 180°$

Taken from DBE/November 2017 Question 8.1.2

Proof:

Let $\hat{A} = x \ldots \ldots (i)$

Then $\hat{O}_1 = 2x \ldots \ldots$ *angle at the centre =*
twice the angle ath the circumference $\ldots \ldots (ii)$

$\hat{O}_2 = 360° -$
$2x \ldots \ldots$ *angles at a point add up to* $360°$ …...(iii)
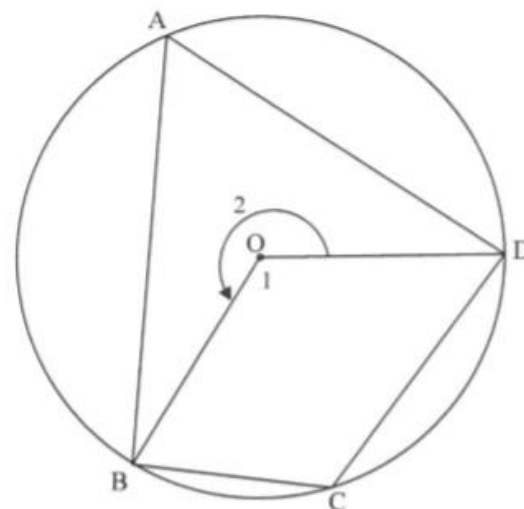
$\hat{C} = 180° - x \ldots..$ *angle at the centre*

$\quad = $ *twice the angle ath the circumference* $\ldots. (iv)$

$\hat{A} + \hat{C} = x + 180° - x = 180°$ ……Proved…...(v)

It can be seen that to prove the above theorem, there is a chain of definitions and theorems combined to make a proof. Statement (iii) is a definition and statements (ii) and (iv) are also theorems.

Converse

A converse of a theorem is just the reverse of that theorem. What was the conclusion of a theorem becomes the premises of the converse and what was the premise of the theorem becomes the conclusion of the converse. An example is given below.

Theorem: A line drawn from the centre of the circle perpendicular to the chord (premise), bisects that chord (conclusion).

Its converse: A line drawn from the centre bisecting the chord (premise), is perpendicular to the that chord (conclusion).

Corollaries

A corollary is a statement that is a direct deduction from a theorem. An example is the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. This is a direct conclusion from the theorem that opposite angles of a cyclic quadrilateral are supplementary.

Axioms, theorem and corollaries together are used to tackle problems in Euclidean geometry. From the researcher's experience as a mathematics teacher, it has been seen that the circle

theorems covered in CAPS can be proved using construction of isosceles triangles or using congruency and some using other theorems.

2.5.1   Role of visualisation in Mathematics

Mathematical visualisation refers to the process of forming images in the mind, by drawing with pencil on paper or with the aid of technology and using the images effectively for discovery of mathematical concepts and understanding (Jones & Bills, 1998). According to Nelson (1983:54), "visualisation is an effective technique for determining just what a problem is asking you to find. If you can picture in your mind's eye what facts are present and which are missing, it is easier to decide what steps to take to find the missing facts".

Visualisation does not only help to sort available data in meaningful structures, but it helps in the development analytical steps to a solution and acts as a catalyst in the interaction between imagery and concept development. "Visualisation, may be mental or physical, and imagery, which may be pictorial" (Jones and Bills, 1998:125). It is further argued that visual representations must be available in all mathematics classrooms (Jones & Bills, 1998).

According to Duval (1998), geometrical reasoning involves three kinds of cognitive functions, which are visualisation, construction and reasoning process. Construction is where diagrams are drawn using tools and reasoning is when there are discussions for the extension of knowledge, clarity and proofs. The three processes can occur independently though they can influence each other. Their interaction is required for effective learning of geometry (Duval, 1998). There are several questions that can be asked when dealing with visualisation. The other question is whether seeing a picture is more important in visualisation than feeling the object and the other question is whether there is something called a picture that can exist in the mind (Duval, 1998). Some examples of visual images include pictures, recurring patterns and memories from past experiences.

2.5.2   Language barriers

The language of learning and teaching (LOLT) can be a source of problems for learners if it is not their mother tongue. The education system in South Africa is failing most of the youth, as the majority of the learners cannot read at the expected grade level. In most schools the LOLT is English, which is usually not their mother tongue (Spaull, 2013). Most reading material is

also in a foreign language for many learners. This means that second language readers will struggle academically if subject material is written in a second language

Mastropieri, Sruggs and Graetz (2006) found that second language readers at secondary school level had challenges in reading in order to succeed. Their research looked at students with learning disabilities and those without. The research was not limited to subject English only but also looked at reading comprehension in other subjects like. Science. They observed differences in students' reading ability and the prescribed reading materials in high school as barriers to reading comprehension.

The next section highlights the importance of proof and reasoning in mathematics in relation to my research on the learning and teaching of grade 11 geometry.

2.6.0   Proof and reasoning in mathematics

For the purpose of this study on the impact of technology in learning circle geometry, it was important to have a deeper understanding of proofs and reasoning as these form the foundation of Euclidean geometry. In their paper 'Students' understanding of the structure of deductive proof', Miyazaki et al. (2017) focused on the structure of deductive proofs in geometry. They showed that geometry is made up mainly of deductive proofs. They acknowledged that teaching and learning of proofs is a major challenge internationally but observed that students do not have a clue of what is expected when asked to 'prove' conjectures, theorems and riders. Most learners resort to using empirical examples when the question requires them to prove a mathematical concept.

According to Miyazaki et al. (2017) there are three aspects to consider when working with proofs. The first is that it is important to see a proof as a structural object. Duval (2002) explains that 'structural object' refers to being able to see a proof as organising premises, conclusions and theorems. These connect to form a proof. Learners must be able to distinguish between a valid reason and a non-valid one because only valid reasons may be used to support arguments. The following example illustrates valid and non-valid reasons

In the diagram below, two circles have a common tangent TAB. PT is a tangent to the smaller circle. PAQ, QRT and NAR are straight lines.

Let $\hat{Q} = x$.
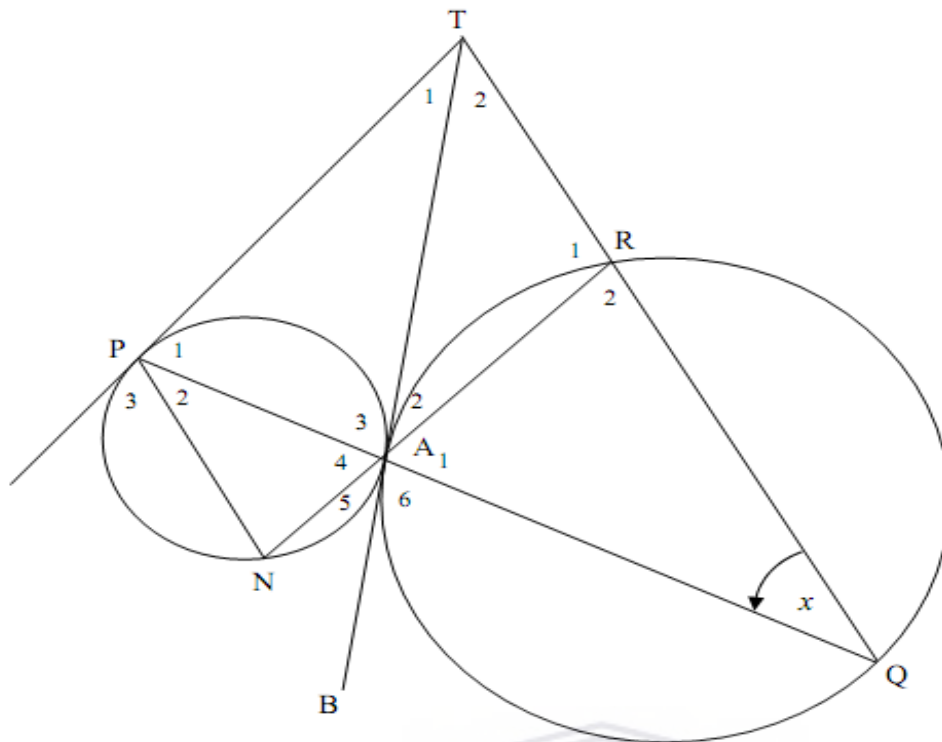
Name, with reasons, any other angle equal to x. (2)

Solution

$\hat{A}_2 = x$ (tangent chord theorem) ……………valid reason …………. …...(i)

$\hat{A}_2 = x$ (corresponding angles equal) …………. non-valid reason…………. (ii)

Justification.

Whilst it is true that $\hat{A}_2 = x$, the given reason in statement (ii) is not a valid reason as the two angles $\hat{A}_2 \; and \; \hat{Q}$ are not corresponding angles by definition and in particular they are not equal as they are not enclosed/formed by a set of parallel lines.

The second aspect is to view a proof as an intellectual activity. This involves identifying, or differentiating between, that which is involved in the proving and that which supports the steps. The third aspect requires identifying components of a proof which include the premise, the conclusion and the related theorems.

Proofs can range from non-mathematical ones to those that are mathematical. According to de Villiers and Hanna (2008), the basic principle at the centre of all proofs is that clear assumptions followed by valid reasoning is essential for one to come to an appropriate argument or conclusion. They state that "mathematical proofs consist ...of explicit chains of inference following agreed rules of deduction, and ... (are) often characterised by the use of

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formal notation, syntax and rules of manipulation" (de Villiers et al., 2008:330). Proofs form the foundation of human reasoning and according to Miyazaki et al. (2017), the concept of proof is internationally recognised as key to mathematics. Yet unfortunately it has been observed that learners struggle with proofs, particularly in geometry. The example that follows illustrates the deductive nature of a proof in geometry as the next statement is directly built from the previous one.

Question 1

In the figure, TP and TS are tangents to the given circle. R is a point on the circumference. Q is a point on PR such that $\widehat{Q_1} = \widehat{P_1}$. SQ is drawn.

Let $\widehat{P_1} = x$.
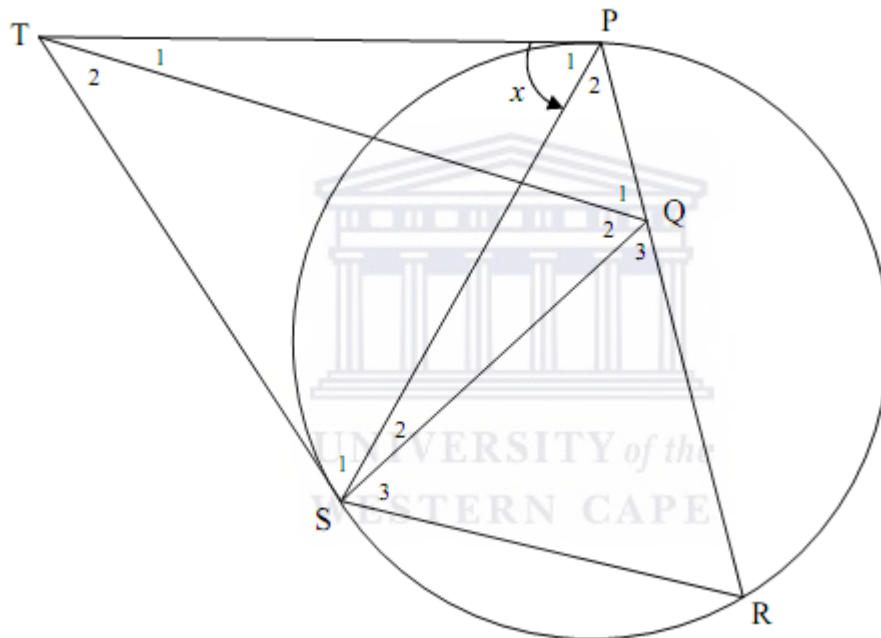
Prove that:

TQ ∥ SR   (4)

Solution 1

$\widehat{P_1} = \widehat{Q_1} = x$ .... (given) ✓   ………………………..(i)
and $\widehat{P_1} = \widehat{R} = x$ ....(tan-chord theorem) ✓ …………….(ii)
∴ $\widehat{Q_1} = \widehat{R} = x$ ✓ …………………………………………………..…….(iii)
∴ TQ ∥ SR ………………….... (corresponding angles are equal) ✓…. (iv)

The conclusion that the two lines are parallel has been reached deductively. That is, if the corresponding angles are equal, then the two lines are parallel. In other words, 'deductively' means that in the calculations, the next stage is supposed to follow logically from the preceding stage. Statement (i) is formulated on the grounds of given information and forms part of the

chain of premises. Statement (ii) is a deduction coming from a valid theorem, and forms part of the chain of premises. Statement 3 is a conclusion logically following from statements (i) & (ii), which are the premises of the argument. Now statement (iii) serves as the premise for making the conclusion in statement (iv), and this completes the proof. It must be noted that there can be several conclusions in between to complete a single proof.

At school level geometry, proofs are normally laid down in a formalised, "two-column format" (Miyazaki et al., 2017). This format is designed to capture a statement supported by a reason. However, this approach does not serve the level of reasoning required in understanding proofs in detail. Miyazaki et al. (2017) argue that teaching must be focused on giving the actual structure of a proof as this becomes more important for reasoning when learners move from elementary levels through high school and beyond. Specifically, structure of deductive proofs is rated higher than other types of proofs as studies have indicated that deductive proofs enable learners to gain a deeper understanding of the construction of proofs.

According to Miyazaki et al. (2017) there are three elements central to proofs and proving. The first is that it is important to understand a proof as a structural object made up of parts. The second is that a proof must be seen as an intellectual activity. The third is that it is important to understand the function and purpose of proofs and proving. It is thus vital for learners to distinguish between valid and non-valid reasoning when attempting to support an argument.

A deductive proof will consist of singular and universal propositions linked by appropriate connectives. The singular propositions are made up of premises and conclusions. An example of this is that *a line drawn perpendicular to a chord (premise) bisects the chord (conclusion)*. The entire statement becomes a theorem.

Extrapolating this assertion, Heinze, Cheng, Ufer, Lin and Reiss (2008) state that a geometrical proof with valid reasoning is made up of the components outlined in the conditions that follow; they state that it is important:

- to understand the information given;
- to see the connections from the premise to the conclusion through proper argument; and,
- to connect a multi-step proof with proper bridging, based on an understanding that a conclusion is reached from premise, and that definitions and theorems help in making a logical argument without any assumptions.

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At the start of learning proofs, the reasoning required is done in an informal way before moving to developmental proofs. Developmental proofs increase "in sophistication as the learner matures towards coherent conception" (de Villiers et al., 2008: 329). This simply means that by building on learners' knowledge proofs become increasingly sophisticated as learning progresses. Developmental proof is thus a concept made up of three features which include the fact that proof and proving strengthens reasoning and understanding of mathematical concepts.

As stated, proofs and proving moves from elementary to complex, and learners must gradually move from one stage to the next. This is why proofs have a long-term link with mathematics. While proofs are vital in mathematics classrooms, their actual role depends on how the teacher infuses them into teaching to get the maximum benefit.

Proofs constitute more than just a sequence of correct and acceptable steps in mathematics; they enhance learning by showing why something is acceptable, and by justifying its acceptability. The moment a learner sees the source of an argument, he or she will be able to apply the same argument in his/her learning more widely (de Villiers et al., 2008).

2.6.1   Importance of proofs

Thus proofs play different roles in mathematics learning. For instance, they have a verification role just like proofs in geometry. If different examples lead to the development of a particular conjecture, it is a proof that will verify the conjecture. As stated earlier, de Villiers et al. (2008) proclaim that proofs' roles include explanation of concept, discovery and cognitive challenge. New discoveries can also result from proofs as we move deductively and make generalisations.

In mathematics not all discoveries come from experimentation. Proofs are used to connect theorems and axioms deductively to make new knowledge. The educator should design tasks for learners that will foster appreciation of the role played by experimentation and proofs. They should learn that the concept of mathematical proof has proved to be very important in empirical sciences where hypotheses are made and need to be proved. Mathematical proofs are used in these fields to decide whether to accept or reject a hypothesis.

The importance of proofs and logic bears further emphasis. Reiss, Hellmich and Reiss (2002) have reiterated that proofs and logical argumentation are vital concepts in mathematics that we cannot do without. Reiss et al. (2002) further argue that mathematics can be regarded as a subject for proving science. According to the National Council of Teachers of Mathematics, NCTM (2000), understanding proofs is considered to be an important component of

mathematics, one which enhances competence in the subject. NCTM (2000) advocate for learning to reason and to construct proofs as a vital dimension to learning mathematics so that learners are taught to understand mathematical arguments and proofs (NCTM,2000). This component of mathematics can actually be the foundation of Euclidean geometry.

2.6.2   Studies on geometrical proofs

Reiss et al. (2002) conducted a study on grade 8 students, testing their ability to carry out a proof. The intention was to use the proving sequence to assess learners' ability to justify and reason. The students were arranged in three equal groups according to their level of understanding. Learners in group three were the top ones who were able to order their arguments consistently and who performed better across all the types of questions given in the assessment.

Both high performing and low achieving classes performed well in simple tasks. However, low achieving classes had hardly any correct solutions in questions where argumentation was required (Reiss et al., 2002). According to Baumert, Lehmann, Lehrke, Schmitz, Clausen, Hosenfeld and Neubrand(1997), these differences in performance might have been due to teaching methods used. The results of the study showed that learners performed much better in mathematical propositions and concepts but that they struggled with problems requiring them to reason mathematically, comprehend proofs and justify statements.

In another research project to gauge learners' understanding of proofs, it was observed that about 67% of 700 grade 8 learners from a German school were able to classify correct proofs as correct and 35 % were able to classify incorrect proofs as incorrect (Reiss et al., 2003). While these differences in performance may also be due to the teachers' actions in the classroom, and while the teaching style forms an important aspect to consider in efforts to improve learning, the study also exposed learners' basic insecurity in fully comprehending geometry proofs.

The research showed that it is easy for learners to identify a correct proof as correct, but it is difficult for learners to see an error in an incorrect proof (Reiss, Hellmich & Thomas, 2002). It was also concluded that learners require a certain basic knowledge of mathematical facts and argumentation. Certain higher order skills are needed to enable learners to apply their knowledge in the context of proofs. Without these skills, as well as those of scientific reasoning, it is clearly difficult to work with proofs.

Another study which investigated this aspect of proofs, was one done by Healy and Hoyles (1998) who reported on how grade 10 learners respond to proofs in a multiple-choice test. Learners were given four options as proofs to a simple mathematical statement.

It was observed that generally learners were able to identify correct proofs. Learners were of the view that formal proofs where they show working will give them more marks from the teacher than narrative proofs where they just explain the reasoning behind. Surprisingly though, learners often used narrative style when writing down their proofs.

In a research similar to that of Healy and Hoyles (1998), Heinze and Reiss (2003) probed cognitive and non-cognitive factors affecting learners' competence in proofs. Their investigation was premised on the idea that mathematics is a scientific discipline specialising in proofs and the proving part is what makes it different from other scientific disciplines. Proofs in the classroom must therefore be taught and learnt according to the standard acceptable in the mathematics community. According to Heinze and Reiss (2003) knowledge of theorems and rules does not sufficiently equip learners to prove theorems. It is understanding and knowledge of correct mathematical steps that constitutes a critical prerequisite.

As stated earlier, the methodological knowledge of a proof comprises of three aspects which are independent of each other, namely are proof scheme, proof structure and chain of conclusions. The proof scheme simply describes the reasoning steps followed in executing and chaining a proof. A mathematical proof scheme postulates a proof follows a deductive reasoning pattern. That is to say, for each next step in a proof there must be a supporting reason which is acceptable in the mathematics community. The proof structure describes the composition of a proof in other words, that a proof is made up of a premise and conclusion. Proof starts with the premise which comprises what is given. It is this premise that we start with and move towards the specific assertion or conclusion in a deductive manner. There must be no gaps in the logical flow of a proof and lastly, the chain of conclusions refers to the fact that each step of a proof must be supported by the previous step (Heinze & Reiss, 2003).

Different studies were conducted to decipher learners' proof schemes, proof structure and conclusion chain. Harel and Sowder (1998) devised three categories of proof scheme. The first is the scheme in which there is an external conviction that involves referencing a higher authority. The second is the empirical proof scheme which is when induction or perceptual arguments are used. The third is the analytical schema when learners use the deductive approach.

In the research by Harel and Sowder (1998), many students were comfortable with deductive as well as inductive reasoning yet the proof structure itself was not obvious to all learners. Selden and Selden (1995), concluded that failure by some learners to see the logical structure in a proof simply means the same learners are not able to construct a proof structure for the given statements.

By applying this conclusion on proof structure to circle geometry, the researcher can also conclude that if a learner fails to follow particular arguments in a deductive proof, then it will be impossible for that learner to prove the riders deductively.

The chain of conclusions is the difficult part to check when trying to measure the understanding of learners in mathematical proofs. The conclusion of a proof must logically follow from the preceding steps. If there is an error along the way, then the conclusion will also be affected by that error. The difficulty lies in ascertaining the source of the error. It may be a result of shortcomings in conceptual knowledge; or it may just be a calculation error. Another source of difficulty for learners is to have a logical flow from premise to conclusion if the problem is far-fetched or else removed from the learner's realistic context (Heinze and Reiss, 2003).

In their study of the same topic, de Villiers et al. (2008) clarified the relationship between argumentation and mathematical proof. They proposed that the former refers only to using plausible arguments whereas the latter is made up of well-connected deductive statements. From a tender age learners have the ability to reason and justify their arguments in social situations, however, this does not easily convert to understanding mathematical proofs. It is the duty of the educator to introduce deductive reasoning to learners to enable them to appreciate its role in mathematical proofs.

They suggest that proofs can be classified in terms of techniques used or claims made, by the proof. The different techniques include proof by induction, by exhaustions and lastly by contradiction. The claims made by a proof include the existence of proofs.

A report by Miyazaki et al. (2017) is in even closer alignment with the research I elected to undertake. They wrote about how students in a Japanese secondary school dealt with deductive proof. This was helpful for me to understand before carrying out my research on using technology in the context of cooperative learning in the learning of grade 11 geometry, as I will explain in due course.

Regarding their context, Fujita and Jones (2014) point out that there is no official syllabus in Japan showing how deductive proofs in geometry must be taught. Textbooks have a general sequencing in grade 8 that educators simply follow. Students in grade 8 in Japan are introduced to deductive proofs through learning the properties of shapes like triangles and quadrilaterals. The properties of straight lines are also learnt at this stage.

As congruency is one of the concepts used to show deductive proofs, Grade 8 learners are given a chance to justify why a particular written proof is correct, using learnt properties. In time, it is surmised, this would enable learners to write their own proofs competently. Proving as an explorative activity has been stressed in the teaching and assessment of geometry. This is where planning – including extending a proof – has been added to the construction of proofs (Miyazaki & Fujita, 2015).

In Miyazaki and Fujita (2015), Japanese learners were exposed to visual proofs in the form of flow charts. These flow charts are a good introduction to the concept of deductive proofs as they show learners the proper connections in a proof from premise to conclusion. There are other merits of flow charts that include the role of linking natural and functional language, (Balacheff, 1987). The lessons on visual proofs in flow charts were outlined in three phases as listed in the sequence of stages that follows:

- Constructing a flow chart proof for open situations (in about four lessons);
- Deriving a paragraph proof from the flow chart (in about three lessons); and
- Refining the paragraph proof into flow-chart format for closed situations.

The foregoing structure helped learners to scaffold their understanding of deductive proofs. At their early age, the structure of deductive proofs might hold no meaning to learners. It may appear like a mere chain of geometrical symbols that can only make sense if connected in a visual flow chart (Miyazaki et al., 2017). There is therefore a need for the teacher to use teaching methods by which learners can actually see how facts are connected.

I have also observed in my own teaching that showing visual diagrams to learners concretises concepts. There are times when new knowledge becomes far-fetched to learners if conveyed in ways they can neither figure out nor connect to. Thus, the learning of proofs is enhanced by geometrical software and technology has contributed greatly to the learning and understanding of proofs.

The dragging opportunities afforded by software like GSP helps by giving an infinite number of simulations supporting a particular conjecture. Although this is not enough to prove the conjecture correct, it becomes easy to prove the conjecture wrong. According to de Villiers et al. (2008:332), "dragging makes the relationship between geometric objects accessible at several levels: perceptual, logical and algebraic". Dynamic geometry software helps in the transition from object to mathematical proofs. The software and proofs are closely linked as the latter helps to bring the required cognitive explanation for why the conviction gained from dragging exists. Learners might be convinced with observations from simulations but still lack a mathematical understanding as to why these obtain. Proof will provide clarity and a better understanding potentially.

Understanding more about proofs certainly helped me in my research as proofs are a major component of circle geometry.

2.7   Curriculum 2005 (C2005)

This was the curriculum designed in 1997 after democracy in South Africa to redress the impact of an uneven society left by apartheid. It was important for the researcher to understand this curriculum as it formed the foundation of other curricula that followed in the post-independence South Africa. The teaching approach (OBE) accompanying this curriculum (C2005) was met with many challenges in its implementation (Chisholm, 2015). C2005 was replaced by the Revised National Curriculum Statement (RNCS).

According to Christie (1999), OBE was an approach borrowed from Australia, which emphasised on outcomes as opposed to content and objectives as emphasised in the RNCS. At its launch, OBE promised to be the solution to South African education's problems, but certain educationists had already predicted its failure (Jansen, 1998).

Under OBE, the old traditional chalk-and-talk method was to be dropped, and it gave power to learners to learn by discovery as they went through prepared tasks. According to Sayed (1997: 17), "curriculum, teaching and assessment were to be organized with the purpose that learners successfully completed and performed predetermined learning tasks and goals". OBE placed the emphasis on skills, knowledge and attitudes according to criteria established by the South African Qualifications Authority (SAQA). According to SAQA learning could only be measured against set standards at the end of a learning programme. For all this to happen a

wide range of resources was needed for effective learning, and unfortunately these resources were lacking in most schools.

2.8   The van Hiele theory of geometric thinking

The other theory used was the van Hiele theory of geometric thinking. This was used because of its relevance to the topic under scrutiny, which is geometry. Also, other studies have concluded that grade 11 circle geometry falls within levels three and four of the van Hiele theory. It was thus important to work within this theoretical paradigm (Miyazaki et al., 2017).

This model was developed to boost the understanding of geometry and was developed in a classroom setting (Meng & Sam, 2013). It was designed by a Dutch couple, P.M. and D. van Hiele to improve the performance of learners in geometry at the secondary school where they were teaching (van Hiele, 1986). The model incorporates different levels and their properties plus the movement of learners along the different levels, as will be explained in detail later. It has become widely used internationally. According to Meng and Sam (2013), most of the geometrical thinking studies have been conducted on the basis of this model. For example, the geometry curriculum of countries like Taiwan is based on the van Hiele model.

The van Hiele model is comprised of five stages involved in geometric thinking. These stages describe how learners view and interpret geometric shapes. According to Luneta (2015), these levels are recognition (Level 1), analysis (Level 2), informal deduction (Level 3), formal deduction (Level 4) and rigor (Level 5). These levels were developed using some ideas borrowed from Piaget although the two theorists differ on a number of aspects.

It was vital for me to understand this model as I carried out my research on the impact of using technology through cooperative learning on learners. This is because the van Hiele model of geometric thinking suits my research in different ways. One crucial way is that the van Hiele levels of geometrical thinking explain how learners think in geometry in a classroom situation, and this is directly relevant to my classroom-based study on technology and geometry.

To begin with, it was important for me to ascertain the level of cognition my grade 11 learners were expected to acquire in order for them to be able to understand circle geometry. According to Miyazaki et al. (2017), learners develop a better understanding of deductive proofs required in geometry when they are operating at van Hiele levels 3 and 4. By comparison, in a study by Ma et al. (2015), it became evident that learners at primary school operate at levels 0 to 2.

The significance of ascertaining the learners' level of operation is explored further in the next section.

2.8.1   Recognition level/ visual level: level 1

At the recognition stage, learners are simply expected to recognise objects in terms of appearance only. A figure is thus recognised by its shape only with no properties attached to it. "Although a model has been determined on basis of the characteristics, a person at this level is not yet aware of that characteristic"(Brannon et al., 2018:2). There is very little understanding of the figure as elaborated by van Hiele when he said, "it is possible to see similar triangles, but it is senseless to ask why they are similar. There is no why, one just sees it" (van Hiele, 1986:63, cited in Christman, 2001). Learners do not move from one level to another automatically, but through teaching and learning.

2.8.2   Analysis level/Descriptive level: level 2

At van Hiele's second level of analysis, shapes are distinguished on the basis of their properties although there is no proper connection of properties. A learner operating at this stage must recognise and be able to name the properties of the geometric figure in question. It is thus important that the teacher or facilitator be aware that for learners at this stage it is difficult to realise that a square is a rectangle because properties of shapes are not properly ordered (van Hiele, 1986). On this point Luneta (2015) notes that the logical order of properties is not fully realised, and the relationship between figures cannot be explained in terms of properties.

According to (Ma et al., 2015), at this stage learners are able to describe a shape by its properties. They can tell that opposite sides of a rectangle are equal but they cannot figure out, using properties, why a rectangle differs from a square. The learners can describe with confidence that diagonals of a rectangle are of the same length and describe properties of a rhombus correctly but still cannot conceptualise the relationship between the shapes by using properties

2.8.3   Informal deduction level: level 3

At the third level, students are able to prove relationships and learners are taught to order the properties of geometric figures so that an understanding of each property is established on the basis of an understanding of the preceding properties. This ordering of properties allows

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students to group objects with same properties. For example, the square is recognised as a rectangle because by definition, it satisfies all properties of a rectangle.

The concept of deductive reasoning thus begins and this deductive reasoning helps learners to prove relationships (Christman, 2001). At this stage however, teachers should be aware that students can give the necessary and sufficient conditions in determining properties of a concept but may still find formal deductions a challenge. According to (Ma et al., 2015), this is the maximum level that primary school students reach. But this view is contradicted by Luneta who argues that learners are expected to reach level three only at high school (Luneta, 2015).

2.8.4 Formal deduction level: level 4

According to the van Hiele model, at level 4, learners are able to reason logically and make formal deductions. Learners are expected to work with axioms and theorems.

