Mathematical modelling with simultaneous equations – An analysis of Grade 10 learners' modelling competencies

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Contents

I. Abstract

Mathematical modelling is gaining extensive interest across the schooling sector worldwide, as it is deemed to develop learners with competencies set to deal with the demands of the fourth industrial revolution and being creative problem solvers. As mathematical modelling has only recently gained momentum across the mathematics curricula for schools in South Africa, many teachers may not be aware of the competencies that are needed to be developed in their learners through solving word problems, and even learners may not be aware of these essential modelling competencies. Hence, this mixed-methods approach study adopted a case-study design located within an interpretative paradigm to explore the levels of mathematical modelling competencies a sample of Grade 10 learners attending a Western Cape School demonstrated as they solved a set of word problems associated with the use of simultaneous equations. Additionally, data collected through observations and limited sets of semi-structured interviews were considered in the data analysis processes, which were largely driven by qualitative content analysis methods and supplemented with elementary descriptive statistical methods.

The findings of this study showed that most of the learners demonstrated non-competency in modelling mainly because of their inability to understand the problem as evident in their failure to comprehend the context of a problem, inability to recognise important quantities associated with a problem, and muddled relationships if any. The study conjecture that the use of the English language could have been a barrier to the sample of English second language speakers understanding the problem. However, a very limited number of students showed partial modelling competency, as they were only able to understand the problem and build a correct model to solve the problem. Regrettably, these students lacked the knowledge of the heuristics for solving a system of linear equations correctly and completely and did not check or verify their answers. The extremely small number of learners, who demonstrated sufficient modelling competency, demonstrated sufficient understanding of the problem, built and solved the system of simultaneous linear equations successfully without necessarily checking or testing whether their answers satisfied the conditions of the problem. Hence, this study recommends that adequate focus be given to the role of language in understanding a problem, heuristic competencies to solve a system of linear equations should be strengthened, and the habit of checking the reasonableness of the solution should be encouraged and developed continuously across problem-solving tasks. Studying learners' modelling competencies requires further work to add to the repertoire of this knowledge domain.

Key words: Mathematical model, modelling competencies, simultaneous linear equations

II. Declaration

I, Dzivaidzo Machingura, declare that the work presented in this thesis, with the title “Mathematical modelling with simultaneous equations- An analysis of Grade 10 learners modelling competencies” ”is my own work. It is being submitted in fulfilment of the requirements for the degree of Master of Education to the University of Western Cape. It has never been submitted for any other degree or examinations to any university.

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V. Acknowledgements

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Chapter 1: An Overview of the Study

1.1 Introduction

In this study, 'Mathematical modelling with simultaneous equations – An analysis of Grade 10 learners' modelling competencies', learner's levels of mathematical modelling competency are explored by demonstrating their endeavour to solve a set of word problems invoking simultaneous equations. Highlights of this chapter include the background, the problem statement, purpose of the study, and the research question, which frame the study. Additionally, the significance of this study, its delimitation, including the synopsis of the research design and methodology adopted, are presented. In conclusion, the organization of this study is summarised chapter-wise.

1.2 Background to the study

Highlights of research conducted in Mathematics Education in South Africa, point to the inadequacies in the pedagogical approaches adopted in learning and teaching Mathematics (Volmink, 1994; Vithal, Adler & Keitel, 2004; Taylor, 2008; Graven, 2012). The systemic evaluation conducted by the National Department of Basic Education (2013) confirmed that learning and teaching Mathematics in general education and training (GET) and the Further Education and Training Bands face challenges of underperformance. These particularly include tasks or items that require higher-order thinking and problem-solving. Evidence of this has also been shown in the statistical analysis of Grade 9 performance across the Annual National Assessment tasks set by the Department of Basic Education in South Africa (Simkins & Spaull, 2013). These researchers reported that in a sample of 100, the range of scores obtained by 91,9% of Grade 9 learners was less than 30%, while 3.8% got 30–39%, 2.1% 40–49%, 1.1% 50–59%, and 0.6% 60–69%, whereas only 0,3% acquired scores between 70–79% and 0,2% 80% and above. Underpinning the weak performance of learners in mathematics across all levels is the general lack of learners to solve problems, particularly non-routine and real-world type problems.

Schleicher (2012), cited in Dede and Bukova-Güzel (2018) stated that 21st-century students are expected to understand mathematical concepts, translate new problems into mathematical, make them amenable to mathematical treatment, and identify relevant mathematical knowledge the problems.

Schleicher (2012), cited in Dede and Bukova-Güzel (2018) stated that 21^{st}-century students are expected to understand mathematical concepts, translate new problems into mathematical ones, make them amenable to mathematical treatment, and identify relevant mathematical knowledge in solving these. Furthermore, he evaluated the solution in the context of the original problem. However, Ng, Widjaja, Chan and Seto (2012) used a researcher-designed rubric to assess two groups of primary school learners, age 11 years. The learners' ability to complete a modelling activity was assessed during the first trial. The results showed inexperienced modellers possess the ability to respond to modelling tasks at different levels of competence. Most of the learners did not complete the modelling cycle or exemplified proficient modelling competence. The authors concluded that inexperienced modellers showed their competence is weak in making assumptions, understanding a problem, mathematizing a situation or building a feasible model (Chan, Ng, Widjaja & Seto, 2012).

Leong and Tan (2015) assessed the modelling competencies of fifteen (16-year-old) Form Four (the equivalent of Grade 10 in South Africa) learners from a mixed-ability class at a private secondary school in Malaysia. These learners had some experience in modelling tasks since their mathematics teacher taught the modelling lesson. In particular, mathematical competencies to identify variables make assumptions, and those of reasoning and interpreting solutions were analysed by Leong and Tan (2015). The learners were assessed based on the three mentioned modelling competencies via an analytical rubric, which assessed the learners on a 4-point scale from unsatisfactory, basic, and proficient to distinguish. Their written work and responses were obtained using interviews. Leong and Tan (2015) concluded that the learners showed weaknesses in making assumptions, and often found it difficult to explain why their models worked. Usually, this has to do with understanding the problem. Once learners understand the problem, they can employ different strategies to solve it.

1.3 Problem statement

Mathematics enhances logical reasoning and problem-solving and as such entry to many professions such as engineering, commerce and health sciences. The lowest-performing of the 11 top subjects for Grade 12 National Senior Certificate (NSC) examinations at the Department of Basic Education (DBE), is consistent mathematics. In 2018, only 37% of 270,516 learners passed mathematics with 40% and higher. Previous NSC examinations showed a 30% to 35% constant pass (https://www.news.uct.ac.za/article/-2019-01-07-matric-maths-pass-rate-poses-significant-challenge-for-universities).

As for the present education system, it is widely known that the country is plagued with several challenges affecting the standard and quality the quality of school leavers. As a result, the readiness of these school leavers to pick up vocations in industries which require mathematical skills is affected. In recent years, a high demand exists for mathematical skills in present-day workplaces, particularly in those areas involving responsiveness towards customer needs along with providing solutions to economic, health, environmental, information and communication problems. Our DBE is trying to address and respond to these demands by instituting curriculum reforms such as placing greater focus on problem-solving and emphasis on mathematical modelling within the respective mathematics curricula across all phases. However, learners seemingly continue performing poorly in tasks or items requiring thinking on a higher level and problem-solving. Hence, this study seeks to track and analyse mathematical modelling competencies Grade 10 learners demonstrate as they solve word problems with the ultimate goal to enhance learning and teaching of mathemattics regarding problem-solving through using a modelling approach.

1.4 Purpose of study

Mathematical modelling may be seen as a multi-step process: presentinga real-world question, developing a model, solving the model, checking the reasonableness of the solution, and reporting results or revising the model. Mathematical modelling is increasingly being embraced at the school level to develop our learners with competencies to cope with the demands of the fourth industrial revolution (4IR) and function as problem solvers. In South Africa, mathematical modelling is a somewhat new domain within the primary and secondary school curriculum. Thus teachers might not be aware of the modelling competencies and sub-competencies which they ought to teach their learners. In this study, competency levels of Grade 10 learners' mathematical modelling when solving word problems associated with simultaneous equations are investigated and described to create a more profound exposition and experience.. In this respect, I explored learners' competency to engage in core mathematical processes such as reading and understanding a problem, building and solving a mathematical model, and interpreting solutions. Sub-competencies associated with each of these mentioned core competencies of mathematical modelling will also be investigated.

1.5 Research question

From this study, the aim will be in providing an answer to the following research question:

> *What levels of mathematical modelling competency do Grade 10 learners demonstrate when solving word problems invoking simultaneous equations?*

1.6 The significance of the study

Findings in this study provide insights and plausible explanations for the poor show of modelling competencies exhibited by learners as they endeavoured to solve word problems connected to simultaneous equations. For example, language could have been a barrier to learners' reading and understanding a problem, as English is not their mother tongue. Therefore, this barrier could enable teachers to revisit how they design their teaching and learning activities to develop problem-solving capabilities. It may prompt to design scaffolded activities that support the development and enhancement of particular mathematical modelling competencies and sub-competencies. For example, teachers could design interactive activities to support the mathematisation of a situation, which is central to building a mathematical model to repesent a situation.

The results of this study could enable subject specialists and the DBE to be aware of learners' challenges and success in solving word problems via a modelling approach, and thus plan professional development activities for teachers accordingly.

Resource materials and textbooks used by educators and learners ought to enhance learners' mathematical modelling competency for solving word problems. When developing resource material and textbook, developers should take into cognisance the problems Grade 10 learners experience at each stage of the modelling process as they try to solve word problems. Being aware of this will bolster learner confidence and success in solving problems via the modelling approach.

1.7 Research design and methodology

This study adopted a mixed-methods approach aligned to a case study design, located within an interpretive paradigm, which entails the use of quantitative and qualitative methods in a single study. This approach affords a more comprehensive and complete analysis of the research problem under study within the context of the research questions (Cresswell, 2009 & 2015). The mixed-methods approach was suitable for this study as a possibility to present quantitative and qualitative research findings and allows space to reflect on the findings and provide necessary clarifications. This opportunity was strengthened and enhanced by adopting a descriptive case design making it possible to describe the collected data and zone into the unobstructed phenomena emerging from these data with an interpretative lens (McMillan & Schumacher, 2010).

In this research, the case was the worksheet containing the set of word problems learners were expected to solve, with the unit of analysis, Grade 10 learners from a high school in the Western Cape. The interpretative lens provided the researcher with opportunities to get insight and in-depth information from learners' written responses, on task observations, and subsequent semi-structured interviews (Babbie & Mouton, 1998).

Data were collected on a sample of 20 of these Grade 10 learners via a task-based worksheet which entailed five word problems associated with simultaneous equations. These were supported by additional data collected via observations and semi-structured interviews. Initially, qualitative content analysis methods were used to analyse the data through the lens of analytical rubric demarcating three levels of modelling competency, i.e. not competent, partially competent and proficiently competent.. Elementary descriptive statistics, including tables and column graphs, were used to exemplify the levels of mathematicalmodelling competencies.

1.8 Limitations of this study

This study, which uses a case study design, draws conclusions from a limited number of sites, in this case, 20 Grade 10 learners from one school in one circuit of the Western Cape Province. According to McMillan and Schumacher (2010), research results emanating from a case study design are not generalizable. Consequently, these study recommendations and implications limit the reach.

Furthermore, municipal strikes and taxi violence in the vicinity of the selected school during the data collection phase hampered the completion of the planned semi-structured interviews to some extent. A final limitation to a limited extent was non-completion of the semi-structured interviews depriving the researcher to account and explain particular moves demonstrated by learners in their written submissions.

1.9 Operational definitions

Problem solving involves "engaging in a task for which the solution method is not known in advance. In order to find a solution, learners must draw on their knowledge, and through this process they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so." (NCTM, 2000, p. 52).

A Mathematical Model is "the representation or transformation of a real situation into mathematical terms, in order to understand more precisely, analyse and possibly predict therefrom" (Arora and Rodgerson, 1991:111).

Mathematical modelling can be seen as a multi-step process: posing a real world problem, developing a model, solving the problem, checking the reasonableness of the solution, and reporting results or revising the model. These steps all work together, informing one another, until a satisfactory solution is found. (California Department of Education, 2015).

Modelling competencies include the skills and abilities to perform appropriate and goal-oriented modelling processes along with the willingness to put these into actions *(*Maaß*, 2006).*

1.10 The study organisation

This thesis comprising five chapters, outlining the study in the following way:

Chapter 1: An Overview of the Study

In this chapter, an introduction to the study is given along with reporting on the background, the problem statement, purpose, research question, and its significance. Furthermore, the research design and methodology, study limitations, and operational definitions are discussed.

Chapter 2: Literature Review

This chapter starts with discussing mathematical problem-solving, including those of word problems, then provides an array of definitions of mathematical modelling, argues the need for mathematical modelling, and eventually explores the modelling process of solving problems. Also, the theoretical framework characterised by Blum and Kaiser's mathematical modelling competencies and sub-competencies provides a lens to explore the topic under investigation.

Chapter 3: Methodology

In this chapter, the study's research design and methodology are presented and explained. First, the research paradigm resonating the study purpose is highlighted, then the research approach, population and sampling techniques, and method and procedure of collecting the data are described and discussed. These are followed by strategies used in analysing the data to gain an understanding of Grade 10 learners' mathematical competencies demonstrated through solving word problems associated with simultaneous equations in a Western Cape school. Issues of trustworthiness of the results, along with ethical considerations, are dealt with, to ensure a reliable reflection for the community of researchers and practitioners to use and consider in further studies.

Chapter 4: Data Analysis and Findings

In this chapter, the analysed data results are reported and a detailed discussion provided for the findings according to the literature review and theoretical framework presented in Chapter two.

Chapter 5: Conclusions and Recommendations

Chapter five comprises a summary of the findings, which assisted in answering the research question, the discussion of these findings' implications and recommendations. These are followed by suggestions for future research, acknowledgement of the study limitations, and ultimately the study conclusion.

1.11 References

All the acknowledged sources of information are listed at the end of the research report according to the APA 6th edition referencing style.

1.12 Conclusion

In concluding this chapter, the study background was followed by the problem statement, along with the underpinning research question. Further to this discussion, followed the study's significance, the research design and methodology, and the process followed in conducting this research on a sample of Grade 10 learners from a Western Cape High School. A synopsis of the methods used for collecting and analysing data is shared, and finally, an outline of the various chapters provided.

In the next chapter, the literature review on mathematical modelling issues and the theoretical framework which guided the study design will be presented.

Chapter 2: Literature Review

2.1 Introduction

In the Literature Review, evidence of previous knowledge or information about the subject under research is investigated. According to Strauss and Corbin (1990), there are two distinctive approaches in presenting the literature. These approaches put the study in the correct context, and allow the researcher to i) examine pre-existing literature with general topics and concepts, and ii) identify possible shortcomings in the research field (Strauss and Corbin, 1990). In this literature review, I will first discuss problem-solving and solving word problems in mathematics, then provide an array of definitions of mathematical modelling, argue the need for mathematical modelling, and eventually explore modelling process about solving problems. I will also focus on the theoretical framework incorporating Maaß's mathematical modelling competencies, providing a view of the topic under investigation.

2.2 What is problem-solving in mathematics

The essence of mathematics is problem-solving. Unfortunately, problem-solving does not consist of a sequence of techniques that can be learnt and thus be reduced to a few processes. Concerning problem-solving, learners engage with problems for which a solution strategy is not instantaneous (Booker & Bond, 2007). Problem-solving is facilitated by offering students room to solve unfamiliar problems and then engaging them in a discussion to ascertain the various ways they tried providing solutions and also reflect on the process. The goal in problem-solving is to develop an open mind, inquiry skill, and demanding minds. When students are experienced in problem-solving, they exude/show the confidence in developing skills to creatively approach new situations, through combining, modifying, and adapting their mathematical tools. In doing so, these students are poised to have an explanation for the answers they get before they can accept it as correct. Problem-solving, according to Altun (as cited in Caliskan, Selcuk and Erol, 2010), is the ability to know what and when to do it. On the other hand, Toluk and Olkun (2004) define problem-solving as being a cognitive process that requires recalling and using a selection of applicable activities in systemic order. Problem-solving is very complex and some experts, like Gorge Poyla, suggested that it should be done in stages.

2.3 Solving word problems in mathematics

Teaching word problems, one needs to select appropriate calculation strategies. For example, carrying out mathematical calculations in word problems, learners should select appropriate calculation strategies. Being able to solve problems with a narrative requires being knowledgeable in semantic construction/comprehension, mathematical associations, and elementary numerical skills and strategies. Semantic construction is a concept to formalise the meaning of languages using different syntactic structures, such as for mathematical objects to explain their meaning and distinguish these language expressions. If learners applied the wrong strategy, this would mean they will fail to interpret the answer. Verschaffel, de Corte and Lasure (1994) argue that when given word problems, learners do not think about real-life issues and limitations. Therefore, it is crucial if learners derive the formula by themselves. Kilpatrick et al. (2001) had the same idea when they stated that children must use adaptive reasoning in mathematics, which means they need to think logically about conceptual associations. Furthermore, problem-solving requires teachers and learners to use specific approaches in mathematics. These are developing analytic ability and logic, building confidence, communication. Representation of word problems modelling mathematical thinking, solving problems, I explain these below as sub-topics.

2.3.1 Developing analytic ability and logic

Manning (2014) suggested that analytical thinking skills are critical, as they assist learners in acquiring information, articulate, visualise and solve complex problems. A learner who possesses analytical and reasoning skills is more self-reliant and secure. Understanding the subject matter along with developing logical reasoning should always be emphasised. Learners are expected to give strong arguments and reasons of how they understand a mathematical proof which comprises a significant sequence of implications. Learners are also expected to ask why one statement follows another. A classroom alive with communication skills, with a focus on argumentative discussion, has the ability to develop analytic and logic reasoning. This type of interaction allows better understanding and use of mathematical applications over a broad spectrum..

2.3.2 Building confidence (problem-solving)

Students with negative attitudes toward mathematics are at risk of weak performances which could lead to mathematic anxiety and or avoiding classes (Taylor, Lyn, Brooks & Kathryn, 1986). Students, who experience persistent success in mathematics, will as such be expected to build confidence through mathematical questioning and investigation. Therefore, they should be supported and rewarded when they are curious, adventurous and for not relenting in their search for solutions which they understand. Environments which bring about these mentioned qualities are more likely to instil confidence in students' mathematical ability.

2.3.3 Communication

Solutions to problems are very important, likewise the approaches to these solutions and the reasoning behind them. It is expected for learners to be able to communicate all the steps taken to arrive at a conclusion but this is sometimes hampered as they fail to develop the skill on time. Learners need to be proficient in the English language, to help them understand mathematics vocabulary. They also are required to read and understand mathematics instructions. Developing of skills is done in a bid for learners to be at ease to discuss their mathematics with care and accuracy. Furthermore, using the right terms to communicate mathematical meaning is regarded as an integral part of mathematical reasoning.

2.3.4 Representation of word problems

Word problems are mathematical exercises in which details of the problems are presented as text rather than in notation. To understand the story first before attempting a question is very critical for learners from Grade 9 as they will formulate equations out of the story. To correctly convey the words to text, one is required to study and be acquainted with the language used. Students find it difficult to solve word problems; therefore, they need to come up with mathematical equations to the problems. Thus, in providing answers to the equations, students would have solved the word problem and. When carrying out mathematical calculations in word problems one should select appropriate calculation strategies.

Anderson (2018) explained that for solving word problems, one needs to ask these questions, (i) what am I looking for, (ii) what do I need to get the answer, and (iii) what do I already have? Word problems are not different from arithmetic, but they require an understanding of the real-life problem. Palm (as cited in Sepeng, 2012) supported this idea when he asserted that when solving mathematical word problems learners need to relate to the problem in a real-world situation and 'undress' these tasks and solve them. Learners should solve the real problems mathematically, provide solutions and convert these to real-life solutions. However, the goal in problem-solving is to develop an open mind, inquiry skills, and demanding minds. Students' ability to solve problems increases their confidence and tactics to assess new situations in a more creative manner. They can modify, adapt, and combine their mathematical tools, which gives them the zeal to always provide explanations for answers before accepting these.

2.4 Learners experience in solving word problems

Word problems are mathematical problems presented in the form of text rather than in mathematical notation. They involve a combination of numbers and words for students to apply mathematics instruction to solve them. Word problems are designed to assist students in applying mathematics concepts to real-life situations. Even though the problems resemble the day-to-day experienced stories, most of the learners have difficulty word problems because they fail to understand what they are required to do. The most challenging part in solving mathematical word problems is to understand a problem and decide what method to follow (Sepeng and Madzorera 2014). Considering this, it is important for mathematics teachers to present algebraic word problems to learners and enable them to get used to the solutions (Seifi, Haghverdi, and Azizmohamadi, 2012). By so doing, learners' word problem-solving skills will be improved and with increasing experience can become conversant with the different terminologies used in various scenarios.

However, to solve mathematical word problems requires mental representation and reading comprehension skills. Learners need to read and understand the question first before attempting it. Otherwise, to solve word problems without understanding, communication or reasoning will make no sense regarding the problem statement. In other words, learners have to use a problem-model strategy to translate the problem statement into a mental representation of situation hidden in the text (Boonen, de Koning, Jolles and van der Schoot, 2016). Therefore, teachers should permit learners to seek assistance whenever they do not know how to deal with a word problem. Word problem activities should not be given to learners as classwork activities or homework without teaching strategies. Otherwise, learners can be good problem solvers if they have correct guidance.

2.5 Impact of language in solving mathematics word problems

In learning mathematics, language plays an important role. Therefore, learners have to read, write, and talk about mathematical concepts correctly. Riccomin, Smith, Hughes and Fries (2015) are of the same idea when they say understanding mathematics vocabulary is a major contributor in solving mathematics problems. They emphasise that teaching and learning the mathematics language develops mathematical proficiency. Allowing students to participate in mathematical discussions and conversations in the classrooms can help them make sense of mathematics but usually, they do this in their first language. Furthermore, mathematics has its own specialised language and grammatical rules. So since learners are learning English, they also need to learn the unique English meanings in a mathematical context. Thus, learners whose second language is English, have difficulty to focus on mathematical content as well as the related language skills that underpin mathematical comprehension. Moreover, most of the examination boards use the English language in their question papers. Therefore, learners fail to comprehend word problems because of the language barrier.

Zerafa (2016) did research in Malta. She investigated learners who speak Maltese as their first language on how they perform arithmetic word problems presented in English. The study aim was to investigate whether learners' performance in arithmetic word problems is influenced by language. This study comprised 30 Grade 3 children (aged 8 to 9 years) from the three Maltese education sectors, Government, Catholic Church, and Independent system. They were divided into two sections, with one group using Maltese as their first language, and the other group English. Participants had to memorise and solve two multi-levelled sets of word problems, either in Maltese or in English. Then the completed non-verbal computations were compared to the word problems. It was concluded that problems were understood and solved using the correct operation when offered in the learner's first language.

Furthermore, some learners learn mathematics in their first and second languages which are called combined classes. Van Rinveld, Schiltz, Brunner, Landrel and Ugen (2016) investigated the effect of the second language in learning mathematics. This study, done in Luxembourg, used language switches between German and French in the school curriculum as instruction. The study aimed to emphasize the role of language in learning and solving mathematical problems. Participants proficient in German and French were recruited for the study. They were given arithmetic problems presented under two different conditions, i.e. using semantic judgment or without using an additional language context. Van Rinveld et al. (2016) found that mathematical additions were performed faster in the condition with an additional language context in the French session, but with no effect of the context in the German session. In conclusion, language context enhanced arithmetic performances in the second instruction language of bilingual students. Overall, the instruction language influences the results.

Raoano (2016) conducted a study on Grade 6 learners' mathematical word problem skills in South Africa. He found that students with English 2nd-language mathematics as instruction found it difficult to attach meaning to word problems written in English. Moreover, the learners could only write number sentences, without being able to demonstrate how they arrived at their answers, although they were allowed to use their mother tongue in explaining these. In another study by Adamu (2019), conducted in English as a second language, Nigerian engineering students indicated they were taught in English. The effects of English comprehension and achievement were examined on engineering mathematics. The findings showed that there was a difference between worded and non-worded problems in the average performance of the students. The lower performance was seen in worded problems as opposed to non-worded problems. However, no significant relationship was shown between English language comprehension and the performance of engineering mathematics in non-worded problems (Adamu, 2019).

Moreover, Reynders (2014) conducted a study in South Africa on Grade 4 learners where the medium of instruction was English that was not their home language. Findings indicated that learners, who were taught in English, found it easier to discuss and understand word problems using their mother tongue as opposed to those trying to solve the word problems in English. Besides, learners found it difficult to rewrite word problems in their own words, and struggled to find alternative English words to simplify the word problem, suggesting that they read without understanding. Having a profound understanding of the text is crucial so that learners know what mathematical operation should be used to solve their word problems (Reynders, 2014).

2.6 Pólya's problem-solving model

Pólya (1957) presented four phases or areas of problem-solving, which have been recommended as the framework for teaching and assessing problem-solving skills. These steps are:

1. Understanding the problem;
2. Devising a plan to solve the problem;
3. Implementing the plan;
4. Reflecting on the problem(Poyla ,1957).

In Pólya's (1957) model, the first step is to understand the problem. Therefore, learners need to understand what they are being asked to uncover. They can work in groups to share what they think the problem means and then move to the second step to devise a plan to solve the problem. Each member in a group can suggest a solution and try to work out what the best solution is, and use that one as agreed. After that, they can implement that plan to see if the answer corresponds to the different steps followed. Finally, they can reflect on other similar problems and check the answers.

Cooperating five intertwining strands will ensure that learning and teaching mathematics are done successfully. The five-strand diagram intertwined by Kilpatrick et al. (2001) is illustrated in fig. 2.1.

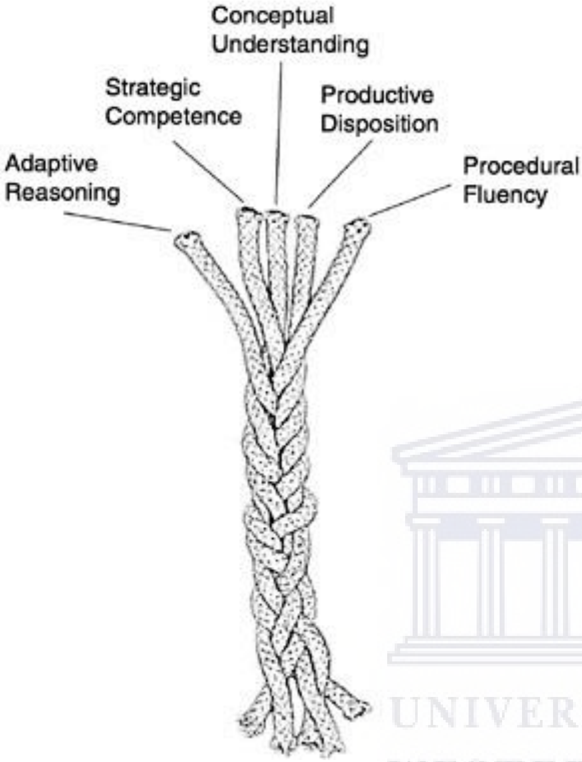


Figure 2.1: Five strands intertwined (Kilpatrick et al., 2001, p.116)

Kilpatrick et al. (2001) suggested that i) conceptual understanding deals with the concept, operation and relations of mathematics; ii) procedural fluency is the skill the learners have to accurately and appropriately carry out procedures; iii) strategic competence is the ability of learners to formulate, represent and solve mathematical problems; iv) adaptive reasoning is when learners can reason logically through reflection and justification; and v) productive disposition is when learners see the importance of learning mathematics. These five strands collectively working together will develop the learners' competencies. Groves (2012) illustrated the problem-solving framework in the form of a graph, which she adapted from the Ministry of Singapore (2006), see fig. 2.2.

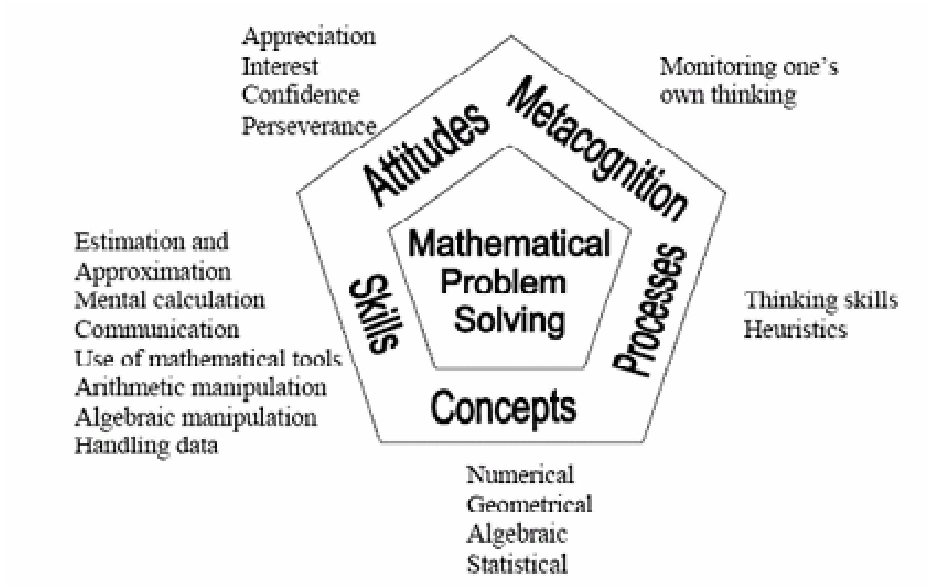


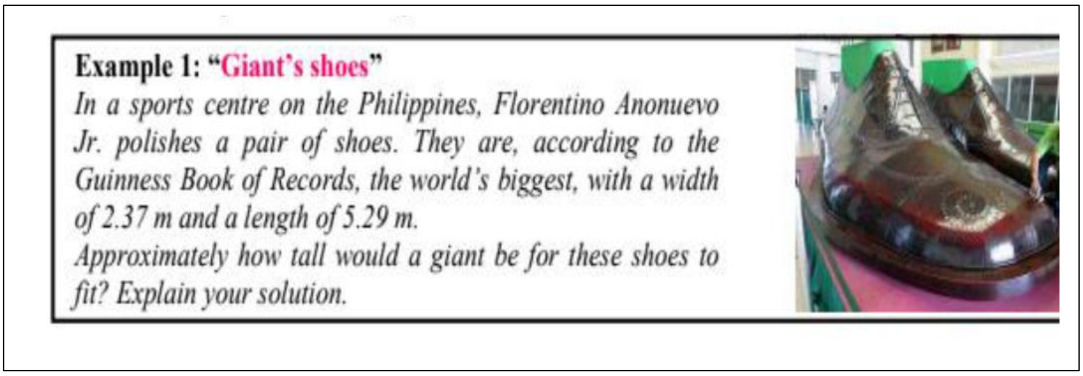
Figure 2.2: The Singapore Mathematics Framework (source: Ministry of Education, Singapore, 2006, p.6)

Problem-solving is the base of attitudes, skills, metacognition, process and concepts as shown in the diagram. Chamberlin and Moon (2008) described mathematical modelling and problem-based learning as integrating pedagogies in improving learners' mathematical thinking. Furthermore, to improve learners' structured problem-solving performance, scaffolding is useful to promote cognitive and metacognitive processes. A scaffold is a support structure that is put in place to enhance a child's learning, which can be removed as needed when the child becomes successful with a particular task. Vygotsky's (1978) concept of the zone of proximal development is a construct which is critical for instructing scaffolding. Vygotsky believes scaffolding is most effective when the support is matched to the needs of the learner. Therefore, it is ideal to use in problem-solving. As mentioned above, mathematical modelling and problem-based learning go in hand in hand. Thus the definition of mathematical modelling follows.

2.7 Definition of mathematical modelling

Mathematical modelling is the representation of real-life problems into mathematical problems. Blum and Ferri (2009) agreed when they described mathematical modelling as translating the real-world to the mathematical world. Thus, meaning word problems should be practical problems, which include everyday life situations that are solved mathematically. In an attempt to explain mathematical modelling, Blum and Ferri (2009) provided an example

Of a task, as shown in fig. 2.3.



Example 1: "Giant's shoes"
In a sports centre on the Philippines, Florentino Anonuevo Jr. polishes a pair of shoes. They are, according to the Guinness Book of Records, the world's biggest, with a width of 2.37 m and a length of 5.29 m.
Approximately how tall would a giant be for these shoes to fit? Explain your solution.

Figure 2.3: Example 1: "Giant's shoes" (Blum & Ferri 2009, p.45)

The word problem above includes a real-life situation. Learners know the shoes they wear everyday need to be polished meaning this is the real-life experience, as the shoe's length and width dimensions are given in meters. Therefore, this will force a learner to solve the problem mathematically and interpret the answer to the real-life situation. Furthermore, a few researchers agreed on describing mathematical modelling in the form of a cycle (Blum and Leiß, 2007; Blum & Ferri, 2009). See the example in fig. 2.4.

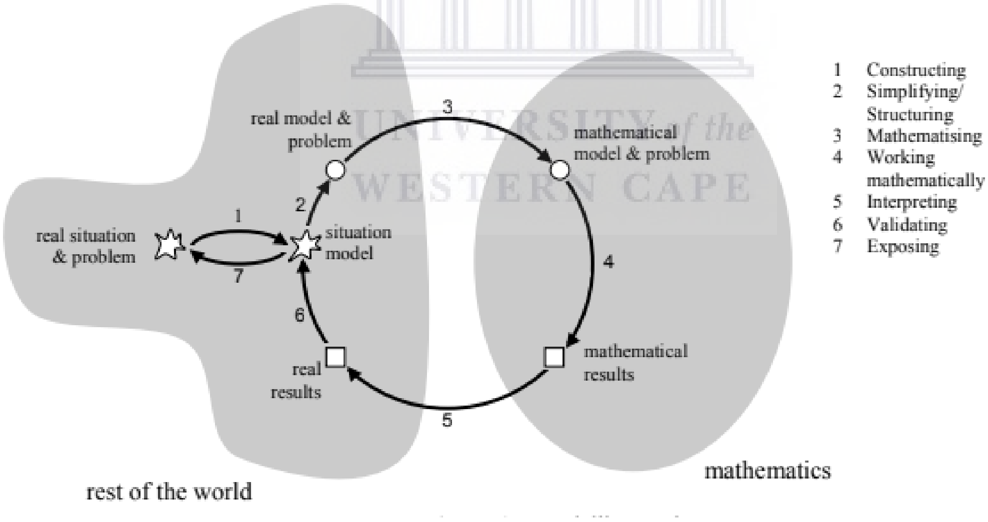


Figure 2.4: The mathematical cycle (Blum & Leiß, as cited in Blum and Ferri, 2009, p.46)

In fig. 2.4, mathematical modelling, whereby a real situation problem is solved mathematically, is illustrated. In stages 1 and 2 on the cycle, the real problem should be constructed, simplified and structured. Stages 3 and 4 are where equations are formed to solve the problem, by mathematization and working out mathematically. In the last stages 5, 6, 7, the mathematics answer will then be interpreted to the real-life solution, and the validation will be done to prove whether the answer is true. However, Blum and Ferri (2009) concluded that mathematical modelling intends to:

- help students' to better understand the world,
- support mathematics learning (motivation, concept formation, comprehension, retaining), and
- Contribute to an adequate picture of mathematics (p 47).

While Bandura (as cited in Davida, Schuman & Relihan 1990), described modelling as an effective teaching tool, it is "increasingly becoming part of an instructional approach deemed to develop students with competencies to function as 21st-century learners and problem solvers" (Ng, Widjaja, Chan and Seto, 2012, p. 146). Ng et al. (2012) further describe mathematical modelling as an important tool to develop the concepts, methods and competence of the learner. While Blum and Ferri (2009, p. 45) argue, "In classroom practice all over the world, however, modelling still has a far less prominent role than desirable." That means a gap exists in solving mathematical word problems using modelling.

Blum and Leiß (2007), and Poyla (1969) advocate the approach called modelling as a means of translating abstract mathematical concepts and their application into the real world. The approach proposes the presentation of mathematics conceptual knowledge in stages and in the form of cycles in teaching mathematics. According to the initiators of the modelling approach, the values of teaching mathematical knowledge and skills are reasoning skills and problem-solving skills. Also, learners acquire mathematical literacy, for instance, vocabulary required for understanding mathematics operations, symbols and formulas (Poyla, 1969; Blomhoj & Jensen, 2006).

Furthermore, Julie (as cited in Eric and Pui Yee, 2013) described modelling as content which emphasizes the development of competencies of the real-world situation to teach mathematics concepts. Thus, it means that the stages of modelling facilitate the development of competencies and problem-solving. However, mathematical modelling has been defined differently by different researchers, i.e. Doerr and English (2003) and Lesh and Doerr (as cited in Eric & Pui Yee, 2013). The authors viewed mathematical modelling as problem-solving as it places more emphasis on the solving process other than the result. Chan (as cited in Eric and Pui Yee, 2013) also mentioned that it shifts the focus towards the flexible use of mathematical ideas and exercise. Thus, mathematical modelling enhances the teaching and learning process. Therefore, mathematical modelling is discussed under point 2.5.

2.8 The need for mathematical modelling

Since mathematical modelling is critical for learners to work out complex mathematics like word problems, Ärlebäck (2010) designed modules on the parameters of Activity Theory introducing the concept in secondary schools. Gareth (2014) and Araújo (2010) also thought that mathematical modelling is critical when they stated that modelling is part of curriculum programmes in different countries. Furthermore, mathematics word problems which lead to simultaneous equations force learners to use modelling as a strategy. Learners who could follow every stage of the modelling cycles while working out word problems can perform better. Blomhoj and Jensen (2003) suggested the following six sub-processes be pursued when working out mathematical models:

(a) Formulation of a task (more or less explicit) that guides you to identify the characteristics of the perceived reality that is to be modelled.

(b) Selection of the relevant objects and relations, from the resulting domain of inquiry, and idealisation of these to make possible a mathematical representation.

(c) Translation of these objects and relations from their initial mode of appearance in mathematics.

(d) Use of mathematical methods to achieve accurate results and conclusions.

(e) Interpretation of these as results and conclusions regarding the initiating domain of inquiry.

(f) Evaluation of the validity of the model by comparison with observed or predicted data or with theoretically based knowledge. (Blomhoj & Jensen 2003, p. 125)

Once learners have engaged in these six processes and completed all aspects, it means they are insightfully ready to tackle any challenge of any given situation (Jorgensen, 2014). Mathematics is used in everyday life, by the working force, such as professionals in engineering science, economics and other fields for solving real-world problems (Burkhardt,2006). However, "Mathematisation transforms the real model into a *mathematical model* which consists of certain equations, worked mathematically (calculating, solving the equations, etc.) yields *mathematical results,* which are interpreted in the real world as *real results,*" (Blum & Ferri 2009, p. 46). Therefore, there is a need for learners to be taught mathematical modelling to be better citizens in future. Blum and Ferri (2009) agreed when they asserted that mathematical modelling prepares students to be responsible for citizenship and effectively participate in societal developments. Furthermore, they stated that by doing modelling makes mathematics easier and meaningful to learners, especially in secondary school.

2.9 An exploration of the mathematical modelling processes

The transformation of problems from the real-world situation to the mathematical world and solved mathematically, is described in the cycle form by Blum and Leiß (2007). These researchers created mathematical modelling cycles with more steps which Xueying (2012) described as unique and critical in teaching and learning mathematics. In fig. 2.5, the modelling process extracted from Blum and Leiß (2007) is illustrated.

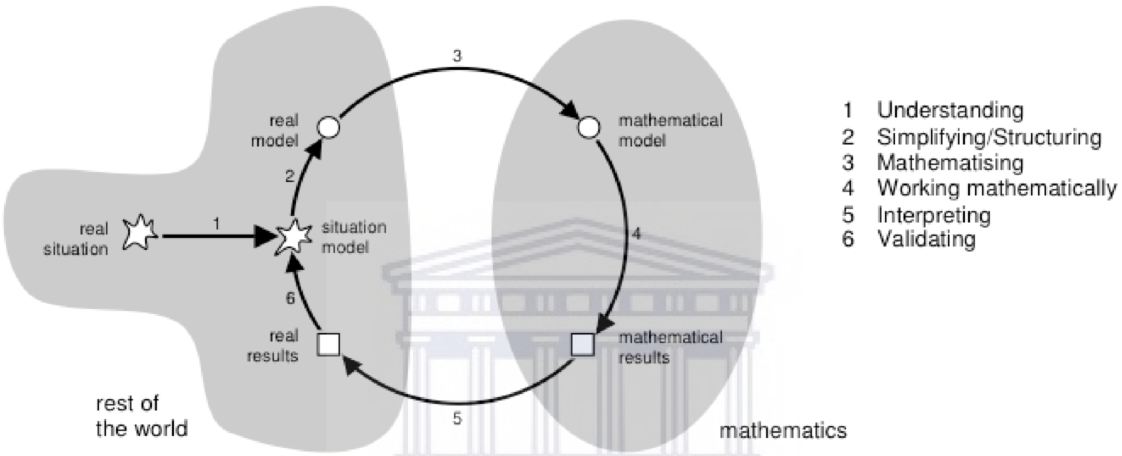


Figure 2.5: Modelling cycle under cognitive perspective (Ferri, 2006a, p.92)

In the diagram, six stages are shown, which are understanding the problem fully, designing a structure on how to solve it, applying mathematics in solving, working mathematically, interpreting the results, and validating if these work, and subsequently implementing them into the real world. The discussed cycles are similar to those of Blum & Leiß (2007) in fig. 2.4. In the same rhetoric, researchers in Mathematics Education, Blum and Leiß (2007) and Poyla (1969), referred to modelling as a way of explaining abstract mathematical concepts and their application into the real world. The approach illustrates the presentation of mathematics' conceptual knowledge in stages and in the form of cycles in the teaching and learning mathematics. According to the pioneers of the modelling approach, modelling is very important in teaching mathematical knowledge and skills, for example, reasoning skills as well as problem-solving skills. In addition, learners can learn mathematical literacy better, for instance, if they understand the vocabulary required in mathematics (Poyla, 1969; Blomhoj & Jensen, 2006).

The skills, which are developed in the stages (fig. 2.4) or processes, can be explained as follow. 1.) Understanding a problem, this is known to be the first stage of all problem-solving and modelling processes. Understanding happens when the problem has been figured out by the solver. Pólya (1973) agreed when he said that all about understanding the problem is identifying what it is and figured out proper solution to solve it. 2.) The problem solver will then simplify the problem by structuring it to a more precise representation. During this process, learners will make assumptions, recognising and identifying key variables that are essential for constructing relationships between them. In this stage 2) competence dealing with information plays an important role" (Sekerák, 2010). 3.) The next stage 4) is constructing a mathematical model where all the gained information from the first stage is converted to a mathematical language known as mathematization (Sekerák, 2010). In particular, mathematization means working out word problems mathematically, which is calculating, solving the equations and getting mathematical results. Sekerák (2010) echoed that the results are presentations of different types of equations and inequalities, propositional functions, graphs, and geometric figures, among other things. Therefore, by using mathematics inside the mathematical model, the mathematical results are solved. The subsequent stage 5) is the verification of the built model. In this stage, a model is verified to see whether it corresponds to the given situation by Sekerák (2010). The verification stage is reached when learners compare their models to assert they have attained an effective model which satisfies the problem requirement. (Eric & Pui Yee, 2013). "Model cannot be in controversy with [the] real situation and everything in the model must be held within mathematical rules and it must adequately describe [the] original situation" (Sekerák, 2010, p. 106). In this stage, the results must be interpreted according to the model's solution, along with an explanation of gained solutions in the language the problem was originally presented (Sekerák 2010). 6.) In the final validation stage, the learners are expected to scrutinize the solution. They should check for errors which might have occurred during the process. Furthermore, a few researchers agreed on describing mathematical modelling in the form of a cycle (Blum and Leiß, 2007; Blum & Ferri, 2009). (See example in fig. 2.4).

2.10 Mathematical modelling competencies

The concept "competence or competencies" is widely used in educational and curriculum research to refer to the capabilities individuals demonstrate in processing knowledge. According to Killen (2015, p. 419), the definition of competence embodies the idea of a person, who adequately demonstrates knowledge, skills and abilities. Eraut (1998, p.128) defined competence as an ability to perform atask and the roles required for the expected standard. Killen (2015) proposed that assessing competencies differ from content-driven assessment in that it requires an assessor to have an explicit description of the performance. Hence, analytic rubrics are suggested. Therefore, the learner may not accurately provide the answers, though finding an appropriate one through courage, determination and pursuit does count in the assessment. This study shares a similar conceptualisation of assessing competences.