In this study, the researcher expected grade 11 learners to be operating at Levels 3 and 4 in order to be able to cope with circle geometry. The reasons for my assumption relate to the point that this is when they are required to work with theorems, and therefore these are the levels at which learners should be able to prove relationships.

2.8.5 Rigor level: level 5

Level 5 is the stage at which students can compare systems with no concrete models available. This is the final stage when axiomatic systems in geometry are well understood. As stated earlier, according to the van Hiele model, learning is facilitated as one level builds on what was done in previous level (Christman, 2001). It is thus not possible for learners to understand work at the next level if their reasoning at the prior level is not sufficient. Van Hiele (1986:66, cited in Christman, 2001) says, "this is the most important cause of bad results in the education of mathematics".

The point is that it is vital for students to pass through all the learning phases in the given sequence for proper understanding of geometry to take place. It is therefore important for teachers to understand this model before engaging learners in geometry. It was equally important for me to understand this model fully before introducing circle geometry to the classes for my research. It was hoped that technology through cooperative learning would help in moving through the levels of the model smoothly.

According to (Dindyal, 2007), there is need to have a framework that addresses some other weakness of the van Hiele levels of geometrical thinking. This theory has a number of weakness that were cited, which include the following:

- The theory was developed when Euclidean geometry was the only component of high school geometry. In the 1960s, other forms of geometry like analytical, vector and transformation geometry were introduced. This introduction of other forms of geometry, which rely on algebraic manipulations means that the van Hiele levels must be revisited as there is a strong connection with algebra.

- The levels of van Hiele are too distinct and do not give room to the fact that some levels can take place at the same time.

- The invent of technology which is very important for the visualisation and simulations might require a revisit of the van Hiele levels and come up with another framework.

The next section uncovers the gap identified for the research.

2.9   The gap identified through this literature review

The literature review informed the researcher of the different angles other researchers have pursued using technology and cooperative learning in teaching. The results showed that using technology and cooperative learning improved motivation and performance of learners significantly when they were used separately.

As already stated, the researcher elected to investigate how learning circle geometry through technology might impact on grade 11 learners' performance in South Africa when used in the context of cooperative learning. This was predicated on the fact that all the studies I encountered examined these methods separately. The comments in the diagnostic reports showed very low average percentages in learner performances in geometry. If a solution could be found to help learners to respond to the solving of circle geometry problems in nuanced ways then it is likely that learners' average performance in geometry could improve. Thus the purpose of this study was to find out if performance improved significantly when technology was used through cooperative learning in the teaching of grade 11 circle theorems.

2.10   Theoretical Framework

As with any study, the theoretical framework constitutes a roadmap for the progression of the research. Importantly it gives guidance and averts digressions or loss of focus in the evolution of the research. The theoretical framework has become the building plan of the study in terms of how the data was collected and analysed. It has also given the researcher hints on possible threats that might be encountered in the survey which might have affected the validity of the conclusion reached.

In this research, constructivism was chosen because it is learner centred. The researcher was mainly interested in an approach that would facilitate learning. Social constructivism by Vygotsky (1978) and cognitive constructivism by Piaget are the main theoretical positions used. These two theories are rooted in the belief that learners are ultimately responsible for their own learning.

2.10.1 Constructivism

Constructivism is a theory about how people learn. It is based on observation and scientific study. According to Brooks and Brooks (1999), constructivism is a theory about knowledge and learning. It is a theory that says individuals construct their own knowledge as they interact with their environment and experiences. Knowledge is built up on existing knowledge and one's experiences are vital in the process. Learners thus create their own understanding by synthesising new experiences into their prior knowledge. In the process, their mind may either accept the new concept if it is in line with what they already know, or it may struggle to accept the new idea. This new idea can later be assimilated if it is compatible with prior knowledge. It is possible too that the new idea is completely thrown away if it fails to link with what the learner already knows. In such an instance no new learning will have taken place.

There are different types of constructivism. There is cognitive, social and radical constructivism to list only a few. These versions of constructivism have similarities and differences. They are similar in that they all agree that teachers should use inquiry methods to facilitate learning and that learners create their own knowledge. An inquiring environment must be created in the classroom and the role of the teacher must be that of a facilitator. It is this kind of environment that is advocated for teaching geometry (NSC diagnostic 2015). For the purposes of this study I have considered to embrace cognitive constructivism and social constructivism.

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Cognitive constructivism is rooted in Piaget's theory whereas social constructivism was founded by Vygotsky. According to Powell and Kalina (2009), the two theories differ in their positions on language development. For cognitive constructivists thinking precedes language whereas for social constructivists language precedes thinking. Each of these is described in more detail in the sections that follow.

2.10.1.1   Cognitive constructivism

Cognitive constructivism is rooted in the theory of Piaget which deals with human adaptation of knowledge and the impossibility of knowing the real world outside our experiences (Von Glasersfeld, 1994). It proposes that knowledge is used to organise experience. As explained by Phillips and Soltis (2004:7), "Piaget viewed learning as an adaptive function of an organism". This means that an organism develops techniques for dealing with and understanding the environment. For Piaget, "learning is an individual's construction and modification of structures for dealing successfully with the world". In this paradigm, it is of paramount importance to have classrooms and activities that support the individual construction of knowledge.

It is my supposition that technology through cooperative learning will facilitate the process of knowledge construction as learners use simulation in learning. By simulation, learners are able to visualise the concept they must learn and construct their own learning in the process.

According to Piaget, human beings cannot be given new information, which they can readily understand and use. They must process the information and construct their own knowledge from it (Piaget,1953). The idea here is that knowledge is built through assimilation and accommodation as humans progress through the four developmental stages Piaget has described. These stages are the sensorimotor, the preoperational, the concrete operational and the formal operational stages.

As children move from one stage to another, the process of assimilation and accommodation is accompanied by cognitive conflict as they try to make meaning of new information. This cognitive conflict will lead to a state of disequilibrium or a state of being uncomfortable as they try to adjust their thinking to fit the new concepts. This process forms the basis of cognitive constructivism. Learners are responsible for making their own knowledge and ultimately are experts in their own learning.

The simulation facility in technology and the collaboration in cooperative learning will certainly help in restoring equilibrium and hence the construction of knowledge.

2.10.1.2   Social constructivism

For social constructivism, knowledge is found in linguistic texts, documents and other articles, which can be communicated on platforms like discussions and lectures. Meaning is achieved through the efforts of the people concerned. Hence social constructivism is about social relationships and meaning is embedded in context. Psychologist Lev Vygotsky (1978) is considered to be the main contributor to the theory of social constructivism. He argued that knowledge is constructed by learners as they attempt to make sense of their experiences.

There is subjective meaning in how individuals interpret the language they read or hear so it is vital for educators to know that the experiential reality of their learners is not the same as theirs (Von Glasersfeld, 1994). Within this reality social interactions are accompanied by personal critical thinking that brings about learning to an individual.

It is my view that Vygotsky's social constructivism requires a classroom setting where interaction is prominent and encouraged as it would occur when cooperative learning and technology are used collaboratively. In indirect support of this point, Lam and Li (2013) aver that social constructivism is the main theory underpinning cooperative learning.

According to Powell and Kalina (2009), social constructivism is founded on Vygotsky's theory of the zone of proximal development (ZPD). Vygotsky describes this as the zone where learning happens with assistance. In other words, ZPD refers to "the discrepancy between a child's actual mental age and the level a child could reach with assistance through a cognitive experience" (Biggie & Shermis, 2004 cited in Powell and Kalina, 2009: 247). Assistance is brought about through interaction with the teacher and with fellow learners. This assistance must be at a level that the student can handle to facilitate learning. As students learn with assistance, their ZPD grows and allows them to learn further. Students will start an activity on their own first, and with the assistance of the teacher later, they will be able to learn new concepts based on what they were initially doing individually.

This reinforces the point that cooperative learning is vital for a constructivist classroom. Scaffolding is also an assisted learning process that allows learners to learn more effectively when there is support from others. According to Vygotsky (1978), students must learn from each other and in the process create own knowledge. Even if a task is completed by a group,

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internalisation of knowledge occurs at a different pace for each learner and is dependent on their different experiences.

The internalisation of knowledge is hence, more rapid where there is social interaction (Powell & Kalina, 2009). Vygotsky believes that social interaction and background have a large bearing on the learning of students as they influence one another's life experiences. The educator must therefore take into consideration this diversity of backgrounds when planning their lessons. Another concurring perspective on this point is that of Woolfolk (2014) who notes that a student will first have to understand themselves and the people around them before learning can occur.

Cooperative learning stems from the theory of social constructivism. Furthermore, social constructivism places great emphasis on the role of social interaction in the learning process (Vygotsky, 1978). Therefore, this theory links to cooperative learning as learners from various backgrounds, with different skills and abilities need to work together to generate knowledge which in turn solves a problem. As far back as 1938, Dewey (1938) suggested that knowledge occurs in situations in which learners have to create meaningful experiences. He also stated that these situations have to be integrated into the social context like in a classroom where students are able to collaborate with each other and construct knowledge together. Additionally, students should be able to learn not only within the classroom but outside of it as well.

The application of social constructivist pedagogies in contemporary formal learning contexts has to increasingly embrace the application of technologies such as computers, the internet and social media applications when teaching the current generation of young students described by Prensky (2001) as digital natives. Moreover, the advancement of technology, has further impacted on the social interaction component of social constructivism. It has changed what it means to create, store, retrieve, access and distribute information. (Desai, Hart & Richards, 2008). Applications such as Facebook, Twitter and WhatsApp as well as educational software, enable the trend of "open content", wherein learners become the producers of their own learning materials (Attwell,2007). As technology becomes integrated into the teaching/learning process, the role of the classroom teacher changes. Classroom teachers become facilitators who assist students in constructing their own understanding and capabilities as they learn through technology (Atwell, 2007)

The next section deals with the cognitive levels in the South African curriculum.

2.10.2　　　　　Cognitive levels in Mathematics

The cognitive levels spell out the level of difficulty at which questions must be pitched in teaching and assessments in the NCS curriculum. After going through the literature on the van Hiele levels of geometric thinking, it became important for the researcher to re-examine the cognitive level of thinking required by the curriculum in mathematics. These cognitive levels in CAPS replaced what had been the learning outcomes in the previous NCS Grades R-12 (DoE, 2011).

Interestingly the CAPS clarifications for mathematics classify mathematics concepts on four levels. These levels, in order of difficulty are: knowledge (K), routine procedure (R), complex procedure (C) and problem solving (P). Knowledge procedures are the less demanding ones; while problem solving demands much of learners (DoE, 2011).

The Department of Basic Education Examination Guidelines (DBE, 2017) for mathematics provides the scope and depth of the content to be tested at grade 12 NSC examinations. These examination guidelines clarify expectations at each of the four cognitive levels and give an approximate percentage distribution of marks for each level in mathematics examinations. It is therefore necessary that examination guidelines for mathematics "must be read in conjunction with The NCS Curriculum and Assessment Policy Statement (CAPS)" (DBE, 2017:3)

2.10.2.1　　　　Knowledge level (K)

According to the DBE (2017) for mathematics, at knowledge level (K) learners are expected to be able to simply recall a formula. An example of such a formula is Pythagoras' theorem, commonly used in Euclidean geometry. At this cognitive level, learners are expected to use estimation, and to round off to a certain number of decimal places. The ability to use correct mathematical vocabulary and reading values directly from diagrams is also tested at this level.

In circle geometry this level will include learners' capacity to state learnt theorems and to write acceptable reasons as they solve riders deductively. Learners must be able to write the acceptable reasons, since not all reasons are acceptable. The DBE (2017) for mathematics stipulates the reasons that are acceptable in examinations. The questions on knowledge level in every assessment must add up to about 20 percent of the total paper. This is to cater for learners of different academic abilities. The following is an example and justification for a question under knowledge level.

<u>Question 8.1.1, DBE Mathematics 2015, Paper 2</u>

Complete the following statement:

The angle subtended by a chord at the centre of a circle… the angle subtended by the same chord at the circumference of the circle.

<u>Justification for the cognitive level.</u>

This is a knowledge question as requires straight recall of a previously learnt definition (SIR Unit, 2018).

2.10.2.2   Routine procedure (R)

The routine procedure (R) is when learners are expected to follow a few prescribed steps on working out a solution to a problem. These prescribed steps for solving riders include using theorems and their converses, where these exist. Examples of theorems to be used in solving riders include, but not limited to the theorems in the list that follows.

- An angle subtended at the centre by a chord is two times the angle subtended by the same chord at the circumference.
- The line drawn from the centre, perpendicular to the chord, bisects the chord.
- The angle between a tangent and chord is equal to the angle subtended by that chord at the circumference in the alternate segment.

The riders that fall under routine procedure use one or two theorems at a time and the learner is required to identify and apply the relevant theorems to solve it. Under routine procedure competence is required in several ways. These include simple application of mathematical concepts involving a few steps, and the identification of the necessary formula from the information sheet where the subject of the formula needs to be changed. The problems grouped under routine procedures are almost similar to those encountered every day in classes. Learners are expected to prove prescribed theorems and derive formulae.

Routine procedures must have a weighting of 35 percent in assessments as prescribed by the examination guidelines (DBE, 2017). The following is an example and justification of a question under routine level.

<u>Question 8.1.2, DBE Mathematics 2015, Paper 2</u>

In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.
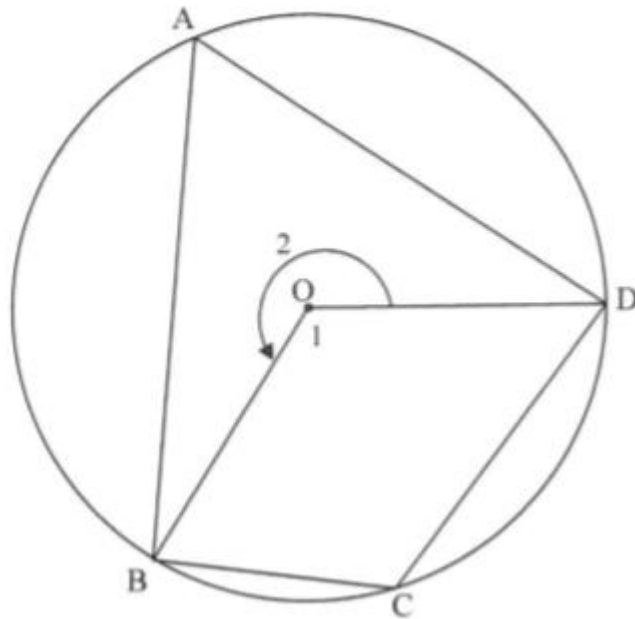
Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180°$.

<u>Justification for the cognitive level.</u>

This is a routine question as requires proof of a previously learnt theorem. The proof of the theorem follows prescribed steps that learners have learnt in class with their teacher. Most prescribed text books also have this proof with the same steps that learners must just understand and be able to follow in assessments (SIR Unit, 2018).

2.10.2.3   Complex procedures (C)

Complex procedures (C) require a deeper understanding and a capacity on the part of the learner to connect concepts. This cognitive level involves complex calculations and reasoning at a higher level. It will involve unseen problems that are not very obvious and real to the learners. There would be no single obvious way to get to a solution. Different topics and concepts need to be integrated to find the way to a solution as stated in the examination guidelines (DBE, 2017). An example of this is a question where one can end up getting different unknowns by applying different circle theorems. A question might require learners to use two or more theorems in solving a problem as illustrated here:

- Tan-chord theorem first;
- then angle at centre $= 2 \times \angle$ at circumference;
- and finally, get the other solution by $\angle s\ opposite = sides$.

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An example of a question that falls into this category is the following question 10.2. from the NSC examination for March 2017:

<u>Question 10.2, DBE Mathematics 2017, Paper 2</u>

In the diagram, O is the centre of circle and P, Q, S and R are points on the circle. $PQ = QS$ and $Q\hat{R}S = y$.

The tangent P meets SQ produced at T. OQ intersects PS at A.
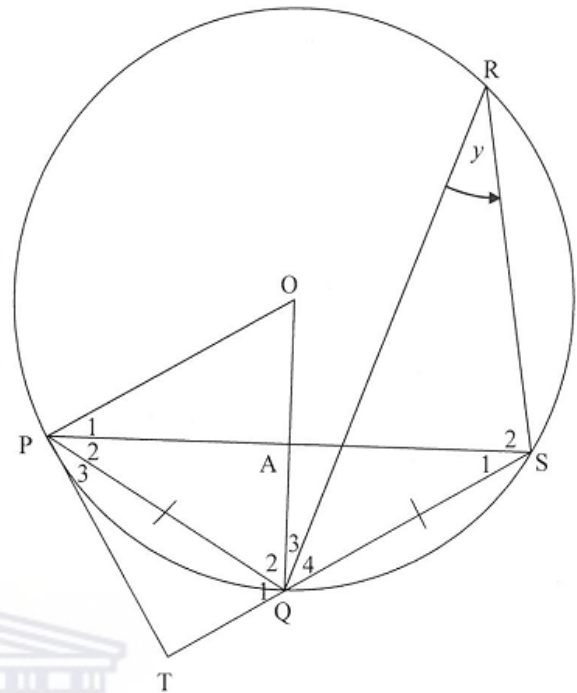
10.2.1 Give a reason why $\hat{P}_2 = y$.   (1)

10.2.2 Prove that PQ bisects $T\hat{P}S$.   (4)

Question taken from (DBE, 2017)

<u>Solution</u>

10.2.1	$\angle s$ in same segment	✓R
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ ….$\angle s\ opp\ equal\ sides$	✓S✓R
	$\hat{S}_1 = \hat{P}_3 = y$ … …… tan-chord theorem	✓S✓R
	$\therefore \hat{P}_2 = \hat{P}_3 = y$	
	$\therefore PQ\ bisects\ T\hat{P}S$	

<u>Justification for the cognitive level.</u>

In the above scenario, the final solution cannot be found if the other pre-requisite solutions are not found. Learners need to have a strategy of showing that a line bisects an angle. They must use theorems to show that the required angles are equal. This introduces the idea of complex procedure to the solution. The process will have a weighting of 30% in assessments. According

to CAPS (DBE, 2011), most examples given under complex procedure involve proving that two lines are parallel, and solving riders requiring more than one theorem.

2.10.2.4 Problem solving

The last and most demanding cognitive level is problem solving. This level involves non-routine problems that are mainly unfamiliar and not necessarily very difficult as the reasoning required is at a higher level. Learners are expected to break the question into different parts making it possible to come up with the solution.

An example of problem solving in circle geometry is when one is asked to prove that a line is a tangent to a circle.

To do this:

- A learner must know what a tangent to a circle is and know the theorems about tangents.
- A learner must also know what must be shown to prove that the line is a tangent.
- A learner must realize that this problem is broken into small pieces of finding necessary angles.

The following question, which is an extension of the question in section 2.6.3 illustrates problem solving.

Question

10.2.4 Prove that PT is a tangent to a circle that passes through points P, O and A. (4)

Solution

| 10.2.4 | $P\hat{O}Q = 2\hat{S}_1 = 2y \ldots \ldots \angle at\ centre = 2 \times$ $\angle at\ circumference$ $T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$…proved in 10.2.2 above $\therefore T\hat{P}A = P\hat{O}Q$ $\therefore PT = tangent \ldots. converse\ of\ tan$ $- chord\ theorem$ | ✓S✓R $T\hat{P}A = P\hat{O}Q$✓ ✓R |

<u>Justification for the cognitive level.</u>

To prove that the line is a tangent, some angles that were not asked about had to be found for the learner to be able to come to a conclusion. The angle in the first line of the solution had to be found first. Once the angles are found, the learner must give an acceptable reason why he/she has arrived at a particular conclusion.

According to CAPS (DBE, 2011), certain problems given under problem solving require learners to prove that:

- A line is a tangent.
- Two line segments are equal.
- A quadrilateral is a parallelogram.

Problem solving must have a weighting of 15% in assessments (DBE, 2017).

For the purpose of this study, the researcher used a lower weighting on knowledge questions because in examinations, most questions on circle geometry are from level two upwards.

In this literature review much reading was done in order to gain a deeper understanding of the concepts in geometry relating to cooperative learning and the use of technology in the classroom. The literature review in this chapter has covered all the information that the researcher gathered about the topic under study.

The next chapter deals with the research methodology in which a number of concepts relevant to the chapter will be discussed.

CHAPTER 3: RESEARCH METHODOLOGY

3.0   Introduction

In the earlier sections of this thesis, studies by different researchers were described and analysed to highlight similarities and differences in their findings. The theoretical framework was also presented, and an explanation was given as to why social and cognitive constructivism are relevant to this study. This chapter mainly looks at how the data for the study was collected, and the methods of data analysis

Research methodology thus refers to the paradigm, research approach, design, method of data collection and the analysis of data that will be used (Kuhn, 1977). The various sections of Chapter 3 on methodology will be defined accordingly. By reviewing relevant literature the researcher was able to identify and discuss research designs such as quasi-experimental as was used by Tieng and Eu (2013) and Bhagat and Chang (2015) in similar studies for the purpose of this study. The manner by which data is collected and analysed has a bearing on the conclusion reached. This is why it was of paramount importance that this research be conducted in the best possible way to come up with reliable, trustworthy and credible results.

The requirement for accuracy in this section of the thesis was also inspired by scholars like De Vos, Strydom, Fouche and Delport (2005), who advise that the research methodology must be described fully to give confidence to readers that the research was carried out using the right methods. They propose that to this end the purpose for which the data was collected, the limitations and research design must be clearly defined. These references helped the researcher to gain a deeper understanding of what is expected in a good study.

3.1   Research paradigm

According to Kuhn (1977), paradigm refers to a research culture, which incorporates a set of beliefs, values, and assumptions that a community of research has in common regarding the nature and conduct of research. It is the philosophy followed in a given research project. It sets the limits to what the researcher can and cannot do in the process of the research.

In this research, the positivist paradigm was selected. A positivist paradigm helps to ensure that the research is performed in a scientific way, in other words that the researcher remain neutral and objective in uncovering truthfully what it is they set out to ascertain. This implies the need for proof and evidence to support a claim. The researcher maintained objectivity by presenting

the lessons to both groups in ways that would not compromise the results obtained. The researcher in this study, thus, emphasised rigor, validity, reliability and objectivity as explained in later sections of this chapter.

Positivists believe that by using an instrument a single truth can be measured (Babbie & Mouton, 1998). In this study, the researcher tried to uncover a truth in a scientific way, while allowing for the possibility of a generalisation of the results. The researcher was thus objective to avoid influencing the results in any way.

This paradigm was chosen because the aim of the research was to gather objective information and then to describe as well as to explain the impact of using technology and cooperative learning in the teaching of grade 11 circle geometry. This research used experimental-control with a quantitative approach.

According to Babbie and Mouton (1998) the quantitative researcher believes that the best way of measuring properties of a phenomenon can be achieved by using quantitative measurement only. Hence only numbers were used in all measurements, while statistical methods made it possible to analyse and make meaning of the collected data as the researcher tried to establish relationships between variables.

3.2   Research approach

There are three main types of approaches in research. These are quantitative, qualitative, mixed methods. For this study the researcher used the convergent parallel mixed method approach in the collection of data, with a greater use of the quantitative methods of data collection and analysis. Cresswell (2014:15) defines convergent parallel mixed methods as follows: "…is a form of mixed methods design in which the researcher converges or merges quantitative and qualitative data in order to provide a comprehensive analysis of the research problem. In the design, the investigator typically collects both forms of data roughly at the same time and then integrates the information in the interpretation of the overall results. Contradictions or incongruent findings are explained or further probed in the design".

The quantitative leg of the mixed methods approach allows for measuring and analysing causal relationships. Using the quantitative lens, the researcher is objective, value free and carries out the research from an outsider perspective (Babbie & Mouton, 1998). Embracing the quantitative leg of the mixed methods approach was deemed suitable because it required of the researcher that his behaviour and attitude not affect the outcome of the study in any way.

Hypotheses and statistical tests were used to find whether a relationship existed between the causal and dependent variables. Data was collected scientifically and was captured in numerical form to facilitate reliable statistical analysis. The tests and questionnaire were used as the instruments by which to gather the data. The questionnaire was used to gather the qualitative information required to answer research question 2. Questions from the questionnaire that were checking understanding and motivation to learn collected qualitative data that was later converted to quantitative data for the purpose of analysis.

Table 3: Summary of research approach for this study

Aspects of the research	Research approach used
Research paradigm	Positivism and interpretivism
Theoretical Framework	Constructivism Cognitive levels (collapsed Blooms)
Research design	Quasi-experimental
Collection of data	Tests Questionnaires
Interpretation of data	t- Distribution, analysis of questionnaires and qualitative content analysis
Trustworthiness	Triangulation with tests and questionnaire
Ethical issues	Informed consent Assent, confidentiality

For the other qualitative part of the mixed methods approach, the researcher used the qualitative content analysis to learners' responses to the pre-test and post- test items. Content analysis is a process of looking at data from different angles with view to identifying keys in the text that will help us understand and interpret the raw data. According to Zhang and Wildemuth (2009:319), qualitative content analysis 'pays attention to unique themes that illustrate the range of meanings of the phenomenon rather than the statistical significance of the particular texts or concepts. Qualitative content analysis can be conducted inductively or deductively. Deductive analysis is where one moves from general theories forming part of the theoretical framework to explain particular observations in the research. The reverse is true for inductive analysis. In this study, the researcher used inductive analysis where raw data was collected and

analysed to identify common errors from learners as they were answering the pre-test and post-test. Table 3 summarises the research approach used in this study.

3.3 Research design

Research design is simply the logic or the structure of a research project, which shows how the study will be conducted. The researcher used quasi-experimental design, replicating the design of another study of the same nature conducted by Bhagat and Chang (2015). Shadaan and Kwan Eu (2013) and Rutten et al. (2012) also used quasi-experimental design in research which was aimed at investigating the effectiveness or impact of a particular approach.

The researcher opted for purposive sampling, which enables sampling to be done in a non-random manner. The sampling style was good enough to provide the required information for the study and was more practical in a school situation where classes could be chosen to belong to a certain group. The sample comprised of two of the four grade 11 classes.

As stated earlier, the control group was not given any additional support during the intervention period with the experimental group. The experimental class was thus taught using technology and cooperative learning, and the control group was taught using the conventional method of chalk and talk by the teacher. The intervention with the experimental group took place in the second and third week of September 2018 during the mathematics class periods at Khumbulani High School.

In this study technology through cooperative learning meant that learners had individual computers in doing their group work. They were required to work on a task as individuals and then come together to complete the group task. Any rewards were for the group's achievement (Lam & Li, 2013). As Bertram and Christiansen (2014) put it, the researcher tried to keep all other factors that could affect results as constant as possible. The experimental research style was used to establish whether any causal relationships existed.

In having chosen quasi-experimental design, the researcher took into account the shortfalls it presents when compared to randomised sampling. Quasi-experimental design was however very convenient in a school situation where there were two classes that were almost equal in most aspects as shown by the pre-tests results in Table 3.3 and 3.4.

This research was aimed at finding out if the independent variable namely, technology through cooperative learning, has any influence on the dependent variable namely, learner performance on grade 11 circle geometry.

3.4 Sample and sampling strategy

According to Bertram and Christiansen (2014:59), "sampling involves making decisions about which people, settings, events or behaviours to include in the study. Researchers need to decide how many individuals, groups or objects will be observed".

It is thus important to choose a sample that is representative of the population. 'Representative' is a concept considered important in the positivist paradigm used in this study. Its aptness for this study will be explained later.

Also, the size of the sample is equally important. The sample for this study was made up of 53 learners with 27 in the experimental group and 26 in the control group. This sample size was good enough to allow for the analysis of data using statistical methods. According to Bertram and Christiansen (2014:63), "a sample size of 30 is the minimum of cases" if statistical analysis of data is to be used.

The composition of the sample is shown in Table 3.1

Table 3.1: Composition of sample

Gender composition	Group of students	Number in group	Percentages
12 boys, 15 girls	Experimental	27	50.9%
12 boys, 14 girls	Control	26	49.1%

The sample was made up of the classes that were being taught by the researcher, namely 11E and 11C where E and C are pseudonyms. The two classes were almost equivalent in features like gender distribution and academic performance. The equivalence of the academic performance of the learners in 11E and 11C was shown by their comparable performance in the June examination results. Class 11E was the experimental group and class 11C was the control group. These two classes were given a pre-test to verify the equivalence of their

academic performance in assessments. Assigning classes 11E and 11C to the experimental (E) and control(C) groups respectively was done using purposive sampling.

Purposive sampling refers to sampling where the researcher selects specific elements for the sample. This sample was chosen because the researcher felt it was representative of the population at Khumbulani High School as it represented two out of the four grade 11 classes at the school, with a total of 115 learners. According to Bertram and Christiansen (2014: 61) "a case can be chosen because it is considered to be representative for the population."

Previous studies have used similar techniques and almost the same number of participants in their samples. A similar research project had the experimental group and control group made up of 16 and 15 learners respectively (Tieng & Eu, 2013). Another study using quasi-experimental design was carried out by Bhagat and Chang (2015) with a sample of only 50 students in which the experimental and control groups were made up of 25 learners each. I therefore found it meaningful to conduct my research with sample of 53 learners with composition shown in Table 3.1.