Blomhoj (2003, p. 126) described competence in mathematical modelling as the ability to "autonomously and insightfully carry through all aspects of [the] mathematical modelling process in a certain context." While mathematical modelling is referred to as a "creation and use of a mathematical model consisting of six sub-processes by (Blomhoj & Jansen, 2006. These were described as: (a) the formulation of task that will guide you to identify the characteristics of the perceived reality that should be modelled, (b) selection of the relevant objects basing on the resulting domain in order to make a good mathematical representation,(c) the translation of objects and relations from their original mode to mathematics, (d) making use of mathematical methods to achieve mathematical results and conclusions(e), interpreting results conclusion regarding the initiation of the domain, (f) the evaluation of the validity of the model by comparing it to the predicted data". Many authors agreed that the development of mathematical modelling competence consists of all six sub-processes. Blomhoj (2003, p. 129) argued that "working with full-scale mathematical modelling is [a] time consuming way of learning." In another school of thought, the authors studied two frameworks for mathematical modelling competence (Neumann, Duchardt, Grüßing, Heinze, Knopp and Ehmke, 2013). These are the Programme for International Student Assessment (PISA) framework for mathematical literacy representing the perspective of relevance to everyday life, and the German Mathematics Education Standards' framework for mathematical competence representing the school-curriculum perspective. These frameworks promote the development of competence equipping learners with everyday life skills because, in modelling cycles, several things become clear, such as translation skills (Burkhardt, 2018).

A 2012 PISA study stated eight important competencies, which are covered in mathematical literacy these are communication, mathematizing, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, and using mathematical tools. Word problems allow all these competencies to be developed. However, Maaß (2006) specified the term modelling competency, in general, and suggested that several definitions for competencies, originating from terms from several of the science branches exist, including differences in competency types. Frey (1999, p. 109) asserts that "Competence is the ability of a person … to check and to judge the factual correctness respectively the adequacy of statements and tasks personally and to transfer them into action." Niss (2003 though,) describes mathematical competence as the ability to understand, judge, do, and use mathematics, in different intra- and extra-mathematical contexts. In contrast, competencies not only comprise abilities and skills but also are a reflection of life and putting these into action (Maaß, 2006). According to Tanner and Jones (as cited in Maaß, 2006), successful modelling requires students not to only use their acquired knowledge but also to monitor the process. In most cases, there is no conscious monitoring of processes (Blum & Ferri, 2009). Learners' competencies can be improved if the whole process could be followed. That is why modelling and competencies are inseparable because understanding modelling competencies and skills is closely related to the definition of the modelling process.

Apart from that, Blum and Kaiser (1997) define modelling competencies in terms of sub-competencies in relation to the modelling process. These authors suggested that when a learner displays abilities or skills to comprehend the real problem and to establish a model based on reality, the learner has (1) to make assumptions for the problem and simplify the situation, recognize quantities that influence the situation, (2) name and identify key variables, constructing relations between the variables, (3) look for available information and to differentiate between relevant and irrelevant information. They went on saying if the learner displayed competencies to set up a mathematical model taken from the real model, it meant the learner has been able to (a) mathematize quantities and their relations, (b) simplify quantities and their relations, if necessary, and lessen their number and complexity, and (c) to choose relevant mathematical symbols and to represent situations graphically. Finally, they suggested that if a learner has competencies to solve mathematical questions within this mathematical model, it meant that the learner has been able to (1) use heuristic strategies such as breaking up the problem into parts and identifying relations to similar or analogy problems, (2) rephrasing the problem or changing the way the problem is viewed (3) varying the quantities. Profke (2000) cited in Maaß (2006) tends to differ when he created his sub-competencies which do not mention skills to interpret and validate but emphasizes general skills like being curios. In connection to the development of competencies, Maaßb (2006) suggested that there is a need for the development of metacognition. Metacognition was defined as knowledge about one's own thinking up to self-regulation in problem-solving (Schoenfeld 1992). While Sjuts (2003) define metacognitive as is the thinking about one's own thinking and management of one's own thinking. Therefore, Metacognition is the fundamental competency that is relevant for other critical competencies such as independent handling with problems or self-regulated learning (Sjuts 2003).

2.11 Relevant studies on modelling

Mathematical modelling has been discussed by many authors but never put into practice at large. Although it might be mentioned in the curriculum, only a few practically exercise it (Galbraith & Clatworthy, 1990; Blum & Ferri, 2009). The reason might be because most teachers do not have the knowledge of mathematical modelling; therefore, they lack teaching strategies. Burkhard (2018) published an article describing research projects on teaching modelling and analysis skills. The reason why he was motivated in writing this article was that authors had observed that teaching of mathematics at British universities and high schools focused on models not modelling. Burkhard (2017) spent most of his working life doing modelling applications, modifying and creating mathematical models of real-life situations. He echoed that the exclusion of mathematical modelling does not develop mathematics or mathematical literacy, which affects future citizens. He studied ways of teaching modelling for 50 years and concluded that there is a lack of attention in modelling situations.

On that note, Galbraith and Clatworthy (1990) argued that modelling is not fully included in the curriculum. Therefore, they did a project which incorporated mathematical modelling. They suggested that there is a gap in most courses and books because the content only emphasises mathematical models. Burkhardt (2018) (as cited in Galbraith & Clatworthy 1990) similarly asserts that the modelling definition is known in schools and colleges at the undergraduate level, but the courses and books put stress more emphasis on models rather than modelling. Galbraith and Clatworthy (1990), over an extended time of two years, implemented a project at a secondary school as a pre-university subject in preparation of students for tertiary studies in science and engineering. Their project emphasised developing modelling skills using qualitative research methods to collect data. Their programme was evaluated according to their aims, which were to develop the confidence and ability of senior school students in:

1. Applying mathematics to unstructured problems and real-life situations;
2. Developing skills of [the] individual and team participation in the solution of problems;
3. Communicating and evaluating the results of a project."

(Galbraith & Clatworthy, 1990:138).

(1) Applying mathematics to unstructured problems and real-life situations (2) developing skills of individual and team participation in the solution of problems (3) communicating and evaluating the results of a project.

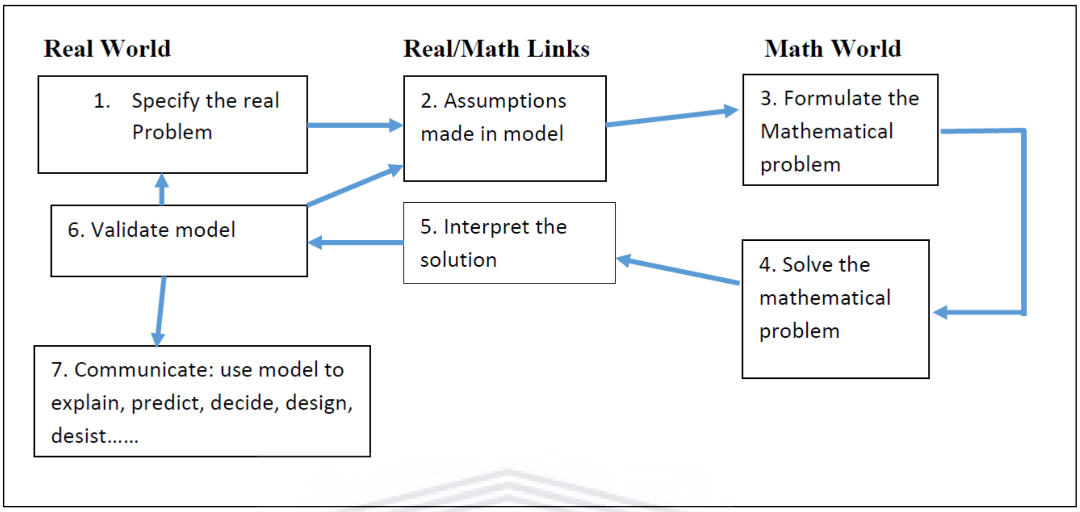


Figure 2.6: Modelling Process- Student engagement in the programme (Gailbrath and Clatworthy 1990, p.139)

The categories depicted in the diagram (fig. 2.6) were assessed according to the assessment criteria shown in table 2.1. Their findings were that their aims were successfully achieved.

Table 2.1: Assessment criteria or standards grid (Galbrath & Clatworthy, 1990, p.140)

CRITERIA	STANDARD 1	STANDARD 2	STANDARD 3
C1. Ability to specify problem clearly	Is able to proceed only when clues are given	Can extract clues from information and translate these into a clear expression of the problem to be solved	Is able to perform as for S2 and in addition can clarify a problem when information is I. Open ended II. Insufficient III. Redundant
C2. Ability to formulate an appropriate model:	Is able to proceed only when clues are provided	Is able to determine important factors and develop relationships	Is able to determine important factors and develop relationships

choose variables and find relations		with a minimum of assistance	independently where no clues exist
C3. Ability to solve the mathematical problem including ❖ Mathematical solution ❖ Interpretation ❖ Validation ❖ Evaluation/refinement	Is able to solve mathematical problem given substantial assistance through clues and hints	Is able to solve the basic problem with little or no assistance. Generally unable to refine model	Is able to solve the basic problem independently. Is able to evaluate and refine the model
C4. Ability to communicate results in a written and oral form	Is able to communicate reasonably in regard to ❖ Layout (including use of visuals) ❖ Presentation ❖ Conciseness ❖ Orally (with some prompting)	Is able to communicate clearly with good use of aids and without prompting	Is able to communicate clearly with outstanding presentation including innovative creative features

In a quasi-experimental study on over 600 students, Blum and Ferri (2009, p. 55), on the other hand, used quantitative (tests and questionnaires) and qualitative (videos) data. They investigated how students and teachers deal with demanding modelling tasks. According to ratings, the best results were achieved in classes where, “the balance between students’ independence and teacher’s guidance was realised best, with a mixture of different kinds of adaptive interventions.” The aim of their study on mathematical modelling was to improve teaching and learning methods. Their study was intended to answer questions, such as, what mathematical modelling is, and what for; how students deal with modelling tasks; how teachers handle modelling in the classroom; and how modelling can appropriately be taught?

However, in their study, Blum and Ferri (2009) attributed the inclusion of mathematical modelling in the curriculum by many countries. In the new national standards of mathematics, Germany was used as an example for considering mathematical modelling as one of the six compulsory competences. Furthermore, Blum and Ferri (2009) stated that routinely, teaching in most countries only included little mathematics modelling.. With exercising mathematics as the essential aim, mathematics word problems are asked, after undressing the context for facilitation purposes.. Therefore, "there is this gap between educational debate (and even official curricula), on the one hand, and classroom practice, on the other hand? The main reason is that modelling is difficult also for teachers, for real-world knowledge is needed, and teaching becomes more open and less predictable." I agree with them because, in most schools, teachers avoid teaching word problems and preferably do pure mathematics.

Additionally, the PISA-2006 results (OECD, 2007) provided the same notion revealing that throughout the world students have problems with modelling tasks. The PISA Mathematics Expert Group carried out analyses, revealing that the most difficult modelling tasks could be taught through inherent cognitive ones. These tasks require high competence from learners. Therefore, they should be taught by someone who knows all the stages of modelling, and who is proficient enough for them to acquire high competencies.

Blum and Ferri (2009) also stated that the gap exists because teachers and children find it hard to understand modelling. Furthermore, there are debates concerning the teaching and learning of mathematical modelling. According to the analysis done by Kaiser and Sriraman (2006, p. 302), most journals illustrate that there is no "homogeneous understanding of modelling and its epistemological backgrounds within the international discussion on modelling." Earlier debates on mathematical modelling resulted in consideration of two perspectives, which are the pragmatic perspective and the scientific, humanistic perspective. The pragmatic perspective is the ability of the learner to use mathematics to solve a practical problem, and the scientific, humanistic perspective focuses on the ability to create relations between mathematics and reality (Kaiser, as cited in Kaiser & Sriraman, 2006). These perspectives seem to fail because Kaiser and Sriraman (2006) further discuss the current approaches used.

Related to the science, humanistic perspective, epistemological goals were considered, which emphasize the development of mathematical theory. Furthermore, Kaiser and Sriraman (2006) suggest that an attempt of André Revuz (1971) to use a triple situation model theory should be considered in mathematics teaching. The triple model theory implies that the model is constructed from the situation, which leads to developing a mathematical theory. Therefore, Kaiser (as cited in Kaiser & Sriraman (2006) decided to practise the pragmatic perspective.

However, teaching mathematical modelling demands the teacher to understand the mathematical modelling approaches first. Although modelling problems are complex, understanding the content is vital (Maaß, 2006). Kazemi and Stipek's (as cited in Groves, 2012, p. 219) encouraged teachers to give a "high level of [the] conceptual press in order to stimulate children's conceptual understanding." When teaching word problems, for example, carrying out mathematical calculations in word problems learners fail to select appropriate calculation strategies.

Furthermore, Maaß (2006) did research to add to former descriptions of modelling competencies based on empirical data. Her empirical study was performed with the aim of showing the effects of the integration of modelling tasks into day-to-day math classes.

However, Maaß's (2006) study was based on the theoretical framework, which later transferred into practice. Regarding her findings, she analysed the students' abilities and found that concerning the concept of modelling competencies, their mistakes lead to more insight. Finally, Maaß (2006) discovered that students who were in the lower secondary level were able to develop modelling competencies. In conclusion, almost all the students were capable of modelling problems with known and unknown contexts. Students could conduct the entire modelling processes independently, although these were not always correct. Then if Maaß (2006) noticed that learners were not always correct, it meant there was still a gap in mathematical modelling. She then analysed lots of mistakes, which were done by students during the process, and noticed that some students could not describe setting up the real model, while others made incorrect assumptions which distorted reality. Hence, the gap in mathematical modelling needing attention is further described, which means that mathematical modelling is critical in day-to-day math classes. Therefore, the importance of modelling is discussed under the next sub-heading.

2.12 The importance of mathematical modelling in schools

Teaching and learning mathematics require the learner and the teacher to understand the subject matter. The teacher is expected to understand the curriculum before breaking it down to subject matter or content. Epstein (2007) agreed when she deemed that an effective teacher should know and understand the subject matter covered in their curriculum. In an investigation by Hill et al. (as cited in Coe, Aloisi, Higgins & Major, 2014, p. 19), about the importance of teachers' "pedagogical content knowledge in mathematics", the teachers' level of understanding was shown to correspond to how students effectively learn. However, Askew et al. (as cited in Coe et al. 2014) had a different idea, believing that highly effective mathematics teachers are identified by particular beliefs which correspond to teaching approaches. Biccard and Wessels

(2011) had a similar idea when they used a modelling approach to investigate the disclosure of beliefs on a group of 32 learners for 12 weeks. Interaction of learners and also three modelling problems, which were solved by 12 children in a group, showed that beliefs improved drastically through modelling. Therefore, modelling helps us to translate beliefs into mathematical language.

Furthermore, Arrieta and Díaz, and Cantoeal (as cited in Eric and Pui Yee, 2013) assert that ‘school mathematics’ and ‘mathematics of the real world’ are different. In teaching and learning, it is essential to know both worlds; modelling can be used to merge mathematics, while mathematics assists in the real world. So, modelling is very important in schools. Galbraith and Stillman (2006) echoed that modelling is a framework used to identify challenges faced by students in the transitions between the modelling stages. Mathematical modelling is used to enhance teachers’ perception to identify where students may have challenges and how to overcome these. Moreover, it enhances the identification of activities that modellers need to have the competence to be successful in applying mathematics. Galbraith and Stillman (2006, p. 75) assert “that an understanding of the blockages and competencies contributes to the planning of teaching towards the identification of the necessary pre-requisite knowledge and skills as preparation for intervention at significant junctures of the modelling process.” Therefore, modelling needs to be taken seriously in schools.

More so, mathematical modelling has proven to enhance and broaden learners reasoning, communication and problem-solving competencies throughout the world. The Curriculum and Assessment Policy Statement (CAPS, 2011) described mathematical modelling as a critical component of the curriculum and advocated that real-life problems be incorporated where appropriate. However, since 2011, modelling is a prescribed theme in the CAPS document. According to the National CAPS, there are two essential abilities that Grades R–12 (Department of Education, 2012) mathematics learners should gradually develop. These are 1.) Identifying, solving problems, and making decisions using critical and creative thinking; and 2.) Demonstrating an understanding of the world as a set of related systems by recognising that problem-solving contexts works together with all other skills. Relevant modelling tasks are precisely the kind of exposure that learners need to empower them in striving to attain the two mentioned CAPS ideals.

2.12.1 Solving word problems through using the modelling approach in schools

"Many teachers and researchers are already familiar with the characterization that mathematical modelling activities are based on translating the real world and mathematics in both directions" (de Almeida 2018). In mathematical modelling activities, many stages need to be included for success. De Almeida (2018) suggested that when given a mathematical problem, it is best to regard modelling as a way to deal with the problem mathematics, to strengthen the use of mathematical modelling activities in the classroom. Furthermore, from de Almeida's (2018, p. 19) viewpoint, the introduction of classes should take the following modelling activities into account. "(a) The mathematics used may not be previously chosen or defined; rather, the required mathematics emerges from the problem and its specificities. (b) Different perceptions of a messy world situation and different criteria for what constitutes an acceptable solution may arise in almost any situation." However, Burkem (2018), in a group of 118 17-year-old students, measured their modelling ability in the classroom. These students were selected on the grounds of high-performance in mathematics. The aim was to compare their performance in solving modelling tasks which test skills of model formulation. These skills included:

GV: Generating variables—the ability to generate the variables or factors that might be pertinent to the problem situation.

SV: Selecting variables—the ability to distinguish the relative importance of variables in the building of a good model.

SQ: Specifying questions—the ability to identify the specific questions crucial to the, typically ill-defined, realistic problem.

GR: Generating relationships—the ability to identify relationships between the variables inherent in the problem situation.

SR: Selecting relationships—the ability to distinguish the applicability of possible relationships to the problem situation. (Burkem 2018, p. 63),

The learners were not taught mathematical modelling beforehand. Therefore, they were asked to attend to the following mathematical modelling tasks:

PROBLEM 1

You are considering driving an ice cream van during the summer break. Your friend, who "knows everything", says that "it's easy money." You make a few enquiries and find that *the van costs £60 per week to hire. Typical selling data is that one can sell an average of 30 ice creams per hour, each costing 5p to make and each selling for 15p.*

How hard will you have to work in order to make this "easy money"? (Explain your reasoning clearly.)

PROBLEM 2

Terry is soon to go to secondary school. The bus trip to school costs 5p and Terry's parents are

Considering the alternative of buying a bicycle.
Help Terry's parents decide what to do by carefully working out the relative merits of the two alternatives.

PROBLEM 3

A new set of traffic lights has been installed at an intersection formed by the crossing of two roads.

Right turns are NOT permitted at this intersection.

For how long should each road be shown the green light? Explain your reasoning clearly. (Burkhardt, 2018, p. 63)

The findings revealed that most of the learners made a reasonable attempt because they had never been taught mathematical modelling, and in most schools, mathematical modelling has not been fully considered (Burkhardt, 2018). Erickson (1999) (as cited in Eric & Pui Yee, 2013), agree with that, when he say that in mathematics education, teaching modelling particularly is scarce, although the practice is emphasised as reform efforts in the mathematics classroom.

This gap has not been filled until now because of obstacles during transitions (Galbraith & Stillman, 2006). Transition is the change from one position in the modelling cycle to another, for example, from a real-life situation into a mathematical problem. That is why Eric and Pui Yee (2013) suggested that when pupils work on a modelling task, there should be an interaction between the pupils (pupil-pupil interaction) with relation to the modelling task (pupil-task interaction). Also, interaction with the teacher (teacher-pupil interaction) concerning the task should generate discussion about problem interpretation, variables, and strategies towards solving it. “The pupils ‘cognitive processing that occurs throughout the interaction is manifested as the pupils’ mathematical problem-solving behaviours” (Eric & Pui Yee, 2013, p. 7).

Therefore, as learners collaborate among one another and with the teacher in solving word problems because most of their modelling tasks require them to think profoundly, it is very important using the modelling approach in schools (Eric & Pui Yee, 2013).

Furthermore, students should transcend the real-world situation in where a problem is formulated by converting the real situation into the organised structure of mathematical language when working out mathematical problems in classrooms. Blum and Ferri (2009) stated, that in the classroom when teaching and learning mathematical modelling, one must first understand the problem situation so that the situation model can be constructed. They provided an example of a modelling problem, as shown in fig. 2.7.

Example 2: "Filling up"
Mrs. Stone lives in Trier, 20 km away from the border of Luxemburg. To fill up her VW Golf she drives to Luxemburg where immediately behind the border there is a petrol station. There you have to pay 1.10 Euro for one litre of petrol whereas in Trier you have to pay 1.35 Euro.
Is it worthwhile for Mrs. Stone to drive to Luxemburg? Give reasons for your answer.

Figure 2.7: Example 2: "Filling up" (Blum and Ferri 2009, p.46)

According to Blum and Ferri (2009), the problem has to be simplified, structured and made more precise, a real model of the situation. They suggested using models in the classroom when teaching and learning mathematical modelling, which learners should follow. These steps are potentially cognitive barriers for students and also essential stages in actual modelling processes, which can necessarily be in any order (Blum & Ferri, 2009; Blum & Leiß, 2007; Matos & Carreira, as cited in Blum & Ferri, 2009).

Furthermore, Blum and Ferri (2009) investigated how students and teachers treat mathematical modelling in classrooms. They revealed that the PISA-2006 results (OECD, 2007) indicated students throughout the world have problems with modelling tasks. According to Blum and Ferri (2009), the analysis of the PISA Mathematics Expert Group reveal the difficulty of modelling tasks can be explained substantially by the inherent cognitive complexity of these tasks, specifically by the demands on students' competencies. On the other hand, to produce quality teaching, a balance should be maintained between "(minimal) teacher's guidance and (maximal) students' independence" as Blum and Ferri (2009, p. 52) suggested.

2.13 Mathematical modelling using equations and simultaneous equations

Poyla (as cited in Jensen, 2006), used models to illustrate the implications of modelling in teaching various skills and concepts, which scientifically had proven to be challenging to teaching and learning learners in mathematics. For instance, the concept of equations and their formulas, to calculate these, require higher-order thinking and problem-solving skills.

The following model extracted from Pólya (as cited in Jensen, 2006), illustrates modelling in mathematics teaching (fig. 2.8).