3.5   Research setting
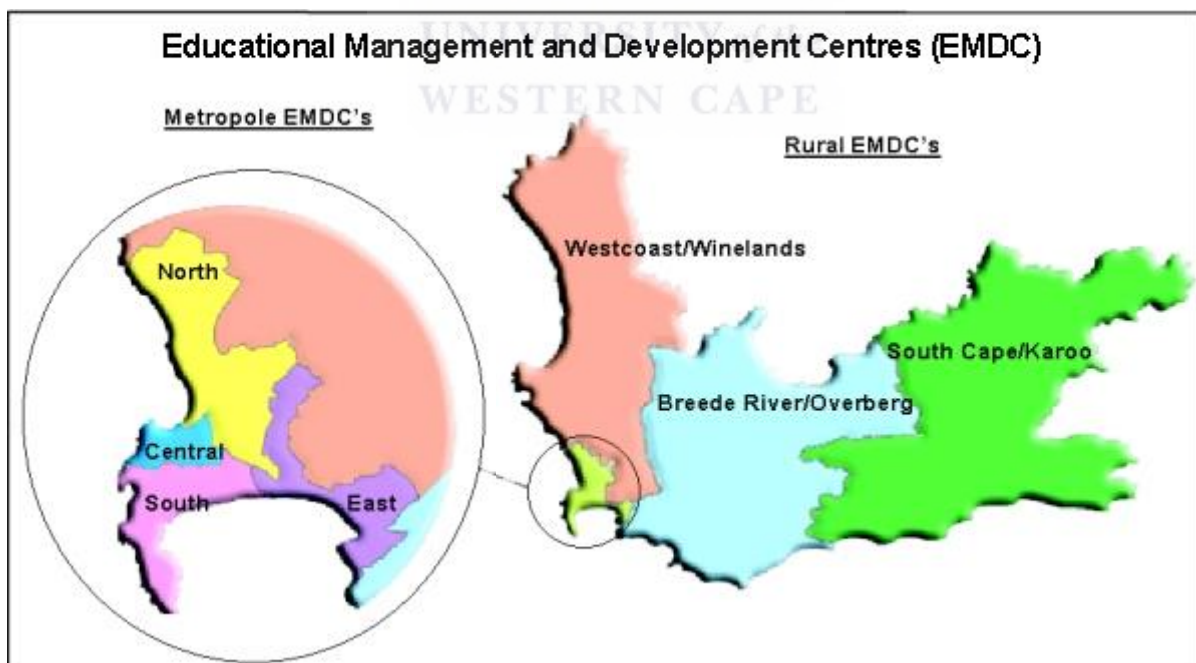
Figure 3.1: Educational Management and Development Centres

(Source: https://wcedonline.westerncape.gov.za/branchIDC/introemdc.html)

The study took place at Khumbulani High School (not its real name) in the Western Cape. Khumbulani High School is a quintile three school which falls under the Metropole East District of the Western Cape Education Department. This school was chosen because it was where the researcher was teaching and it became easy to conduct the study at the school. The school was also chosen for financial reasons as no travelling costs were involved, thus making it easy for the researcher to carry out a thorough study.

Figure 3.1 provides an overview of the location of the Educational Management and Development Centres (EMDCs) in the Western Cape Province. On the map, Khumbulani High School falls in the East District. All the learners in this township had IsiXhosa as their home language with English as the first additional language of the school as well as the language of teaching and learning.

This school has enjoyed very good pass rates in the NSC examinations with an average of above 90 percent since its inception about 20 years ago. The pass rate for mathematics at the school was equally good averaging above 90 percent. The staff establishment of the school is not particularly large; there were about 22 teachers in total for the enrolment of 560 learners. The mathematics department consisted of 5 teachers of which 3 were teaching the subject full time. All learners at the school were doing mathematics as there was no option to do mathematical literacy since this was a mathematics and science focus-school. The teachers in the mathematics department all had degrees and were appropriately qualified to teach the subject.

In terms of infrastructure, Khumbulani High School was better equipped than several schools in the township. The school has fully equipped computer laboratories for Information Technology. The school was also privileged to have the Western Cape Government Schools internet connection. However, this internet link was very slow and unreliable, making teaching and learning with technology difficult. Getting access to computer laboratories was very difficult for other subjects as the laboratories were meant only for Information Technology.

3.6   Pilot test results

The instruments for the research had to be tested using a different sample, to check whether they were of the standard required to collect the information the researcher intends to get. According to Bertram and Christiansen (2014: 49), the pilot study is normally done on a smaller

group "to see if the questions are understood in the intended way" and to identify the weaknesses and the strengths in them.

For the tests administered, it was good to have the trial run to check the cognitive level of demand and suitability to the grade 11 learners. This also served to uncover any ambiguity in the questioning techniques. The pilot tests also helped to gauge the suitability of questions used in the questionnaires.

The researcher therefore conducted a pilot study at a school in the same locality which is similar to Khumbulani High School in a number of respects. The purpose was to check the suitability of the questionnaire and tests used to gather data for this study. Like Khumbulani High School, that school also has computer laboratories with internet access provided by the Education Department for Western Cape Government Schools.

The pilot studies on my pre-test and post-test revealed that the tests were a bit long for the allocated time. I had to reduce the length of the test and distribute the marks accordingly on the remaining questions so that the tests remained out of 50 marks. The questionnaire had to be trimmed again and the number of open-ended questions had to be reduced from 4 to 2. Questions that had been intentionally asked in two different ways to check for consistency in answering were reduced.

The next section that follows explains in detail how the actual data collection took place at Khumbulani High School.

3.7. Data collection

The researcher wanted to be able to answer the research questions on the impact of using technology through cooperative learning on learners' understanding of grade 11 circle geometry. According to Bertram and Christiansen (2014), 'data collection methods' refers to how data or information required to answer the research questions was obtained. As indicated, in this research, triangulation with tests and questionnaires were used. The researcher wanted to collect data that assesses whether intervention given to the experimental group made a difference or not thereby answering research question 1, and for this purpose a pre-test (see Appendix 10a and 10b) and a post-test (Appendix 11a and 11b) were used. The questionnaire (see Appendix 9) was used to collect data to answer research question 2.

To this end it was a requirement to check the equivalence between the abilities of the control (C) group and experimental (E) group to be able to comment on the effectiveness of the treatment given to the E group. The rationale was that it would have been statistically impossible to draw conclusions on the impact of teaching with technology in the context of cooperative learning if the two groups were not certified as equivalent at the start of the study. Figure 3.2 shows confirmation of equivalence between the two groups by using statistical hypothesis testing with data from Microsoft Excel. It was then possible to collect data for the study. Testing was used to check learning that took place during the teaching of geometry for both groups, whist the questionnaire was used only in the case of the experimental group to determine their levels of motivation to learn geometry.

However, every method of information gathering in the sample has its weaknesses (Bertram & Christiansen, 2014). The researcher therefore chose testing and questionnaire as the most effective methods for obtaining information, which could help establish answers to posited research questions. In a related study on the effectiveness of using geogebra, Shadaan et al. (2013) also used testing and a questionnaire. Table 3.2 provides an overview of the data collection methods and accompanying instruments that will be used to collect necessary data that could enable the researcher to answer each research question, and ensure nececessary triangulation of data.

Table 3.2: Triangulation summary

Research Question	Data collection Method	Instrument
Research Question 1 How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 circle geometry?	Pre-test	Test on application of first 4 theorems, converse and corollaries including: • line from centre ⊥ to chord • line from centre to midpt of chord • perp bisector of chord • ∠ at centre = 2 ×∠ at circumference • ∠s in the same seg • equal chords; equal ∠s • ∠s in semi-circle

	Post-test	Test on all 8 theorems, converses and corollaries including: • opp ∠s of cyclic quad • converse opp ∠s of cyclic quad • ext ∠ of cyclic quad • Tans from same pt • tan chord theorem • converse tan chord theorem • converse ext ∠ of cyclic quad
Research Question 2 To what extent, if at all, do learners exhibit changes in motivation to participate in answering grade 11 circle geometry questions when using technology within a cooperative learning context?	Questionnaire	Questionnaire (See Appendix 9)

3.7.1 Pre -testing

The testing method was selected to enable the researcher to check the relationship between the independent and dependent variables. The independent variable was teaching grade 11 Euclidean geometry using technology in the context of cooperative learning. The dependent variables were the impact on academic performance and level of motivation to learn. Testing was used to gather data to answer the first research question was:

- How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 circle geometry?

Tieng and Eu (2013), who did a similar study, used three pre-tests to confirm that their sample comprised of equivalent groups. Bhagat and Chang (2015) also used testing for similar research. With these insights I proceeded to rely on the June results and a pre-test (Appendix 10a) to inform me of whether my two samples were similar or not. The June Mathematics Paper Two and pre-test results for the control and experimental groups are shown in Tables 3.5 and 3.6 respectively.

The pre-test used covered the first four theorems of Euclidean geometry, one converse and a corollary of the fourth theorem. Before the pre-test, both groups were taught the work covered using the conventional method, where the teacher used chalk and talk, question and answer, followed by written work from textbooks. The parts of a circle were explained to the learners. The theorems, and associated converses and corollaries in the list that follows were covered as stipulated in the CAPS document for mathematics.

- The line drawn from the centre of a circle, perpendicular to the chord, bisects the chord (theorem)
- The line drawn from the centre of a circle to the midpoint of a chord, is perpendicular to the chord **(converse of theorem)**.
- The perpendicular bisector of a chord passes through the centre of a circle**(theorem).** The angle subtended by a chord at the centre of a circle is twice the angle subtended by the same chord at the circumference **(theorem)**
- Angles subtended by same chord are equal at the circumference (theorem)
- Equal chords subtend equal angles at circumference (DBE, 2011**) (corollary)**

Learners were taught how to prove the theorems and how to use the theorems in answering the questions. The acceptable reason for each theorem was given. Learners were not given the pre-test before being taught the first four theorems as there was not much to test since the learners had limited knowledge of circle geometry. It therefore made sense for the researcher to give a pre-test covering the concepts of the first four theorems only.

Before the pre-test was given, the June Mathematics Paper 2 results had shown an average of 54.23% and 54.3% for the control and experimental groups respectively. The two groups differed by 0.07%, which was negligible. The Mathematics Paper 2 results were used because

geometry is a topic covered in Mathematics Paper 2 as by prescribed in the examination guidelines (DBE, 2017). The pre-test also showed that the two groups were almost at the same level, with averages of 30.54% and 30.3% for control and experimental groups respectively and a difference of 0.24%. The t-test had to be used to assess whether this difference 0.24% was statistically significant or whether it could be ignored.

The null (H_0) and alternative (H_1) hypothesis were set up. The test statistic chosen was the one where the population standard deviation was not known. The sample standard deviation had to be used instead (Walpole & Myers, 1985). A two-tailed t-distribution test was used with $51(n_1 + n_2 - 2)$ degrees of freedom where n_1 and n_2 are the sample sizes for the control and experimental group respectively. The June and pre-test results for control and experimental groups are shown in Excel Tables 3.5 and 3.6 respectively. Tables 3.3 and 3.4 show cognitive classification of the given pre-test and percentage of mark allocation in pre-test respectively.

Table 3.3 Cognitive classification of questions in pre-test

Question	Cognitive level	Reason	Marks
1.1	Routine Procedure (R)	Simple application and calculations requiring few steps. There are specific steps to follow	(5)
1.2	Routine Procedure (R)	Proof of a prescribed theorem	(5)
1.3	Routine Procedure (R)	Simple application and calculations requiring few steps. There are specific steps to follow	(5)
2.1	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts	(2)
2.2	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts	(2)
2.3	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts	(2)
2.4	Routine Procedure (R)	Simple application and calculations requiring few steps. There are specific steps to follow	(2)
3.1	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts	(1)

3.2.1	Routine Procedure (R)	There is a simple application of mathematical concepts involving few steps	(2)
3.2.2	Routine Procedure (R	There is a simple application of mathematical concepts involving few steps	(2)
3.2.3(a)	Routine Procedure (R	There is a simple application of mathematical concepts involving few steps	(2)
3.2.3(b)	Complex Procedure (C)	Significant connections between different theorems	(4)
4.1	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts	(1)
4.2	Knowledge (K)	Straight recall (or reproduction) of previously learnt facts.	(2)
4.3	Routine Procedure (R	There is a simple application of mathematical concepts involving few steps.	(2)
4.4	Routine Procedure (R	There is a simple application of mathematical concepts involving few steps.	(2)
5.1.1	Knowledge (K)	Simple recall and use of a single theorem	(1)
5.1.2	Knowledge (K)	Simple recall and use of a single theorem	(1)
5.2.1	Routine Procedure (R)	There is simple application of mathematical concepts, involving few steps	(3)
5.2.2	Routine Procedure (R)	There is simple application of mathematical concepts involving few steps	(2)
5.2.3	Routine Procedure (R)	There is simple application of mathematical concepts involving few steps	(2)

Table 3.4: Percentage of mark allocation in pre-test per cognitive level

Cognitive level	Total mark allocation	Percentage mark allocation
Knowledge	12/50	24
Routine	34/50	68
Complex	4/50	8
Problem solving	0/50	0

Table 3.5: Control group marks from Microsoft Excel

	CONTROL GROUP: Gr 11	2018	2018
		June Paper 2	Pre-test
	Name/Possible mark	100	50
1	C1	62	18
2	C2	58	5
3	C3	68	23
4	C4	42	23
5	C5	44	11
6	C6	66	4
7	C7	76	24
8	C8	69	26
9	C9	37	16
10	C10	82	25
11	C11	65	14
12	C12	59	18
13	C13	28	3
14	C14	24	6
15	C15	52	5
16	C16	70	13
17	C17	84	27
18	C18	62	22
19	C19	48	5
20	C20	38	11
21	C21	48	17
22	C22	41	17
23	C23	22	2
24	C24	35	15
25	C25	40	20
26	C26	90	27
	AVERAGE	54,231	15,27

Table 3.6: Experimental group marks from Microsoft Excel

	EXPERIMENTAL GROUP: Gr 11	Paper 2	pre-test
	Task		
	Name/ Possible mark	100	50
1	E1	47	15
2	E2	48	14
3	E3	71	22
4	E4	44	5
5	E5	44	0
6	E6	72	20
7	E7	61	9
8	E8	28	19
9	E9	13	7
10	E10	55	19
11	E11	40	25
12	E12	73	18
13	E13	76	17
14	E14	34	15
15	E15	76	20
16	E16	49	11
17	E17	64	22
18	E18	16	21
19	E19	74	26
20	E20	54	14
21	E21	57	13
22	E22	73	14
23	E23	54	19
24	E24	59	6
25	E25	86	21
26	E26	70	10
27	E27	28	7
	AVERAGE	54,3	15,15

The t-test was used to check whether the mean for the control and experimental group were statistically not different and would allow the two groups to be comparable in the study. The calculations for the t-test used are set out as follows in Figure 3.2:

The null (H_0) and alternative (H_1) hypotheses were set as follows:

H_0: the means are equal: $\overline{x_1} = \overline{x_2}$

H_1: Experimental group mean $\neq$ Control group mean: $\overline{x_1} \neq \overline{x_2}$

The test statistic used was $t = \dfrac{\overline{x_1}-\overline{x_2}}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}\left(\dfrac{\alpha}{2}\right)$

where $s_p = \sqrt{\dfrac{(n_1-1){S_1}^2+(n_2-1){S_2}^2}{n_1+n_2-2}}$ and ${S_1}^2 = \sum_{i=1}^{n}\left(\dfrac{(x_i-\bar{x})^2}{n-1}\right)$ is the sample

standard deviation since the population standard deviation is not known(Walpole and Myers,1985).

Criteria: reject H_0 if $t_{calculated} > 1.997$

Now, $\overline{x_1} = 15.27$; $\overline{x_2} = 15.15$; ${S_2}^2 = 43.05$; ${S_1}^2 = 66.76$; $n_1 = 26$; $n_2 = 27$

$$s_p = \sqrt{\dfrac{(26-1)66.76+(27-1)43.05}{27+26-2}} = 7.39$$

$$\therefore t = \dfrac{\overline{x_1}-\overline{x_2}}{s_p\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}} \sim t_{(n_1+n_2-2)}\left(\dfrac{\alpha}{2}\right.$$

$$= \dfrac{15.27-15.15}{7.39\sqrt{\dfrac{1}{26}+\dfrac{1}{27}}} \sim t_{51)}0.25 = 0.06$$

Figure 3.2: Hypothesis testing on the difference between the pre-test means

Decision:

Since $t_{calculated} = 0.06 < 1.997 = t_{table}$, the researcher failed to reject H_0 and concluded that at 5% level of significance, the two means do not differ significantly. Therefore, the two groups were comparable and it was now possible to give the intervention to the E group as explained in the next section.

3.7.2   Post- Testing

After establishing that the two samples were similar, the control group, 11C was taught using the conventional teaching method whereas the experimental group, 11E was taught using technology through cooperative learning. Both groups were taught the next four theorems, converses and corollaries on cyclic quadrilaterals and tangents of which some are outlined in the list that follows:

- Opposite angles of a cyclic quadrilateral are supplementary.

- The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

- The angle between a tangent and a chord to a circle is equal to the angle made by that chord at the circumference in the alternate segment.

- Tangents from a common point to a circle are equal in length.

Learners in the experimental group were given some group task. Each learner had a tablet and would go to the internet to access the online version of Heymath. This version would allow simulations (see Figures 3.3 to 3.6 in this chapter) to take place in the learning process. Learners would be exposed to online assessments with solutions. There were simulations available for all the theorems, converses and corollaries covered in this research as shown in Figure 3.4. The group would share the given task amongst themselves and later discuss observations and what was learnt from the different activities. This was done particularly to promote student engagement as advocated by (Preciado Babb, Saar, Marcotte, Brandon, & Friesen, 2013). This was identified as positive behaviour for learning at high school.

After sharing observations from Heymath in their groups, learners would move on to answer some online practice questions, still as a group. Learners were also given some informal tasks that they had to answer as a group to bring in the concept of cooperative learning. Marks were awarded to the group depending on each individual's understanding of the concept, using the rubric in Table 3.7. The marks obtained from group work using the rubric were used as part of the mid-term reports for learners. Group members received the same mark for the group task. This was done to bring in the concept of accountability that each member must have to the group's performance, as advocated by Slavin (2010). In the formal tests individual work was allocated to each participant. Table 3.7 is an example of the rubric used to award marks for the E groups. This rubric was used only in informal tests and marks obtained were not used anywhere in this research.

The rubric for assessing the group task in experimental group was designed in such a way as to promote some aspects of group work. It was important that every member of the group was contributing and held accountable for the success of the group in general, and for themselves as individuals in particular. The aspects in the first column of the rubric are the ones valued in cooperative learning and, allocating them some points meant that learners were going to consider them as they engaged in their group tasks.

Table 3.7: Rubric for cooperative learning

Concept explanation and individual understanding	1 mark Attempts to explain concepts, but does so wrongly	2 marks Explains concepts, but with some errors; does not fully understand the theorems	3 marks Partially explains theorems and relationships	4 marks Explains concepts fully; understands theorems and connections
Distribution of task to group members	1 mark Some members not given work at all	2 marks Some members have more work than others	3 marks Task equally distributed to all members	
Collaboration within the group	1 mark Learners working as individuals; no consultation	2 marks There is some evidence of collaboration	3 marks Good collaboration	4 marks High level of collaboration
Learner engagement	1 mark Some learners not busy with given task	2 marks Learners not fully engaged with task	3 marks Learners are engaged with the task	4 marks Learners fully engaged with task
Accountability to group	1 mark No evidence of accountability to group; all individual answers wrong	2 marks Some evidence of accountability to the group; light sanctions imposed; some individual answers correct	3 marks Fully accountable to group; harsh sanction imposed; most individual answers correct	
Work ethic	1 mark Negative work ethic	2 marks Positive work ethic		
Total				/20

The rubric shown in Table 3.7, was designed in a way that promotes some aspects of cooperative learning like group work, accountability and collaboration. It was found from the rubric that learners were cooperative in doing group task. Learners became very responsible when they realised that the group activities were to be marked according to the criteria in the rubric. This was done to avoid a situation where few learners would do the work for the entire group. However, these rubric scores were not used directly to answer the research questions.

Learners in the experimental group were each given the username and password to be able to log on to Heymath. Once the lesson had started, the teacher would teach the concept of the day. Soon after, learners would be given an opportunity to discover more by themselves by using the Heymath circle tool. This tool has a dragging facility that allows simulation to take place and this was very helpful for conceptual understanding and improving the learning of students. It is like a game that learners play and learn from in the process. After experimental group made some discoveries with the circle tool, the teacher and the class would prove the theorems deductively using the conventional method of question and answer. The teacher would be leading the discussion for learners to have a mathematical understanding to reinforce the observations made.

For the control group, learners would be taught using the conventional method: the educator would prove the theorems and apply them in solving riders with learners. This group of learners had no opportunity to visualise by animation what the theorems actually mean in reality. The Heymath Circles Tool is shown in Figure 3.3.

Figure 3.3: Heymath Circles Tool

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When learners in the experimental group were working with the Heymath Circles Tool, they would come to the page shown in Figure 3.4 and click any circle to investigate, as shown in the figure. After clicking the theorem, the Circle Tool would allow them to drag the points on the circle enabling discovery learning to take place.
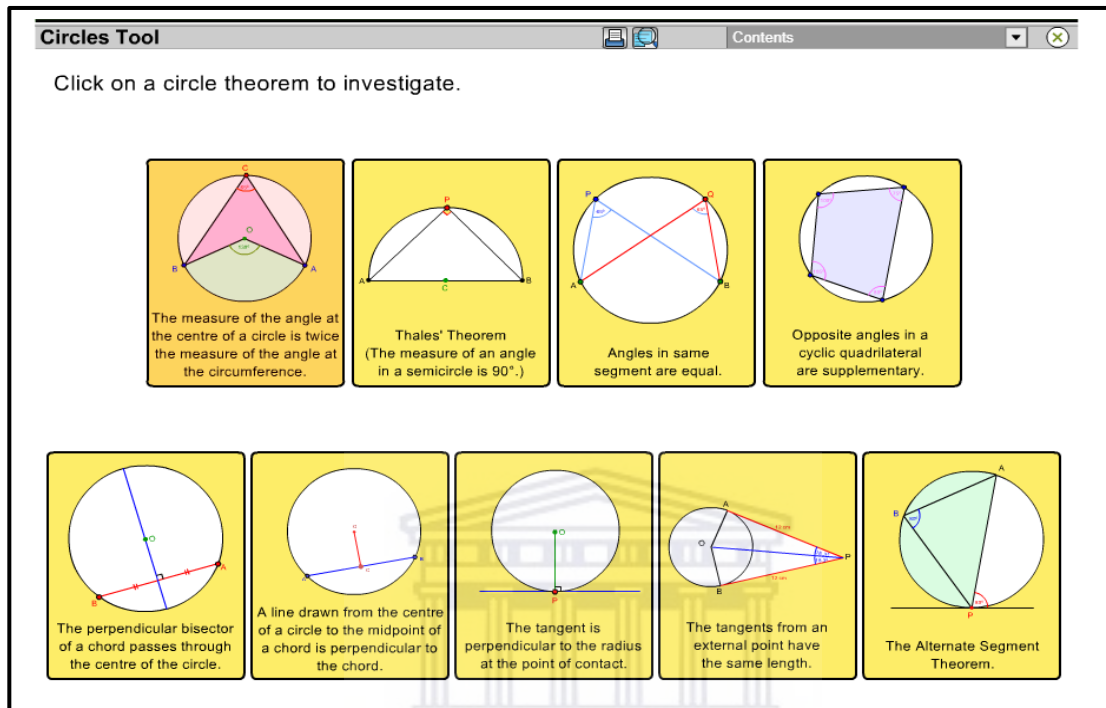
Figure 3.4: Different circle theorems to investigate

Figure 3.5 shows the interface screen that allows dragging on one cyclic quadrilateral theorem.
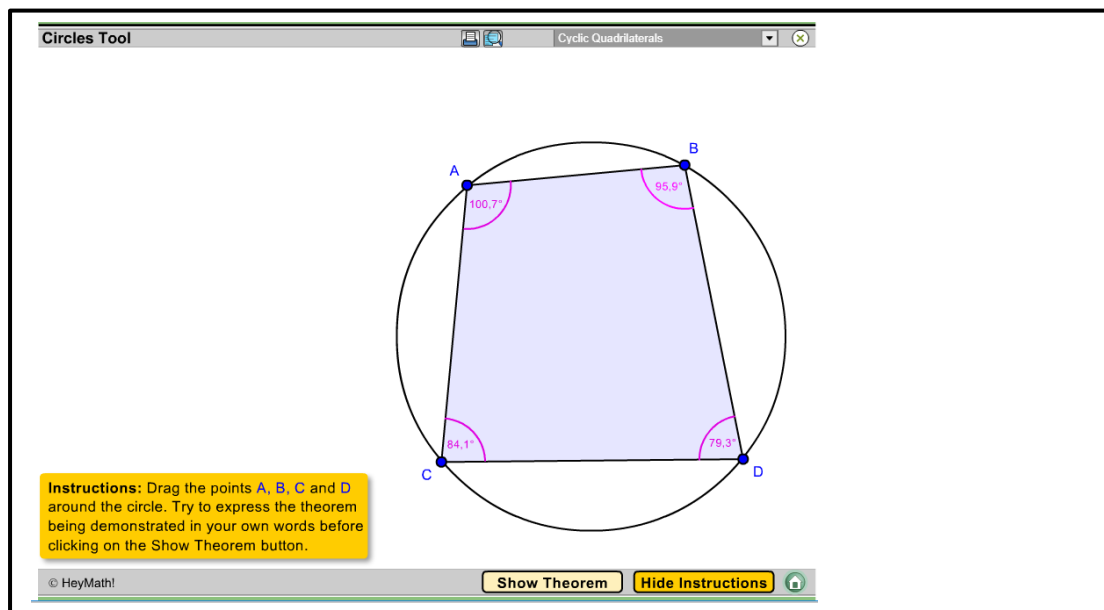
Figure 3.5: Interface screen with dragging facility

Every theorem has its own diagram to drag that could be accessed by clicking the drop-down arrow in Figure 3.5 and choosing the theorem or clicking the theorem from Figure 3.4. By dragging any point, A, B, C or D along the circumference as shown in the instructions, learners could see that opposite angles of a cyclic quadrilateral are supplementary. They could see it no matter which point was dragged. Every theorem on circle geometry could be shown by dragging after clicking the relevant theorem from Figure 3.4. This proved to be a very useful tool that promotes discovery learning. The instructions for every theorem are very clear on the platform and appeared to be very straightforward for learners as they played with the tool and learnt mathematics in the process. There are options to hide instructions or theorems.
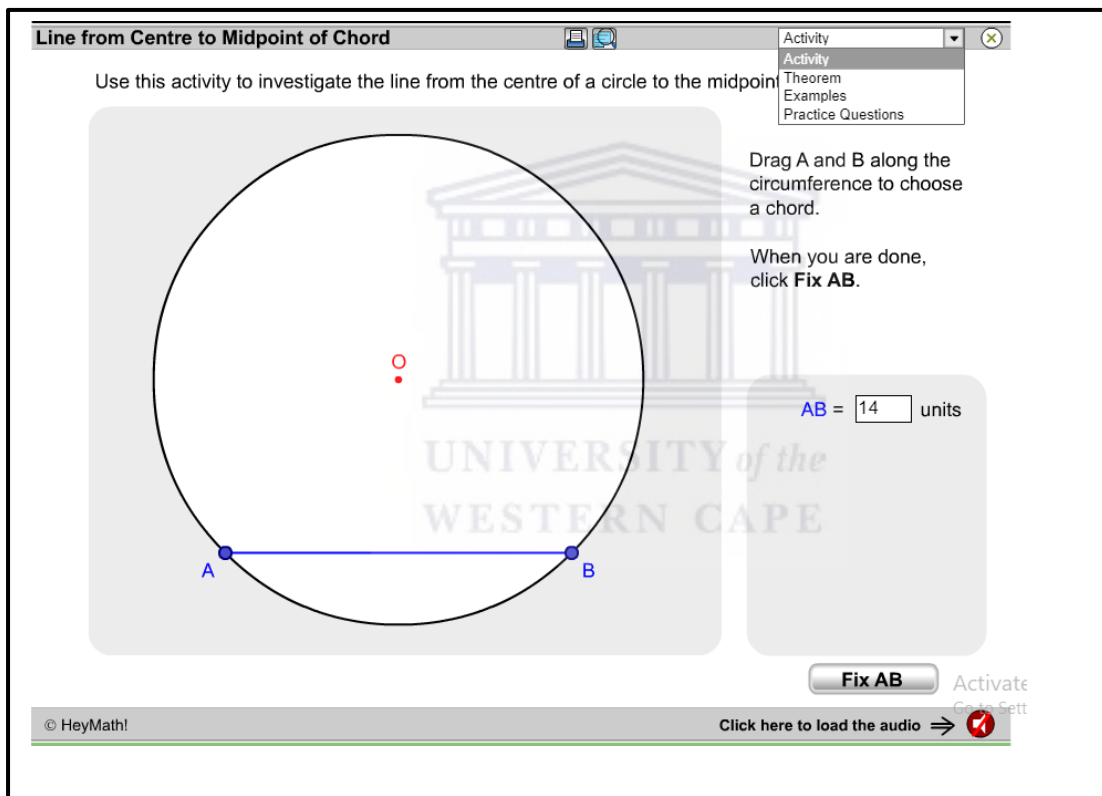
Figure 3.6: Concepts under drop down arrow shown

Figure 3.6 shows the different options found in Heymath. Learners had the option to listen to some audios explaining the different concepts. Learners could also load or remove audio as shown at the bottom of Figure 3.6. They had the option of following the different examples according to how they distributed the task in their group.

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Once learners had completed the examples and activities for a particular theorem, they would find the practice question when they clicked the drop-down arrow. The concepts under the drop-down arrow are clearly shown in Figure 3.6. There are activities, theorems, examples and practice questions. All these were additional, helping learners to broaden their understanding besides the dragging facility that allows for simulations.

The practice questions for the experimental group and the control group were the same. The only difference was that the experimental group's questions were online and interactive as shown in Figure 3.7, whereas the control group was taught using the conventional method and the textbook. The worksheet with practice questions was also part of the learning material used by the control group. The experimental group could click on the question, work it out and then click on the answer button to see the answers as shown in Figure 3.7. There was the option for the experimental group to go to the next or the previous question. This made it easy for experimental group to use technology in the context of cooperative learning. They would divide questions amongst themselves in the group and then discuss solutions.

Figure 3.7: Interactive platform used by experimental group

Clicking the next button allowed them to move from question 1 up to question 43. The print questions option allowed the researcher to print papers for the control group.

After the topic had been completed, learners were given an opportunity to revise their work for the coming post-test. The researcher then gave a post-test (Appendix 11a) to both groups. The test was marked using the same marking memorandum – for consistency in marking. The post-test results were recorded, and the researcher ended up with two sets of data that could be used to test the different set hypotheses. The data obtained was used to test the differences between means as part of analysis of data.