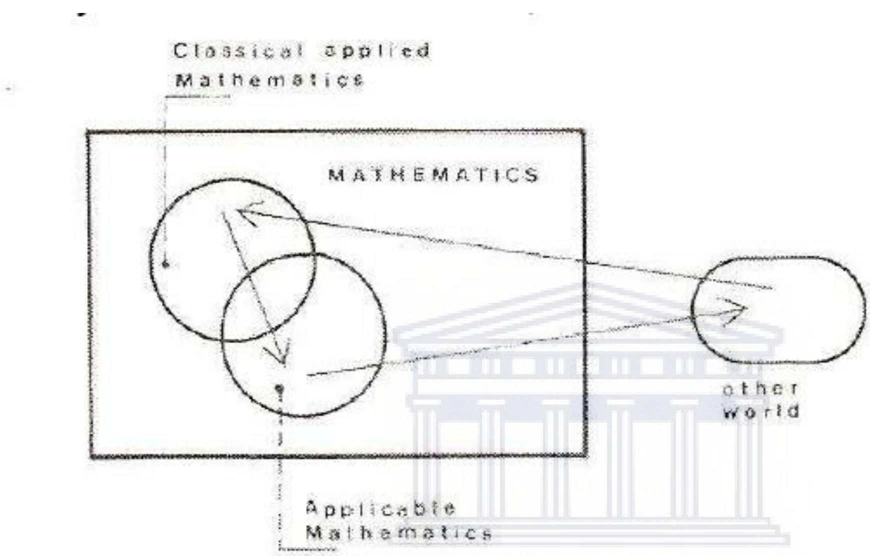


Figure 2.8: Modelling cycle adopted from Pólya (1969, p.147)

The cycle described the transformation of problems from the real situation to the mathematical world and solved mathematically, and then applied in the real world. By following this process, learners develop mathematical thinking. Blum (as cited in Maaß, 2006), suggested a mathematical modelling cycle with competences and sub-competencies in an attempt to define modelling, as shown in fig. 2.9.

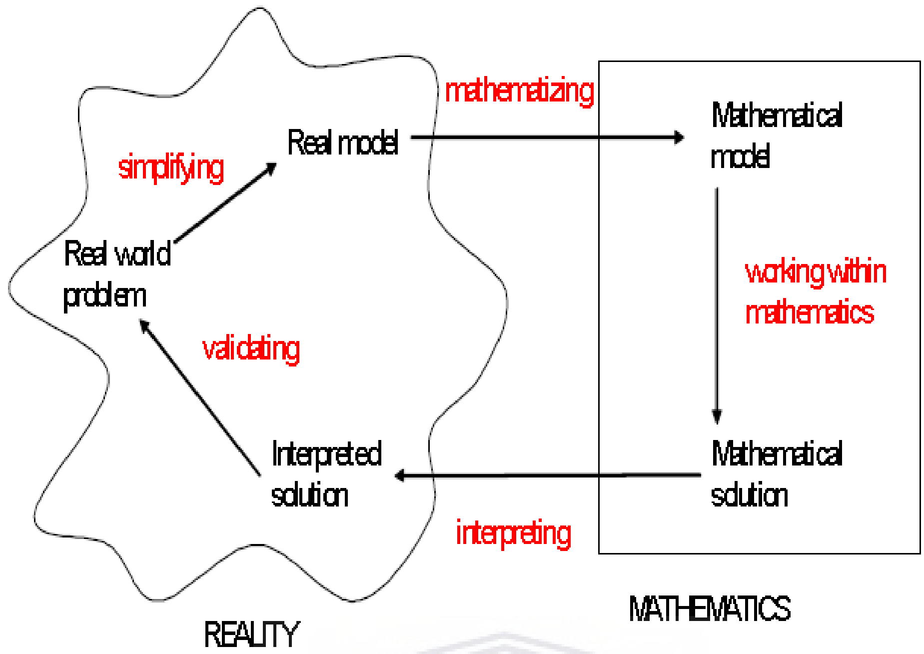


Figure 2.9: Modelling process (Blum, 1996, p.18, as cited in Maaß, 2006, p.115)

These steps have competencies and sub-competencies are all those in red and the competencies are all in black. The cycle has clear steps, which describe the modelling stages. These steps have competencies and sub-competencies, which are all those in red and the competencies, are all in black. The real-world problem, which is the competence, is the starting point because learners will read and understand the real problem. By exercising the sub-competence, simplifying, structuring and idealising this problem, you get a real model which is the competence. If the sub-competence, mathematizing, is applied to the real model, the competence, mathematical solution, will be found. The interpretation of a solution, which is a sub-competence, can be validated, but the interpreted solution is competence. Generally, this means that mathematical modelling cannot be separated from mathematical competencies (Niss, 2003; Blum & Ferri, 2009).

An important equation for solving word problems is the simultaneous equation. This equation usually involves two equations with two or more unknowns that should have the same values in each equation. Generally, if there are unknown variables, then independent equations are needed to obtain a value for each of the number of variables.

Pólya (as cited in Sternder, 2017), suggested that trial and error is a basic approach to solve problems, for example, in an equation. He further suggested that in trial and error, one can try a list of numbers and match the solution. Similarly, Sternder (2017) agreed that trial and error helps you to explore and to have a better understanding of the problem. Furthermore, Blum and Ferri (2009) echoed that the process of mathematization changes the real model into a mathematical model forming particular equations. Therefore, working mathematically (e.g., calculating, solving the equations) yields mathematical results, which are interpreted in the real world as real results.

However, to derive simultaneous equations, one must have two independent equations to solve for two unknown variables. For example, $is\, x + y = -1,\ 3 = y - 2x$. Simultaneous equations that can be solved algebraically using substitution and elimination methods or graphically.

Example of solving by substitution and elimination

1. Use the simplest of the two given equations to express one of the variables in terms of the other.
2. Substitute into the second equation. By doing this you reduce the number of equations and the number of variables by one.
3. You can now have one equation with one unknown variable which can be solved.
4. Use the solution to substitute back into the first equation to find the value of the other unknown variable.

The solution to the example

I have given an example of a simultaneous equation above that is:

$$x + y = -1 \qquad (1)$$

$$3 = y - 2x \qquad (2)$$

So one can use equation (1) to express x in terms of y:

$$x = y + 1$$

Now x can be substituted in equation 2

$$3 = y - 2(y + 1)$$

$$3 = y - 2y - 2$$

$$5 = -y$$

$$y = -5$$

Substitute back into equation (1) and solve for x

$$x = (-5) + 1$$

$$x = -4$$

To see if the answer is correct, learners can check their answers by substituting these back into both original equations. In this research, I used the elimination method in the worksheets of participants.

<u>Solving by elimination</u>

The elimination method is used to solve linear systems. In the elimination method, you either add or subtract the equations to get an equation in one variable.

<u>For example</u>

$$3x + y = 2 \ (1)$$

$$6x - y = 25 \ (2)$$

<u>Solution</u>

Make the coefficients of one of the variables the same in both equations.

The coefficients of y in the given equations are 1 and -1.

Eliminate the variable y by adding equation (1) and equation (2) together

$$3x + y = 2$$

$$\underline{+ \ 6x - y = 25}$$

$$\underline{9x + 0 = 27}$$

To find x, the learners must simplify and solve for x

$$9x = 27$$

$$x = 3$$

Now the learners can substitute x back into either original equation and solve for$y$. You can substitute in equation (1)

$$3\,(3) + y = 2$$

$$y = 2 - 9$$

$$y = -7$$

You can check if the answer satisfies the equation.

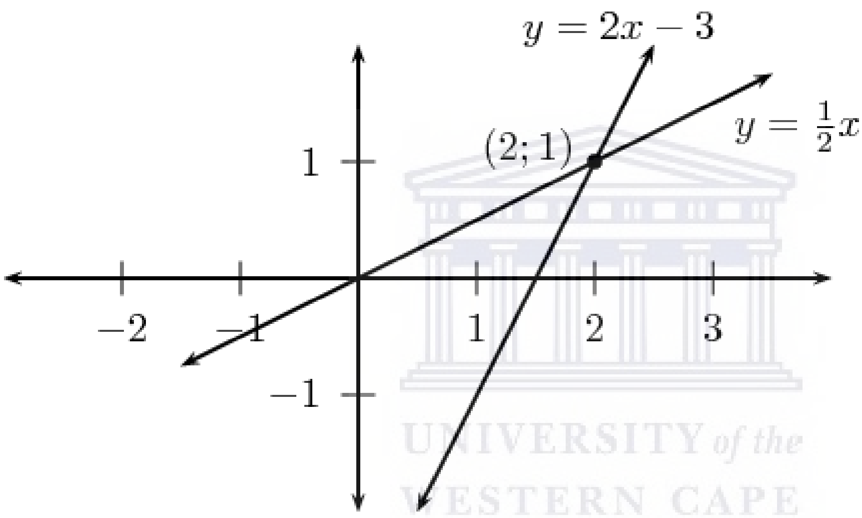
I have mentioned the graphical solution above. Simultaneous equations can also be solved graphically. If the graphs of each linear equation are drawn, then the solution to the system of simultaneous equations is the coordinate of the point at which the two graphs intersect.

<u>For example</u>

$$x = 2y \ (1)$$

$$y = 2x - 3 \ (2)$$

The graphs of the two equations are shown below.



The intersection of the two graphs is (2, 1). So the solution to the system of simultaneous equations is $x = 2$ and $y = 1$

Substitute equation (1) into equation (2)

$$x = 2y$$

$$y = 2\,(2y) - 3$$

Then solve for y:

$$y - 4y = -3$$

$$-3y = -3$$

$$y = 1$$

Substitute the value of y back into equation (1)

$$x = 2(1)$$

$$x = 2$$

2.14 Learners experience in solving simultaneous linear equations

Solving simultaneous equations requires one to be at the stage of understanding the problem by presenting information that is known, which is done in the form of symbol representation or by formulating a mathematical model. Information can be presented in the form of verbal representation or by using written text. When solving simultaneous equations with two unknown variables, learners can solve it in different ways, i.e. linear, geometry and graphically. Solving simultaneous equations equip learners with valuable problem-solving skills, as well as a wide understanding of the variable concept, of substituting values for variables in simple formulae expressions and equations. Thus, learners can translate the word problems with variables into number sentences. Furthermore, curriculum and assessment policy statement (CAPS, 2012) emphasises the development of skills when solving a simultaneous equation. Main objectives are to help learners recognise the representation of different concepts as well as the same concept in algebraic expressions, to describe situations in algebraic language, formulae and expressions, to analyse and interpret equations, and to communicate effectively mathematical ideas in visually, symbolically and linguistically.

However, when solving simultaneous equations solution of problems are not immediate. That is why word problems leading to simultaneous equations are designed to train students to spend effort in obtaining a mathematical understanding of a problem to allow its solution. If such understanding has been achieved, then it is easier for learners to tackle simultaneous equations. Since it is a long strategy which demands a lot of concentration and understanding of concepts, many errors may occur during the solving process. For example, learners struggle with the generation of equations to represent the relationship between the quantities (Kieren, 2007). Learners must be very careful during the solving process. Drijvers, Goddijn, and Kindt (2011) suggest that many errors occur when changing arithmetic to algebra during solving algebra. Sometimes errors encountered during solving equations are because of a lack of understanding. If learners do not understand the concept, errors are likely to happen, which means that there may be some misconceptions of concepts. Usually, this can be caused by misapplication of a mathematical rule. For example, French (2002) explains that some learners perceive the equation

$(a + b)\ 2$ as $a\ 2 + b\ 2$ to be correct, which means the rule of removing brackets has been misunderstood.

2.15 Theoretical framework

A theoretical framework is defined by Grant and Osanloo (2014, p.12) as the foundation from which all knowledge is constructed for a research study. This foundation serves as the 'blueprint' for or the structure that guides the entire research inquiry. The design of this research study, the data collection techniques, and methods of data analysis, and latter interpretation and discussion of findings have all been pivoted around the modelling competency framework assimilated by Blum and Kaiser (1997, p. 9 as cited in Maaß, 2006, pp. 116-117). For this study, mathematical modelling competency is defined as one's ability to traverse across each of the respective phases as articulated in fig. 2.10, which constitute the modelling process, when solving a given problem (Govender, 2018).

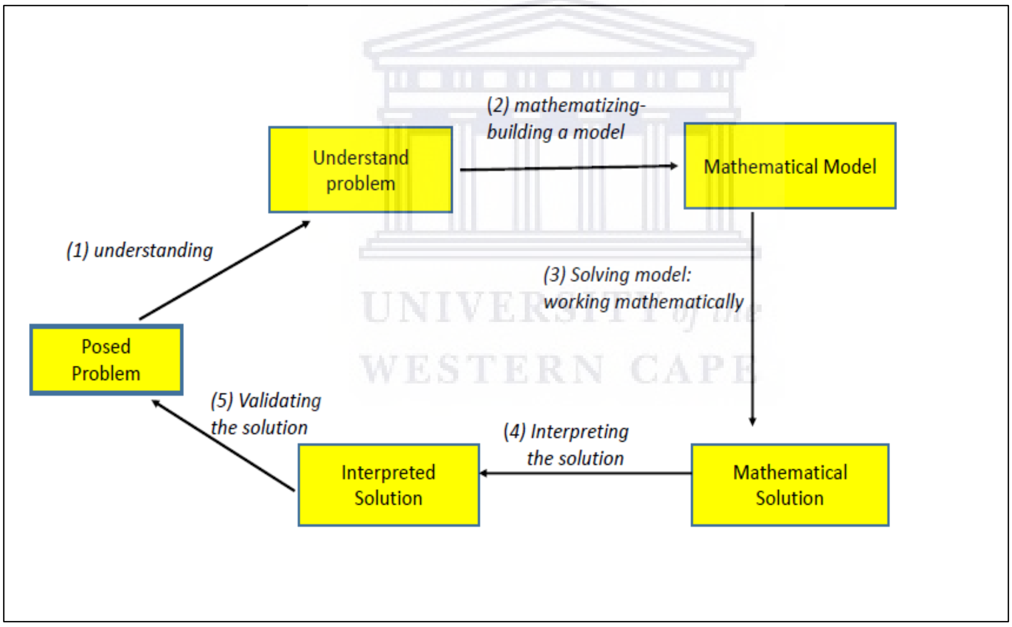


Figure 2.10: Modelling cycle illustrating a well-defined modelling process (adpated from Govender, 2018)

As illustrated in fig. 2.10, the mathematical modelling process consists of a few stages, which must be followed to have a proper solution to a practical problem. These stages are, (1) Reading and understanding a problem (2) mathematizing the situation to build a mathematical model; (3) working mathematically within the context of the mathematical model to generate a mathematical solution; (4) interpreting the solution ; (5) validating the interpreted solution.

Understanding a problem is the first stage of all problem solving and modelling processes, it demand the learner to figure out what the problem is about and understand what question or problem is being posed (Pólya, 1973). The learner will then simplify the situation by arranging and structuring the problem. Simplifying involves making assumptions, recognizing and naming quantities. Learners are also required to identify and construct relationships between key variables (Blum and Kaiser, 1997). Also, Schaap, Vos, & Goedhart (2011) suggest that understanding a problem requires competencies of visualizing and drawing to simplifying the situation and overcome blockages while building a mathematical model.

Proper structuring of problems and visualizing enables a learner to choose and identify the relevant quantities associated with the practical problem that can be transformed into mathematics. Translating a real problem into mathematical structures and formulas is called mathematizing (Brady, 2018; Niss, 2015). Therefore, learners should do proper selected mathematical knowledge, techniques; heuristic strategies and tools to work upon mathematically to produce a mathematical solution. Therefore, a mathematical solution must be interpreted regarding the conditions and parameters governing the real practical problem and translated back to the language of practice to provide a solution for the original problem. This procedure includes finding a solution in a manner that is comprehensible and commonly understood by the relevant community of practice.

Finally, in the validating process, the student is expected to show and prove the solution in real terms or else expected to check if an error has occurred through the initial assumptions or fault in the mathematical work if the solution fits the situation. Validation can be used to simulate the original practical problem through a set of varying assumptions and parameters affecting it (Oberholzer, 1992). Also, the process of validating makes space for one to realize alternate ways of solving the same problem (Blum & (Kaiser, 1997, as cited in Maaß, 2006).

However, the modelling process may appear to be one-directional and sequential, though, in reality, it is not. Niss (2015, p.1445) says: “An actual solution model may go through steps in a different order and the strips may be intrinsically entangled.” Equivalently, Ferri (2006) affirms that students do not move through the steps of the cycle sequentially, but “they some-times jump directly from the real situation to the mathematical model or go forth and back several times between the real world and mathematics.” In this study, it is envisaged that as mathematics educators we will be made aware of the critical aspects of the modelling process inclusive of the associated skills and abilities.

According to Blum and Kaiser (1997, p. 9 as cited in Maaß, 2006, pp. 116-117), essentially, the five broad mathematical competencies can be tabulated as follow:

A. Competency to understand the real problem and to set up a model based on reality.

B. Competency to set up a mathematical model from reality.

C. Competency to solve mathematical questions within this mathematical model.

D. Competency to interpret mathematical results in a real situation.

E. Competency to validate the solution.

Specific sub-competencies (sub-skills and abilities) exist for each modelling competency (A-E). Blum and Kaiser (1997, p. 9 as cited in Maaß, 2006, pp. 116-117) provide the following detailed list of sub-competencies that permeates each phase (or competency) of the modelling process:

A. Sub-competencies to understand the real problem and to set up a model based on reality.

- To make assumptions for the problem and simplify the situation;
- To recognise quantities that influence the situation, to name them and identify the key variables;
- To construct relationships between variables;
- To look for available information and to differentiate between relevant and irrelevant information.

B. Sub-competencies to set up a mathematical model from reality

- To mathematize relevant quantities and their relations;
- To simplify relevant quantities and their relations if necessary and to reduce their number and complexity;
- To choose appropriate mathematical notations and to represent situations graphically.

C. Sub-competencies to solve mathematical questions within this mathematical model:

- To use heuristic strategies such as division of the problem into part problems, establishing relations to a similar or analog problem, viewing the problem in a different form, varying the quantities or the available data, etc.;
- To use mathematical knowledge to solve the problem.

D. Sub-competencies to interpret mathematical results in a real situation:

- To interpret mathematical results in extra-mathematical contexts;
- To generalize solutions that were developed for a special situation;
- To view solutions to a problem by using appropriate mathematical language and/or communicate about the solutions.

E. Sub-competencies to validate the solution:

- To critically check and reflect on found solutions;
- To gain review some parts of the model or again go through the modelling process if solutions do not fit the situation;
- To reflect on other ways of solving the problem or if solutions can be developed differently;
- To generally question the model.

2.13 Summary – Chapter 2

In this chapter, I begin by discussing problem-solving in mathematics, solving word problems in mathematics, then provide an array of definitions of mathematical modelling, arguments for the need for mathematical modelling, and eventually explore modelling process about solving problems. Additionally, the theoretical framework characterised by Blum and Kaiser's mathematical modelling competencies and sub-competencies are discussed, which provides a view to explore the topic under investigation.

Chapter 3: Methodology

3.1 Introduction

The focus in the previous chapter was on the literature on solving word problems, mathematical modelling processes and competencies. This chapter presents and explains the research design and methodology used in this study. It starts with the research paradigm that resonates with the purpose of this study, then describes and discusses the research approach, population and sampling techniques, data collection methods and procedures, These are followed by data analysis strategies used to gain an understanding of the mathematical competencies Grade 10 learners in a Western Cape school demonstrated through solving word problems associated with simultaneous equations. The issues of trustworthiness and as well as ethical considerations are discussed in this chapter to enable the results to be considered by the community of researchers and practitioners as worthy for reflection, use and consideration in further studies.

3.2 The Research Paradigm

According to Kuhn (1962), as cited in Flick (2009:69), a paradigm is defined as, "an integrated cluster of substantive concepts, variables and problems attached with corresponding methodological approach and tools." This provides a frame of reference that could be used to organise observations and reasoning, and a plethora of research affirms that three core research paradigms underpin human social sciences research in general. According to (Hussain, Elyas, & Nasseef, 2013), these include the positivist, interpretive and critical paradigms. For this study, the researcher selected the interpretative paradigm, as it provides the researcher with opportunities to get insight and in-depth information from responses and interactions of participants in a given activity (Babbie & Mouton, 1998). According to Cresswell (2009), the interpretive paradigm provides the lens through which the researcher can describe, analyse, and interpret features of a specific situation, preserving its complexity and communicating the perspectives of the participants.

3.3 Research Approach

A research approach is a logical plan that enables a researcher to provide answers to the research questions associated with a research problem (Yin, 2009). Generally, the research approaches that can be used to obtain the necessary information to sufficiently answer the main research questions include a quantitative approach, a qualitative approach, and a mixed-methods approach

(McMillan and Schumacher, 2010). Often the research problem signals the kinds of research approach that a researcher should adopt to conduct a study to realize a plausible set of answers to posited research questions. Cresswell (2009 & 2015) discusses each of these approaches as follows:

- A quantitative approach entails the maintenance of objectivity in measuring and describing a phenomenon. Quantitative inquiry entails the researcher following a logical model formulated by the hypothesis which provides the space for specific expectations to be developed. The focus is not on experiences, ideas and solutions are expressed by research participants but rather on numerical data.
- A qualitative approach focuses on non-numerical data and the research information is gathered from the phenomenon being explored, investigated or examined. In a qualitative study, the researcher seeks meaningful understanding of the phenomenon from bounded settings through analysing participants' experiences, views and responses.
- A mixed-methods approach entails the use of quantitative and qualitative methods in a single study to afford a more comprehensive analysis of the research problem under study within the context of the research questions. The use of the mixed methods approach makes it possible to present the research findings quantitatively and also provide opportunities for the researcher to interrogate such findings to explain why such findings/results occur.

Data for this study were collected using the mixed-methods approach, where the collection of qualitative and quantitative data was pivoted around a set of word problems that were attempted by a group of Grade 10 learners (Gray, 2011). According to Schumacher and MacMillan (2010), mixed-methods are useful to collect objective and subjective data. As numerical data collected through objective measures do not have an explicit explanation of other issues related to the phenomenon under investigation, the non-numerical data collected through subjective measures enables the researcher to gain an in-depth understanding of learners' mathematical modelling competencies with an appropriate explanation of adequate scope and quality.

3.4 Research design

The research design refers to a plan for selecting subjects and data collection procedures to answer the research questions (McMillan and Schumacher, 2010). As this study adopted a mixed-methods approach with a greater reliance on qualitative data, the researcher adopted a case study design. According to Cresswell (2015), a case study involves a profound and thorough interrogation and examination of an authentic phenomenon in a natural context. The information generated from case studies leads to rich and in-depth insight into the cases in question (Yin, 2009). A descriptive case study was chosen for this study, as it provides the opportunity to describe the unobstructed phenomena emerging from the collected data, including a description of the data collected (McMillan and Schumacher, 2012). In this research, the case was the worksheet containing the set of word problems that learners were expected to solve. Yin (2009) stated that in a descriptive case study, before starting the study, the researcher must determine the unit of analysis of the study. The unit of analysis in this study was the Grade 10 learners from a Western Cape high school.

3.5 Research Setting

The research was done at a Technical High School X in the Western Cape Province, situated 5 km from the University of the Western Cape. The school offers dual-medium education in English and Afrikaans, along with academic and technical subjects from Grades 8 to 12. The school offers academic subjects for Grades 8 to 9 but academic and technical subjects for Grades 10 to 12. Each grade has six classes with an average of 40 students per class from Grades 8 to 12. The school has a total of 1200 learners.

3.6 Sample and Sampling strategy

For this study, the non-probability sample was used. In non-probability sampling, samples are selected based on the subjective judgement of the researcher, instead of a random selection. Many types of non-probability sampling are available. The researcher used an uncontrolled quota sampling method, which is a non-probability type of sampling to select subjects. In an uncontrolled quota sample, the researcher used a convenience sample which was three (3) classes of Grade 10 learners. These three classes of Grade 10 learners were doing technical mathematics. Classes comprised 23 learners each. These classes were $10A^1$, $10A^2$ and $10A^3$. Learners in $10A^1$ were receiving technical mathematics in Afrikaans only, while those in $10A^2$ and $10A^3$ used English as the instruction medium. The researcher was teaching $10A^3$ – the reason why it was very convenient to use them. The researcher used learners belonging to her and her colleagues' classes $10A^1$ and $10A^2$ as well.

Learners were asked to volunteer to participate. Volunteering is whereby a person willingly gives their time to help an organisation or an individual without payment. From the group of 69 learners, a sample of 20 learners volunteered to participate in this study. Gay and Airasian (2003) described a sample as a part of the population intended to be used. The population is the group of interest for whom the results will be ideally generalised, and from where information will be collected and conclusions are drawn. The sample of 20 learners consisted of 12 girls and eight boys, and they were mixed Xhosa home language speaking and Afrikaans home language speaking learners. Their ages ranged from 15 to 17 years. During the research process, the researcher arranged for learners, who were not part of the group to join another colleague's class to avoid disruptions.

3.7 Data Collection

To provide an answer to the main research question, written feedbacks, to task-based activities, explanations and descriptions are some of the tools to elicit data from the research participants. According to McMillan and Schumacher (2010) strategies for collecting data such as worksheets, observations and semi-structured interviews, makes it possible for researchers to gather data needed for a particular study.

In this study, a worksheet, observation and semi-structured interviews were used to collect the relevant data needed to provide answers to the research question. The worksheet (see Appendix 1) in the context of this study was deemed to be a sheet of paper with a set of word problems associated with simultaneous equations, which leaners had to solve in double mathematics period of 80 minutes. The purpose of the worksheet sheet was to determine learners' modelling competencies, ascertain how they solved problems pivoted around simultaneous equations using modelling steps, and identify challenges and difficulties learners' experienced in solving such problems. These five problems were selected from school textbooks with the aim to allow learners to read and understand the problem, select a strategy to solve the problem (includes building a mathematical model), solve the problem (solve their mathematical mode), and then interpret their solutions to see if it makes sense. As learners were working through each problem individually, the researcher observed learners' moves in attempting to solve the word problems using a structured observation schedule (see Appendix 2)

According to McLeod (2015), a structured observation schedule is used in controlled observations, and it is the duty of the researcher to brief research participants of the study aims so to let them know why they are being observed. In controlled observations, the researcher usually avoids any direct contact with the group, also known as non-participation. Hence, in this research study, the researcher acted as non-participant observer and used the observational checklist was used to record to systematically record results of the behaviour of learners as they worked through each of the word problems. The intention of the observation was to supplement the information provided by the participants to get credible information with regard to their levels of mathematical modelling competencies in solving word problems.

Semi-structured interviews was a third strategy that was planned to be used to collect data The interviews with the learners were scheduled to be completed after school hours for about 45 minutes for 5 days over a week. The reason for conducting the interviews after school hours was due to the fact the researcher did not want to interrupt the teaching and learning process. The interview with each learner was to be about 10 minutes. The researcher designed an interview schedule (see Appendix 3) to ensure focus without imposing too much pressure on participants. Qualitative research allows interaction between the researcher and participants as it provides data collection based on human experiences. For example, the learners expressed how they felt about mathematics after the intervention of intentional teaching strategy. Furthermore, the qualitative approach creates openness during research. The researcher asked open-ended questions, and learners were given the freedom to answer them in any way they could. As part of the interviewing process, participants had the choice to answer questions they wished to at various stages of the interview as recommended by Cresswell (2015). However, these interview plans were not fully realized, as during the time of data collection, there were municipal protests and taxi protests. This created fear in the learners and uncertainty about staying at school after hours, as most of them travelled to school using public transport. Hence, very few leaners were interviewed with a limited number of questions posed.

The use of participants' written responses to the worksheet items, non-participant observation, and limited semi-structured interviews aided the triangulation of data collection methods so as to get the most important information from the data during the analysis.

3.7 Data Analysis

There is a range of processes of data analysis. In this study, we used the data collected from participants' responses to the word problems in the worksheet, observations and limited semi-structured interview to do the necessary analysis gain a better understanding of the information to discern appropriate answers to the posited research question. To facilitate this process, the researcher first marked each learner response in terms of the expected solution presented in Appendix 2 to gain a 'bird's eye' picture of learner performance in problem-solving. The marked responses were then classified into three categories, incorrect, partially correct, and completely correct. This classification then paved the way for qualitative content analysis on each categorical response problem-wise. According to Zhang and Wildemuth (2009:319), qualitative content analysis centres on characteristic themes which describe a phenomenon rather than the statistical significance of the particular texts or concepts.

Qualitative analysis can be deductive when the researcher argues from the general to specific or inductive or when the reverse happens (Du Plooy, Davis and Bezuidenhout, 2014). In qualitative data analysis, data sets are grouped into chunks, which are later assigned to broader categories of related meanings (Maree, 2007). In doing so, the data sets are coded (like levels of performance in this study) and themes, which can be applied to all the textual information (like the responses from learners to each problem in the worksheet). Following these themes, the researcher can identify possible patterns in the data set. The act of grouping data into certain categories is referred to coding, and this, in turn, simplifies and enhances the management of complex data sets (Du Plooy, Davis and Bezuidenhout, 2014). Coding can also be applied to written texts, including observational notes and interview transcripts (Maree, 2007).

For this study, the researcher conducted a deductive qualitative analysis. To assist with this, the researcher constructed an analytical framework (see Table 3.2) using the literature on mathematical modelling and the theoretical framework on mathematical modelling competencies as articulated Blum and Kaiser (as cited in Maaß, 2006) and discussed in Chapter two. To facilitate the construction of the analytical frameworks (shown in Table 3.2 and Table 3.3) in the form of a rubric, the researcher used the generic analytical rubric template shown in Table 3.1 as a guide.

Table 3.1: Generic structure for development of analytical rubric

		Performance Ratings		
		Performance Level 1	Performance Level 2	Performance Level 3
Criteria	[Criterion 1]	Performance descriptor	Performance descriptor	Performance descriptor

	[Criterion 2]	Performance descriptor	Performance descriptor	Performance descriptor
	[Criterion 3]	Performance descriptor	Performance descriptor	Performance descriptor

The generic analytical rubric structure (as indicated in Table 3.1), made provision for three important elements that characterize a rubric, namely: performance rating, criterion, and performance descriptor. As indicated in Table 3.2, we developed the modelling competencies' criteria for assessing learners' responses with reference to understanding the problem, building a mathematical model and solving the mathematical model for successful completion of the modelling problem. We took into consideration that there is a broader spectrum of modelling competencies, which includes interpreting results and validating results. However, since the Grade 10 learners have not been specifically and directly exposed to model eliciting tasks previously, it was decided to assess competencies that could permeate normal teacher-led mathematics lessons, for example, understanding the problem, building a model and solving the model. These core steps of the modelling processes were used to construct and frame each performance descriptor in terms of the set of modelling competencies and sub-competencies, that was used to **code** learners' responses into levels 1-3 as indicated in Table 3.2. While competencies of interpreting and validating results were not considered in coding learners responses into levels 1-3, they were nonetheless given due consideration in the analysis through the use of the analytical framework given in Table 3.3.

Table 3.2: Mathematical modelling competency- analytical framework 1 - understanding, building a model, and solving a model

Competencies	Level 1 Not competent	Level 2 Partially competent	Level 3 Proficiently Competent
Understanding the problem	• No comprehension of the context/given information • No assumptions made/ incorrect notions of assumption. • No key words listed • Does not recognize quantities associated with problem. • No variables • No evidence of any relationship or relationship is muddled and does not make sense • Includes the expressions showing that s/he did not understand the problem, did not determine the givens and goals, and did not form, or mistakenly formed, a relationship between them.	• Some aspects of the context/given information are comprehended • Assumptions stated are irrelevant or partially relevant to model • Recognize most quantities associated with problem, names them and identifies variables to some extent. In some cases less than 2 variables listed based on interpretation of task. • Constructs relationships between identified/generated variables that are correct to some extent or incorrect to some extent or irrelevant or has missing parts. • Includes the expressions showing that s/he understood the problem to some extent or completely, determined the givens and goals to some extent but did not form, or mistakenly formed, a relationship between them.	• Pertinent aspects of the context or given information are comprehended • Assumptions stated are relevant to model • Recognize all quantities associated with problem, names them and identifies all key variables • Constructs correct meaningful relationships between identified/generated variables • Includes the expressions showing that s/he understood the problem completely, determined the givens and goals, and formed a relationship between them.
Building Mathematical Model	• Uses no mathematical language or notation • No evidence of the use of representations • No evidence of mathematizing • Never show attempt to develop a mathematical model or mistakenly constructs mathematical model/s. • No evidence of system of simultaneous linear equations.	• Some attempt made to use language and notation • Attempts to use representations in the presentations towards building the model • Makes some attempt to mathematize identified quantities and their relations but not completely successful. • Show little attempt to build a mathematical model or constructs correct mathematical model/s based on partly- acceptable assumptions or constructs incomplete/wrong mathematical model/s based on realistic assumptions and relating them to one another. • Model is represented by a system of simultaneous linear equations which is wrong or partially correct.	• Uses appropriate and correct mathematical language and notations • Uses representations appropriately and effectively (e.g, diagrams, graphs, notations) • Mathematizes relevant quantities and their relations efficiently • Develop a perfect mathematical model: correctly construct the needed mathematical model/s according to realistic assumptions, explaining model/s, and relating them to one another. • Model is represented by a perfect system of simultaneous equations (two correct linear equations).
Solving model	• Shows limited or no understanding of mathematical concepts • Mathematical reasoning is not logical • Inappropriate use of mathematics • Errors shown in computation • Not presenting a mathematical solution, wrongly solving the constructed models or trying to solve the wrong mathematical model.	• Shows some understanding of mathematical concepts • Mathematical reasoning is somewhat logical • Incomplete response and /or Inappropriate use of mathematics • Minor errors shown in computation or includes deficiencies or mistakes in the solution of the correctly constructed mathematical models. • Correctly solves the mathematical models that were incompletely/wrongly constructed.	• Shows clear understanding of relevant mathematical concepts • Mathematical reasoning sis logical • Correct use of mathematics • Computation is clear and correct • Achieving the correct mathematical solution by solving the correctly constructed mathematical models

Table 3.3: Mathematical modelling competency- analytical framework 2 - interpreting and validating a solution

Competencies	Not achieved	Partially competent	Proficiently Competent
Interpreting solution	• No attempt to use mathematical language, communication is poor • Misinterpreting, or not interpreting, the obtained mathematical solution in given context	• Some attempt to use mathematical language, and ideas are not clearly communicated. • Correctly interpreting the erroneous or incomplete mathematical solution in given context. • Incompletely interpreting the obtained correct mathematical solution in given context	• Correct use of mathematical language, clear appropriate explanations. • Correctly interpreting the obtained correct mathematical solution in given context.
Validating solution	• Does not critically check and reflect on found solutions • Not validating or making an invalid validation	• Critically check and reflect on found solutions to some extent • Validating completely and not correcting the determined mistakes • Validating completely and correcting the determined mistakes to some extent	• Critically check and reflect on found solutions • Validating completely and correcting the determined mistakes.

The numeric data, showing the number of learners, who were coded as demonstrating level 1, and level 3 performances, were assimilated into a table for each problem and amplified using simple descriptive statistical displays such as table and column graphs.

The qualitative data collected via observations and semi-structured interviews were coded and used to augment, substantiate, explain and justify the levels of performance generated through the levels of performance that emanated through analysis of data via the rubric shown in Table 3.2 and Table 3.3, wherever applicable.

3.8 Validity and reliability concerns

Through sharing activities with my supervisor and colleagues, who teach Grade 9 and Grade 10, I validated the instruments used in the data collection. Performing this activity ensured that tasks were free of ambiguity and that the level of the language used in the instruction was at the learners' level of understanding. In the process, I allowed for member checking, which is a quality control procedure to improve the accuracy, credibility and validity of the captured records (Cresswell, 2015). As respondent validation, both the researcher and participants verified that captured records were accurate. All participants verified the information as accurate, which served as an assurance for validity.

According to McMillan and Schumacher (2010), the reliability measurement procedure is solidity or consistency of the measurement. This means that, if the same variable is measured under the same conditions, a reliable measurement procedure will produce identical or nearly identical results. Thus, to establish reliability, data were collected from two Grade 10 learners, who provided answers to the problems and indicated areas of difficulty regarding the research tools used. Issues raised by the subject head, colleagues, and the two learners were considered to ensure reliability of the instruments that were used in the data collection process.

3.9 Ethical considerations

A clearance letter from the Department of Education and ethical clearance from the University of the Western Cape were obtained before data collection commenced. The researcher used the letter to request permission to conduct the research at the proposed site. Ethical considerations are important (Singer & Vinson, 2002). Permission is required, even though a research study is built on trust between the researcher and the case (Andrews & Pradhan, 2001). Explicit measures were adhered to prevent problems. Participants were informed about the research process that was to be conducted, and given the opportunity to exit the research activities at any stage that they felt uncomfortable. Pseudonyms were used to ensure that participants' names would not be published in the manuscript, and also the name of the school is not disclosed in this manuscript.

All consent forms obtained from participating learners and their parents were filled by the stakeholders mentioned and signed (see Appendices b3 to b7). The researcher created a comfortable environment for the participants during the class activity and interviews and made sure that everybody understood the whole process. Moreover, the researcher maintained confidentiality by not declaring information about respondents to anyone – all information was kept private.

3.10 Summary – Chapter 3

All the information about the process, procedures and measures taken during the empirical study has been recorded in this chapter. The discussion in this chapter elaborates on the research design and methodology selected for soliciting data through qualitative and quantitative methods. The analytical measures to determine the validity of the research instruments and reliability of data collected from the sampled participants have also been explained. The discussion indicated how the data collected were analysed and coded using a mathematical modelling competency rubric. The importance of ethical issues was also highlighted in this study.

Chapter 4: Data Analysis and Findings

4.1 Introduction

Chapter three offered an overview of the design, methodology and approach that was applied to answer the critical questions of this study. A comprehensive description of the data collection tools and how they were used to collect data was discussed in detail. Furthermore, validity, ethical issues, and data analysis methods were explained. An analysis of the Grade 10 learners' written responses to five modelling type questions invoking simultaneous equations, is provided in this chapter and the main findings summarized. The analysis was carried out based on learners' written work regarding the criteria outlined in the analytical rubrics. For each question, learners' responses were interrogated to establish how they solved word problems through modelling with simultaneous equations and the kinds of mathematical modelling competencies they demonstrate.

4.2 Analysis of data collected using the Worksheet

The activities in the worksheet (Appendix 1) required learners to solve mathematical problems individually. The worksheet included five-word problems, which learners were expected to solve at their own pace. The researcher supervised and controlled the problem-solving activities, ensuring that learners did not seek any assistance from each other. After the learners completed the task and submitted their written work, the researcher marked the answer sheets as per designed marking guideline and provided feedback and remarks to participating learners.

In this section, the data collected from the worksheets as completed by 20 Grade 10 Mathematics learners, are analysed item by item through qualitative content analysis procedures. The items in the worksheet were designed to provide an opportunities for learners to read and understand the problem, generate variables, select relationships, generate relationships, model situations through a system of simultaneous equations, use mathematical techniques to solve the system of equations, and ultimately look back at their solutions to see if it makes sense and modify it is deemed necessary. In doing so, it was envisaged that the learner could traverse the core steps of the modelling process, namely:

- Getting to know the problem and understanding it;
- Building a mathematical model
- Solving the mathematical model

- Interpreting results

As illustrated in the analytical rubric in Table 3.2, the modelling competencies demonstrated by the learners for each task will be characterised into three levels (level 1, level 2, and level 3). The lowest is level 1 suggesting the lack of the competencies and the highest level 3 the explicit demonstration of modelling competencies. A selection of learners' responses to each question is presented to exemplify the level of modelling competence exhibited by learners, including how they solved the word problems. The analysis will be furnished question wise. Before proceeding with the analysis, the problem will be re-stated, and an expected solution will be provided to give a general sense and feel of the nature of the modelling competencies the problem and this will be followed by the exemplification of each level modelling competence exhibited. Analytical framework 3B will be used to provide additional insight into learners modelling competency regarding interpreting and validating results.

4.2.1 Analysis of modelling competencies: Problem 1 in the worksheet

Problem 1 in the worksheet reads as follows:

Half the sum of two numbers is 27 and their difference is 6. What are the numbers?

An expected solution for Problem 1 is as follows:

Let the larger number be x *and the smaller number be*y.

Half their sum = 27: therefore $\frac{1}{2}(x + y) = 27$ ……. *(1)*

Their difference = 6: therefore $x - y = 6$ …… *(2)*

Multiply equation (1) by (2)

Therefore, the answer will be $x + y = 54$ ……. *(3)*

Now add equation (2) and equation (3) to get rid of 1 variable (namely, y)

$x - y = 6$

$x + y = 54$

$2x + 0 = 60$ …… *(4)*

$\therefore x = 30$

Now substitute $x = 30$ *in equation (3),*

30 + y = 54

$\therefore y = 24$

So the numbers are 30 and 24.

Check: Difference: 30-24 =6

Half the sum of two numbers is 27: $\frac{1}{2}\ (30 + 24) = \frac{1}{2}(54) = 27$

As illustrated in Figure 4.1, analysis of learners' responses to Problem 1 showed that three learners were assessed to be at level 1 modelling competence, 12 learners at level 2 modelling competence, and five learners at level 3 modelling competence.

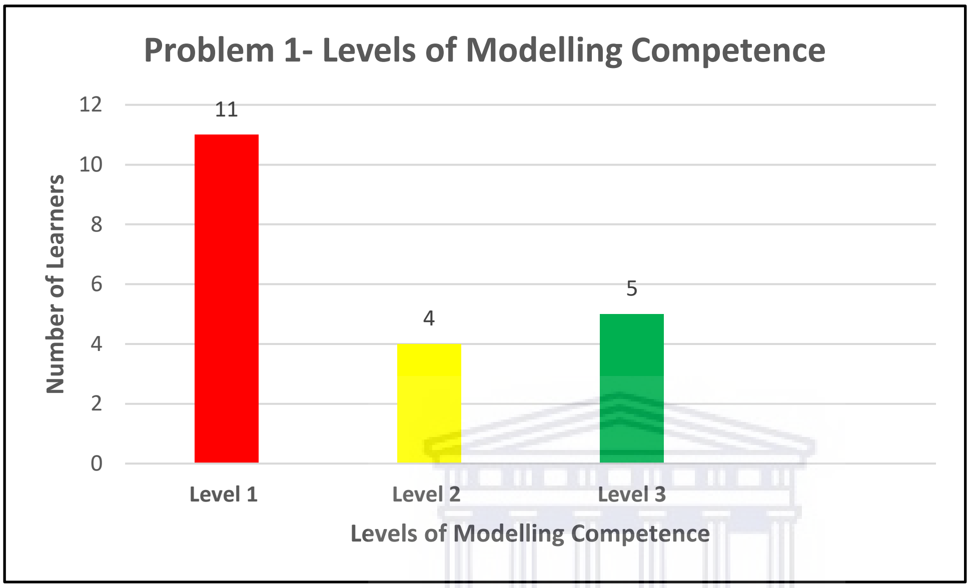


Figure 4.1: Problem 1- Levels of Modelling Competence

4.2.2 Exemplification of Level 1 Modelling Competence

In this set of eleven learners, who demonstrated level 1 modelling competence, five learners attempted to solve the problem through doing various kinds of incorrect mathematical manipulations and calculations, which resulted in the productions of incorrect answers. However, the other six learners just wrote down the correct answers 24 and 30 in one line without showing any calculations, written out work or reasoning.

The five learners, who showed incorrect mathematical manipulations and calculations, demonstrated no comprehension of what was given and what was required to calculate as evident. None of the learners stated any assumptions governing the problem associated with the sum and difference of two numbers. They did not consider the use of variables and muddled up connections between 27 and 6 and the concepts of sum and difference. For example, all of this is evident in the response of learner X, as shown in Figure 4.2.

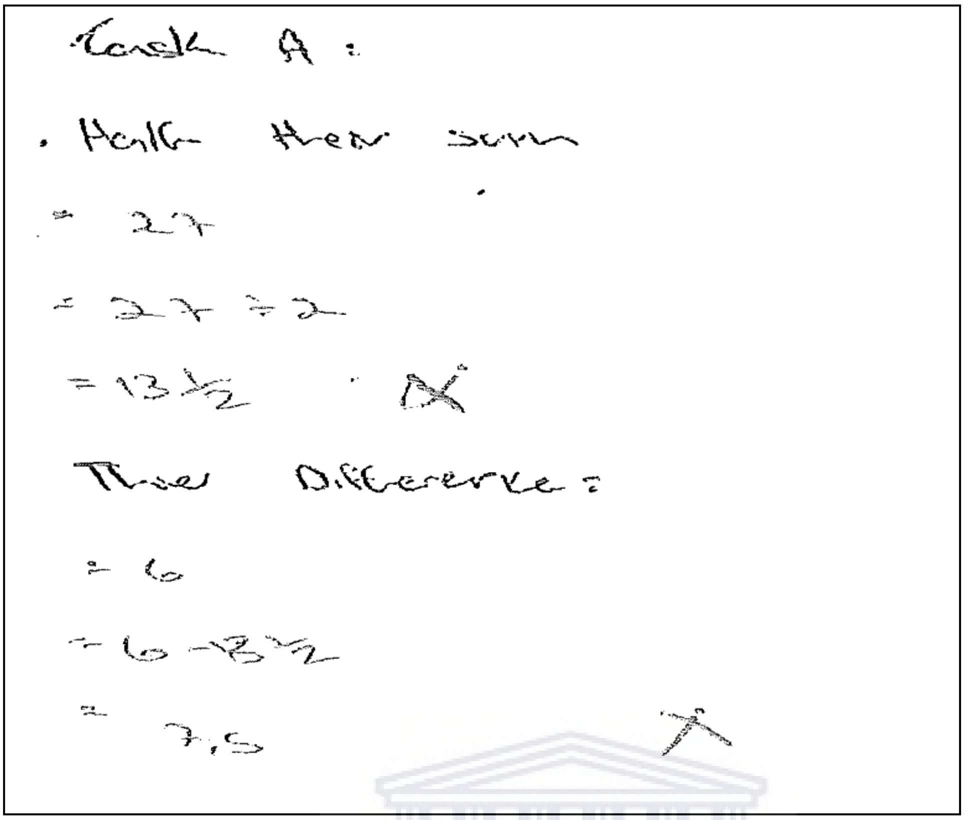


Figure 4.2: Learner X response to Problem 1

In particular, learner X did not realize that 27 is actually the answer to the sum of the two given numbers that is posited in the problem but instead just proceeded to find half of 27. When the researcher probed learner X during the interview, as to why s/he found half of 27, learner X responded as follows: "*I did not read the question well*".