However, this testing method has its weakness. According to Bertram and Christiansen (2014), it is difficult to have the right type of questions in a test. A test that is too difficult or too easy will not measure what it is intended to measure. That meant the researcher would end up with validity problems. To overcome this problem, I gave a pilot run first. I tried out the test with a different group and made adjustments accordingly. There was another question with missing information making it impossible to get a solution. The other question was not clear to learners in terms of wording used. This had to be fixed before the actual research. I also used the required percentages for cognitive levels required in the examination guidelines for tests. This meant that the test set was of the required standard. (see Table 3.3 and 3.9)
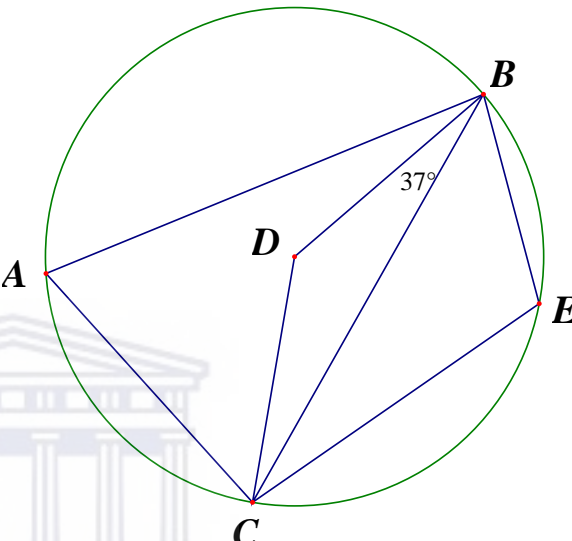
The other weakness is that of the results obtained from multiple choice questions. Some answers in multiple choice questions are found by guessing and this makes it difficult to account for these responses when analysing data. Multiple choice questions were not used in this research because of the stated weaknesses associated with them. On the other hand, open ended questions allow learners to express their thoughts in different ways making it difficult to code these different responses for analysis purposes.

Keeping all other factors constant is also not easy when dealing with human beings. There may be other factors accounting for the results observed which include, but not limited to absenteeism from school. This is an important validity issue in all experimental research. Knowing all the stated weaknesses before the study prepared the researcher to be able to overcome them in devising valid testing instruments.

To have a valid post-test I used the cognitive levels given in examination guidelines (DBE, 2017) and SIR Unit (2018) to set a test that meets the required standard. These guidelines give approximate percentage weightings that must be used in assessment. The knowledge weighting was reduced as most questions under circle geometry will be on level two upwards. Table 3.6

is a summary of levels and weighting of the questions in the post-test. The full discussion is given in Table 3.8 followed by the summary classification into cognitive levels in Table 3.9 and finally Table 3.10 summarises the percentage of mark allocation in the post test per each cognitive level.

Table 3.8: Full discussion on cognitive classification of post-test questions according to SIR unit (2018)

Question 1

D is the centre of a circle.

A, B, E, C are points on the circle.

Given $C\hat{B}D = 37°$, find:

1.1 $B\hat{A}C$ (5)
1.2 $B\hat{E}C$ (2)

Discussion for question 1.1

This is a routine question as it requires simple application of mathematical concepts involving few steps. Learners are expected to recognise the theorem of angle at the centre being twice the angle at the circumfrence. They must combine this theorem with the one on angles opposite equal sides of an isosceles triangle. These are questions that are normally exposed to learners in class and it becomes a routine question.

Memorandum for question 1.1

$BD = DC$ ..... both radii✓

$\hat{C} = 37°$....∠ opposite = sides✓

$B\hat{D}C = 180° - 2 \times 37° = 106°$ ... sum of $∠ \, of \, Δ$ ✓

$\therefore B\hat{A}C = \frac{1}{2} \times 106° = 53°$✓ ...$∠ \, at \, centre = 2 \times ∠ \, at \, circum$✓

Discussion for question 1.2

This is a knowledge question as it requires simple recall and use of a single theorem. Learners are expected to recall that opposite angles of a cyclic quadrilateral are supplementary. This is a theorem that learners would have done in class.

Memorandum for question 1.2

$B\hat{E}C = 180° - 53° = 127°$✓ ... opp ∠ of cyclic quad✓

Question 2

D is the centre of a circle.

F, G, H, I are points on the circle.

$F\hat{D}H = 84°$ and $FH \parallel DI$.

$F\hat{I}H = z$, $H\hat{F}I = x$ and $F\hat{G}H = y$.

Find the size of angles x, y and z.
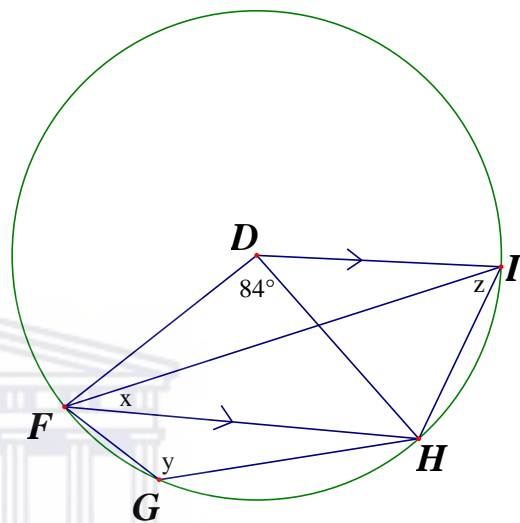
Discussion for question 2

This is a complex question as it requires significant connections between different theorems. Four theorems are used here. It is vital for learners to see the relationship between the angles. Learners must find z using the angle at the centre theorem. They must also use theorems on cyclic quadrilaterals, isosceles triangles and parallel lines to find the other angles. There is no obvious starting point as this can only be seen in working the problem. In this case, it is easy to first find z.

Memorandum for question 2

$F\hat{D}H = 2F\hat{I}H = 2z$ ... ∠ at centre = 2 × ∠ at circum✓

$z = F\hat{I}H = \frac{84°}{2} = 42°$✓

$y = F\hat{G}H = 180° - 42°$...opp ∠ of cyclic quad✓

$\qquad\qquad\qquad = 138°$✓

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To get x

$F\hat{H}D = \frac{180° - 84°}{2} = 48°$....∠ opposite = sides✓

$H\hat{D}I = 48°$ alternating∠, $DI \parallel FH$✓

$H\hat{D}I = 48° = 2 \times x$ ... ∠ at centre = 2 × ∠ at circum✓

$$x = 24°✓$$

Question 3

J is the centre of a circle with diameter NK and tangent ON that touches the circle at point N.

Given $N\hat{L}J = 24°$ and $M\hat{J}L = 56°$, find:

3.1 $N\hat{K}L$ (5)
3.2 $M\hat{N}L$ (2)
3.3 $O\hat{N}M$ (3)

[10]

Discussion for question 3.1

This is a complex question as it requires significant connections between different theorems. Theorems of isosceles triangles, angle in a semi-circle and sum of angles of a triangle must be connected. This requires higher order thinking as there is a single theorem that must be used before applying other theorems. It is not a difficult question though. According to SIR unit (2018), the level of difficulty is independent on the cognitive level of a question.

Memorandum for question 3.1

$L\hat{N}K = 24°$✓ ....∠ opposite = sides

$N\hat{L}K = 90°$✓ ....∠ in a semi circle✓

∴ $N\hat{K}L = 180° - (90° + 24°) = 66°$✓ .... sum of ∠ in Δ✓

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Discussion for question 3.2
This is a knowledge question as it requires simple recall and use of a single theorem. Learners are expected to recall that the angle at the centre is twice the angle at the circumference. This is a theorem that learners must have done in class.

Memorandum for question 3.2
$M\hat{N}L = \frac{1}{2} \times 56° \dots \angle\ at\ centre = 2 \times \angle\ at\ circum$✓ $= 28°$✓

Discussion for question 3.3
This is a complex question as it requires significant connections between different theorems if the obvious alternative route of finding $N\hat{J}M$ first is used. This alternative would require learners to connect different theorems including the tan-chord theorem, angle at the centre and angle in a semi-circle theorem to come to the solution. Learners who can quickly see the $tan \perp rad$ theorem in the question can use the solution below. From my experience as a mathematics teacher, the $tan \perp rad$ theorem is not common in assessments and learners will not easily see and use it in a question.

Memorandum for question 3.3
$O\hat{N}J = 90°$✓ $\dots. tan \perp rad$✓ $O\hat{N}M = 90° - (24° + 28°) = 38°$✓

Question 4   ABCE is a cyclic quadrilateral on a circle with centre F. The produced lines AE and BC intersect at D. Given $A\hat{B}E = 60°, B\hat{C}A = 47°$ and $C\hat{E}D = 70°$, find: 4.1 $\quad E\hat{B}C$ (2)   4.2 $\quad B\hat{A}E.$ (3)
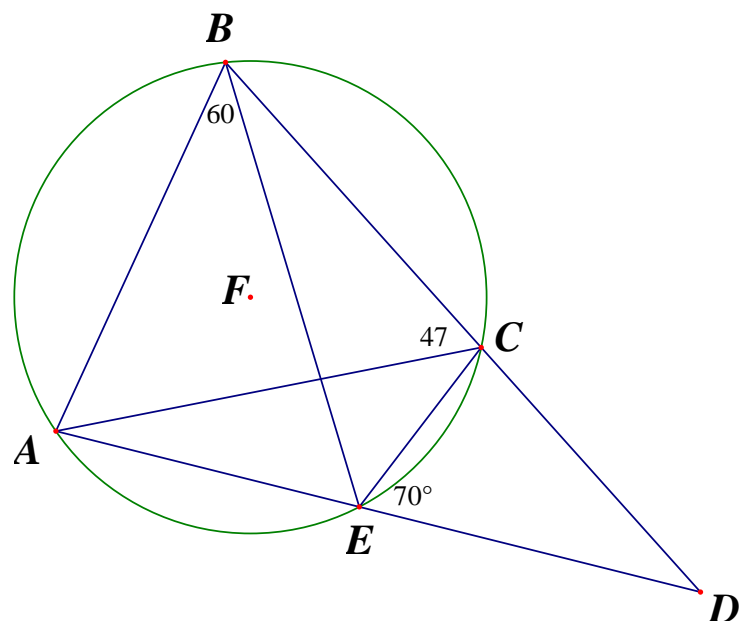
Discussion for question 4.1

This is a routine question as it requires simple application of mathematical concepts involving few steps. Learners must identify the cyclic quadrilateral ABCE first. They must then use the theorem on exterior anlge of a cyclic quadrilateral

Memorandum for question 4.1

$A\hat{B}C = 70°..$ ext ∠ of cyclic quad✓

$E\hat{B}C = 70° - 60° = 10°$✓

Discussion for question 4.2

This is a routine question as it requires simple application of mathematical concepts involving few steps. Learners must first use the theorem on angles in the same segment and then theorem on opposite angles of a cyclic quadrilateral.

Memorandum for question 4.2

$A\hat{C}E = 60°..$ same segment AE✓

$E\hat{C}B + B\hat{A}E = 180°$ *opp ∠ of cyclic quad*✓

$$B\hat{A}E = 180° - (47° + 60°) = 73°✓$$

Question 4.3
4.3   In the diagram JI is a diameter of the circle with centre K. MHN is a tangent to the circle at H. L is a point on HI and $LK \perp JI$. $K\hat{J}H = y$.
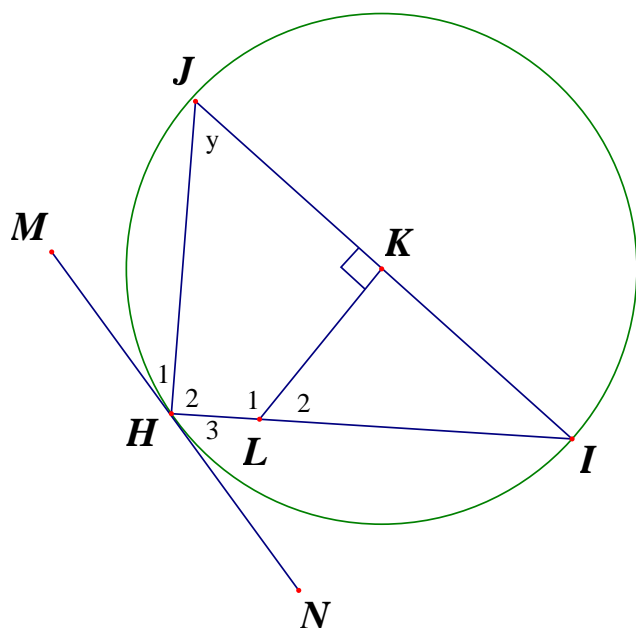
4.3.1   Prove that LKJH is a cyclic quadrilateral.   (3)

4.3.2   Determine, giving reasons, the size of $\hat{H}_1$ in terms of y.   (3)

Discussion for question 4.3.1

This is a problem solving question as it requires higher order reasoning. There is no obvious starting point or route to the solution. Learners need to know what must be shown to prove that a quadrilateral is cyclic. In the end, the converse theorem will be applied.

Memorandum for question 4.3.1

$L\hat{K}J = 90° \dots given$ ✓

$\hat{H}_2 = 90° \dots \angle\ in\ a\ semi\ circle$ ✓

$\therefore L\hat{K}J + \hat{H}_2 = 180°$

$\therefore LKJH$ is cyclic quadrilateral$\dots opp \angle\ supplementary$ ✓

Discussion for question 4.3.2

This is a complex question as it requires significant connections between different theorems and there is no obvious direct route to the solution. Learner must be able to use findings in 4.3.1 that LKJH is a cyclic quadrilateral. They must then use theorems on cyclic quadrilateral together with the tan-chord theorem and sum of angles of a triangle.

Memorandum for question 4.3.2

$\hat{L}_2 = K\hat{J}H = y \ldots$ ext $\angle$ of cyclic quad$\checkmark$

$\hat{I} = 90° - y \ldots$ sum of $\angle$ in Δ

$\hat{H}_1 = \hat{I} = 90° - y\checkmark \ldots$ tan *chord theorem* $\checkmark$

Question 5

In the diagram below, T is the centre of the circle and O, U, R and V are points on the circle. $OU = UR$ and $U\hat{V}R = x$. The tangent at O meets RU produced at W. TU intersects OR at B.
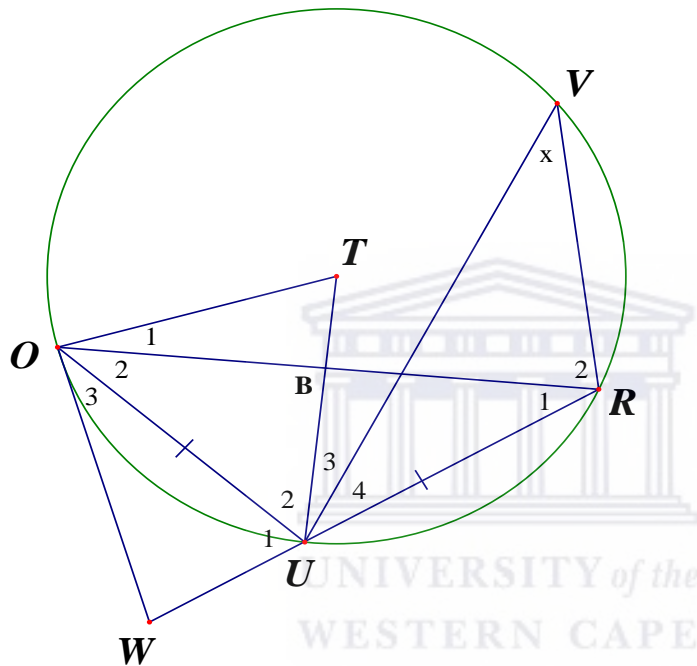
5.1 Give a reason why $\hat{O}_2 = x$. (1)

5.2 Prove that OU bisects $W\hat{O}R$. (4)

5.3 Determine $O\hat{T}U$ in terms of x. (2)

5.4 Prove that OW is a tangent to the circle that passes through points O, T and B. (2)

5.5 Prove that $T\hat{B}O = 90°$. (5)

                            [14]

Discussion for question 5.1

This is a knowledge question as it requires simple recall and use of a single theorem. Learners are expected to recall the angles in the same segment theorem.

Memorandum for question 5.1
∠ in the same segment✓

Discussion for question 5.2
This is a problem solving question as it requires higher order reasoning. There is no obvious starting point or route to the solution. Learners need to know what to show when a line bisects the angle. They must show that $\hat{O}_2 = \hat{O}_3$. They must use the theorem on isosceles triangles and the tan-chord theorem to prove.

Memorandum for question 5.2
$\hat{O}_2 = \hat{R}_1 = x$✓ ....∠ opposite = sides✓ $\hat{R}_1 = \hat{O}_3 = x$✓ ... $\tan chord\ theorem$✓ $\hat{O}_2 = \hat{O}_3$ $\therefore OU\ bisects\ W\hat{O}R$

Discussion for question 5.3
This is a routine question as it requires simple application of mathematical concepts involving few steps. Learners must first use the theorem on angles from equal segment and then theorem on angle at the centre equals wice the angle at the circumference.

Memorandum for question 5.3
$x = \hat{R}_1 ... ∠ from\ equal\ segments$ ✓ $O\hat{T}U = 2\hat{R}_1 = 2x ... ∠ at\ centre = 2 × ∠ at\ circum$✓

Discussion for question 5.4
This is a complex question as it requires proving and there is no obvious direct route to the solution. Learners need to know what to show when a line is a tangent. They must use the two angles proved in 5.2 and 5.3. If learners fail to link this question with preceding parts, then it can become a problem solving one as it will involve breaking the question into so many parts. They must use the converse of tan-chord theorem to prove.

Memorandum for question 5.4	
$W\hat{O}B = \hat{O}_2 + \hat{O}_3 = 2x ..... proved\ in\ 5.2$	$W\hat{O}B = O\hat{T}U..$✓

$W\hat{O}B = O\hat{T}U..\,proved\;in\;5.3$ $\therefore OW =$ $tangent\;....Converse\;of\;\tan\;chord\;theorem$	R✓
Discussion for question 5.5 This is a problem solving question as it requires higher order reasoning. There is no obvious starting point or route to the solution. Learners must be able to identify the $tan \perp rad$ theorem. There are different theorems on tangents and learners must firstly be able to identify the relevant theorem and use it accordingly.	
Memorandum for question 5.5 $T\hat{O}W = 90° ....\tan \perp rad$ $\hat{O}_1 = 90° - 2x$ $\therefore \hat{O}_1 + \hat{T} + T\hat{B}O = 180° ...\text{sum of } \angle \text{ in } \Delta$ $(90° - 2x) + 2x + T\hat{B}O = 180°$ $\therefore T\hat{B}O = 90°$	S✓R✓ S✓ S✓ S✓

Table 3.9: Summary classification of Post-test into cognitive levels

Question	Cognitive level	Reason	Marks
1.1	Routine Procedure (R)	Simple application and calculations requiring few steps. There are specific steps to follow	(5)
1.2	Knowledge (K)	Simple recall and use of a single theorem	(2)
2	Complex Procedure (C)	Significant connections between different theorems	(8)
3.1	Complex Procedure (C)	Significant connections between different theorems	(5)
3.2	Knowledge (K)	Simple recall and use of a single theorem	(2)
3.3	Complex Procedure (C)	Significant connections between different theorems	(3)

4.1	Routine Procedure (R)	There is simple application of mathematical concepts involving few steps	(2)
4.2	Routine Procedure (R)	There is simple application of mathematical concepts involving few steps	(3)
4.3.1	Problem-solving	Non-routine, higher order reasoning	(3)
4.3.2	Complex Procedure (C)	Significant connections between different theorems	(3)
5.1	Knowledge (K)	Simple recall and use of a single theorem	(1)
5.2	Problem-solving	Non-routine, higher order reasoning	(4)
5.3	Routine Procedure (R)	There is simple application of mathematical concepts involving few steps	(2)
5.4	Complex Procedure (C)	Significant connections between different theorems	(2)
5.5	Problem-solving	Non-routine, higher order reasoning	(5)

Table 3.10: Percentage of mark allocation in post-test per cognitive level

Cognitive level	Total mark allocation	Percentage mark allocation
Knowledge	5/50	10
Routine	12/50	24
Complex	21/50	42
Problem solving	12/50	24

3.7.3. Questionnaires

The other instrument (shown in Appendix 9) that was used in this research was the questionnaire focusing on motivation. The questionnaire gathered information for research question 2 and it was also used to find out more about the level of motivation and participation on the topic by the experimental group. The researcher wanted to find out the percentage of learners in the experimental group who were positively influenced by the intervention. The

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results in levels of motivation to learning helped me formulate the conclusion of my research. The aspects of validity and reliability of the questionnaire had to be observed for it to render information that was useful to the study. Validity refers to the extent to which it measures what it is intended to measure. Reliability gives the extent to which the instrument will give consistent results if it is used at another time. These qualities must be met by the questionnaire before it is taken out to collect data. In this study, the researcher obviated the problem by giving the instrument a trial run before taking it out into the research field.

The questionnaire was also used to check views of the experimental group on the topic after the teaching had taken place. To do this I consulted the reports of two other relevant studies. Herrmann (2013) had conducted a similar quasi-experimental research project and had used questionnaires to glean students' perceptions of cooperative learning. Likewise, Eu (2002) had used questionnaires to gain an understanding of students' perceptions and beliefs about using GSP.

By using the pre-test, post-test and the questionnaire, the researcher was able to address some problems associated with using only one instrument in a research. Bertram and Christiansen (2014) asserts that for questionnaires to serve the desired purpose respondents should be literate. This did not necessarily pose a problem in thus as most of the sample of grade 11 learners were able to read and write However, it is plausible that being second English language speakers could have interfered to some extent their understanding of the given information and the required to do aspects in a question. To minimise such interferences, the researcher was available to clarify any misunderstandings learners experienced. The researcher's presence helped to avert the challenge of some questionnaires not being returned

The questionnaires were given to learners after the intervention and the post-test were administered. The researcher wanted to find out the impact of the treatment on some other qualitative aspects like motivation to learn. After the concept of the day was taught, the researcher used part of the mathematics lesson to have the questionnaires completed. Learners who were given the questionnaires were asked to complete them individually without influencing each other. Learners were asked to consult the teacher if anything required clarity. After completing the exercise, the questionnaires were immediately collected and the information was later collated and put into tables by the researcher for purpose of data analysis. The questions in the questionnaire could be classified into four categories. Question 1 and 3 were checking on how friendly the Heymath programme was to the learners. Question 4 and 5

were checking learner understanding of the topic. The third category was questions 6 and 7 on how beneficial the simulations to the learners were. Question 2, 8 and 9 were checking on levels of motivation to learn. The responses for each question were counted using the tally method and the numbers recorded for analysis purposes.

3.8.   Important aspects of study

Aspects that are vital in any study for it to be regarded as authentic are validity, reliability and trustworthiness of instruments. Therefore, in general, it is worth establishing their soundness before any collection of data is done.

3.8.1   Validity and Reliability

Validity and reliability are two aspects that are very important in any research. If both are not satisfied, then the results of the study are useless and cannot be relied upon. In this study, validity and reliability checks were done in the pilot study.

3.8.1.1 Validity

According to Fischer and Etchegaray (2010), validity is concerned with whether the research questions and data collection tools measure what the researcher intends to find. If the tools used to collect the information for the research are not designed to get the data necessary to achieve a relevant conclusion to the research problem, then the research items are not valid. It is thus vital to check the wording of research items to see if they measure what is intended to be measured. There are different types of validity which include face validity, content, criterion and construct validity.

Face validity is achieved by not looking at the content of the research tools in detail to check if the wording or pictures used are universal. At a glance, the research tools might appear to measure what they are intended to measure and yet they are not. Content validity concerns the content of the research items to ensure these cover the domain of the research. In this study the domain is using technology in the context of cooperative learning. This means any input measured must be within this domain if it is to reveal the relevant output, to justify a conclusion.

Coupled with content validity is construct validity which focuses on whether what we think we are measuring is what we are really measuring (Fischer & Etchegaray, 2010). The weight of any claim in research thus depends on the validity of data used (Roberts, 2016).

A number of steps were taken in this study to ensure that my research instruments were valid. The first was familiarising myself with literature that helped me to understand validity.

With respect to the test used, the questions were taken mainly from past NSC mathematics examinations. This ensured that the questions used were suitable and relevant for grade 11 learners. Also, the content covered was related to the topic under scrutiny. By classifying the questions in the tests into different cognitive levels as required by the CAPS curriculum, the researcher was able to produce a valid test instrument. The consultation sessions with the supervisor helped in eliminating questions that were not relevant to the study.

3.8.1.2 Reliability

Reliability is concerned with consistency of results obtained using the same tool over space and time (Cohen, et al., 2007). As Fischer and Etchegaray state, "reliability is focused on the extent to which responses to a survey's items are consistent", (Fischer & Etchegaray, 2010:133). This is normally checked by asking the same question in different ways to see if responses to that question are consistent.

In this study, the questionnaire in Appendix 9 contained a number of questions asked in duplicate, but differently. Questions 4 and 5 are both focussed on understanding, whereas Questions 6 and 7 deal with the effect of simulations in using Heymath.

3.8.2   Trustworthiness

In research, trustworthiness is a term used to describe the credibility of results obtained from the study. Different approaches may yield different results because every research approach has strengths and shortcomings. In this research, triangulation through tests and a questionnaire ensured credibility in the conclusions drawn.

3.9   Data analysis

In quantitative data analysis, the collected numerical data is used to answer a research question

The results from the pre-test and post-test were numerical already, and the post test results were used in the t-test to test the hypothesis on the difference between two means as shown later in Figure 4.3 of the data analysis in the next chapter.

For statistical analysis of the post-test results, the t-distribution was used to test the effectiveness of the intervention given to the experimental group. The f-test was used to check if the population variances for the two samples were the same to allow the researcher to use the t-test.

For question 2, the tally system was used to count the number of different responses per question in the questionnaire. The totals obtained were then used for analysis by converting them to percentages first.

3.9.1 Calculation of statistics

A statistic is any calculation done for a sample. A calculation for a population is called a parameter. In this research, the mean and standard deviations from the sample were calculated using Microsoft Excel and used in the test statistic. The standard deviations and means for the experimental and control groups were calculated.

3.9.2 Student t-distribution

The calculated mean marks of the two samples were very useful. The researcher used the t-distribution to test if the difference between the two mean marks was significant statistically. A two-tailed test was used to check whether the experimental group mean was equal to the control group mean. The f-test was used to establish that the population variances for the experimental group was the same as that of the control group so that the t-test could be used as the t-test can only be used on the assumption that the population variances were the same.

According to Walpole and Myers (1985) the t-distribution is used when the population variance is not known, and the sample size one group is less than 30. If the sample size is more than 30 and population standard deviation is not known then normal distribution can be used (Walpole & Myers, 1985). In this case the standard deviation calculated from the sample was used.

The alternative hypothesis set was that using technology through cooperative learning will improve the learning of grade 11 circle geometry. This hypothesis was tested against the null hypothesis stating that there is no difference between the two methods. The difference between the mean marks was calculated and the test statistic used to check if the alternative hypothesis could be rejected. According to Walpole and Myers (1985:260), "the rejection of a hypothesis is to conclude that it is false, while the acceptance of a hypothesis merely implies that we have insufficient evidence to believe otherwise". For this study, acceptance of the null hypothesis meant that the treatment had no positive impact on the experimental group; and rejection of the null hypothesis meant that the treatment had a positive impact.

The researcher was aware of the two types of errors that could result in this research. Type I error is when the researcher declares the two methods are different when they are actually the same. The probability of making a Type I error is the level of significance represented by the

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Greek symbol α, and is used in the test statistic (Walpole & Myers, 1985). Type II error is when the researcher says the methods are the same when they are actually different. These errors must be avoided at all costs.

The researcher understood that poor data analysis could destroy very good research. As a result, the researcher sought expert opinion whenever he was unsure of a method. The researcher was aware of ethical considerations and took precautions not to manipulate data to get desired results. The conclusion was reached only through proper data handling.

3.10 Ethical issues

There are ethical issues that have to be observed in any study. According Bertram and Christiansen (2014:65), "ethics has to do with behaviour that is considered right or wrong". Learners have rights and these must be observed. According to the Economic and Social Research Council in the U.K (as cited in Dowling and Brown, 2010), the Research Ethics Framework has stipulated some guidelines for acceptable ethical practices that must be adopted by institutions of higher learning.

The University of the Western Cape requires that the Research Proposal and all letters of permission in the appendices be submitted to the WCED Research Ethics Committee for approval. The researcher must strive to work for the integrity and quality of the research through different reviews and by abiding by all issues ethical. It is a requirement in the Research Ethics Framework that participants be fully informed of all information relating to the research. They must be made aware of what is expected of them in the research and must decide if they still want to participate in it.

Hence, the purpose of the research was explained to participants in the introductory part of the questionnaire and their consent was secured before the study began.

Confidentiality is another element within the scope of the Research Ethics Framework that must be observed (Dowling & Brown, 2010). Participants must be promised confidentiality regarding all information they will supply and insist on anonymity.

Furthermore, in this study questions asking for confidential information were avoided to ensure that participants felt free in the process. The right to withdraw from the study at any point without victimisation was made clear to participants and this was carefully explained in the second last paragraph of the permission letters (Appendices 2 to 6) as well as in the consent

and assent letters for the parents and learners respectively. The questionnaire contained the same clause to inform participants of their rights in the study.

There are also ethical issues concerning the treatment people as experimental objects in research. According to Bertram and Christiansen (2014), there is insufficient justification for why participants might be put in an experimental group if it yields negative results for them. Likewise, neither is it possible to justify to participants the reason why they were allocated to a control group if this has negatively affected their achievement in learning.

Also, parents must be made aware of what will be done in the group affected and why it is necessary to do such a study with their children's participation. It was ascertained already that this study would not harm participants. One method would just be better than the other without any serious harm to them. It was important for the researcher to know these ethical considerations to be able to carry out the study in an acceptable environment.

Anonymity was also retained throughout the research process. Pseudonyms were used on questionnaires and in the recording of marks. As indicated earlier, 'Khumbulani High School' in the study is not the school's real name.

On the part of the researcher, honesty in the collection and analysis of data was vital as the conclusion of this study could influence future actions. All the foregoing ethical concerns were thus observed and addressed.

In this chapter, the researcher explained the procedure by which the data for the study was collected. The research design and research methods employed were detailed in the various sections of the chapter. It was specified that pre-test, post-test and questionnaires were the only instruments used to collect data for this study. Ethical issues considered in this study were explained and it was made clear that the researcher, retaining a neutral position throughout, used the scientific approach in collecting data. The manner in which the actual intervention was administered was explained clearly in Section 3.7.2. Conventional teaching methods were used for the control group.

The next chapter is on data analysis, involving the use of statistical calculations to interpret the scientific meaning of the collected data.

Chapter 4: DATA ANALYSIS, FINDINGS AND DISCUSSION

4.0   Introduction

Once the data had been collected, the researcher organised it in tabular form on Microsoft Excel in such a way that calculations could be done to interpret the data. The researcher was thus able to analyse the collected data using statistical tests for question 1. The paired observation test was also done for question 1 to check if there was a significant improvement from pre-test to post-test for each of the two groups. For question 2, tallying method was used to have totals for the different responses from the questionnaire for analysis purposes. These totals were converted to percentages to make meaning of the data. This is in accordance with the advice of Bertram and Christiansen (2014), who recommend that research findings must be presented in a way that makes sense to readers.

In Chapter 4 the research findings will be presented according to the research questions, followed by a discussion of the findings.

4.1.   Research Question One:

How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 geometry?

The June Mathematics Paper 2 results were used to ascertain whether the two groups were operating at the same level academically. From Tables 4.1 and 4.2 it can be seen that the June Mathematics Paper 2 examination average was 54.23% and 54.3% for the control and experimental groups respectively prior to start of intervention, hence conforming that both groups were of comparable capabilities. In order to answer research question 1, the pre and post test results were used. The results obtained in the two tests are shown in Tables 4.1 and 4.2 for the control and experimental groups respectively. As with the June Mathematics Paper 2 examination, the pre-test results for the control and experimental groups stood at 30.54% and 30.3 % respectively, indicating that the pre-test means for E and C groups were statistically the same – as explained in Figure 3.2 in the previous chapter in Section 3.7.1.

This question was answered using the pre-test and post-test. The pre-test was covering work on the first four theorems, a converse theorem and a corollary. The post-test was covering the entire topic of grade 11 Euclidean geometry with emphasis on the last four theorems, their corollaries and accompanying converses. The experimental group was exposed to an

intervention in teaching using Heymath in the context of cooperative learning whilst the control group was taught using the conventional method of chalk and talk from the teacher together with the question and answer method. Tables 4.1 and 4.2 show the results in the post-test for the control and experimental group respectively.

Table 4.1: Test results from Microsoft Excel for control group

	CONTROL GROUP	GRADE 11C: 2018	A	B	C	D	E
		June Paper2	Pre-test	Pre-test %	post-test	Post –test %	Post-pretest
	Name/Possible mark	100	50	100	50	100	d_1
1	C1	62	18	36	4	8	-14
2	C2	58	5	10	1	2	-4
3	C3	68	23	43	1	2	-22
4	C4	42	23	46	11	22	-12
5	C5	44	11	22	14	28	3
6	C6	66	4	8	3	6	-1
7	C7	76	24	48	23	46	-1
8	C8	69	26	52	8	16	-18
9	C9	37	16	32	9	18	-7
10	C10	82	25	50	15	30	-10
11	C11	65	14	28	10	20	-4
12	C12	59	18	36	20	40	2
13	C13	28	3	6	1	2	-2
14	C14	24	6	12	5	10	-1
15	C15	52	5	10	9	18	-4
16	C16	70	13	26	7	14	-6
17	C17	84	27	54	20	40	-7
18	C18	62	22	44	9	18	-13
19	C19	48	5	10	5	10	0
20	C20	38	11	22	9	18	-2
21	C21	48	17	34	8	16	-9
22	C22	41	17	34	10	20	-7
23	C23	22	2	4	2	4	0
24	C24	35	15	30	6	12	-9
25	C25	40	20	40	18	36	-2
26	C26	90	27	54	32	64	5
	AVERAGE	54,231	15,27	30.54	10	20	-5.58

$\overline{x_1} = 10; S_1{}^2 = 57.52; n_1 = 26$

Table 4.2: Test results from Microsoft Excel for experimental group

	EXPERIMENTAL GROUP	Grade 11 E	F	G	H	I	J
	Task	Paper 2	Pre-test	Pre-test%	Post-test	Post-test %	posttest-pretest
	Name/ Possible mark	100	50	100	50	100	d_2
1	E1	47	15	30	19	38	4
2	E2	48	14	28	20	40	6
3	E3	71	22	44	24	48	2
4	E4	44	5	10	21	42	16
5	E5	44	0	0	15	30	15
6	E6	72	20	40	27	54	7
7	E7	61	9	18	10	20	1
8	E8	28	19	38	11	22	-8
9	E9	13	7	14	3	6	-4
10	E10	55	19	36	28	56	9
11	E11	40	25	50	10	20	-15
12	E12	73	18	36	26	52	8
13	E13	76	17	34	17	34	0
14	E14	34	15	30	28	56	13
15	E15	76	20	40	25	50	5
16	E16	49	11	22	17	34	6
17	E17	64	22	44	17	34	-5
18	E18	16	21	42	9	18	-12
19	E19	74	26	52	24	48	-2
20	E20	54	14	28	15	30	1
21	E21	57	13	26	35	70	22
22	E22	73	14	28	16	32	2
23	E23	54	19	38	15	30	-4
24	E24	59	6	12	7	14	1
25	E25	86	21	42	5	10	-16
26	E26	70	10	20	12	24	2
27	E27	28	7	14	11	22	4
	AVERAGE	54,3	15,15	30.3	17,3	34.6	2.15

Tables 4.1 and 4.2 have columns showing the post-test results. Column C in Table 4.1 and column H in Table 4.2 show the results as raw marks out of 50, whereas columns D and I in Tables 4.1 and 4.2 respectively show these marks converted to percentages. The average mark for the post-test was 20% for the control group and 34.6% for the experimental groups. These two averages will be discussed further in the next section (Section 4.1.3).

Within the South African Schooling System, a learner passes a subject if he or she scores 30% or more in the examination or test. In the control group, only 6 of 26 learners passed the post-test giving a pass rate of 7.7%. The quality was so poor in that only 1 learner (i.e. 3.8%) passed the post-test with more than 50%. The experimental group was somehow different with 18 of 27 learners passing with at least scoring 30% or more, thus giving a pass rate of 66.7%. This pass rate may look impressive on paper, but the quality of the results was very poor as only 6 (22.2%) learners passed the post-test with more than 50% showing that the intervention was not as effective as was anticipated.

4.1.1 Pre-test results presentation

Qualitative content analysis of the learners' responses in the pre-test, revealed that proving a theorem presented a challenge for both groups, even though this was a mere routine procedure type of question as stipulated by the NSC examination guidelines (DBE, 2017). Table 4.3 indicated that the two prominent errors associated with proving a theorem are: avoidance of using sketch diagram and the use of the conclusion as a starting point to prove the given theorem.

Table 4.3: Most common Pre-test errors

Question	Common error	Frequency C-group	%	Frequency E-group	%
1.2	No sketch diagram	11	42.3	20	70.1
	Used conclusion to prove theorem	10	38.5	9	33.3

In particular, Q 1.2 required learners to prove the angle at centre theorem, which states

The angle subtended by a chord at the centre is twice the angle subtended by that chord at the circumference.

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The common error was students' failure to recognise that in proving this theorem, the premise is used to prove the conclusion of the theorem. Most of the learners were using the conclusion that they were required to prove, which led to a mathematical breakdown. For example, Learner C13's response as shown in Figure 4.1 demonstrates such a breakdown:

C13: "$A\hat{O}B = 2A\hat{P}B \dots \dots \angle$ at centre $= 2 \times \angle$ at circum".

Figure 4.1: C13's response to Q1.2

As in Table 4.3, a total of 10 (38.5%) learners in the control group and 9 (33.3%) learners in the experimental group used the given conclusion to prove the same theorem.

It is customary in geometry, to ensure that a relevant diagram with all the necessary details and constructions accompanies a written proof to enable a reader to make sense of the proof explanation. However, learners from both the experimental group and control group attempted to prove the theorem without drawing a sketch.

For example, as shown in Figure 4.2, C24 answered the question correctly but without a diagram.

Figure 4.2: C24's response to Q1.2

All proofs must be shown with accompanying sketches where all extra constructions are shown. E22 started with making an assumption but then used the conclusion to prove the theorem as illustrated in Figure 4.3
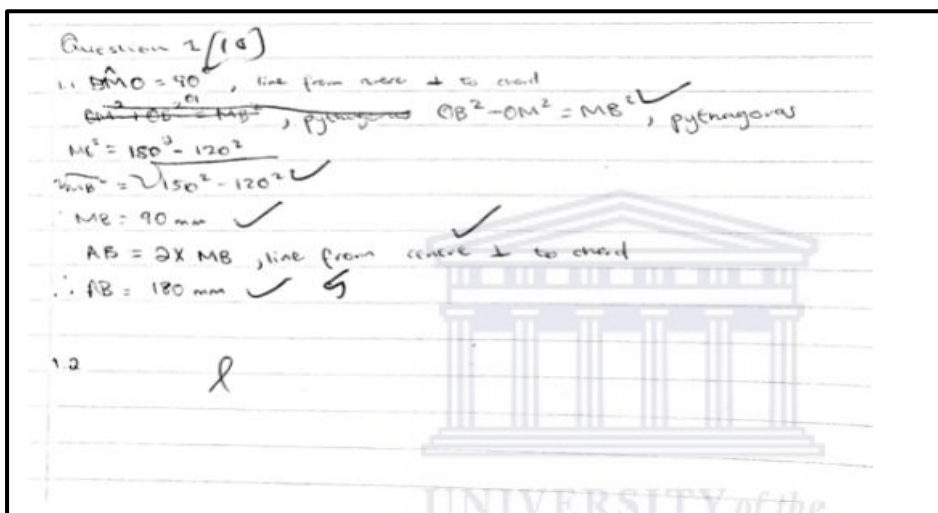
$$\text{let } A\hat{P}B = 2x$$

$$A\hat{O}B = x \ldots \ldots \ldots \angle \text{ at centre} = 2 \times \angle \text{ at circum.}$$

$$\therefore A\hat{O}B = 2A\hat{P}B\text{"}.$$

Figure 4.3: E22's response to Q1.2

In E22's response there is no proving done of the given theorem. Even the concept of which angle is bigger than the other does not exist.

E11, one of the students who got 50% in the pre-test, could not attempt answering question 1.2 on proving a theorem as shown in Figure 4.4.

Figure 4.4: E11's response to Q1.2

A total of 20 (74.1%) learners from the experimental group attempted to prove the theorem without drawing a sketch, compared to 11 (42.3%) learners in the control group who answered without the required diagram. This might be because learners in the experimental group were used to having all diagrams on the computer and this worked against them when diagrams had to be drawn.

Figure 4.5 shows what information learners were given in question 1.1 and what they were required to calculate giving reasons.

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Question 1

1.1　In the diagram alongside, AB is a chord

of a circle with centre O.

ON ⊥ AB and cuts AB at M

and meets the circle at N.

If MN = 30 mm and the radius

of the circle is 150 mm.
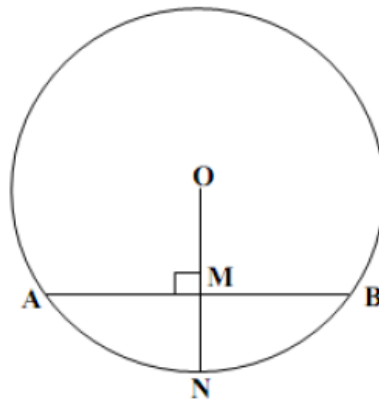
Calculate, with reasons, the length of AB.

Figure 4.5: Q 1.1. in Pre-test

Although some learners from both the control group and experimental group could not prove the theorem in question 1.2, they scored high marks for question 1.1 but failed to provide reasons for some of their essential statements in their calculation of the length of AB. For example, Learner C1 and Learner E1, who obtained 36% and 30% respectively, both got 4/5 marks in question 1.1 but both scored zero for question 1.2 (see Figures 4.6 and 4.7 respectively)
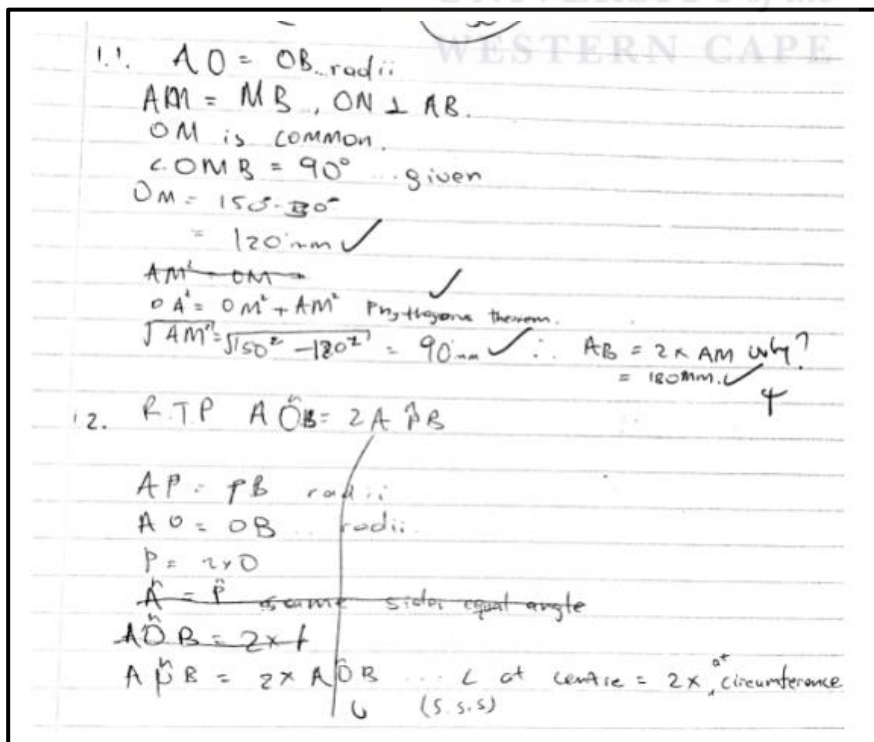
Figure 4.6: C1's response to Q 1.1 and 1.2
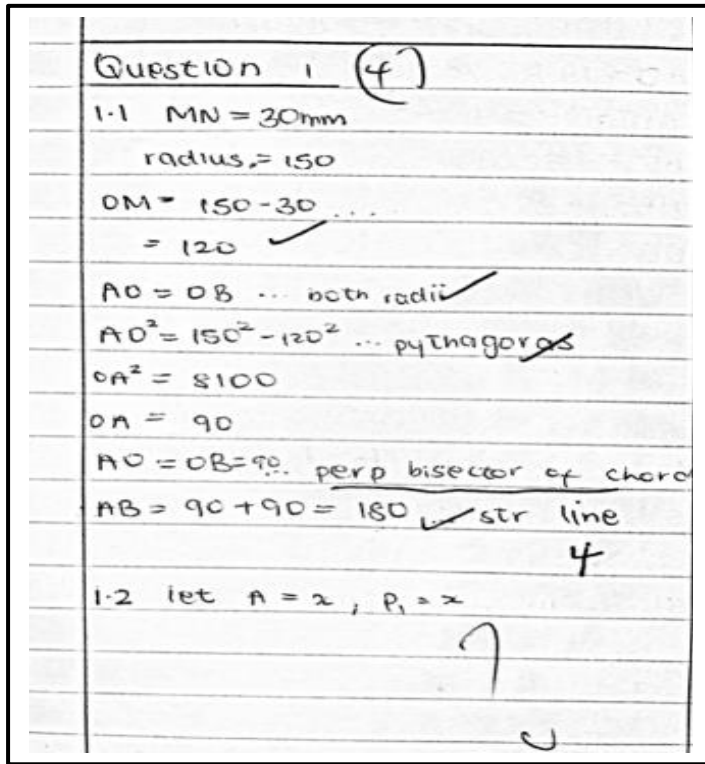
Figure 4.6 E1's Response to Q1.1 and 1.2

Question 1.1 was a routine question testing what learners were exposed to daily in class. This level requires learners to get to a solution by applying few routine steps. Learner C1 did not give the supporting reason as expected in Euclidean geometry and E1 gave a wrong reason. It was suggested that learners be given the acceptable reasons in the examination guidelines to get used to them (DBE, 2018). An explanation to a theorem should be explained using the corresponding diagram.

Even though both learners did not draw the sketch diagram required for proving a theorem in question 1.2, which was a routine question, C1 did some wrong working and not answering the question whilst E1 did basically nothing. According to the marking guideline no marks were awarded for solution if the diagram was not provided. This common error showed that learners in the two groups had the same understanding of Euclidean geometry before giving the intervention to the experimental group. The fact that learners had the errors as shown in Table 4.3 means that this question 1.2 was poorly done. About 70.1% of the learners got nothing by not sketching the graph. Other learners still lost some marks by using the conclusion to prove a theorem instead of starting from premises and move deductively to a conclusion.

4.1.2 Post-test results presentation

In this section the results of the post-test are presented by examining responses obtained from given questions. The major highlight of the post-test results was that the control group trailed behind the experimental group with an average of about 14.6%. While the experimental group had an average of 34.6% in the post-test, the control group's average stood at a staggering 20%. This difference of 14.6% is significant, as will be shown in Section 4.1.3. A snapshot of the errors encountered in the post-test is shown in Table 4.4.

Table 4.4: Post-test errors

Question	Common error	Frequency C-group	%	Frequency E-group	%
2	Same segment instead of: $\angle\ at\ centre = 2 \times \angle\ at\ circum$	12	46.2	3	11.1
3.2	Same segment instead of: $\angle\ at\ centre = 2 \times \angle\ at\ circum$	8	30.8	3	11.1
3.2	Question not answered (left blank)	6	23.1	6	22.2
3.3	Tan-chord theorem given as reason instead of $tan \perp rad$	7	26.9	7	25.9
3.3	Question not answered or no reason given	12	46.2	9	33.3
4.3.1	Not using converse of theorems of cyclic quadrilateral to prove that the quadrilateral is cyclic	10	38.5	14	51.9
5.3	Same segment instead of: $\angle\ at\ centre = 2 \times \angle\ at\ circum$	1	3.8	5	18.5
5.3	Question not answered (left blank)	8	30.8	8	29.6
5.4	Not using converse of theorems of tangents to prove that the line is a tangent	6	23.1	7	25.9
5.4	Question not answered (left blank)	16	61.5	13	48.1

The results of the post-test shown in Table 4.4 are a 'mixed bag' as there are times when the numbers are in favour of the control group and times when the numbers are in favour of the experimental group. For an example, question 4.3.1 was in favour of the control group with 38,5% making the mistakes compared to 51,9% from experimental group with the same error of not using the converse to prove that the quadrilateral was cyclic.

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For question 2 of the post-test 46.2% of learners in the control group used the same segment theorem instead of using 'the angle at the centre' theorem, but only 11.1% of the experimental group expressed this confusion in this question. An analysis of this will follow in the discussion section. Figure 4.8(a) shows this error from learner C4 in the control group and Figure 4.8(b) for learner E16 in experimental group.

Figure 4.8 (a): Response of learner C4 to Q 2

 Figure 4.8 (b): for learner E16 to Q2

This confusion and mixing up of theorems was evident in a number of questions which include questions 2, 3.3 and 5.3. As illustrated in Table 4.4, in question 2, learners C4 and E16 seemed to think that as long as two angles originated from the same chord, then the reason of 'angles in the same segment' would apply despite the fact that one angle is at the centre and the other at the circumference.

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For question 5.3 more errors arose that were related to mixing up the two theorems. Figure 4.9 (a) and Figure 4.9 (b) for question 5.3's solution for learner C3 and E20 from the control group and experimental group are shown.
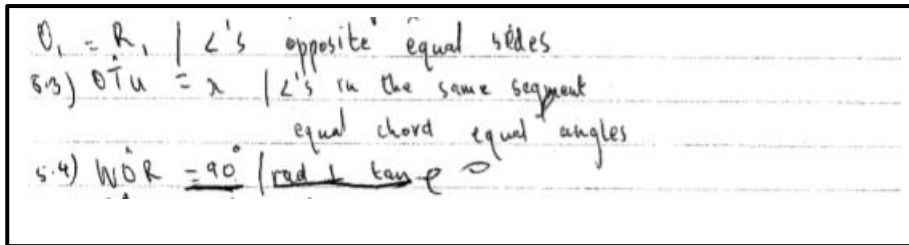
Figure 4.9 (a) Learner C3 Response to Q5.3 and Q5.4
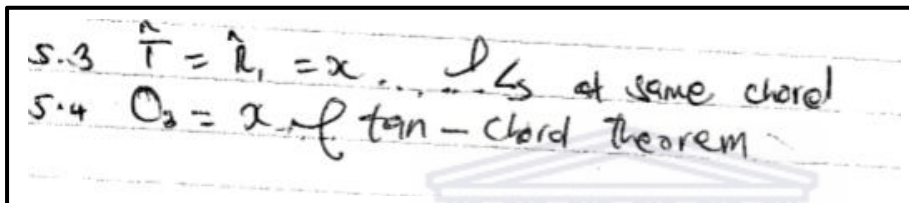
Figure 4.9 (b) Learner E20 Response to Q5.3 and Q5.4

Learners mixed up the angle at the centre theorem with angles in the same segment theorem. This confusion was common in the experimental group with 18,5% of the learners making this error compared to 3,8% from the control group as shown in Table 4.4.

The other common observation was learners' leaving questions unanswered. It was observed that about 23.1% of the control group left question 3.2 unanswered, compared to 22.2% of the experimental group. This observation revealed an additional weakness on top of the 30.8% of learners from the control group who conflated the theorems. In question 3.2, total of 53.9% from the control group did not get this question correct, and this was comprised as follows: omission (23,1%), mixing up theorems (30,8%); while other learners got this question wrong for different reasons like the sum of angles of a triangle as given by learner E15 in Figure 4.10(b). From the experimental group, Learner C8's reason was angles opposite equal sides as shown in Figure 4.10(a).
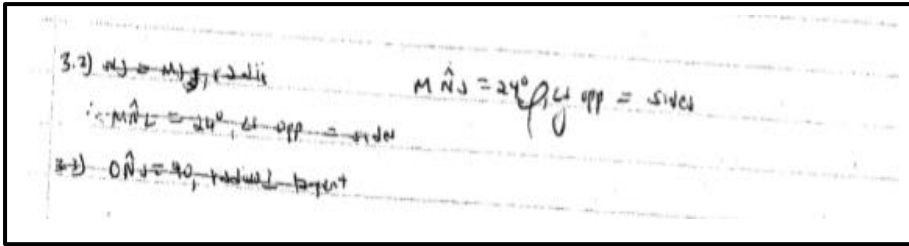
Figure 4.10 (a) Learner C8 Response to Q3.2
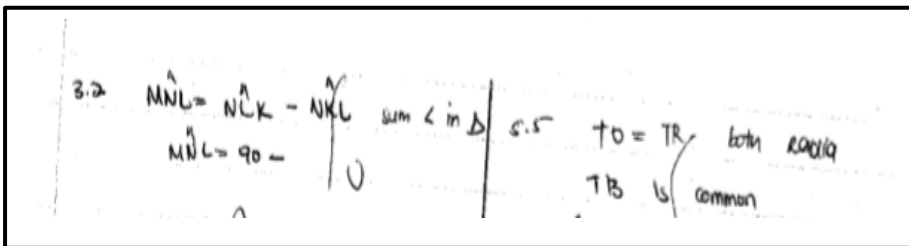
Figure 4.10(b) Learner E15 Response to Q3.2

The common error in question 3.3 was that learners referred to the other theorem of tangents, $tan \perp rad$ as the tan-chord theorem. Learner C6's response is shown in Figure 4.11(a) and Learner E1's response is in Figure 4.11(b).
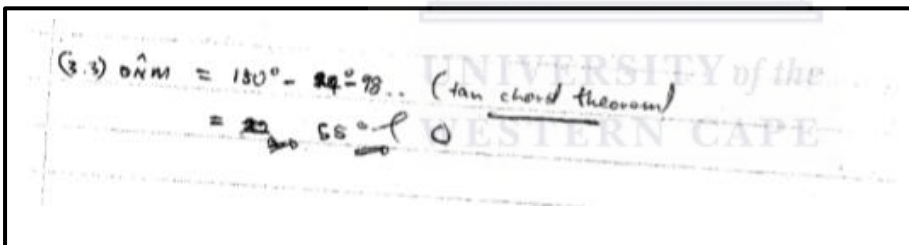
Figure 4.11(a) Learner C6 Response to Q3.3
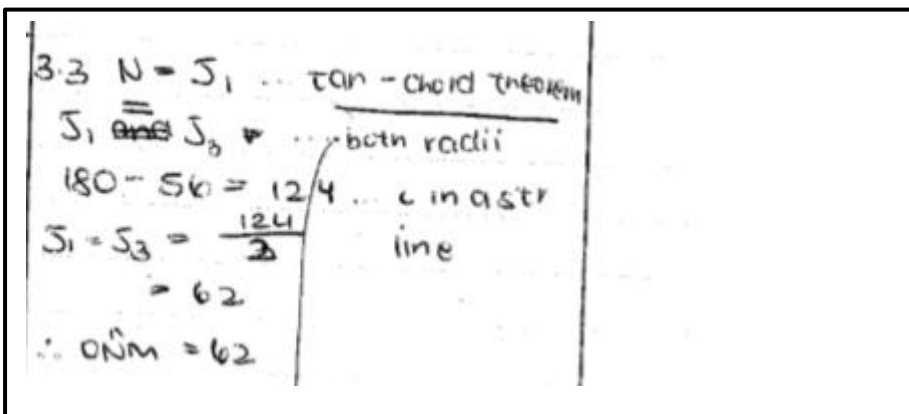
Figure 4.11(b) Learner E1 Response to Q3.3

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The number of learners who used this incorrect reason was 26.9% and 25.9% from the control and experimental groups respectively. Another sizable percentage of learners left question 3.3 blank or answered the question without providing any reasons for the steps used as shown in Table 4.4. This amounted to a breakdown in Euclidean geometry. About 46.2% of the learners from the control group and 33.3% from the experimental group either omitted the question or did not provide reasons.

Another common error emerged in question 4.3.1. Learners were required to prove that a given quadrilateral was cyclic. A very large percentage of the learners assumed that the quadrilateral is already cyclic before proving it. About 38.5% of learners in the control group used theorems of cyclic quadrilaterals to prove that the quadrilateral was cyclic. C4's solution follows.

Key: ✓ Correct ,   .   Wrong

$$L\hat{K}J + J\hat{H}L = 180° \dots opp\angle s \text{ of cyclic quad}$$

$$L\hat{K}J = 90° \dots\dots given$$

$$J\hat{H}L = 90° \dots tan$$

$$90° + 90° = 180°$$

Figure 4.12: **Learner E1 response for Question 4.3.1.** The circled error is what the learner was supposed to prove, and thus could not be used as a reason. From the experimental group, a staggering 51.9% made this wrong assumption. E18 wrote: $H_2 = k = 90° \dots\dots opp \angle s \text{ of cyclic quads}$, E12 had the first part done well and the wrong assumption later used. The solution written by E12 is shown in Figure 4.13.

Key: ✓ correct, ⬭ wrong

$$L\hat{K}J = 90° \dots. given ✓$$

$$H_2 = 90° \dots.. \angle s \ in \ a \ semi \ circle ✓$$

$$L\hat{K}J + H_2 = 180° \dots. \boxed{opp \ \angle s \ of \ cyclic}$$

$$\therefore LKJH \ is \ a \ cyclic \ quad$$

Figure 4.13: Expected solution for question 4.3.1

For question 5.3, about 30.8% and 29.6% of the control and experimental groups left this question unanswered. It implied that proving a property presented a challenge. This percentage, combined with that of learners who made assumption errors, meant that the majority of the learners got the wrong answer for this question.

The error of assumption was common for questions in proving a particular property. It was also found that about 23.1% of learners in the control group did not use the converse of tangent theorems to prove that a line was a tangent in question 5.4. Sample solutions for Learners C12 and E20 are shown in Figures 4.14 (a) and 4.14(b) respectively. Both learners used the tan-chord theorem when they were supposed to prove that a line is a tangent.
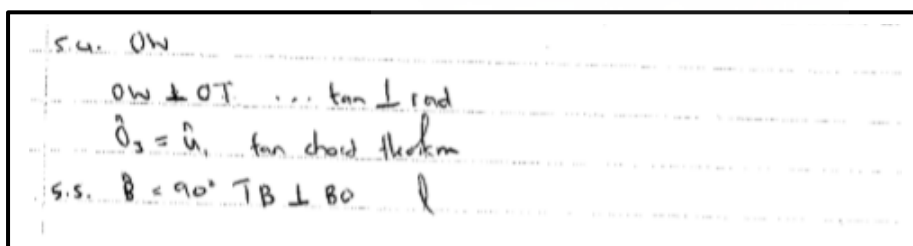
Figure 4.14(a): Learner C12 responses to Q5.4 and Q5.5
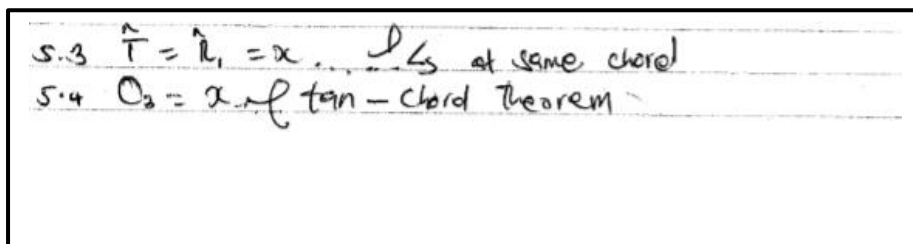
Figure 4.14(b) Learner E20 responses to Q5.3 and Q5.4

For the same question, about 25.9% from the experimental group also did not use the converse of theorems. This question on proving tangency was left blank by 61.5% of the control group and 48.1% of the experimental group. A deeper analysis follows in the next section of statistical analysis and the discussion.