Furthermore, learner X could do not fathom out the quantities between which a difference of 6 exists, and hence wrote down a completely wrong number sentence, namely '$= 6 - 13\frac{1}{2}$'. When the researcher probed learner X during the interview as to why she 'found half of 27' and wrote down '$= 6 - 13\frac{1}{2}$' Learner X responded as follows: "*I did not read the question well".* All of this demonstrates a **lack of understanding** of the problem.

The dilemma of not reading and comprehending the problem has led learner Y to merely write down two numbers, as shown in Figure 4.3.

Figure 4.3: Learner Y response to Problem 1

When the researcher probed learner [Y] as to how she arrived at her answer. Learner Y responded as follows:

"Half of 20 is 10 and 27 take away 20 is 7, so I added 7 to 10 to get 17, and half of 10 is 5 plus 6 is 11 in the Equation".

Like learner X, it is evident that the learner Y has muddled up the relationships between the given quantities in the problem, which makes no sense. Retrospectively, learner Y like learner X does not demonstrate any evidence of the use of representations and mathematization. In particular learner X, U, V, Y and P, does not show any attempt to develop a mathematical model or build a system of simultaneous linear equations. This shows a **lack of competence in building a mathematical model** that makes sense.

Despite learner U, V, X, Y and P showing no understanding of mathematical concepts within the context of the given problem, their responses itself are symptomatic of illogical mathematical reasoning and inappropriate use of mathematics. Furthermore, learner U, V, X, Y and P, did not attempt to look back at their solution to either interpret/validate it within the given context.

At the Grade 10 level, learners are expected to formulate equations using variables and solve them simultaneously as deemed necessary. Besides, they are expected to show the relevant steps they followed when working out their solutions to the problem. To the contrary, as indicated earlier, six out of the 11 learners, just wrote down the correct answers 24 and 30. Figure 4.4 illustrates such a one-line response written by learner G.

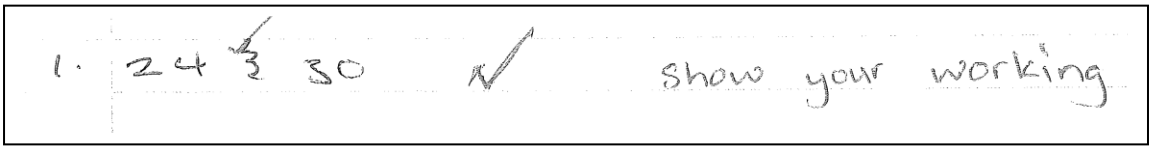


Figure 4.4: Learner G response to Question 1

When the researcher asked learner G how she came up with the two answers, 24 and 30 that are correct, she answered as follows:

I multiplied 27 by 2 and got 54 and then subtracted 24 from 54 to get 30, I also subtracted 3 from 27 and I got 24.

When the researcher asked why she subtracted 3 from 27, learner G responded as follows:

I do not know.

The step of multiplying 27 by 2 to get 54 shows that the learner did manage to figure out a strategy to obtain the value of the sum of the two unknown numbers. Knowing that the sum of the 2 unknown numbers is 54 is crucial to solving the problem. Although the learner could not explain why she subtracted 3 from 27 to get 24, she could have found half of 6 (which is the difference between the 2 unknown numbers) to get 3 which she subtracted from 27 (which is half the sum of the 2 unknown numbers) to get 24. As to why the learner initially subtracted 24 from 54 to get 30 is not clear. Even though 24 and 30 are the correct answers, the strategies and procedures used to generate 24 and 30 lack mathematical explanation and correctness.

In my observation of the learners, while they were busy attempting the questions, I realised that some of them became overwhelmed by reading the word problem questions. I could see the expression on their faces.

4.2.3 Exemplification of Level 2 Modelling Competence

Four learners, who demonstrated level 2 competences attempted to build a system of linear equations, which was partially correct. Learner C's response, as shown in Figure 4.5, illustrates one such response.

It seems that learner C has comprehended aspects of the given problem to some reasonable extent. This is evident in the assignment of variables x and y to the two numbers mentioned in the problem. The initial algebraic representation of the statement 'half the sum of two numbers is 27' was correctly represented as 'half sum$= 27 \rightarrow \frac{1}{2}(x+y)$' However, the translation of this to an equivalent relationship was wrongly expressed as "∴sum$= 54 \rightarrow 2(x+y)$. This could be attributed to the learner correctly doubling 27 to get 54 but instead of doubling $\frac{1}{2}(x+y)$ to get $(x+y)$ chose to double $(x+y)$ to get$2(x+y)$. However, the interpretation of the statement, '… and their difference is 6' was conceptually corrected as expressed by the linear algebraic equation, $x-y=6$. All of these moves suggest that learner C has recognised the pertinent quantities associated with the problem and connected them to variables x and y, and proceeded to construct relationships between them.

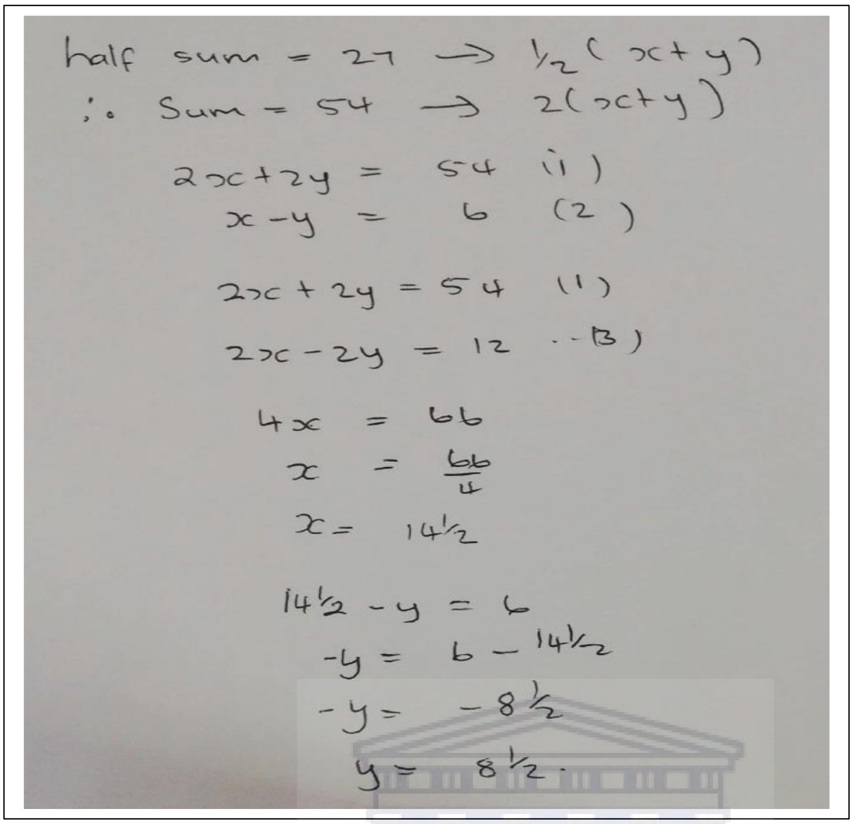


Figure 4.5: Learner C response to Problem 1

With the exception of the error, “ $\therefore$sum= $54 \rightarrow 2(x+y)$, all other relationships were correctly constructed adn assimilated to form the following system of simultaneous linear equations:

$$2x + 2y = 54 \quad \ldots\ldots(1)$$

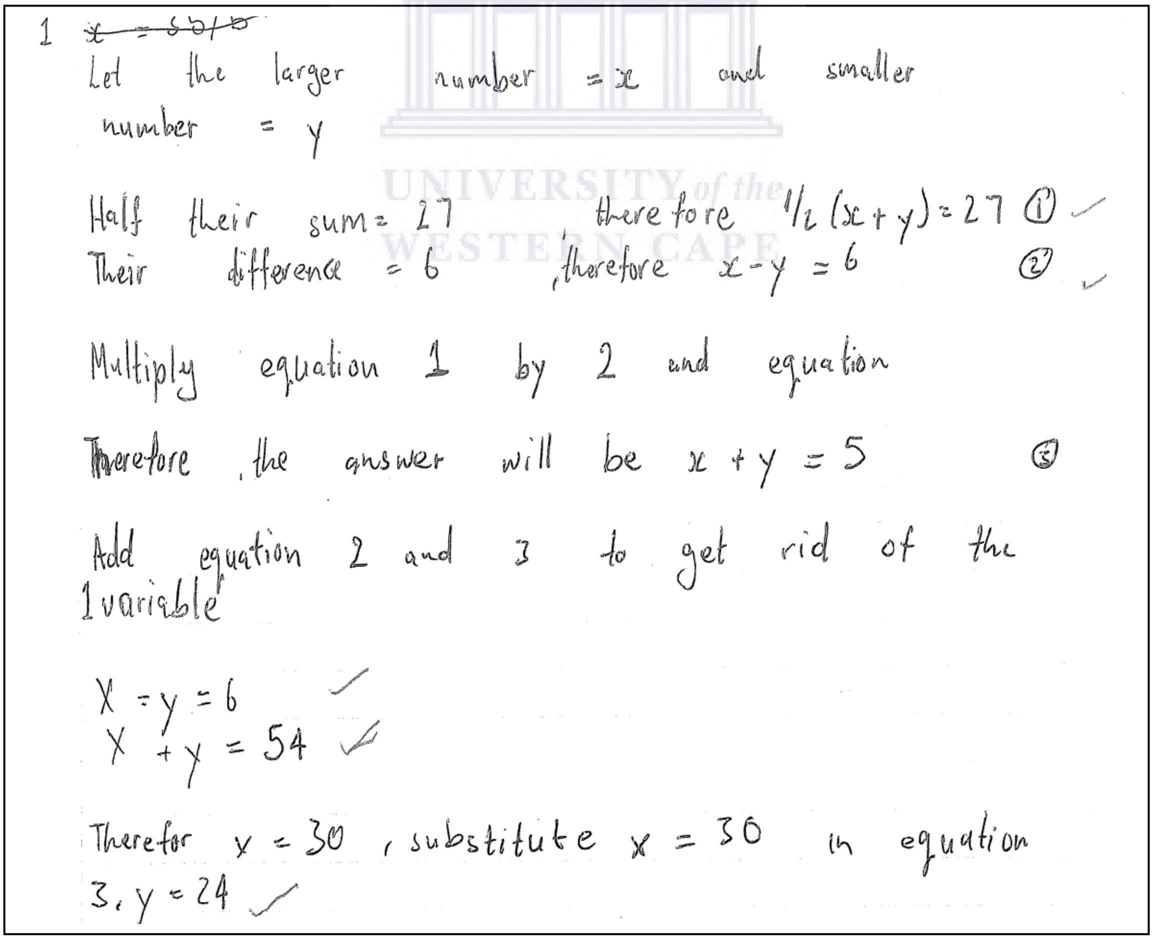
$$x - y = 6 \quad \ldots\ldots \quad (2)$$

The system of simultaneous linear equations assimilated by learner C, shows that he was able to develop a model that could be used find the solution to the given problem, even though the model itself was partially correct.

In solving the system of linear equations simultaneously, the learner made a computational error when simplifying $\frac{66}{4}$ to get $14\frac{1}{2}$ insetad of $16\frac{1}{2}$. This resulted in the learner in solving the system of simultaneous equations to get $x = 14\frac{1}{2}$ instead of $x = 16\frac{1}{2}$. This could be attributed to learner not being proficient in the simplification of fractions. However, this error was carried through consistently without any further computational errors to obtain of $= 8\frac{1}{2}$. So in terms of solving the model, the learner does shows understanding of the salient steps in the procedure to solve a system of linear equations even though a minor error is shown in the computation of the value of x. However, like the other three learners in this group, learner C, made no attempt to interpret/validate the solution within the given context.

4.2.4 Exemplification of Level 3 Modelling Competence

As illustrated in Figure 4.1, five learners exemplified level 3 modelling competence. Their level of comprehension and understanding of the problem was evident in the kinds of assumptions made in relation to the unknown numbers associated with the problem, the corresponding variables assigned to the number, and the construction of meaningful relationships between the variables. Their kinds of mathematizing effort were richer regarding formulating mathematical relationships and building a mathematical model represented by a system of two linear equations. These five learners solved their respective mathematical models to achieve the correct mathematical solutions. Figure 4.6 illustrates the detail response of one such learner Z.


1 ~~x = 55/5~~
Let the larger number = x and smaller number = y

Half their sum = 27, therefore 1/2 (x + y) = 27 ①
Their difference = 6, therefore x - y = 6 ②

Multiply equation 1 by 2 and equation

Therefore, the answer will be x + y = 5 ③

Add equation 2 and 3 to get rid of the 1 variable

X = y = 6
X + y = 54

Therefor x = 30, substitute x = 30 in equation 3, y = 24

Figure 4.6: Learner Z response to Problem 1

The very first statement, ‘*let the larger number be* $\boldsymbol{x}$ *and the smaller number be* $\boldsymbol{y}$*’*, are credible assumptions showing firstly that the learner has realized that since there exists a difference between the numbers, the two numbers are not equal and consequently one number should be bigger than the other to get a difference of 6. This has led the student to assign variables to the quantities identified, namely the larger number to be x and the smaller number to be y. The assumptions made were followed up in the mathematisation process which culminated in the construction of key linear relationships as articulated in the following system of linear equations:

$$\frac{1}{2}(x+y) = 27 \dots\dots\dots (1)$$

$$(x-y) = 6 \qquad \dots\dots\dots (2)$$

In building up the linear equations $\frac{1}{2}(x+y) = 27$ and $(x-y) = 6$ the learner has respectively demonstrated the necessary conceptual understanding of the statements ‘*half their sum is equal to 27*’ and ‘their *difference is 6*’. This suggests that the learner has the necessary competence to build and use representations appropriately in the form of algebraic equations, and was successful in building a mathematical model that could assist in solving the given problem.

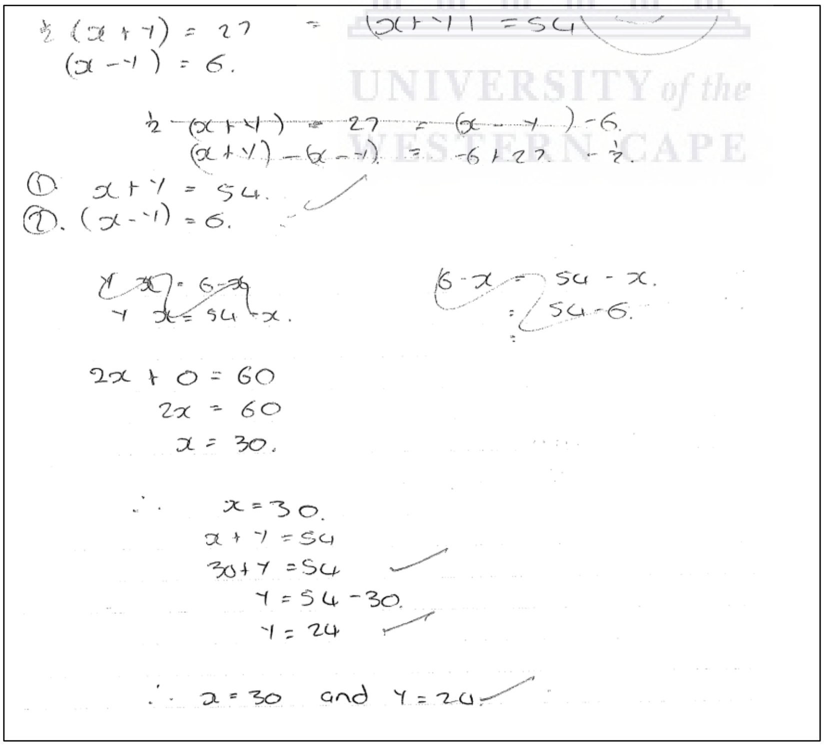


Figure 4.7 illustrates the detail response of a learner H

Although there is a small slip (namely writing down $x + y = 5$ instead of writing$x + y = 54$) as noted in line 6 of the learner's solution, this is mitigated by the correct equation $x + y = 54$ written down in line 10 of the learner's solution. Despite this minor anomaly, the learner used the elimination method accompanied with logical reasoning and sound use of mathematical procedures and manipulations to obtain the required values of the respective numbers. Regrettably, the learner did not look back and test if the answers 24 and 30 were correct or not.

Although learner H did not explicitly state that x and y represents the two unknown numbers nor which the variables x or y) represent the bigger number or smaller number, it is plausible to assume that the learner assigned the variables x and y to the unknown members as evident in the constructions of two sets of relationships represented by following the system of linear equations:

$\frac{1}{2}(x + y) = 27 \ \rightarrow (x + y) = 54$

$$(x - y) = 6$$

It seems that initially, the learner tried to solve by setting up $\frac{1}{2}(x + y) - 27 = (x - y) - 6$. Thereafter the learner manipulated it to get to up $(x + y) - (x - y) = -6 + 27 - 1/2$. These moves are mathematically flawed and could not enable the learner to obtain the values of x and y. It is plausible, that the learner on realizing that he/she could not move any further, recalled the procedure to solve a system of simultaneous equations, and hence proceeded to reflect on the initial set of equations rewrite it down as follows:

$x + y = 54 \ldots . \ldots (1)$

$(x - y) = 6 \ldots \ldots (2)$

This system inadvertently characterizes the mathematical model that needs to be solved to find the two given numbers described in this problem. Although the learner did not annotate what he/she was doing in each step in the process of solving the system of simultaneous equations, it can be easily discerned as to what steps and procedures the learner invoked and applied to successfully solve for x and y. For example, to get to $2x + 0 = 60$ it is quite clear that learner added equation (1) to equation (2), and thereafter solved for x to get $x = 30$. In addition, the equation $30 + y = 54$, demonstrates the the learner replaced x by 30 in equation (1), which reads $x + y = 54$. The learner correctly solved $30 + y = 54$ to get $y = 24$. In the final analysis, the learner concludes: $\therefore x = 30$ and $y = 24$. All of this demonstrates that the learner knows and understands the procedure to solve the system of linear equations simultaneously, and is able to apply it correctly to solve the constructed mathematical model. However, like most other learners, there is no written evidence to show that the learner verified and validated his/her answers.

4.2.5 Analysis of modelling competencies: Problem 2 in the worksheet

Problem 2 reads as follows:

If 1 is added to the numerator and 2 to the denominator of fraction, the ratio of the ne numerator to the new denominator is 2:3. If 1 is subtracted from the numerator and 2 from the denominator, the new numerator is equal to new denominator. Find the fraction.

An expected solution for Problem 2 is as follows:

Learners were instructed to follow the instruction which stated, "Using the numerator N and denominator D":

Therefore, $\frac{N+1}{D+2} = \frac{2}{3}$ …………………(1)

$3N + 3 = 2D + 4$

$3N - 2D = 1$ ………… (2)

$N - 1 = D - 2$ … … (3)

$N - D = -1$ … … (4)

Multiply equation (3) by 2 to get:

$2N - 2D = -2$ … … … … … … (5),

Equation (4)- Equation (5):

$3N - 2D = 1$

$\underline{2N - 2D = -2}$

$N = 3$

Substitute $N = 3$ in equation 4:

$3 - D = -1$

$\therefore D = 4$

Figure 4.8 represents the number of learners demonstrating particular levels of associated modelling competencies. Fifteen out of 20 learners (75%) struggled to make any headway in responding to the problem, while three learners showed some competency in recognizing quantities linked to the problem and assigning variables to them to the extent of having two sets of linear equations and not proceeding further. Only two learners (10%) were able able to determine the respective values of the numerator and denominator that makes up the fraction.

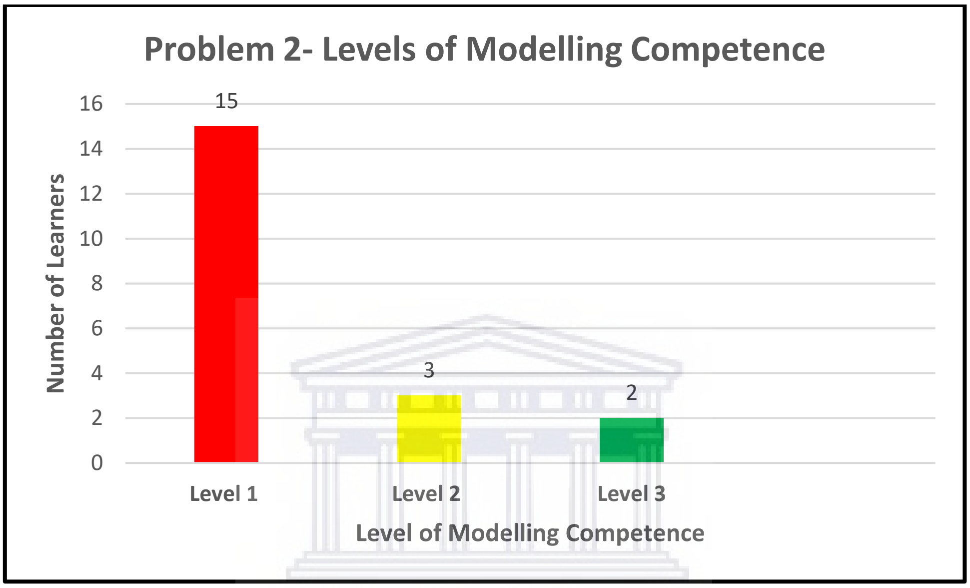


Figure 4.8: Problem 2- Levels of Modelling Competence

4.3. Exemplification of Level 1 Modelling Competence

Examples of learner’s work showing a lack of any modelling competencies are illustrated in Figure 4.9 and Figure 4.10. It is quite evident, that while these learners had some literal conception of what a fraction is, they were not able to comprehend what each of the given conditions in the problem implied.