4.1.3 Statistical Analysis of post-test data and interpretation of results

The analysis of data was done firstly using the t-distribution to test the significance of the difference between the two means, that is, the means of the control and experimental groups in the post-test. The aim was to check whether the two means are statistically the same or different. The observed data and calculations using values obtained through Microsoft Excel are shown in Figure 4.15.

$\overline{x_2} = 17,3; S_2^2 = 63.37; n_2 = 27$

The null (H_0) and alternative (H_1) hypotheses were set as follows:

H_0: The means are equal: $\bar{x}_1 = \bar{x}_2$

H_1: Experimental group mean ($\bar{x}_2$) $\neq$ control group mean($\bar{x}_1$): $\bar{x}_1 = \bar{x}_2$

The test statistic used was $t = \dfrac{\overline{x_1} - \overline{x_2}}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{(n_1+n_2-2)}\left(\frac{\alpha}{2}\right)$

where $s_p = \sqrt{\dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$ and $S_1^2 = \sum_{i=1}^{n}\left(\dfrac{(x_i-\bar{x})^2}{n-1}\right)$ is the sample standard deviation since the population standard deviation is not known (Walpole and Myers,1985).

Criteria: reject H_0 if $t_{calculated} > 1.997$

Now, $\overline{x_1} = 10; S_1^2 = 57.52; n_1 = 26; \overline{x_2} = 17,3; S_2^2 = 63.37; n_2 = 27$

$$s_p = \sqrt{\frac{(26-1)57.52 + (27-1)63.37}{27+26-2}} = 7.78$$

$$t = \frac{\overline{x_1} - \overline{x_2}}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{17.3 - 10}{7.78\sqrt{\frac{1}{26} + \frac{1}{27}}} = 3.41$$

Figure 4.15: Test statistics and Calculations

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Decision:

Since $t_{calculated} = 3.41 > 1.997 = t_{table}$, the researcher rejected H_0 and concluded that at 5% level of significance, the means differ significantly. From the calculation shown it became clear that the intervention improved the results significantly as the mean for the experimental group was higher than that of the control group. Put differently, learners in the experimental group performed much better compared to the control group.

It was also necessary for the researcher to check – using the f-test for variances – whether the assumption was correct in choosing the foregoing test statistic in Figure 4.15 for difference between two means. This t-test could be used only on the assumption that the population variances for the control group and experimental group were the same. The null (H_0) and alternative (H_1) hypotheses were set as follows:

H_0: The variances were the same: $\sigma_1{}^2 = \sigma_2{}^2$

H_1: The variances were not the same: $\sigma_1{}^2 \neq \sigma_2{}^2$ where $\sigma_1{}^2$ and $\sigma_2{}^2$ are the population variances for the control and experimental groups respectively. The $\alpha = 0.1$ was used and the critical region was calculated as follows in Figure 4.16.

$$f_{0.05}(26,27) \approx 1.95$$

and lower critical value calculated as

$$f_{0.95}(26,27) \approx \frac{1}{f_{0.05}(27,26)} = \frac{1}{1.96} = 0.51$$

Test statistic: $f = \dfrac{S_1{}^2}{S_2{}^2}$ with (26, 27) degrees of freedom (Walpole and Myers, 1985).

$f = \dfrac{57.52}{63.37} = 0.91$. Criteria: Reject H_0 if $f < 0.51$ or $f > 1.95$

Figure 4.16: $f - test$ for population variance

Decision: the researcher failed to reject H_0 and concluded that the population variances were the same. Therefore, the test statistics used to test the difference of the two means was justified and was correct.

The researcher also looked at paired observations for both the control and experimental groups for the pre-test and post-test. The researcher used results of the paired scores to test if either group registered a significant improvement from pre-test to post-test. The t-test was also used

here to check if the marks for pre-test and post-test were significantly different as shown in Figure 4.17.

The statistics found for control group were: $\overline{d_1} = -5.58;\ S_{d_1} = 6.49;\ n_1 = 26$

The statistics found for control group were: $\overline{d_2} = 2.15;\ S_{d_2} = 8.98;\ n_2 = 27$

The test statistic used

$$t = \frac{\bar{d} - d_0}{s_d \sqrt{n}};\ v = n - 1$$

H_0: The mean score difference equals zero: $u_D = 0$

H_1: Experimental group mean $u_D \neq 0$

The statics found are shown in Table 4.3.
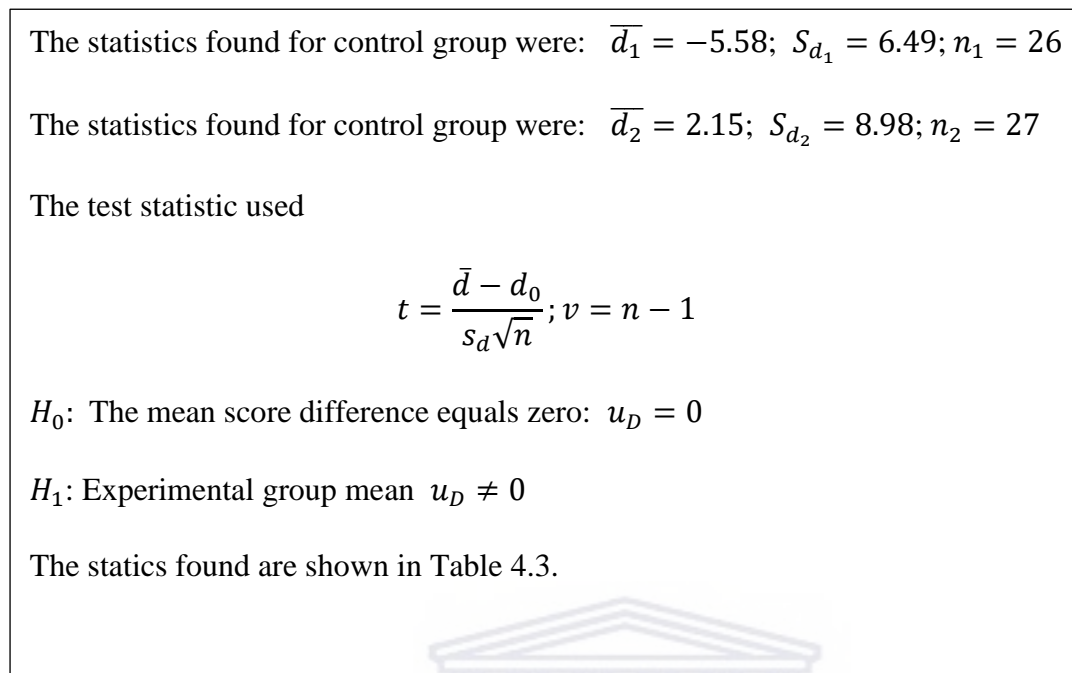
Figure 4.17: Calculations for paired observations

The results obtained from the calculations using formulae in Figure 4.17 are summarised in Table 4.5 below. These calculations were comparing the difference between the mean mark obtained in the pre-test and that of the post-test for both the control and experimental group.

Table 4.5: Results of paired observation: t-test

Pair		$\bar{d}$	Standard deviation	$t_{calculated}$	$t_{0.025(n-1)}$	decision
$1(n = 26)$	d_1	-5.58	6.49	-0.169	-2.060	Accept
$2(n = 27$	d_2	2.15	8.98	0.046	2.056	Accept

Where

$$d_1 = posttest\ score\ -\ pretest\ score\ for\ control\ group$$

$$d_2 = posttest\ score\ -\ pretest\ score\ for\ experimenal\ group$$

The calculations done on mean score differences for the control group and the experimental group for paired differences between pre-test and post-test-results are shown in Table 4.5. In both controlled and experimental groups, the researcher failed to reject the null hypothesis at

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5% level of significance because the calculated value is not on the tail side of the table value of the t-distribution and concluded that there was no significant difference between the post-test results and pre-test results for either of the two groups.

The control group mean actually came down in the post-test results from 30.5% to 20%, but this decrease was not significant according to Table 4.5. The experimental group had an increase in the mean from 30.3% in the pre-test to 34.6%. for the post-test but this increase in the mean was not significant. These results show that the experimental group average in the post-test did not rise high enough to be regarded as different from that of the pre-test. The important fact to consider here is that the difference between the post-test mean and pre-test mean for the experimental group was positive, indicating that the performance improved in the post-test to some extent.

After the statistical test for the difference between the mean marks, it was interesting to note from Figure 4.15 that performance for the two groups was significantly different in the post-test results, with the experimental group scoring significantly higher than the control group. This meant that teaching using technology in the context of cooperative learning did have a positive impact. This was in alignment with Delgado et al. (2015:397) who claimed that "several meta-analyses showed promising results of the effectiveness of technology in the classroom". The results obtained concur with the outcome of a study by Hutkemri and Effandi (2014) which concluded that technology, in the form of Geogebra, improved the conceptual understanding of learners in the experimental group. Hutkemri and Effandi (2014) further acknowledge that graphical representation made learning easier. This means that technology in the context of cooperative learning can actually be used to improve learning in grade 11 circle geometry.

However, a worrying observation was that the average for the experimental group was still very poor, positioned at 34.6% despite having received the intervention, though it was better than the control group with an average of 20%. The average for the two groups combined was now 27.4%, meaning the conventional method pulled down the average. The poor pass rate confirmed Van de Sandt's (2007) observation that geometry is a difficult topic in South African secondary schools. This might mean that even though technology in the context of cooperative learning was effective, the topic under consideration was generally a challenge to learners, as the poor test results show. The poor results can also be attributed to the common errors made by the experimental group in post-test where 51,9% of the learners did not use the converse of

the theorem as shown in Table 4.4. This corroborates Luneta's assessment that most grade 12 learners in South Africa operate at van Hiele level 2, and grade 11 circle geometry requires learners to be at level 3 or 4 (Luneta, 2015).At van Hiele level 3, learners are expected to be able to work with properties of geometrical shapes in solving problems. This discrepancy remains a major concern in whatever efforts are made to improve performance in grade 11 geometry.

For the control group the average for the post-test (20%) dropped from that obtained in the pre-test (34%). For the experimental group the average rose in the post-test (34.6%) compared to that obtained in the pre-test (30.3%). This small rise might be due to the fact that most questions in the post-test were of higher cognitive levels (see Table 3.9) though these questions are consistent with the nature of questions contained in the CAPS document (DBE, 2011). This small increase of only 4.3% from pre-test average to post–test average in the experimental group showed that impact of the intervention was not great and can also be attributed to the observation by Delgado et al. (2015:397) when he said, "several inherent methodological and study design issues dampen the amount of variance that technology accounts for". In that article, the author was acknowledging that different studies have produced results differing on the effectiveness of using technology in the classroom. At times the impact was very minimal and of variant sizes as indicated in the meta-analyses of the effectiveness of technology (Delgado et al., 2015).

Learners' responses to different questions were of great interest in this study and the researcher analysed learner responses thoroughly as indicated in Table 4.4, to get an understanding of possible causes. The most common error observed was that learners were not able to prove a particular property of circles as shown in Figure 4.18(a) and 4.18(b) where learners ended up mixing up theorems and their converses. For example, questions 4.3.1 and 5.4 in the post-test required learners to prove that a quadrilateral was cyclic and that a line was a tangent to a circle respectively.

As indicated in Table 4.4, most learners in the control group and experimental group could not use the converse of theorems. For example, as shown in Figures 4.18(a) and 4.18(b), a solution of learner E_3 and C_{16} who are comparable in the pre-test is shown in Figure 4.18(a) and 4.18(b) respectively
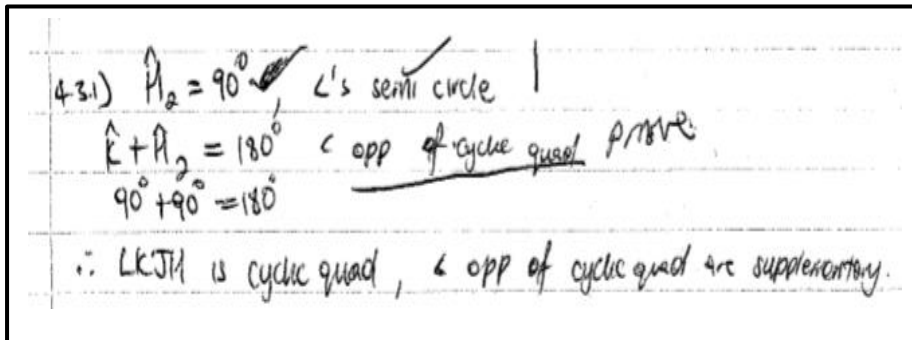
Figure 4.18 (a): Response of Learner C16 to Q4.3
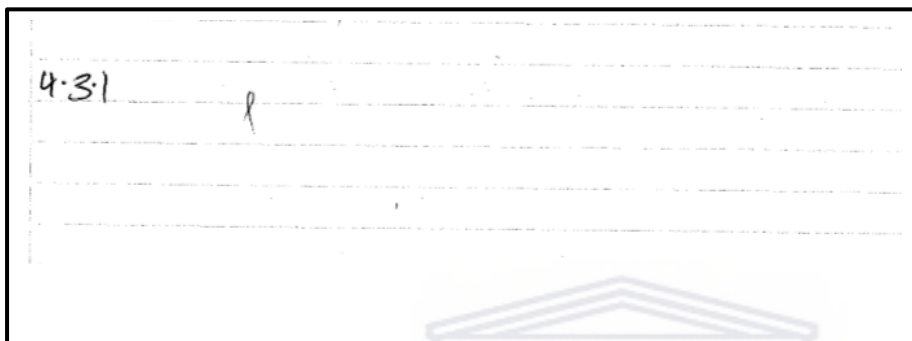
Figure 4.18(b): *Response* of Learner E3 to Q 4.3

The learner E_3 was able to get a mark from this question whereas learner C_{16} left the question blank. This showed that a learner in experimental group was at least able to identify the theorem at play although he did not understand that it was the converse of that theorem that was required. C_{16} had no clue and left the question blank. This observation can be linked to the positive effect of using technology in the context of cooperative learning to the experimental group. E_3 ended up improving in the post-test result to 40% whereas C_{16} dropped in performance to 14%. This is in line with observations of (Almeqdadi, 2000:166) when he said " students in the experimental group gained more scores from pre-test to the post-test, which refers to their use of computers and the GSP program". Learner E_{21} had the biggest jump from pre-test to post-test rising from 26% to 70% respectively and had the highest score in the post-test.

The error by E_3 of not using the converse of the theorem might be attributed to the fact that in teaching cyclic quadrilaterals, the teacher had emphasised the theorem at the expense of its converse. This observation concurs with Luneta's (2015) when he states that teaching a geometrical concept in a particular way incapacitates learners when the orientation of the shape is changed in the assessment. Luneta (2015) researched geometric shapes observing that

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learners understand the relationships of angles in Figure 4.19(b) better than those in Figure 4.19(a) because Figure 4.19(b) is the common orientation used in teaching the theorem of an angle at the centre being twice the angle at the circumference.
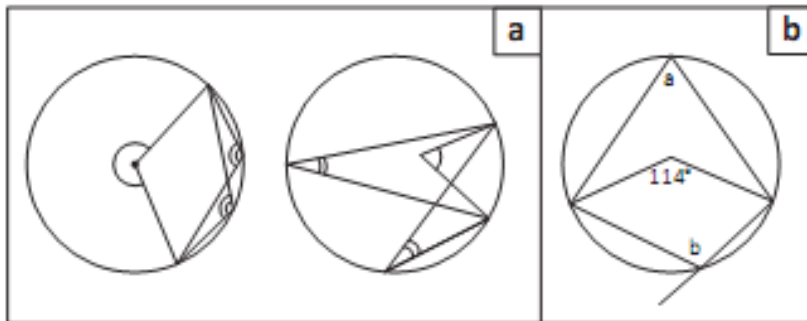
Figure 4.19: The angle subtended by a chord at the centre is twice the angle subtended by the same chord at the circumference.

Lack of mathematical foundation was also linked to this failure to prove a property. According to (van Hiele, 1986), learners can only move from one level to another through proper teaching and learning. In lower grades learners are taught how to prove, for example, they may be given this kind of item to solve:
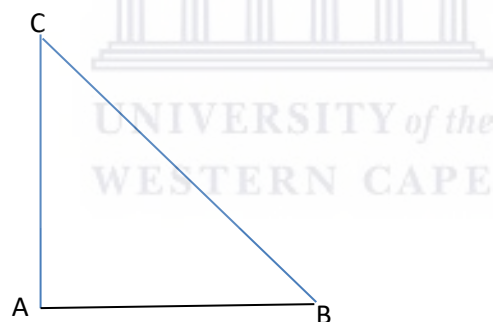
Given that $\hat{A} = 90°$, $AC = 3cm$, $BC = 5cm$, show that $AB = 4cm$.

<u>Solution</u>

In this question, leaners are taught to use the Pythagoras theorem to solve for AB. They are taught not to take the 5 and substitute where there is AB. If learners get this idea in lower grades, then the concept of using the converse to prove properties will make sense when they are now dealing with grade 11 Euclidean geometry.

Proving that a quadrilateral is cyclic can be pitched at cognitive level 3 or 4 depending on the number of connecting steps involved in the proof. (This is shown in the section on concept clarification in the CAPS document (DBE, 2011). There is deductive reasoning required in this

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proof where learners need to know what must be shown to prove that a quadrilateral is cyclic. If it is found, for an example that opposite angles are supplementary, then it can be deduced that the quadrilateral is cyclic because it will have met a sufficient requirement for it to be classified as such. The required reasoning poses a challenge to the majority of learners because of the cognitive deficit (Luneta, 2015). This lack of conceptual understanding was attributed to the way learners were taught in earlier grades.

The other error observed was the mixing up of theorems. Questions 2, 3.2, 3.3 and 5.3 registered this type of error frequently as shown in Table 4.4. For Question 2, 46.2% of control group and 11.1% of experimental group had made this mistake. Learners could not differentiate between angles in the same segment and an angle at the centre that is twice the angle at circumference. The low percentage of learners from experimental group who had this error can be attributed to visualisation that was brought by technology as indicated by Shadaan et al. (2013). Two learners C_3 and E_8 were comparable from the pre-test with scores of 36% and 38% respectively. In question 2 in the post-test, C_3 got 0/8 whereas E_8 got 4/8 marks. Their solutions are shown in Figures 4.20(a) and 4.20(b) respectively.
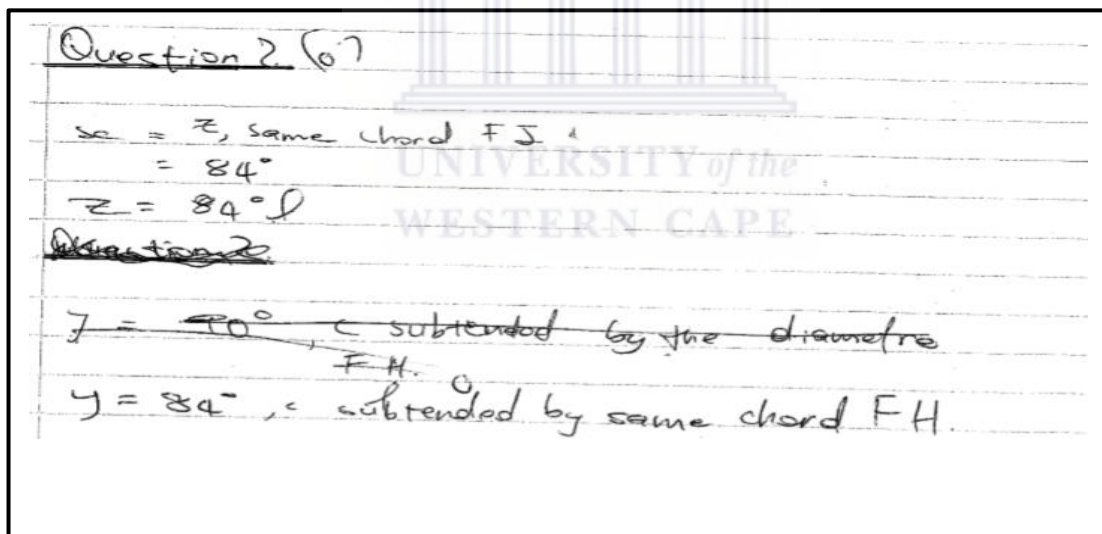

Figure 4.20(a): Response of   Learner C3 to Q 2
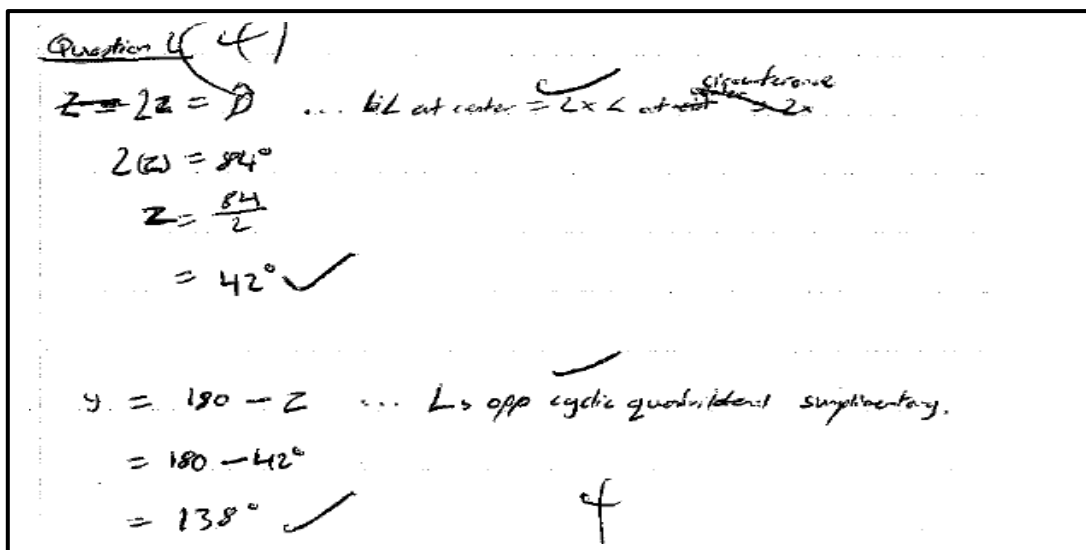
Figure 4.20 (b) Response of Learner E8 to Question 2

A sizable number of learners in the control group (46.2%) viewed any angles coming from the same chord as equal irrespective of whether these were at the centre or on the circumference. This can be attributed to the teaching style used. The fact that only 11.1% of experimental group made the same error meant that using technology in the context of cooperative learning helped learners to understand the differences between the theorems concerned. This was in line with the observation by Bhagat and Chang (2015) that using technology improved performance as learners improved visualisation skills and their ability to reason.

Leaving some questions blank was a common tendency amongst the learners. Questions 3.2, 5.3 and 5.4 were left blank mainly by learners in the control group, as depicted in Table 4.4; while learners in the experimental group fared better with room for improvement. This can also be linked to 2018 diagnostic report were learners could not identify an isosceles triangle in the circle (DBE,2018). Once learners fail to identify an important aspect, they will leave out the question or end up making own assumption leading to a mathematical breakdown (DBE,2018)

Question 3.3 required learners to combine their knowledge on isosceles triangles with the theorem on diameter and tangent. This question required complex procedures where learners were expected to connect a number of steps. This question was left blank or answered with no reasons supplied by 46.2% of the control group and 33.3% of experimental group. The DBE (2018) report highlighted the same observation where learners would not give or give an incomplete reason in answering the questions. Leaving a question blank showed a lack of knowledge for tackling problem solving. Moreover, writing a question and not supplying

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reasons is a procedural error that must be avoided in Euclidean geometry. However, the majority of learners in the control group made this procedural error. The DBE (2018) report suggested that learners must be encouraged to scrutinise diagrams and the information given for clues that can help in solving the problem. It is plausible that the use of technology in the context of cooperative learning helped reduce this type of error among the experimental group of learners. This outcome corroborated Zulnaidi and Zakaria's (2012:105) suggestion that "using Geogebra in the teaching and learning of mathematics could increase conceptual as well as procedural knowledge of students". For example, Heymath would show supporting reasons for every step in the calculation which is a procedure that cannot be omitted in Euclidean geometry. To add to their insight, in cooperative learning, learners would mark each other's work to check the accuracy of calculations and the supporting reasons provided.

In answering research question one, statistical tests led to the conclusion that using technology in the context of cooperative learning helped to improve learners' performance. This was borne out by the fact that the average for the experimental group turned out to be significantly higher than that of the control group average. However, a question by question analysis left the researcher critical of the effectiveness of the experiment, given the errors that emerged in the post-test. Some learners did not attempt to answer other questions and left them blank. Wrong reasons were provided for other questions and the use of converse theorems was a challenge for some. Different possible explanations have been provided to justify the results observed.

In addition to the preceding discussion, the researcher is of the view that the impact of the intervention was not fully effective probably because of the poor internet connection experienced during the research and the language of learning and teaching (LOLT) which is English was a barrier as it was not the home language for the learners. Poor internet connection resulted in the visual to take more than the normal time to upload and this caused learners not to have sufficient time to take full advantage of Heymath. Mastropieri, Scuggs and Graetz (2006) is of the view that second language readers at secondary school level face challenges in reading and comprehension to succeed. This could have impacted negatively to the learners of Khumbulani high school.

Research question two is addressed in the next section.

4.2.   Research Question Two:

To what extent, if at all, do learners exhibit changes in motivation to participate in answering grade 11 circle geometry questions when using technology within cooperative learning context?

This second question was answered using a questionnaire shown in Appendix 9. The questionnaire was given to the experimental group only, to assess their views on the topic after having been exposed to learning through technology in a context of cooperative learning. The researcher used the tally system to count the number of learners in the questionnaires who gave a particular option as their answer, for the purpose of analysis. The questionnaire results were used to answer only the second research question.

4.2.1   Questionnaire results presentation

Data collected from questionnaires was initially analysed using the tally system. This was to allow tabulation for more sophisticated analysis. Some questions in the questionnaire were grouped together for the purpose of analysis if they were testing the same concept in different ways. The results obtained are shown in Table 4.6. From Table 4.6, Questions 1 and 3 were ascertaining whether the Heymath programme was user friendly.

For question 1, about 52% of the learners strongly agreed that the programme was easy to use. No learner disagreed that the programme was user friendly. Question 3 was set to assess the clarity of diagrams in the programme. Forty four percent of the learners strongly agreed (SA) that Heymath makes circles very clear and only 8% of the learners were not happy with Heymath circles. If learners enjoyed using the programme, then they would be motivated to learn using Heymath.

The next group of questions were questions 4 and 5, testing learner understanding. About 48% of learners agreed (A) that the Heymath programme enhanced their understanding, with an additional 32% (SA) strongly agreeing that the programme was good for improving understanding. Only 4% of learners disagreed; claiming that Heymath was not helpful. For question 5 a total of 68% said they understood theorems better with Heymath than when using a textbook. Sixteen percent of the learners were not sure (NS) whether Heymath is different from their usual textbook. A total of 16% of the learners were on the negative side, disagreeing that Heymath is better than a textbook for learning geometry.

Questions 6 and 7 assessed the effect of simulations when using Heymath. 60% of the participants strongly agreed (SA) that the programme allows a dynamic investigation of the properties of circles. Another 32% simply agreed that the programme is good for investigating circles. For question 7, all the learners agreed that Heymath allowed them to see properties changing as they used the dragging facility.

Lastly, questions 2, 8 and 9 probed the level of motivation and confidence gained respectively. For question 8, a total of 68% indicated they had become more confident to participate when using the software, whilst 20% (NS) were not sure, and 12% disagreed that the programme had had an impact on their motivation.

In relation to question 2, a total of 76% acknowledged that Heymath had motivated them in learning circle geometry, compared to only 4% who were not motivated by the programme. For question 9 a total of 80% of learners agreed that the programme had helped them to interact with the teacher and fellow learners and this motivated learners to participate in learning. Only 8% disagreed that the programme helped them improve interaction in class.

Table 4.6: Results from questionnaires by experimental group

	Item	SD		D		NS		A		SA	
		N	%	n	%	n	%	N	%	n	%
1	Heymath was easy to use	0	0	0	0	2	8	10	40	13	52
2	Heymath motivates me when learning circle geometry	0	0	1	4	5	20	11	44	8	32
3	Heymath circles are very clear	0	0	2	8	2	8	10	40	11	44
4	Heymath helps me in understanding circle geometry	0	0	1	4	4	16	12	48	8	32
5	I understand the theorems better with Heymath than when using a textbook	1	4	3	12	4	16	6	24	11	44
6	Heymath allows the dynamic investigation of circle properties	0	0	0	0	2	8	8	32	15	60
7	When I use dragging in Heymath I can see which properties change	0	0	0	0	0	0	5	20	20	80
8	I am more confident to participate during the Heymath lesson than in a conventional lesson	0	0	3	12	5	20	13	52	4	16

9	I was able to interact with my teacher and group members during Heymath lessons	0	0	2	8	3	12	10	40	10	40

Key: SD=strongly disagree; D=disagree; NS= not sure; A= agree; SA= strongly agree

4.2.2   Discussion

Having taken the NS response from the questionnaire as the neutral point, I was able to see that most of the learners are on the positive side insofar as Heymath is concerned. This was in line with observations made by Shadaan et al. (2013) where 93% of the students were happy with student-teacher interactions using Geogebra. Very few learners were on the negative side – SD and D – in all responses to the questions asked. Question 2 was checking on motivation when using Heymath in circle geometry. It was found that 76% of the learners in the experimental group had positive sentiments about using the Heymath software. The other 20% were not sure whether Heymath brought motivation into working on geometry problems. Only 4% of the learners had negative comments about Heymath in bringing any motivation. It is therefore clear that Heymath brought motivation to learners. This observation was consistent with Bhagat and Chang (2015) in that mathematical learning software motivates learners. A similar conclusion was reached by Saha, Ayub and Tarmizi (2010). In addition Bhagat and Chang(2015) found that Geogebra was effective in teaching geometry to grade 9 learners and helped to rebuild lost interest in geometry as a result of poor performance.

The bar graph in Figure 4.21 provides a pictorial representation of the responses in the following Question 2.

Heymath motivates me when learning circle geometry.

☐Strongly do not agree ☐ Do not agree ☐ Not sure ☐Agree ☐Strongly agree

It is very clear from the picture that the majority of the learners in the experimental group agreed or strongly agreed. Therefore, technology through cooperative learning brought about motivation.

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Bar Graph representing responses on question 2
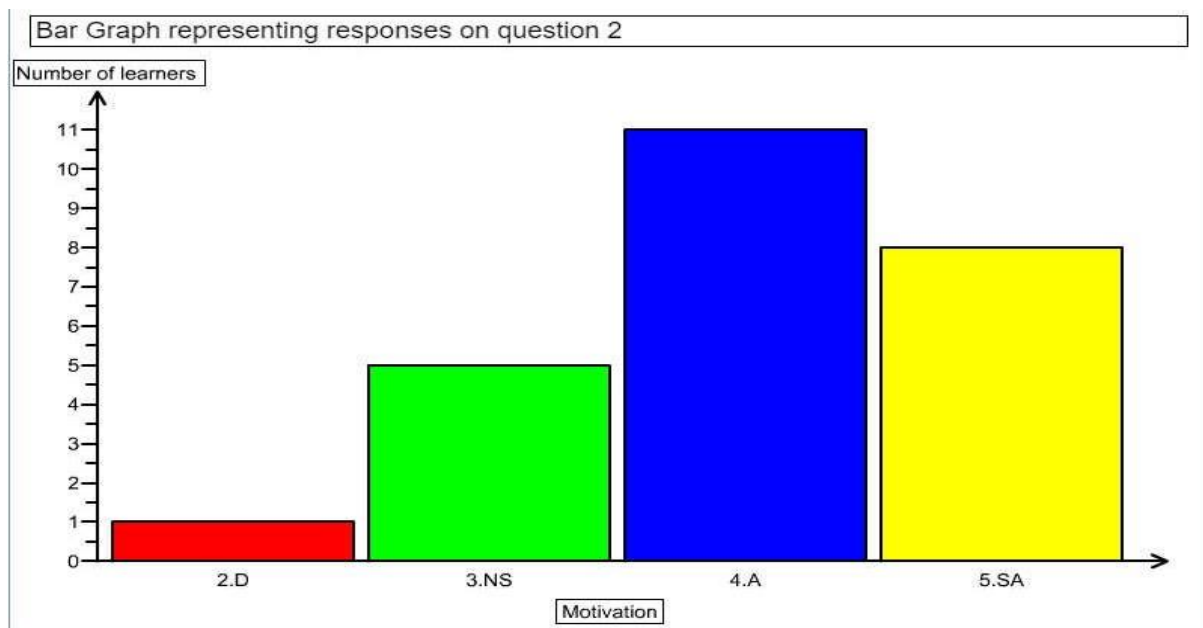
Figure 4.21: Responses in Q 2 of questionnaire

What emerges from Figure 4.21 shows that using technology in teaching and learning is instrumental in bringing motivation to learners maybe due to the rich constructivist learning environment created which is conducive for learning. This conclusion also concurs with Tieng and Eu (2013) when they noted that using GSP helped learners to be more creative in problem solving.

Question 4 on the questionnaire assessed the level of understanding brought about by using the software. It was found that 68% of the learners were happy that Heymath led to a better understanding of the geometry concepts. This improvement in learning could be a result of positive roles brought in by visualisation using technology and scaffolding in learning from cooperative learning. Only 12% had negative comments about Heymath enhancing understanding. The other 16% were not sure. This observation of improved understanding concurred with Tieng and Eu (2013) who observed an improvement in the experimental group after the use of GSP, according to the van Hiele levels of cognition. This meant that technology ultimately improved understanding. Also in relation to concept of understanding, Zulnaidi and Zakaria (2012:105) noted that Geogebra improved conceptual understanding and thereby improved performance.

Questions 8 and 9 gauged confidence and interaction levels affected by the software. From the answers obtained, it was clear that learners were able to interact to a greater extent with the

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educator and with their peers. A similar finding is that of Tieng and Eu (2013) who noted that learners were able to interact more with their counterparts and to share their ideas when they used GSP.

The main reason given by learners in this research was that Heymath allowed them to see the changes because of the dragging facility. Visualisation and experimentation allowed learners to develop analytical skills to a solution as explained by Jones and Bills (1998). Visualisation allowed learners to picture the problem in the mind. Some learners engaged the teacher for clarity on what they had observed. Some interacted with other learners, explaining and helping out with misconceptions, and this brought life to the geometry lesson. From the results observed, it was concluded that Heymath motivates, improves confidence and raises levels of interaction.

CHAPTER 5: CONCLUSION, IMPLICATIONS AND RECOMMENDATIONS

5.0   Introduction

This study was done to investigate the impact of using technology in the context of cooperative learning. Its aim was to find ways to improve the teaching and learning of grade 11 Euclidean Geometry. The researcher anticipated that teaching with technology combined with some aspects of cooperative learning would better results and motivate learners to learn as they progress through the topic. The purpose was to ascertain whether this idea was correct so that a better approach to teaching this topic would be found.

The results obtained in this study are presented in Chapter 4, followed by detailed analysis and discussion. This chapter summarises the findings in relation to the research questions, the discussion and the literature review. Recommendations to various stakeholders in education are proffered as a contribution towards improving the quality of teaching and learning in the field of geometry specifically, and mathematics more generally.

5.1   Main Findings

The findings revealed that learners enjoy working with technology and given access to it, can learn aspects of technology quickly. The use of Heymath in the context of cooperative learning improved performance. It was found that the net performance of learners improved but this improvement was however marginal and not in line with expectations. This limited marginal increase may be due to reasons explained under the findings described in relation to each research question. The intervention improved the level of motivation to learn geometry as shown by positive responses in Figure 4.21. The findings are discussed sequentially.

5.1.1 Research question one

How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 circle geometry?

The findings for this question from testing are summarised as follows:

(a) The t-test showed that the experimental group performed significantly better than the control group. It was noteworthy that both groups had poor averages of 20% and 34.6% for the control and experimental groups respectively.

(b) The intervention with the experimental group reduced the number of procedural errors in the answering of questions. Learners in the experimental group were able to give reasons for their steps more cogently than the learners in the control group were able to. This is in line with what De Villiers (2003) hinted when he said, there is no reason for denying students the chance to explore conjectures and results experimentally when adult mathematicians quite often use such activities in their own research?

(c) The intervention improved learners' ability to differentiate between theorems. The experimental group was able to differentiate the angle at the centre theorem from the angles in the same segment better than the control group.

(d) It was also noticed that between their pre-test and post-test results there was not much difference in the performance of the two groups. The control group declined in performance in the post-test, though this was not statistically significant. The experimental group had a better average in the post-test although this increase too was statistically not significant. This conclusion was drawn from the paired difference test using the t-distribution. The drop in performance of the control group in the post-test was attributed to the fact that the post-test covered concepts in geometry that are known to be difficult for learners. Lack visualisation and experimentation could have played a role in bringing down results in the post-test. Proving a concept is a challenge to learners if they have a poor understanding of the other concepts involved. The experimental group had an increase in percentage for class average in the post-test and this could only be attributed to the use of Heymath that enhanced their understanding.

5.1.2 Research question two

To what extent, if at all, do learners exhibit changes in motivation to participate in answering grade 11 circle geometry questions when using technology within a cooperative learning context?

The findings from questionnaires are summarised in the points that follow.

(a) The learners in the experimental group benefited from using Heymath as a number of them claimed it enhanced their understanding.

(b) It was also found that Heymath motivated learners as they found joy in using the programme.

(c) Technology introduced confidence to interact with the teacher and fellow learners. Interaction was achieved though working in groups to complete a given task. Most learners attributed these developments to the dragging facility in Heymath. For Questions 9 and 10, most learners noted a positive attribute in using Heymath. Most learners claimed that Heymath had allowed them to "see" what the teacher was teaching.

(d) Visualisation allowed learners to connect theorems with reality – to perceive them practically. In response to Question 11, some learners said that they could see, pause, think and continue working with the concept as they interacted with Heymath features.

5.2   Implications of the study for teaching and learning of geometry

5.2.1   Implications for practice

Based on the findings of this research, there are a number of implications that can be drawn for learners, teachers, subject advisors and curriculum planners and the government.

- From 5.1.1 (a), technology helps learners to learn better and as was concluded by Shadaan and Kwan Eu (2013:8) when the said "students experienced a hands on method of learning" after they used Geogebra. This It is important that educators and subject advisors incorporate technology in their planning. The government must make the provision of technology a priority. Government must make sure that internet access and connectivity is be improved in schools.

- 5.1.1 (d) has shown that proof pose a challenge to learners if there are gaps in their understanding. It is vital for teachers to consolidate what has been observed and discussed by learners in their groups to have a deeper understanding of proof and geometry.

- 5.1.2 (d)Teachers and learners should not underestimate roles of visualisation and experimentation in learning as explained by Jones and Bills (1998) when they said visual representations are a requirement of every classroom. Visual representation brings reality in learning through pictures and diagrams and therefore enhances understanding. As learners use the dragging facility in Heymath, they will end up learning a theorem through experimentation and visualisation as suggested in the 2017 NSC report that teaching should achieve real understanding (DBE, 2017).

- From 5.1.2 (b), using technology in the context of cooperative learning brings motivation to learn. This positive attitude will in turn improve academic performance for the learners.

5.2.2 Implications for researchers

There are some implications for further research resulting from this study:

- Researchers must fully understand the strength of internet connectivity for the schools where they want to carry out the study as this will impact on their findings. If this connectivity is not strong, then the offline version of Heymath must be used.

- More time must be given to the research to fully benefit from the many activities in Heymath that cannot be covered over the allocated time in CAPS. This is also in line with recommendations from DBE (2017:173) diagnostic report that " more time needs to be spent on the teaching of Euclidean geometry in all grades".

5.3   Recommendations

5.3.1   For Teachers of Mathematics

There are several suggestions that teachers can draw from this study:

- Proper planning in using technology in the context of cooperative learning is recommended. When learners are in groups, some learners do not cooperate, and the concept of cooperative learning will lose meaning. Learners also end up on social platforms if not properly supervised. This will further waste the limited time available for this topic. This is the same advice given by Herrmann (2013) when he said teachers must be aware that cooperative learning does not imply engagement if not planned properly. According to Herrmann (2013), some learners feel that cooperative learning takes away the teacher from explaining concepts. There the teacher must plan lesson properly to avoid frustrating some other learners.

- Using Heymath in the context of cooperative learning helps learners to construct own learning as advocated for in constructivism and in the results of this study. Teachers are encouraged to use Heymath in the teaching of geometry because it allows for simulation as learners use the dragging facility to discover the invariant which becomes the new learnt theorem.

Teachers can improve classroom management by getting learners involved in the learning by using technology in the context of cooperative learning. This study found that motivation levels improve with this method and thereby increasing concentration resulting in improved learning.

5.3.2 For Curriculum Planners and Subject advisors

- Curriculum planners and subject advisors are also encouraged to use the positive findings from this study to develop Pacesetter for teaching of Euclidean geometry. The three weeks allocated to the teaching of this topic in the CAPS document is not enough as it does not allow learners to spend time on using technology, which may help them to make own discoveries and to enhance their learning within a constructivist environment.

5.3.3 For Further research

This study was done under difficult conditions. Initially, the researcher wanted to use an offline version of Heymath but ended up using the online version because licence for the offline version had expired. This was the major source of problems encountered in the research.

- It is thus recommended that this study be conducted using the offline version as it has all the features found in the online version. Using the offline version would eliminate all the problems of internet connectivity encountered in this research. (The offline version would have allowed learners to continue working at home).

- It is also recommended that more time be given to the experimental group as Heymath activities generally require time. Having limited time to work could have poor impact on using technology through cooperative learning.

- The sample for the research can also be expanded to compare schools in the urban suburb, or even to include the whole country if resources permit. Such a sample will be more representative of the population of South Africa than one limited to learners from a particular community. It might also be of great interest to take the interventions separately to see which one has the greater impact, that is, taking cooperative learning and technology separately to see which one has greater impact so that more effort can be put into the one with greater impact.

5.4 Limitations of the study

As the truth or falsity of a hypothesis will never be known completely unless the whole population is examined (Walpole & Myers, 1985), the researcher knew that even though he had used the scientific approach in the research, he was not able to generalise the findings for

all contexts. This was firstly because the sample was not representative of the entire population of South Africa. It was a sample chosen from a disadvantaged social area in the Cape Flats and thus could not be representative of the whole country.

Secondly, the research was conducted under difficult conditions as the school's internet was very slow, making it difficult to access the online Heymath. Learners were therefore not able to make full use of the programme because of the poor internet connection which also affected the results. Once the mathematics lessons were over, accessing the internet became a challenge for the experimental group. It was even worse when they were at home as they had limited access to both the internet and to devices. Whenever the internet was inaccessible during the Heymath lesson this meant that the objectives for the day were not met. The offline version would have allowed learners to continue working with the programme at home as they do their homework.

According to Delgado et al. (2015), since the beginning of the digital age, there have been efforts to improve learner access to resources. This is because if the number of technological devices in the classroom is limited or their use unduly restricted, the implementation of e-learning becomes very difficult. If every learner has a personal computer, learning will take place efficiently as there will be more time for each learner on the device.

Bebell and Kay (2010) postulate that it is actually impossible to measure the impact of technology on learning if some learners do not have access to the technology on time. This is exactly the situation I encountered in my research where internet and access to tablets was not easy during teaching, and this problem might have impacted negatively on my research

5.5   Conclusion

This convergent parallel mixed methods study, which adopted a quasi-experimental design investigated the impact of the blended use of technology and cooperative learning on learners' performance on grade 11 circle geometry. The research took place at a Khayelitsha school and the scope of technology was limited to using a mathematical computer programme called Heymath. Grounded in social and cognitive constructivism views of mathematics teaching and learning, as well as the DBE cognitive level framework for setting of examination papers, this study through the lens of a positivist paradigm initially subjected two comparable groups (control and experimental) of grade 11 learners to completing a geometry pre-test.

The control group consisted of 26 grade 11 learners and the experimental group consistent of 27 grade 11 learners. Then the experimental group (E) was taught circle geometry using technology in the context of cooperative learning while the control group (C) was taught using conventional methods. Thereafter data was collected via a geometry post- test from both groups. Through statistical analysis (use of t-test) it was found that using technology to teach in a context of cooperative learning improves learners' performance. The qualitative content analysis provided a deeper exposition of pertinent errors such as the non-use of a diagram in the development and presentation of a proof of a theorem.

Finally, the experimental group completed a questionnaire designed to ascertain the extent to which learners exhibit changes in motivation in answering grade 11 circle geometry questions when afforded the use of technology within a cooperative learning environment. The results emanating from the statistical analysis showed positive changes in motivation in learners wanting to participate in answering grade 11 circle geometry questions when using technology within a cooperative learning context. Learners using technology through cooperative learning tended to participate not only more but with a greater sense of confidence. This could be attributed to them having more opportunity to experiment through dragging objects in a dynamic environment, observing visually what relationships remains invariant and what changes with a greater degree of conviction, and finally communicating what they actually saw and not what they only heard.

The results emanating from this study is of great significance. It is suggested that teachers, subject advisors, curriculum planners should take greater responsibility to incorporate the use of technology, like Heymath, jointly with cooperative learning approaches in the facilitation of teaching and learning of circle geometry across our classrooms. It is hoped that technology used in the context of cooperative learning will lead to the improvement of mathematical competence in general and competence in Euclidean geometry in particular.

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APPENDICES

This section contains examples of all instruments that were used in this research. It consists of appendix 1 to appendix 11b. Appendix 1 is an example of the background information sheet and appendix 11b is the post-test marking memorandum used in the research

APPENDIX A: Permission from the Western Cape Education Department

Western Cape Government
Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za
tel: +27 021 467 9272
Fax: 0865902282
Private Bag x9114, Cape Town, 8000
wced.wcape.gov.za

REFERENCE: 20180820–5460
ENQUIRIES: Dr A T Wyngaard

Mr William Shonhiwa
68 Watercrest Street
Watergate Village
Mitchells Plain
7785

Dear Mr William Shonhiwa

RESEARCH PROPOSAL: THE IMPACT OF USING TECHNOLOGY THROUGH COOPERATIVE LEARNING ON LEARNERS' PERFORMANCE IN GRADE 11 CIRCLE GEOMETRY

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **07 September 2018 till 28 September 2018**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

> **The Director: Research Services**
> **Western Cape Education Department**
> **Private Bag X9114**
> **CAPE TOWN**
> **8000**

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 22 August 2018

Lower Parliament Street, Cape Town, 8001
tel: +27 21 467 9272   fax: 0865902282
Safe Schools: 0800 45 46 47

Private Bag X9114, Cape Town, 8000
Employment and salary enquiries: 0861 92 33 22
www.westerncape.gov.za

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APPENDIX B: Permission from UWC Senate Research Committee

OFFICE OF THE DIRECTOR: RESEARCH
RESEARCH AND INNOVATION DIVISION

Private Bag X17, Bellville 7535
South Africa
T: +27 21 959 4111/2948
F: +27 21 959 3170
E: research-ethics@uwc.ac.za
www.uwc.ac.za

10 September 2018

Mr W Shonhiwa
Faculty of Education

Ethics Reference Number: HS18/7/9

Project Title: The impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry at a Khayelistha school

Approval Period: 31 August 2018 - 31 August 2019

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval.

Please remember to submit a progress report in good time for annual renewal.

The Committee must be informed of any serious adverse event and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

PROVISIONAL REC NUMBER - 130416-049

FROM TOPE TO ACTION THROUGH KNOWLEDGE.

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 1: Background information sheet

Dear Sir/Madam

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at a Khayelitsha school.

Research Title: The impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry at a Khayelitsha school.

The research study is guided by the following research questions:

- How does the use of technology within a cooperative learning context impact on learners' understanding of grade 11 circle geometry?
- To what extent, if any, do learners exhibit changes in motivation to participate in answering grade 11 circle geometry questions when using technology within a cooperative learning context?

The research participants will comprise of Grade 11 Mathematics Learners from a secondary school in Western Cape. Data collection will be in the form of testing and questionnaires with 52 grade 11 learners.

Participation in this study is voluntary. Participants have the right to withdraw from the research at any stage of the research process without giving any explanations should they feel uncomfortable with this research. Participants are guaranteed utmost confidentiality regarding all information collected from them. Pseudonyms will be used to protect their identity.

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The researcher will make all the research information and correspondence available to each participant (learner) and their parents in English language.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: ……………………………   Date: …………………

APPENDIX 2: Permission letter

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 2: PERMISSION LETTER

THE Western Cape EDUCATION DEPARTMENT

The Research Director

Western Cape Education Department

P/Bag X9114

Cape Town

Dear Dr Wyngaardt

Re: Permission to conduct research at Centre of Science and Technology

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at a Khayelitsha school.

I would like to request your permission to engage with the learners by testing in the form of a post-test. I will also give them questionnaires to get information regarding their experiences in solving problems using technology in the context of cooperative learning. This is aimed at checking if using technology through cooperative learning raised the level of motivation to participate during the mathematics lesson.

The research will not disrupt the class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes

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other than to understand the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: ………………………………   Date: …………………

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 3: PERMISSION LETTER

The Principal

Centre of Science and Technology
P.O Box 112
Ilitha Park
Khayelitsha
7784
South Africa

Dear Madam

Re: Permission to conduct research at your school

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at Centre of Science and Technology.

I would like to request your permission to engage with the learners by testing in the form of a post-test. I will also give them questionnaires to get information regarding their experiences in solving problems using technology in the context of cooperative learning. This is aimed at checking if using technology through cooperative learning raised the level of motivation to participate during the mathematics lesson.

The research will not disrupt the class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes

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other than to understand the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: ………………………………   Date: …………………

APPENDIX 4: Permission letter

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 4: PERMISSION LETTER

THE HEAD OF DEPARTMENT

Centre of Science and Technology
P.O Box 112
Ilitha Park
Khayelitsha
7784
South Africa

Dear Sir

Re: Permission to conduct research at your school

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at Centre of Science and Technology.

I would like to request your permission to engage with the learners by testing in the form of a post-test. I will also give them questionnaires to get information regarding their experiences in solving problems using technology in the context of cooperative learning. This is aimed at checking if using technology through cooperative learning raised the level of motivation to participate during the mathematics lesson.

The research will not disrupt the class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes

other than to understand the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: ………………………………   Date: …………………

UNIVERSITY *of the*

WESTERN CAPE

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 5: PERMISSION LETTER

GRADE 11 TEACHER(S)

Centre of Science and Technology
P.O Box 112
Ilitha Park
Khayelitsha
7784
South Africa

Dear Sir/Madam

Re: Permission to conduct research in your school

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at Centre of Science and Technology.

I would like to request your permission to engage with the learners by testing in the form of a post-test. I will also give them questionnaires to get information regarding their experiences in solving problems using technology in the context of cooperative learning. This is aimed at checking if using technology through cooperative learning raised the level of motivation to participate during the mathematics lesson.

The research will not disrupt the class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes

other than to understand the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: ………………………………   Date: …………………

UNIVERSITY *of the*

WESTERN CAPE

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FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 6: PERMISSION LETTER

THE PARENTS

Dear Parent/Guardian

Re: Permission for your child's participation in research

My name is **William Shonhiwa,** a Masters student in the Mathematics Education Department of the Faculty of Education at the University of the Western Cape. I am conducting research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. The target group will be Grade 11 mathematics learners at Centre of Science and Technology.

I would like to request your permission to engage with the learners by testing in the form of a post-test. I will also give them questionnaires to get information regarding their experiences in solving problems using technology in the context of cooperative learning. This is aimed at checking if using technology through cooperative learning raised the level of motivation to participate during the mathematics lesson.

The research will not disrupt the class schedules or teaching and learning in the classroom. In addition, participation will be voluntary, so participants will be free to withdraw at any time without giving reasons should they feel uncomfortable with my research. The identity of the learners in the study will remain anonymous. Information received as part of the study will be used for my research purposes only. It will not be used in any public platform for any purposes other than to understand the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry.

Should you wish to find out more about the research, you are welcome to contact my supervisor, **Prof Rajendran Govender,** whose contact details are provided below.

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Yours sincerely,

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

Signature of the researcher: …………………………………   Date: …………………

APPENDIX 7: Parent/Guardian's informed consent form

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 7: PARENT/GUARDIAN'S INFORMED CONSENT FORM

I ……………………………………………………….. agree/disagree to allow my son/daughter to be part of your study. I am aware that my son/daughter's participation in this study is voluntary. If, for any reason, I wish to stop my son/daughter from being part of this study, I may do so without having to give an explanation. I understand the intent and purpose of this study.

I am aware the data will be used for a Master's thesis and research paper. I have the right to review, comment on, and/or withdraw information prior to the paper's submission. The data gathered in this study is confidential and anonymous with respect to my son/daughter's identity, unless I specify or indicate otherwise. In the case of classroom test and questionnaire, I have been promised that my son/daughter's identity and that of the school will be protected, and that my son/daughter's duties will not be disrupted by the researcher.

I have read and understood the above information. I give my consent for my son/daughter to participate in the study.

_____   _____

Parent/Guardian's Signature   Date

_____   _____

Researcher's Signature   Date

Researcher: Mr William Shonhiwa   Supervisor: Dr Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

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APPENDIX 8: Assent letter from learners

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 8: Assent letter from learners

I ………………………………………………………….. agree to be part of the study and I am aware that my participation in this study is voluntary. If, for any reason, I wish to stop being part of this study, I may do so without having to give an explanation. I understand the intent and purpose of this study.

I am aware the data will be used for a Master's thesis and research paper. I have the right to review, comment on, and/or withdraw information prior to the paper's submission. The data gathered in this study is confidential and anonymous with respect to my identity, unless I specify or indicate otherwise. In the case of classroom test and questionnaire, I have been promised that my identity and that of the school will be protected, and that my duties will not be disrupted by the researcher.

I have read and understood the above information. I hereby give my consent to participate in the study.

_____   _____

Learner's Signature   Date

_____   _____

Researcher's Signature   Date

Researcher: Mr William Shonhiwa   Supervisor: Prof Rajendran Govender

Contact number: 076 816 8155   Tel: 021-9592248

Email: williamshonhiwa@yahoo.com   Email: rgovender@uwc.ac.za

FACULTY OF EDUCATION
UNIVERSITY OF THE WESTERN CAPE
Private Bag X17, Bellville, 7535, South Africa

APPENDIX 9: RESEARCH INSTRUMENT – QUESTIONNAIRE

THE QUESTIONNAIRE

I am a Masters student from the University of Western Cape carrying out a research on the impact of using technology through cooperative learning on learners' performance on grade 11 circle geometry. May you please help in the survey by completing the questions below. Your responses will be kept confidential. You can only compete this questionnaire if you have signed the assent letter to participate in the study. You are also free to withdraw from the study at any time should you wish to do so.

Please put a tick (✓) in the box next to the answer of your choice or write in the spaces provided where applicable.

General information

- Gender: Male   ☐   female ☐

- Age: 13-14 ☐   15-16 ☐   17-19 ☐

Questions

1. Heymath was easy to use.
   ☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

2. Heymath motivates me when learning circle geometry.
   ☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

3. Heymath circles are very clear.
   ☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

4. Heymath helps me in understanding circle geometry.
   ☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

5. I understand the theorems better with Heymath than using text book.

☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

6. Heymath allows the dynamic investigation of circle properties.

☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

7. When I use dragging in Heymath, I can see which properties change.

☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

8. I am more confident to participate during the Heymath lesson than in conventional lessons.

☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

9. I was able to interact with my teacher and group members during the Heymath lesson.

☐ Strongly do not agree ☐ Do not agree ☐ Not sure ☐ Agree ☐ Strongly agree

10. If your answer to question 9 is agree or strongly agree. Explain why.

....

……………………………………………………………………………………………

……………………………………………………………………………………………

11. Write down advantages, if any, of the dragging facility in Heymath in learning circle geometry over just using a textbook.

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APPENDIX 10A: Pre-test

Grade 11 Pre-Test   Topic: Circle Geometry

Time: 1 hour   Date: 11 September 2018

Marks: 50

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Instructions

- Show all working
- Use only acceptable reasons

Question 1

1.1   In the diagram alongside, AB is a chord

of a circle with centre O.

ON ⊥ AB and cuts AB at M

and meets the circle at N.

If MN = 30 mm and the radius

of the circle is 150 mm.

Calculate, with reasons, the length of AB.
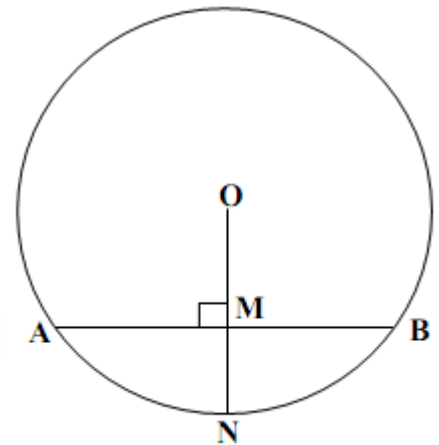
(5)

1.2   In the diagram, O is the centre of the circle and P is a

point on the circumference of the circle. Arc AB

subtends $A\hat{O}B$ at the centre of the circle and $A\hat{P}B$ at the

circumference of the circle.
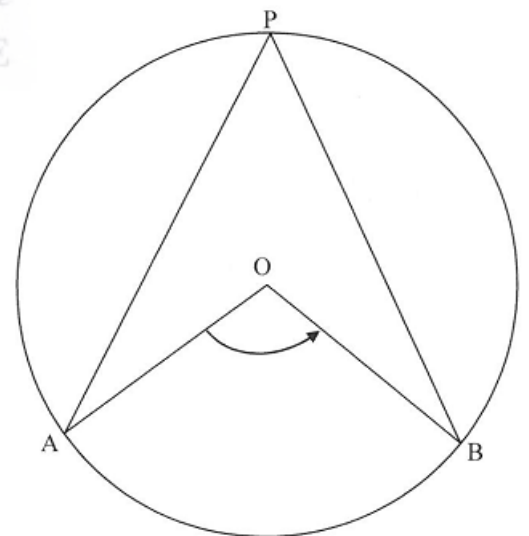
Use the diagram to prove the theorem that states that $A\hat{O}B = 2A\hat{P}B$.   (5)

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1.3   O is the centre of a circle.

A, B, C, D are points on the circle.