Figure 4.9: Learner S response to Problem 1

For example, as can be seen in Figure 4.9, learner S wrote down ‘ $= \frac{1}{1} + \frac{1}{2} = \cdots$’, signalling that he could have thought that the given fraction is ‘ $\frac{1}{1}$’ and not realizing that it is unknown at this stage, and hence should be represented by a set of variables like ‘ $\frac{x}{y}$’. Furthermore, the learner either misconstrued the addition of $\frac{1}{2}$ to $\frac{1}{1}$ as the same as $\frac{1+1}{1+2}$ by writing $= \frac{1}{1} + \frac{1}{2}$. This confirms that learner S did not undertand the given condition, namely: “*If 1 is added to the numerator amd 2 to the denominator*.” The latter condition is seriously misunderstood by learners S and M, as they mererly write the fraction $\frac{1}{2}$ in their very first line and confrim that 1 is the numertaor and 2 is the denominator of the fraction.

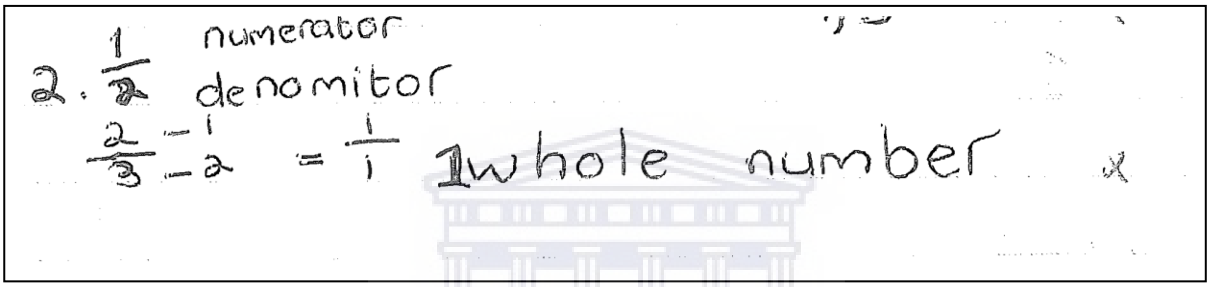


Figure 4.10: Learner M response to Problem 2

Learners S and M, like others in this group, show deficits in understanding the concept of a ratio. The condition, “ … *the ratio of the numerator to the new denominator is 2:3*” has been literally taken to mean the resultant fraction is $\frac{2}{3}$ rather than realizing that the ratio of the numerator to the denominator in the new fraction after adding 1 to the numerator and 2 to the denominator is $\frac{2}{3}$. There is a likelihood that these learners did not realize that the ratio of $\frac{2}{3}$ could imply that numerator and denominator of the new fraction could be 4 and 6, respectively or 6 and 9, respectively, and not simply 2 and 3. They ought to know the concept of equivalent fractions, for example, are $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

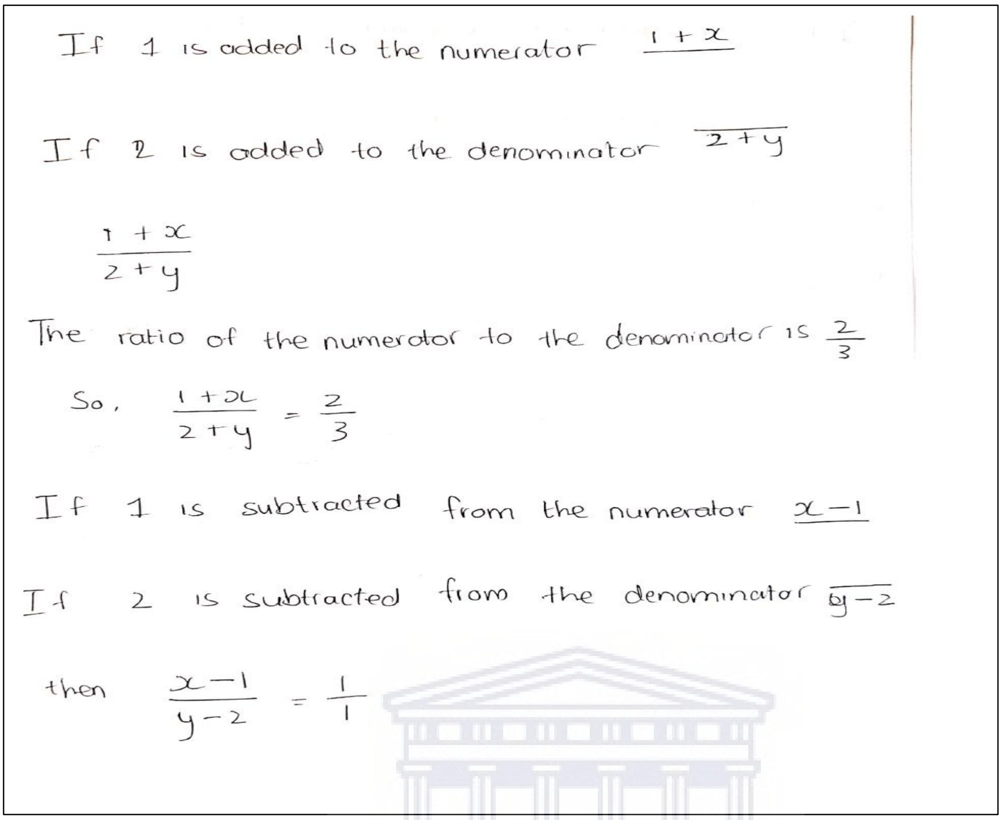
In taking the fraction formed after adding 1 to the numerator and 2 to the denominator to be mistakenly $\frac{2}{3}$, learner S subtracted $\frac{1}{2}$ from $\frac{2}{3}$ whereas learners M explicitly subtracted 1 from 2 (the numerator) and 2 from 3 (the denominator) in response to the given condition: “*If 1 is subtracted from the numerator and 2 is subtracted from the denominator*”. This is indicative of some understanding of the aforementioned condition by learner M even though it was applied to the incorrect fraction as follows: $\frac{2-1}{3-2} = \frac{1}{1}$. However, in the case of learner S, there is a complete conceptual breakdown as the learner first assimilated the fraction $\frac{1}{2}$ and then subtracted it from $\frac{2}{3}$, which shows that he/she did understand the condition. Mathematically learner S's statement $\frac{2}{3} - \frac{1}{2} = \frac{1}{1}$ does not make sense and is mathematically flawed as $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$. It is plausible that learner S intended to subtract 1 from the numerator and 2 from the denominator of the fraction $\frac{2}{3}$, but did not know how to symbolically represent such a relationship. Although by coincidence learners S and M simplification ultimately yields the fraction $\frac{1}{1}$, which signals that the ratio of the numerator to the denominator is 1:1 (i.e equal), this does not in any way lead to finding the correct fraction, where the numerator is 3 and the denominator is 4.

In summary, learners S and M, like the others in this group of 15 learners, did not articulate any set of assumptions, nor assign variables to the unknown numerator and denominator of the unknown fraction at the initial stages of their modelling journey. This negatively impacted on reading the imposed conditions onto a legitimate fraction as expressed through clearly defined variables, and the construction of plausible fractional algebraic relationships which could serve as a mathematical model to assist in finding the values of the numerator and denominator of the required fraction in the given problem. Mathematical reasoning and computation were compromised, for example, as exemplified by treating a ratio of $\frac{2}{3}$ as being the value of the fraction in the pre-ultimate steps.

4.3.1 Exemplification of Level 2 Modelling Competence

Although learner T, as illustrated in Figure 4.11, did not solve the problem, he/she did show some understanding of the given information to some extent by assigning variables to recognised quantities and building two sets of correct linear equations.

A statement written in the *if-then* form is a conditional statement. The part of the statement following *if* is called the *hypothesis*, and the part following *then* is called the conclusion. In the case of the conditional statement, *If 1 is added to the numerator and 2 to the denominator of a fraction, the ratio of the numerator to the denominator is 2:3*, the hypothesis is identified as*' If 1 is added to the numerator and 2 to the denominator of a fraction'* and the conclusion is identified as *'the ratio of the numerator to the denominator is 2:3'.*


If 1 is added to the numerator $\frac{1+x}{\quad}$

If 2 is added to the denominator $\frac{\quad}{2+y}$

$\frac{1+x}{2+y}$

The ratio of the numerator to the denominator is $\frac{2}{3}$

So, $\frac{1+x}{2+y} = \frac{2}{3}$

If 1 is subtracted from the numerator $\frac{x-1}{\quad}$

If 2 is subtracted from the denominator $\frac{\quad}{y-2}$

then $\frac{x-1}{y-2} = \frac{1}{1}$

Figure 4.11: Learner T's response to Problem 2

In the very first line, the learner writes the equivalence of $\frac{1+x}{\quad}$, which confirms that he assigned the variable 'x' to represent the value of the numerator in the unknown fraction. This assignment of variable 'x' is actually an assumption the learner as lately put forth even though not explicitly stated. Likewise, the learner writes $\frac{\quad}{2+y}$, which confirms that he assigned the variable 'y' to represent the value of the denominator in the unknown fraction. These expressions of the numerator and denominator, demonstrate correct comprehension of the hypothesis, 'If *1 is added to the numerator and 2 to the denominator of a fraction*. By writing down $\frac{1+x}{2+y}$ in line 3, the learner has represented the transformation of the unknown faction $\frac{x}{y}$ by interpreting and applying the hypothesis in the conditional statement correctly.

As illustrated in lines 4 and 5 in Figure 4.11, the learner has fully grasped the meaning of the conclusion in the initial given conditional statement by finally writing down $\frac{1+x}{2+y} = \frac{2}{3}$. This formation of correct relationships between variables does demonstrate that the learner is able to make assumptions (expressed in the form of variables), comprehend conditional statements and represent them appropriately through using algebraic symbols and notations.

The learner has shown sufficient competence in comprehending and interpreting the conditional statement, '*If 1 is subtracted from the numerator and 2 is subtracted from the denominator*', as illustrated by considering the initial unknown fraction defined in terms of the variables *x* and *y* and performing the necessary algebraic manipulations. In particular, the learner demonstrates an understanding of the first part of the hypothesis which reads '*If 1 is subtracted from the numerator…"* by writing as follows:

If 1 is subtracted from the numerator → $\frac{x-1}{}$

The learner also shows understanding of the second part of the hypothesis

If 2 is subtracted from the denominator → $\frac{}{y-2}$

By equating the fraction $\frac{x-1}{y-2}$ to the fraction $\frac{1}{1}$ as follows: $\frac{x-1}{y-2} = \frac{1}{1}$, the learner has demonstrated that the ratio of the numerator is equal to the ratio of the denominator, which in this case equivalently means that the numerator is equal to the denominator. Arriving at $\frac{x-1}{y-2} = \frac{1}{1}$ is a pivotal algebraic equation, which is necessary to be used with $\frac{1+x}{2+y} = \frac{2}{3}$ to solve for x and y simultaneously to obtain the original fraction. Regrettably, the learner did not proceed any further with solving for x and y simultaneously.

The researcher on seeing that learner has managed to build a system of linear equations, which is an essential modelling step, inquired from the learner if he has completed with solving the problem. The learner responded, "*yes, I have. I found found the fraction*" and points to $\frac{x-1}{y-2} = \frac{1}{1}$. This suggests that the learner did not fully comprehend what he must actually find, namely the values of x and y, which will give the values of the numerator and denominator of the original fraction.

4.3.2 Exemplification of Level 3 Modelling Competence

Learner R's strategy to build a system of linear equations, which could be solved simultaneously, paved the way for the learner successfully obtain the numerical values of the numerator and denominator of the targeted fraction in the given problem. First, this has been attributed to the learner assigning the variable N to the numerator and the variable D to the denominator of the targeted fraction in the given problem. As evident in line 2 of the solution, in Figure 4.12, the

-2, and calls it equation 3. In principal, the learner has built a system of 2 linear equations, which is a model that was pursued by the learner to solve the problem. As evident in line 2 of the solution, in Figure 4.12, the learner by writing N+1 has shown comprehension and understanding of the first part of the hypothesis of the first conditional statement, namely '*If 1 is added to the numerator… of a fraction'.* Similarly, by writing D+2, the learner understood the second part of the hypothesis of the first conditional statement, namely '*If …added… and 2 to the denominator of a fraction'*.

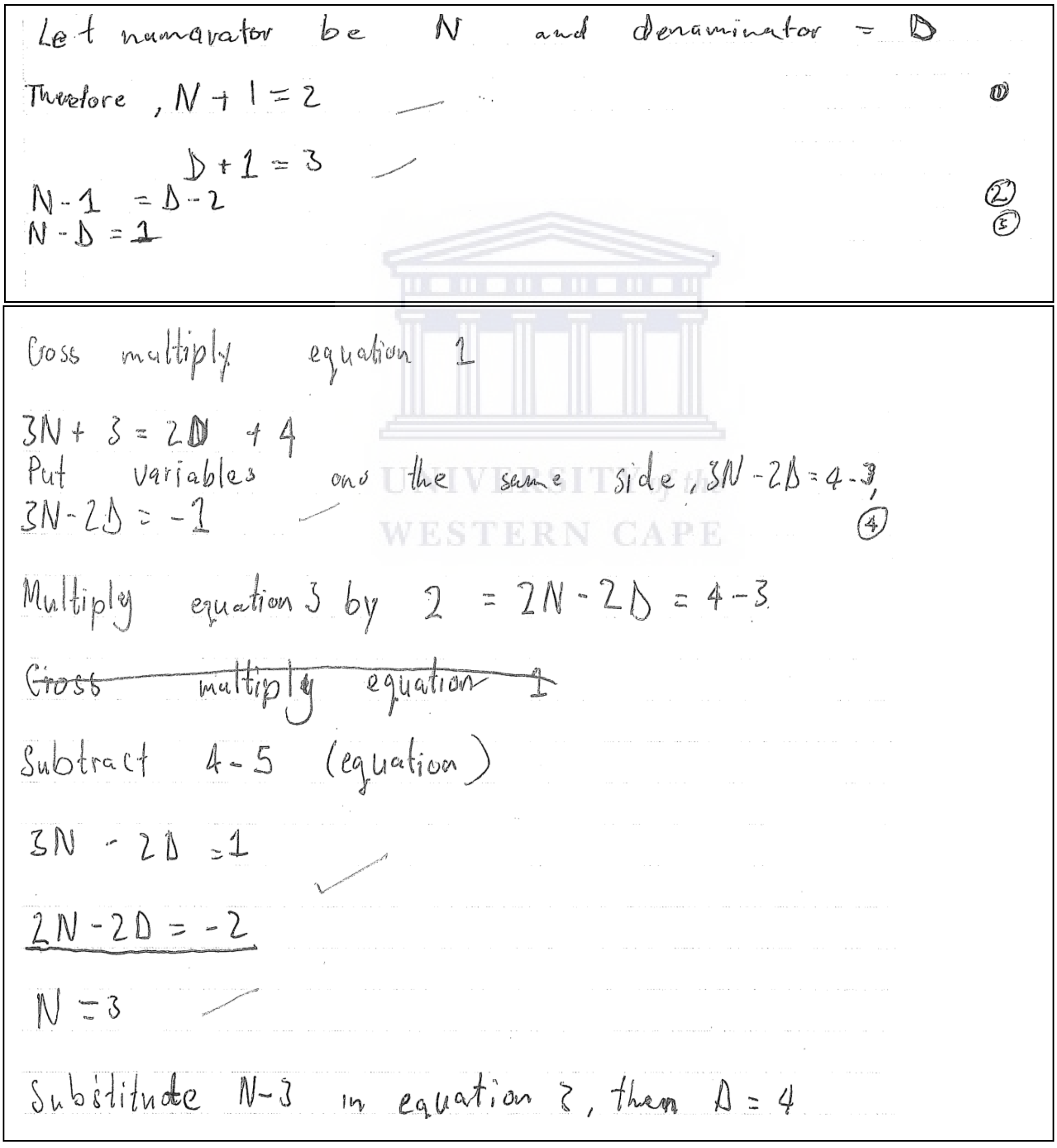

Let numarator be N and denaminator = D
Therefore, N + 1 = 2 ①
D + 1 = 3
N - 1 = D - 2 ②
N - D = 1 ③
Cross multiply equation 1
3N + 3 = 2D + 4
Put variables ono the same side, 3N - 2D = 4 - 3,
3N - 2D = -1 ④
Multiply equation 3 by 2 = 2N - 2D = 4 - 3
~~Cross multiply equation 1~~
Subtract 4 - 5 (equation)
3N - 2D = 1
2N - 2D = -2
N = 3
Subistitude N-3 in equation 3, then D = 4

Figure 4.12: Learner R's response to Problem 2

By writing $\frac{N+1}{D+1} = \frac{2}{3}$, the learner shows clear comprehension and interpretation the hypothesis and conclusion advanced in the conditional statement: '*If 1 is added to the numerator and 2 to the denominator of a fraction, the ratio of the numerator to the denominator is 2:3'*. The learner calls $\frac{N+1}{D+1} = \frac{2}{3}$ equation 1.

Furthermore, in line 3 of the solution in Figure 4.12, the learner correctly formulates a relationship between the variables N and D, namely $N - 1 = D - 2$ (which is called equation 2). This resonates with the hypothesis and conclusion advanced in the conditional statement: '*If 1 is subtracted from the numerator and 2 is subtracted from the denominator, the new numerator is equal to the new denominator'*. The learner appropriately transposes all the variables in the equation $N - 1 = D - 2$ to the left-hand side of the equation to get $N - D =$

The learner looks back at $\frac{N+1}{D+1} = \frac{2}{3}$ (which is equation 1) and proceeds cross multiply to get

$3N + 3 = 2D + 4$. Then the learner proceeds to transpose variables to the left-hand side and constants to the right and side of the equation to obtain equation 4, namely $3N - 2D = 1$.

Being familiar with the procedure to solve a system of linear equations, then learner proceeds to multiply equation 3 by 2 to get $2N - 2D = -2$, which is called equation 5. Working with the system of simplified linear equations, the learner as evident in lines 1-2 in Figure 4.12, solves for N as follows by subtracting equations (5) from equation (4):

$$3N - 2D = 1 \ \ldots\ldots\ldots\ldots \ (4)$$

$$\underline{2N - 2D = -2 \ \ldots\ldots\ldots\ldots \ (5)}$$

$$N = 3$$

The learner efficiently proceeds to substitute N=3 in earlier equation (3) to obtain D =4. All of this suggests that the learner is fully conversant with the procedure of solving a system of linear equations simultaneously, and in particular successfully solved his/her formulated model to get the value of the numerator (N) and denominator (D) of the targeted fraction to be 3 and 4 respectively. However, the learner ought to have shown deeper interpretation of the values of N and D by actually stating that targeted fraction is $\frac{3}{4}$. Moreover, the learner did not check if the values of N and D satisfied the conditional statements presented in the problem.

4.4 Analysis of modelling competencies: Problem 3 in the worksheet

Problem 3 reads as follows:

Andre has more money than Bob. If Andre gave Bob R20, they would have the same amount. While if Bob gave Andre R22, Andre would then have twice as much as Bob. How much does each one actually have?

An expected solution for Problem 3 is as follows:

Let x be the <u>amount of money</u> that Andre has. Let y be the <u>amount</u> that Bob has.

$x - 20 = y + 20 \ldots\ldots$ (1) [If Andre gives Bob R20, they would have the same

$x + 22 = 2(y - 22) \ldots\ldots$ (2) [If Bob gave Andre R22, Andre would then have twice as much as Bob}

From (1), we get $x = y + 40$ …. (3)

Substitute $x = y + 40\ in$ equation (2) :

$y + 40 + 22 = 2(y - 22)$

$\Rightarrow \quad y + 62 = 2y - 44$

$\Rightarrow \quad y = 106$

Substitute $y = 106$ in (3) :

$x = 106 + 40$

$\Rightarrow \quad x = 146$

Therefore, Bob has R106 and Andre has $R146$

Check: LHS = $x - 20$ = 146-20 = 126

⇒Andre will have R126 if he gives Bob R20.

$RHS = y + 20 = 106 + 20 = 126$

⇒Bob will have R126 if he receives R20 from Andre.

As illustrated in Figure 4.13, thirteen out of 20 learners (65% of the sample) demonstrated level 1 modelling competence, 3 out of 20 learners (15%) demonstrated level 2 competence and 4 out of 20 learners (20%) demonstrated level 3 competence.

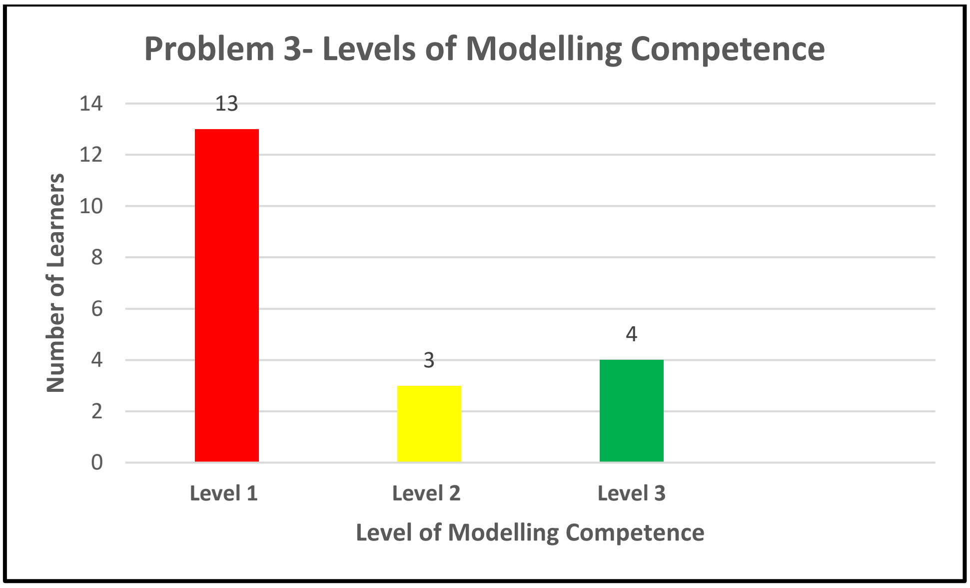


Figure 4.13: Problem 3- Levels of Modelling Competence

4.4.1 Exemplification of Level 1 Modelling Competence

As illustrated in Figure 4.14, learner 14 selected to work only with the second condition in the given problem and wrongfully ignored the first condition in the problem. In working with only the second condition, the learner wrongly assumed from the hypothesis '*Bob gave Andre R22'* that Bob just only had R22. The learner also wrongly interpreted the conclusion of the second condition, namely '*Andre would then have twice as much as Bob'* to literally mean that Andre has $2\ xR22\ =\ R44$. All of this demonstrates that learner 14 did not have the necessary competence to read and understand the problem.