Given $A\hat{C}O = 33°$, find: $A\hat{B}C$ (5)
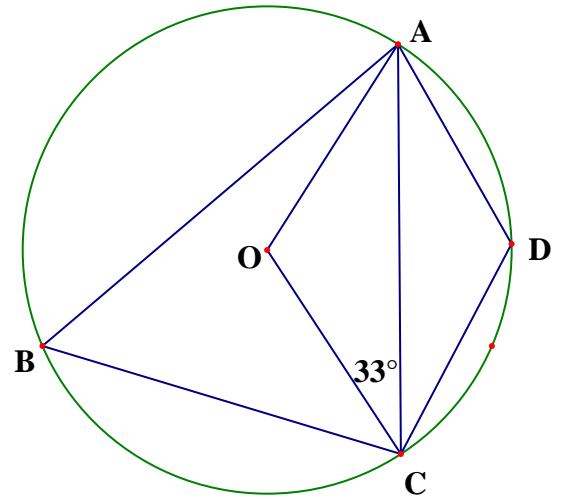
[15]

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Question 2

In the diagram alongside, O is the centre of circle
RMPS. T is the midpoint of RM. $\hat{R} = 50°$
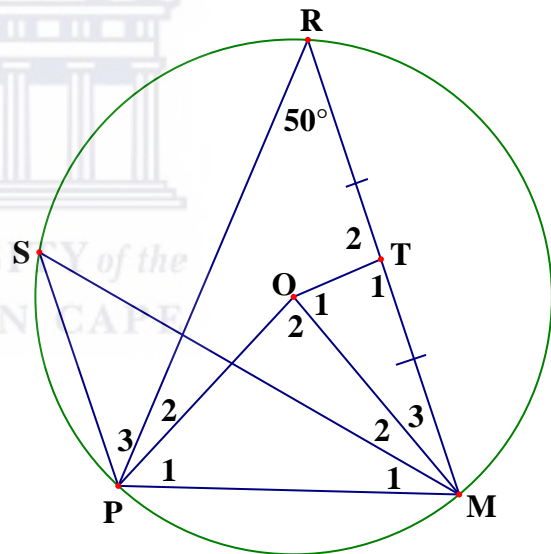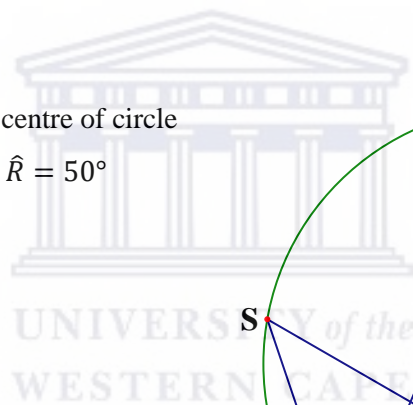
Calculate, with reasons, the value of the
following:

2.1 $\hat{T}_1$ (2)

2.2 $\hat{O}_2$ (2)

2.3 $\hat{S}$ (2)

2.4 $\hat{P}_1$ (2)

[8]

_____

QUESTION 3

3.1 Complete the statement below by filling in the missing word(s) so that the statement

is CORRECT:

The angle subtended by a chord or arc at the centre of a circle is … (1)

3.2 In the figure below, O is the centre of the circle and $PT = PR$.

Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$.
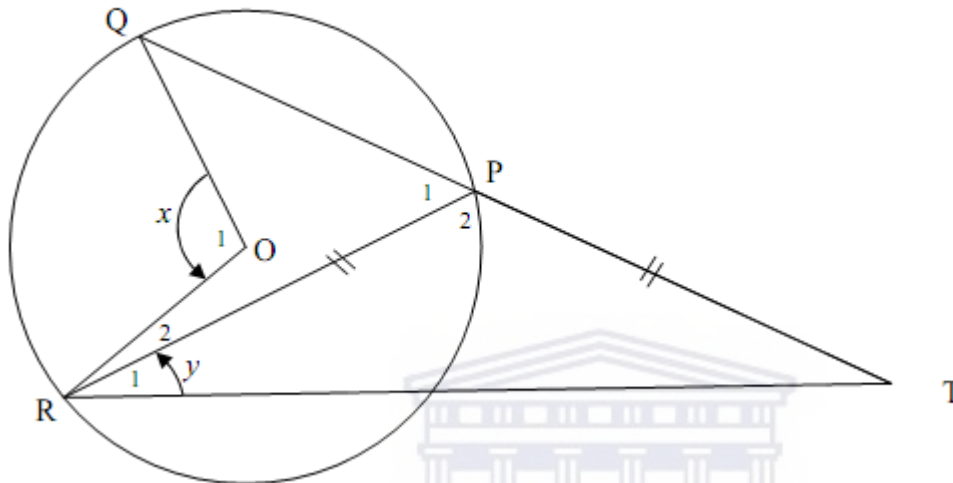
3.2.1 Calculate $\hat{P}_2$ (2)

3.2.2 Express x in terms of y. (2)

3.2.3 If $TQ = TR$ and $x = 120°$, calculate the measure of:
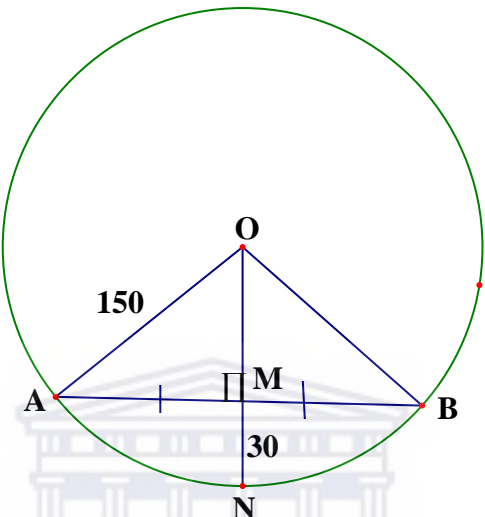
(a) y (2)

(b) $\hat{R}_2$ (Hint: Draw QR) (4)

[11]

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QUESTION 4

In the diagram, O is the centre of the circle. A, B, C and D are points on the

circumference of the circle. Diameter BD bisects chord AC at E. Chords AB, CD and

AD are drawn. $\hat{C} = 43°$.
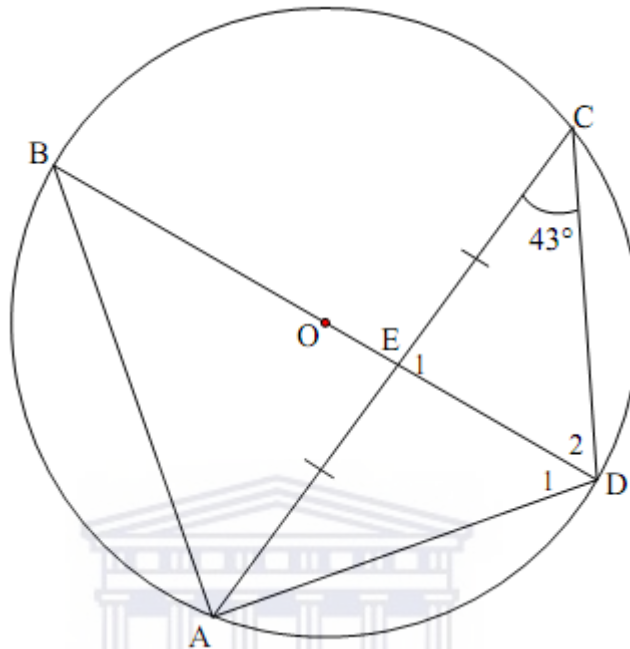
4.1   Give a reason for $DE \perp AC$.   (1)

4.2   Calculate, giving reasons, the size of $\hat{B}$.   (2)

4.3   Prove that $\hat{E}_1 = B\hat{A}D$.   (2)

4.4   The length of the diameter of the circle is 28 units. Calculate the length of AB.   (2)
       .   **[7]**

_____

Question 5

In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle and CB is the diameter of the circle. Chord CA intersect radius OD at E. AB is drawn. $CD \parallel OA$ and $\hat{A}_2 = x$.
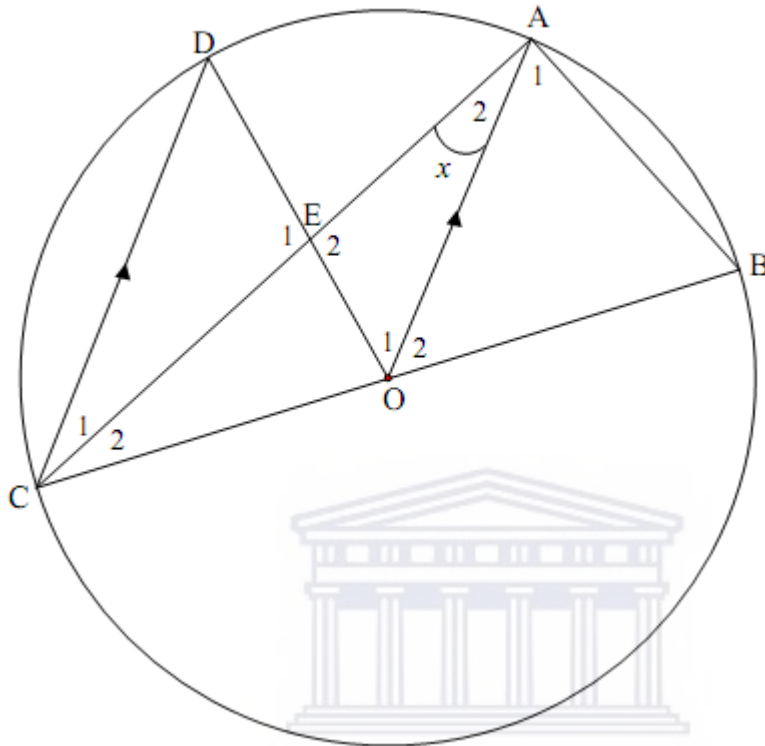
5.1 Give reasons why

     5.1.1 $\hat{C}_1 = x$ (1)

     5.1.2 $\hat{C}_2 = x$ (1)

5.2 Determine, giving reasons, the size of the following angles in terms of x.

     5.2.1 $\hat{A}_1$ (3)

     5.2.2 $\hat{O}_1$ (2)

     5.2.3 $\hat{O}_2$ (2)

                                                                         [9]

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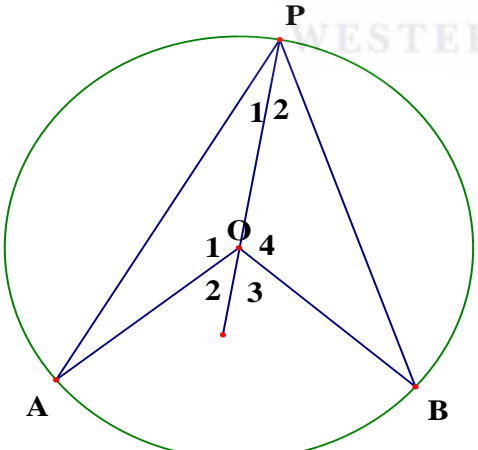
Marks: 50   11 September 2018

Question 1

1.1		
	$ON = radius = 150mm$ $\therefore OM = 120mm$✓ …pythagoras theorem $In\ \Delta AOM:$ $AM^2 = AO^2 - OM^2$✓ $= 150^2 - 120^2$ $\therefore AM = 90m$✓ $AB = 180mm$✓$line\ frm\ centre\ to\ midpt\ of\ chord$✓	
		(5)

In the figure: circle with centre O, radius 150, chord AB with midpoint M, ON = 30, N on circle.

1.2		
	Let $\hat{P}_1 = x$ $\hat{A} = x$….∠ opposite = sides✓ $\hat{O}_2 = 2x$ exterior ∠ $of\ \Delta$✓ $\hat{P}_2 = y$	

	$\hat{B} = y$ ....∠ opposite = sides✓	
	$\hat{O}_3 = 2y$ exterior ∠ $of \Delta$✓	
	∴ $\hat{O}_{2+3} = A\hat{O}B = 2y + 2x = 2(x + y)$✓	
	$= 2\hat{P}$......proved	(5)
1.3	$OA = OC$ ….. both radii✓	
	$\hat{A} = 33°$....∠ opposite = sides✓	
	$A\hat{O}C = 180° - 2 \times 33° = 114°$ … sum of ∠ $of \Delta$ ✓	(5)
	∴ $A\hat{B}C = \frac{1}{2} \times 114° = 57°$✓ ...∠ $at\ centre = 2 \times$ ∠ $at\ circum$✓	. [15]

Question 2

2.1	$T_1 = 90°$✓ line from centre to midpt chord✓	. (2)
2.2	$O_2 = 100°$✓ ∠ at centre✓	. . (2)
2.3	$\hat{S} = \hat{R} = 50°$ ✓ ∠s in same segment ✓	(2)
2.4	$\hat{P}_1 + P\hat{M}0 = 180° - 100°$ ∠s of Δ✓	. (2)
	$2\hat{P}_1 = 80°$ radii	[8]
	$\hat{P}_1 = 40°$✓	

Question 3

3.1	equal to twice the angle subtended by same chord at the circumference.	answer . (1)
3.2.1	$\hat{T} = y$ … … … …∠ $opposite = sides$✓	. (2)
	$\hat{P}_2 = 180° - 2y$ ……. $angle\ sum\ of\ \Delta$✓	
3.2.2	$\hat{P}_1 = 2y$ …….∠ $on\ a\ stright\ line$✓	Angle and reason✓ Answer and reason✓ (2)

	and $\hat{O}_1 = 2\hat{P}_1$ .....(angle at centre......)$i.e \ x = 2(2y) = 4y$✓	
3.2.3(a)	From 6.2.1 $x = 4y = 120°$✓ $\therefore y = 30°$✓	(2)
3.2.3(b)	$Join \ Q \ to \ R \ and \ let \ Q\hat{R}P = \hat{R}_3$ $\hat{T} = y = 30°$✓ $but \ T\hat{Q}R = T\hat{R}Q \ldots\ldots \angle \ opposite = sides$ $T\hat{Q}R = T\hat{R}Q = \dfrac{180° - 30°}{2} = 75°$✓ Now$\hat{R}_1 + \hat{R}_2 + \hat{R}_3 = 75°$✓ $i.e \ 30° + \hat{R}_2 + 30° = 75°$ $\therefore \hat{R}_2 = 15°$✓	(4) **[10]**

Question 4

4.1	Line from centre to midpoint of chord OR Line from centre bisects chord	✓ reason	(1)
4.2	$\hat{B} = 43°$ [∠s in same segment]	✓S ✓R	(2)
4.3	$B\hat{A}D = 90°$ [∠ in semi-circle] $= \hat{E}_1$	✓S ✓R 	(2)
4.4	$BD = 28 \ units$ $\therefore \cos 43° = \dfrac{AB}{28}$ ✓ $\therefore AB = 28\cos 43° = 20.48 \ units$✓	✓correct ratio ✓answer .	(2) **[7]**

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https://etd.uwc.ac.za/

Question 5

5.1.1	alternate ∠s; CD || OA	✓R	(1)
5.1.2	CO = OA [radii] ∠s opp equal sides	✓R	(1)
5.2.1	$C\hat{A}B = 90° \dots.. \angle in\ semi\ circle$ $\therefore\ \hat{A}_1 = 90° - x$	✓S ✓R ✓S	(3)
5.2.2	$\hat{O}_1 = 2x$ [∠ at centre = 2× ∠ at circumference/midpoints∠	✓S ✓R	(2)
5.2.3	$\hat{O}_2 = 2x✓ \dots…. ext\ \angle\ of\ \Delta ACO✓$ OR $O_2 = 2x✓ \dots…. \angle\ at\ centre = 2 \times \angle\ at\ circum✓$	✓S ✓R	(2) [9]

APPENDIX 11a: Post-test

Grade 11 Post-Test   Topic: Circle Geometry

Time: 1 hour   Date: 18 September 2018

Marks: 50

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Instructions

- Show all working
- Use only acceptable reasons

Question 1

D is the centre of a circle.

A, B, E, C are points on the circle.

Given $C\hat{B}D = 37°$, find:

1.1	$B\hat{A}C$	(5)
1.2	$B\hat{E}C$	(2)
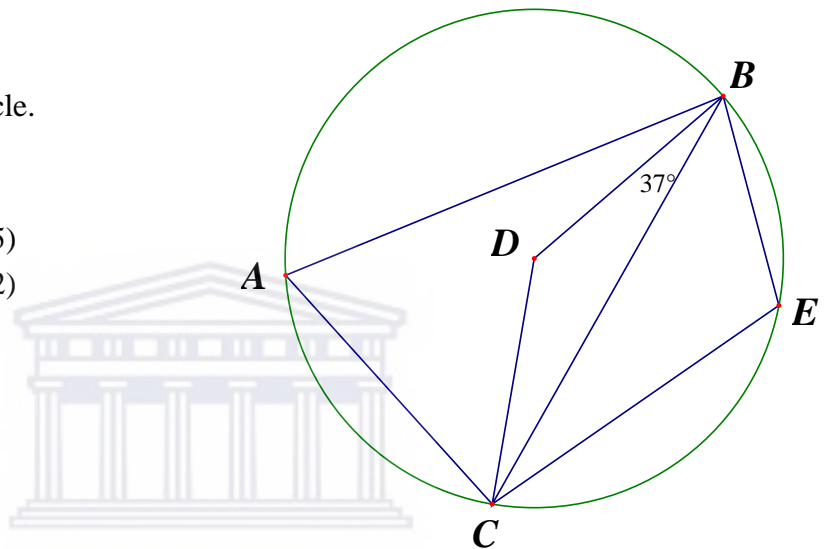
[7]

_____

Question 2

D is the centre of a circle.

F, G, H, I are points on the circle.

$F\hat{D}H = 84°$ and $FH \parallel DI$.

$F\hat{I}H = z$, $H\hat{F}I = x$ and $F\hat{G}H = y$.

Find the size of angles x, y and z.
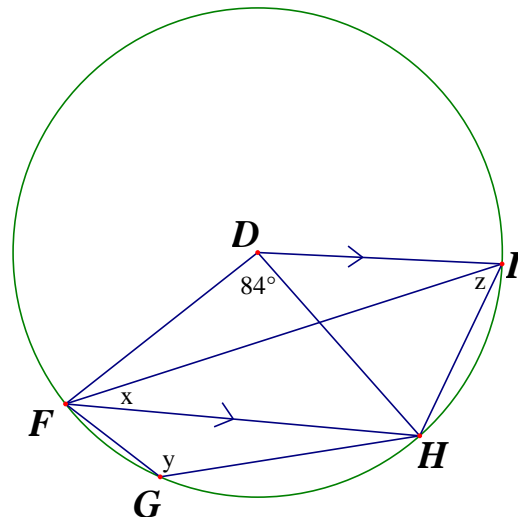
[8]

Question 3

J is the centre of a circle with diameter NK and tangent ON that touches the circle at point N.

Given $N\hat{L}J = 24°$ and $M\hat{J}L = 56°$, find:

3.1 $N\hat{K}L$ (5)
3.2 $M\hat{N}L$ (2)
3.3 $O\hat{N}M$ (3)
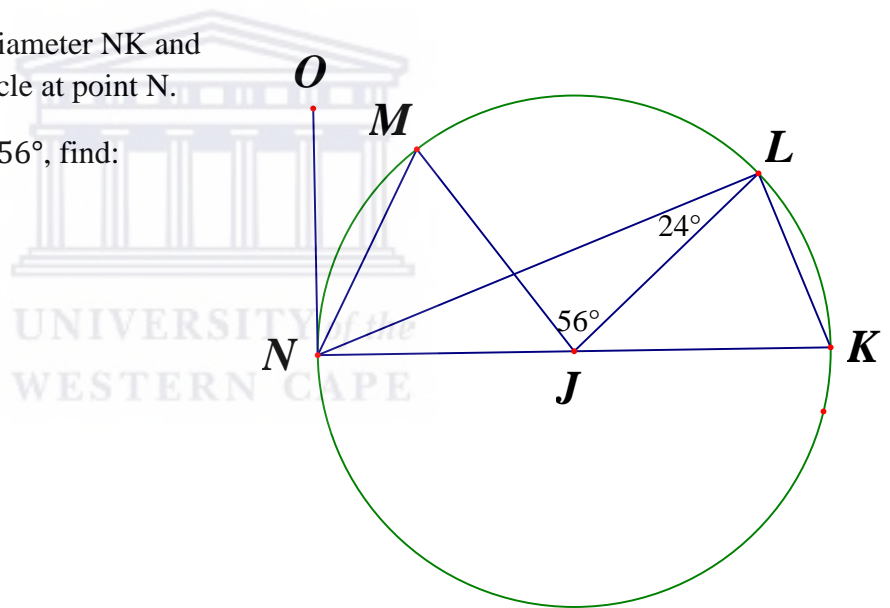
[10]

Question 4
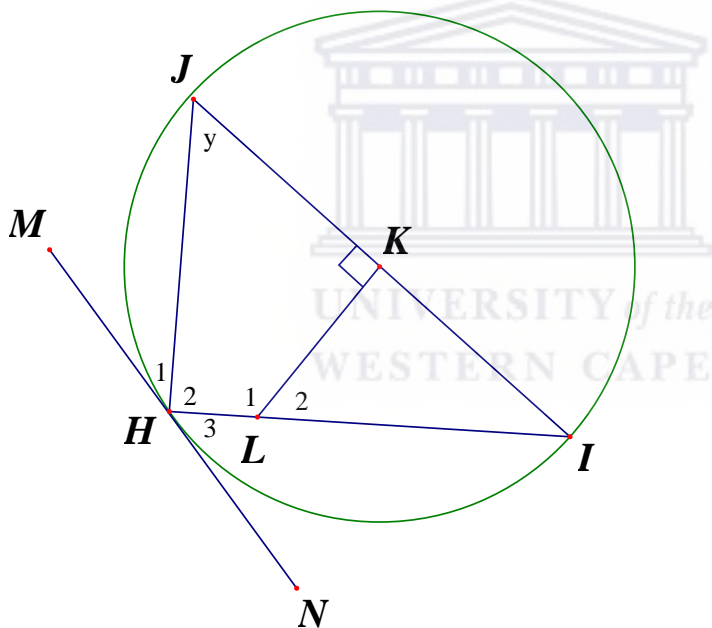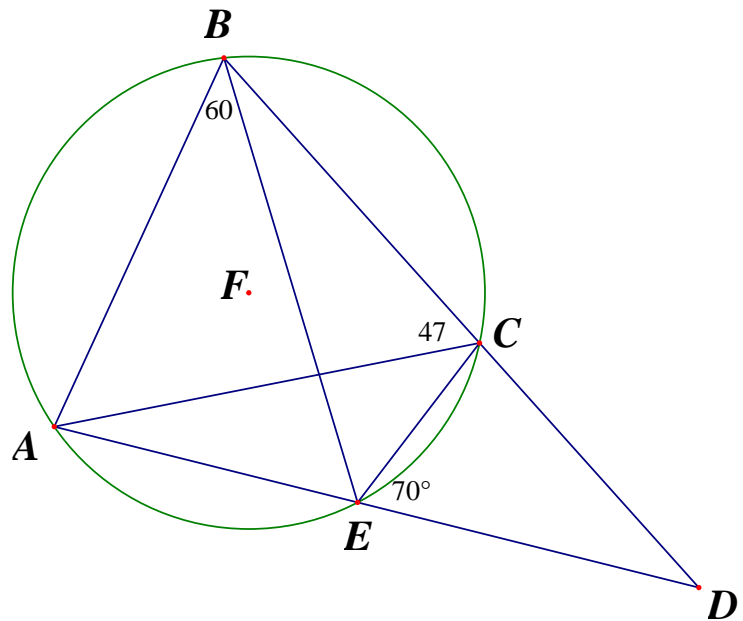
ABCE is a cyclic quadrilateral on a
circle with centre F.
The produced lines AE and BC
intersect at D.
Given $A\hat{B}E = 60°, B\hat{C}A = 47°$ and
$C\hat{E}D = 70°$, find:

4.1 $E\hat{B}C$ (2)
4.2 $B\hat{A}E$.   (3)

4.3   In the diagram JI is a diameter
      of the circle with centre K.
      MHN is a tangent to the circle
      at H. L is a point on HI and
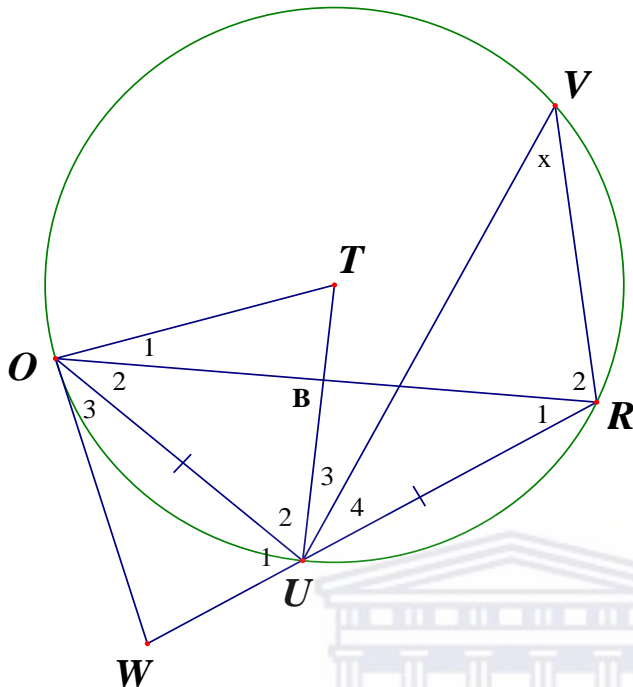 $LK \perp JI$. $K\hat{J}H = y$.

4.3.1   Prove that LKJH is a cyclic quadrilateral.   (3)

4.3.2   Determine, giving reasons, the size of $\hat{H}_1$ in terms of y.   (3)

[11]

Question 5

In the diagram below, T is the centre of the circle and O, U, R and V are points on the circle. $OU = UR$ and $U\hat{V}R = x$. The tangent at O meets RU produced at W. TU intersects OR at B.

5.1   Give a reason why $\hat{O}_2 = x$.   (1)

5.2   Prove that OU bisects $W\hat{O}R$.   (4)

5.3   Determine $O\hat{T}U$ in terms of x.   (2)

5.4   Prove that OW is a tangent to the circle that passes through points O, T and B.   (2)

5.5   Prove that $T\hat{B}O = 90°$.   (5)

                                                                                                              [14]

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APPENDIX 11b: Post-test memorandum

Post-Test Geometry memo   18 September 2018

Marks: 50

Question 1

1.1	$BD = DC$ ….. both radii✓ $\hat{C} = 37°$….∠ opposite = sides✓ $B\hat{D}C = 180° - 2 \times 37° = 106°$ … sum of ∠ of Δ ✓ $\therefore B\hat{A}C = \frac{1}{2} \times 106° = 53°$✓ …∠ at centre = $2 \times ∠$ at circum✓	(5)
1.2	$B\hat{E}C = 180° - 53° = 127°$✓ …opp ∠ of cyclic quad✓	(2) **[7]**

Question 2

2.	$F\hat{D}H = 2F\hat{I}H = 2z$ …∠ at centre = $2 \times$ ∠ at circum✓ $z = F\hat{I}H = \frac{84°}{2} = 42°$✓ $y = F\hat{G}H = 180° - 42°$…opp ∠ of cyclic quad✓ $\qquad\qquad = 138°$✓ To get x $F\hat{H}D = \frac{180° - 84°}{2} = 48°$….∠ opposite = sides✓ $H\hat{D}I = 48°$ alternating∠, $DI \parallel FH$✓ $H\hat{D}I = 48° = 2 \times x$ …∠ at centre = $2 \times$ ∠ at circum✓ $\qquad\qquad x = 24°$✓	(8) **[8]**

Question 3

3.1	$L\hat{N}K = 24°$✓ ....∠ opposite = sides $N\hat{L}K = 90°$✓ ....∠ in a semi circle✓ ∴ $N\hat{K}L = 180° - (90° + 24°) = 66°$✓ .... sum of ∠ in Δ✓	(5)
3.2	$M\hat{N}L = \frac{1}{2} \times 56°$ ...∠ at centre = 2 × ∠ at circum✓ $= 28°$✓	(2)
3.3	$O\hat{N}J = 90°$✓ ....tan ⊥ rad✓ $O\hat{N}M = 90° - (24° + 28°) = 38°$✓	(3) **[10]**

Question 4

4.1	$A\hat{B}C = 70°$.. ext ∠ of cyclic quad✓ $E\hat{B}C = 70° - 60° = 10°$✓	(2)
4.2	$A\hat{C}E = 60°$.. same segment AE✓ $E\hat{C}B + B\hat{A}E = 180°$ opp ∠ of cyclic quad✓ $B\hat{A}E = 180° - (47° + 60°) = 73°$✓	(3)
4.3.1	$L\hat{K}J = 90°$ ... given ✓ $\hat{H}_2 = 90°$ ....∠ in a semi circle✓ ∴ $L\hat{K}J + \hat{H}_2 = 180°$ ∴ LKJH is cyclic quadrilateral....opp ∠ supplementary✓	(3)
4.3.2	$\hat{L}_2 = K\hat{J}H = y$ ....ext ∠ of cyclic quad✓ $\hat{I} = 90° - y$ .. sum of ∠ in Δ $\hat{H}_1 = \hat{I} = 90° - y$✓ ....tan chord theorem ✓	S✓ S✓ R✓ (3) [11]

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Question 5

5.1	∠ in the same segment		(1)
5.2	$\hat{O}_2 = \hat{R}_1 = x$ ....∠ opposite = sides $\hat{R}_1 = \hat{O}_3 = x$ ... $\tan$ *chord theorem* $\hat{O}_2 = \hat{O}_3$ ∴ OU *bisects* $W\hat{O}R$	S✓R✓ S✓R✓ (4)
5.3	$x = \hat{R}_1$ ... ∠ *from equal segments* ✓ $O\hat{T}U = 2\hat{R}_1 = 2x$ ... ∠ *at centre* = 2 × ∠ *at circum*✓	S&R✓ (2) S&R✓
5.4	$W\hat{O}B = \hat{O}_2 + \hat{O}_3 = 2x$ .....*proved in* 5.2 $W\hat{O}B = O\hat{T}U$..*proved in* 5.3✓ ∴ $OW =$ *tangent* ....*Converse of* $\tan$ *chord theorem*✓	 $W\hat{O}B = O\hat{T}U$..✓ R✓ (2)
5.5	$T\hat{O}W = 90°$ ....…..$\tan \perp rad$ $\hat{O}_1 = 90° - 2x$ ∴ $\hat{O}_1 + \hat{T} + T\hat{B}O = 180°$ ... sum of ∠ *in* Δ $(90° - 2x) + 2x + T\hat{B}O = 180°$ ∴ $T\hat{B}O = 90°$	S✓R✓ (5) S✓ S✓ S✓ [14]

APPENDIX 12: Letter from language editor

Dust Jacket

1 Village Square

16 Hampstead Road

Harfield Village

7708

10 October 2019

Dear Sir/Madam

CONFIRMATION OF LANGUAGE EDIT

This is to confirm that I have read William Shonhiwa's thesis for admission to the degree of Masters in Mathematics Education at The University of the Western Cape, and to the best of my knowledge, it is coherent and grammatically correct.

Yours faithfully

Marilyn Braam

Marilyn Braam