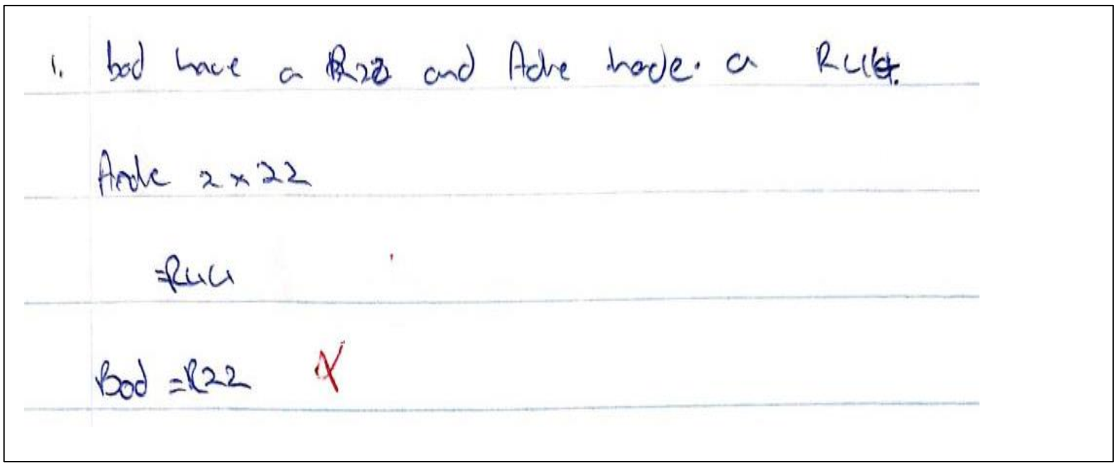


Figure 4.14: Learner 14 Responses to Problem 3

Much like learner 14, learner 20 responded, as shown in Figure 4.15 also signals a lack of understanding of the problem. Learner 20 merely doubled the amount of R22 that Bob gave

Andre to arrive at the amount of R44 that Andre would have initially had in his possession. Even worse, the learner subtracted the amount of R20 that Andre has Bob initially from incorrect calculated amount of R44 that Andre would have initially had. All of this shows that learner 20 like learner 14 has not recognised quantities associated with the problem appropriately and correctly, and expressed calculations are muddled up. Learners 14 and 20, learners showed level 1 competence did not demonstrate any evidence of mathematizing towards building a meaningful mathematical model.

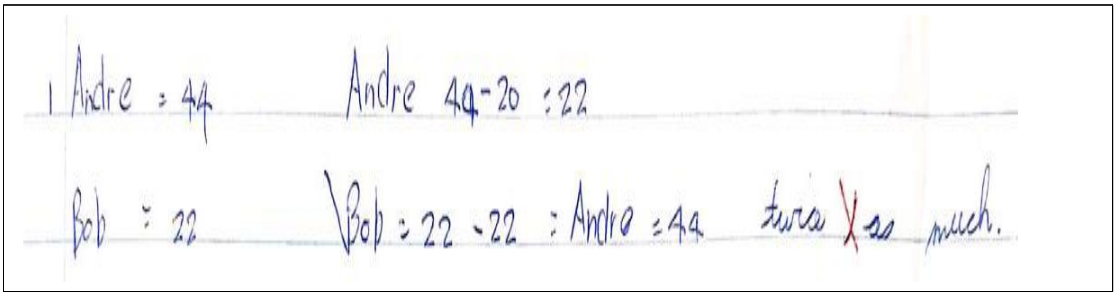


Figure 4.15: Learner 20 Responses to Problem 3

4.4.2 Exemplification of Level 2 Modelling Competence

Learner 15's representation in line 1, signals that he is fully aware that Andre has more than Bob by using the symbol '$>$', which means 'greater than', to indicate that Andre $>$ Bob. This sense that Andre's amount is greater than Bob's amount is further confirmed by the use of variables x and y in line 4 in the form of an inequality $x > y$ wherein x was assigned to Andre and y was assigned to Bob. Reflecting on the learners' statement in line 2 in association with line 1, it is evident that the learner fully understands the relationship between the hypothesis and conclusion in the conditional statement '*If Andre gave Bob R20, they would have the same'.* This is further re-iterated in line 5 by the learners' corresponding algebraic representation, namely: $x + 20 = y - 20$. In effect, this shows that the learner was able to recognize main quantities associated with the problem, generate and assign variables. Indeed, this means that the learner has the potential to use mathematical representations in their presentations towards building a mathematical model.

Although the learners' representation in line 3 demonstrates understanding of the conditional statement, 'I*f Bob gave Andre R22, Andre would then have twice as much as Bob',* the learner incorrectly expressed this relationship using variables x and and y as follows: $x - 22 = 2y$.

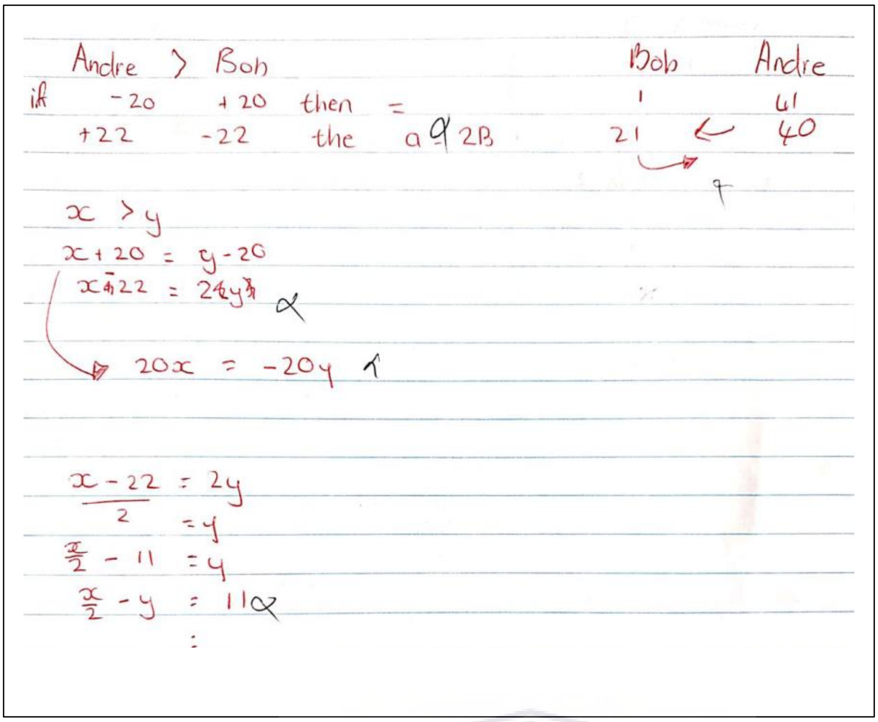


Figure 4.16: Learner 15 Responses to Problem 3

This signals that even though the learner made the necessary attempt to mathematize identified quantities and their relations, he/she was not completely successful, and thus built a mathematical model represented by a system of simultaneous equations which is partially correct as follows:

For $x > y$,

$x + 20 = y - 20$ ………… (1) [This is correct]

$x - 22 = 2y$ … … … … … (2) [This is incorrect]

Although the above system of simultaneous equations is partially correct, the learner was not able to apply the basic procedures to solve this system of equations simultaneously as the next few steps contained algebraic manipulation and computation errors. For example, $20x = -20y$ is an incorrect derivation?

4.4.3 Exemplification of Level 3 Modelling Competence

Learner 12 has appropriately used symbolic representation (namely$\rightarrow, =$) to show that the giving of the R20 by Andre to Bob has resulted in the amount of money Andre to be equal to Bob (i.e. A= B), as illustrated in lines 1 and 2 in Figure 4.17. The conclusion that $A = B$, is a clear indication the learner has fully grasped the meaning of every word in the conditional statement: "*If Andre gave Bob R20, they would have the same*". These understandings have enabled the learner through using variable x and y to represent the meaning of the conditional statement in the form of an algebraic relationship as follows: $x - 20 = y + 20$ (see line 7 in Figure 4.17). Although the learner has not explicitly assigned variable x to represent the amount of money Andre has and the variable y to represent the amount of money Bob has, it is highly likely that is what the learner conceived and intended.

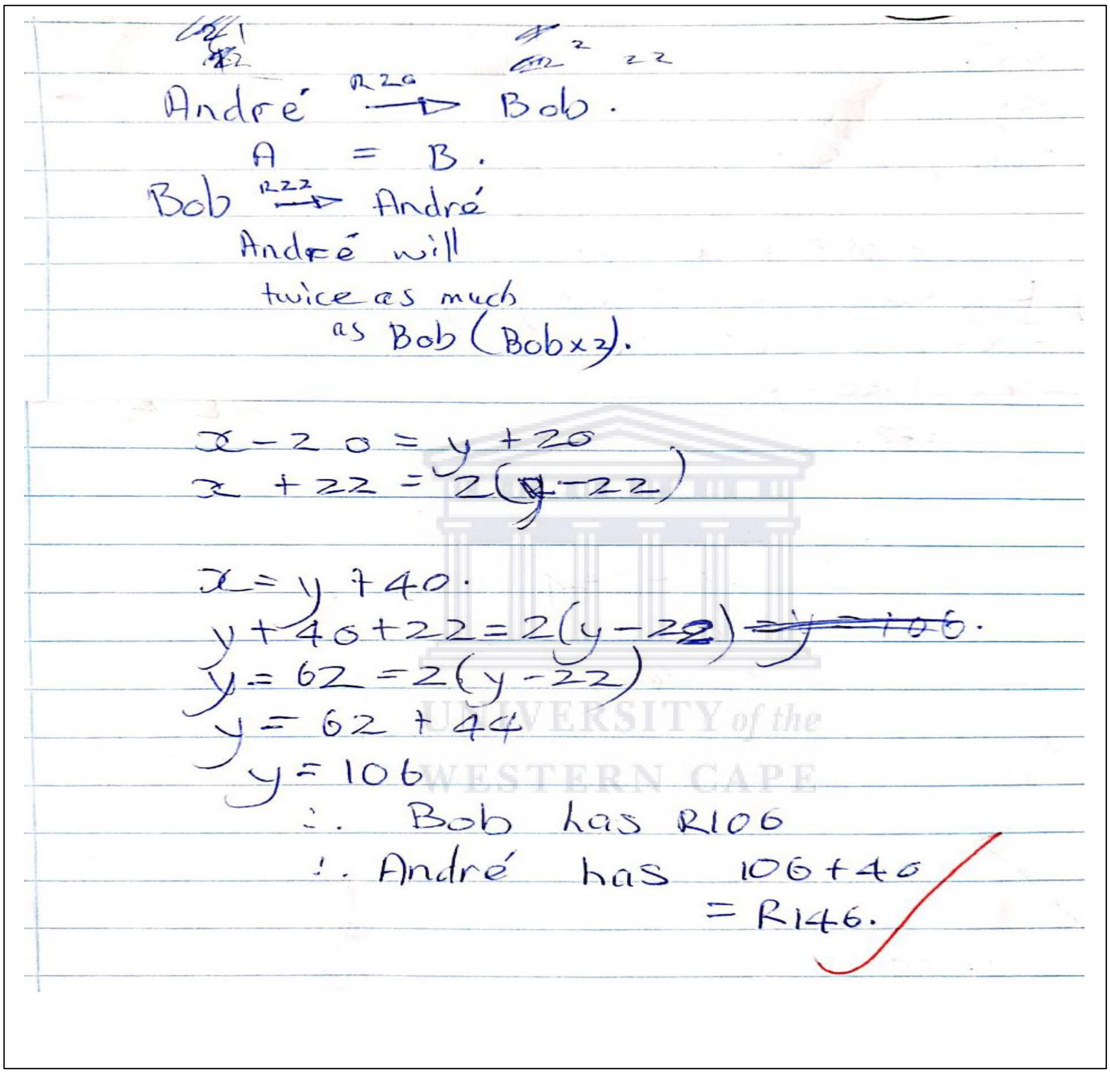


Figure 4.17 Learner 1 Response to Problem 3

The learner's articulations in lines 3 to 6 in Figure 4.17, and culmination in Andre = Bob x 2 shows that he has sufficient understanding of the conditional statement '*If Bob gave Andre R22, Andre would then have twice as much as Bob'.* This profound conceptual understanding is manifested in the learner's correct expression of the ensued relationship using mathematical symbols and notation appropriately to write and construct an algebraic linear equation in terms of the variables x and y as follows: $x + 22 = 2(y - 22)$, as shown in line 8 in Figure 4.17. Hence, it is evident that the learner has mathematized relevant quantities and relations defiantly, and managed to develop a perfect mathematical model as represented by the following system of linear equations:

$$x - 20 = y + 20$$

$$x + 22 = 2(y - 22)$$

As illustrated in Figure 4.17, the learner has applied the procedures to solve his system of linear equations (i.e. the built mathematical model) correctly to arrive at the logical conclusion that initially, Bob has R106 and Andre had R146. In this sense, it is prudent to say that the learner has solved his mathematical model amicably. However, the learner, like the majority of the other learners, did not attempt to check if his answers satisfied the conditions of the given problem nor did any other form of interpretation.

As illustrated in Figure 4.18, learner 1, as a starting point, assigned variables x *and* y to represent the money Andre and Bob respectively have in their initial possession.

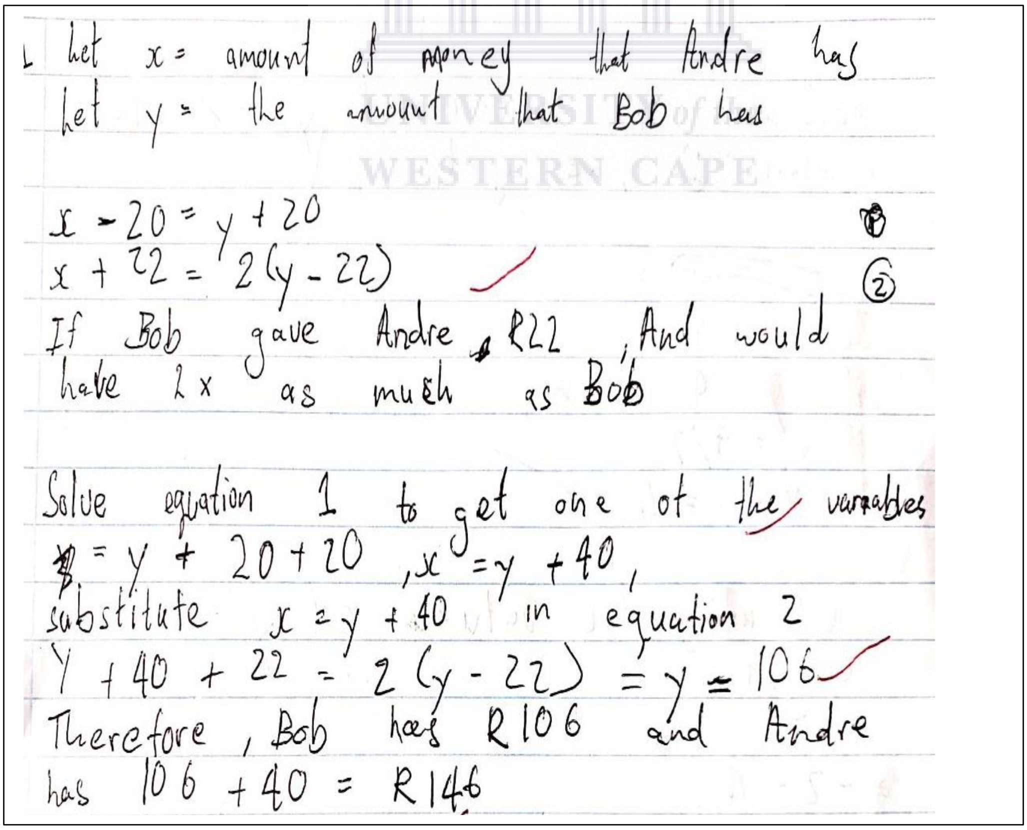

Let x = amount of money that Andre has
Let y = the amount that Bob has
x - 20 = y + 20 ①
x + 22 = 2(y - 22) ②
If Bob gave Andre R22, And would have 2x as much as Bob
Solve equation 1 to get one of the variables
x = y + 20 + 20, x = y + 40,
substitute x = y + 40 in equation 2
y + 40 + 22 = 2(y - 22) = y = 106
Therefore, Bob has R106 and Andre has 106 + 40 = R146

Figure 4.18: Learner 1 Response to Problem 3

Using these variables, the learner has mathematized the relevant quantities appropriately to build a mathematical model defined by the system of 2 linear equations (see lines 3 and 4 in Figure 4.18) as follows:

$$x - 20 = y + 20 \ldots\ldots\ldots.(1)$$

$$x + 22 = 2(y - 22) \quad \ldots\ldots\ldots \quad (2)$$

The learner successfully manipulated the system of learner equations in solving them simultaneously to correctly realize the amounts that Bob and Andre initially had in their possession. However, evidence of attempts to test and check if their solutions make sense was not pursued.

4.4.4 Analysis of modelling competencies: Problem 4 in the worksheet

Problem 4 reads as follows:

In a two-digit number, the unit digit is thrice the tens digit. If 36 is added to the number, the digits interchange their place. Find the number.

An expected solution for Problem 4 is as follows:

Let the digit in the unit place be x and the digit in the tens place be y.

Then $x = 3y$ …………(1), and the number is $10y + x$

The number obtained by reversing the digits is $10x + y$.

If 36 is added to the number, digits interchange their places,

Therefore, we have $10y + x + 36 = 10x + y$

$$9x - 9y = 36$$

$$x - y = 4 \ldots\ldots\ldots.(2)$$

Substituting the value of $x = 3y$ in equation (2), we get

$3y - y = 4$

$2y = 4$

$y = 2$

substituting the value of $y = 2$ in equation (1), we get

$x - 2 = 4$

$x = 6$

Therefore, the number is $10(2) + 6 = 26$

Check: The units digits is 6 and the tens digit is 2. Since the units digit $6 = 3x2,$ it is three times the tens digit which is 2.

As illustrated in Figure 4.19, twelve out of 20 learners (60% of the sample) demonstrated level 1 modelling competence, 6 out of 20 learners 30%) demonstrated level 2 competence and 2 out of 20 learners (10%) demonstrated level 3 competence.

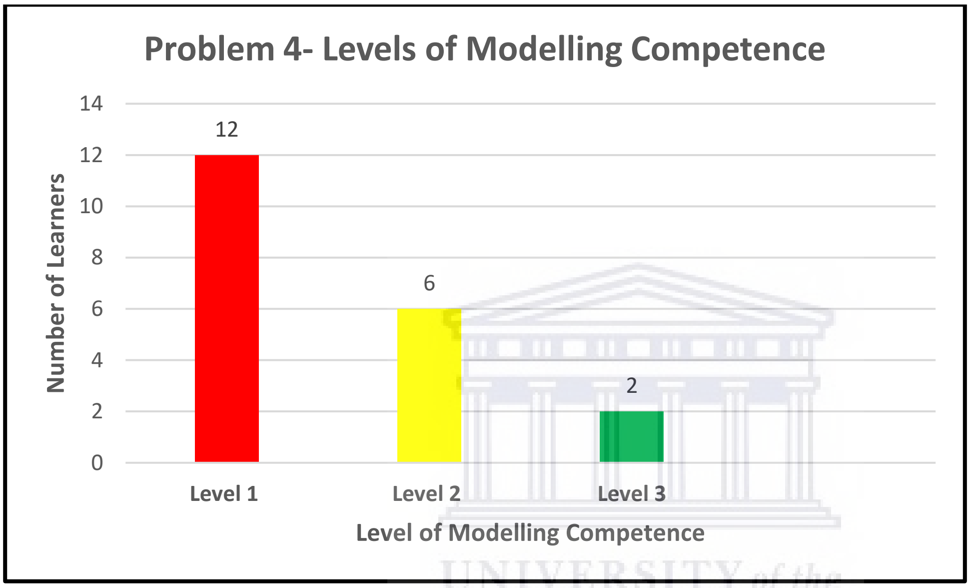


Figure 4.19: Problem 4 - Levels of Modelling Competence

4.5 Exemplification of Level 1 Modelling Competence

Most of the learners in the sample (60%), as shown in Figure 4.19, seemed not to have comprehended the given information in the given problem. For example, learner 3, in line 1 literally latched on to 36 (which is a number clearly defined in the problem) and incorrectly considered 2 as the '2-digit' number, which was subtracted from 36 to give 34. It seems that the learner on reading the statement 'the unit digit is thrice the tens digit' decided to wrongfully subtract $3x10$ from 36 (as indicated in line 2 in Figure 4.20) and even get that computation incorrect by writing '$36 = 3x10 = 43'$. All of this demonstrates a lack that the learner, as most of the others in this sample, did not recognise quantities associated with problems and was muddled up in his/her thinking. The expressions and relationships make no sense and are devoid of the use of many variables.

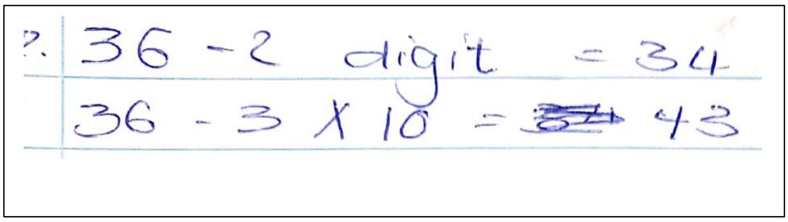


Figure 4.20: Learner 3 Responses to Problem 4

4.5.1 Exemplification of Level 2 Modelling Competence

As illustrated in Figure 4.21, learner 13 seemed to have assigned the variable x to the units digit and y to the tens digit as evident in the algebraic representation '$x = 3y'$ in line 1. By actually writing down '$x = 3y'$, the learner shows necessary understanding of the given statement: '*In a two-digit number, the unit digit is thrice the tens digit*'. In line 2, the learner wrote down '$= 10y + x$', and seemed to have overlooked the meaning of the '$='$ symbol as '$x = 3y \neq 10y + 3x'$. However, we may easily regard this as a slip since it is evident in line 3 that the learner actually represented the 2-digit number in the form '$10y + x$'.

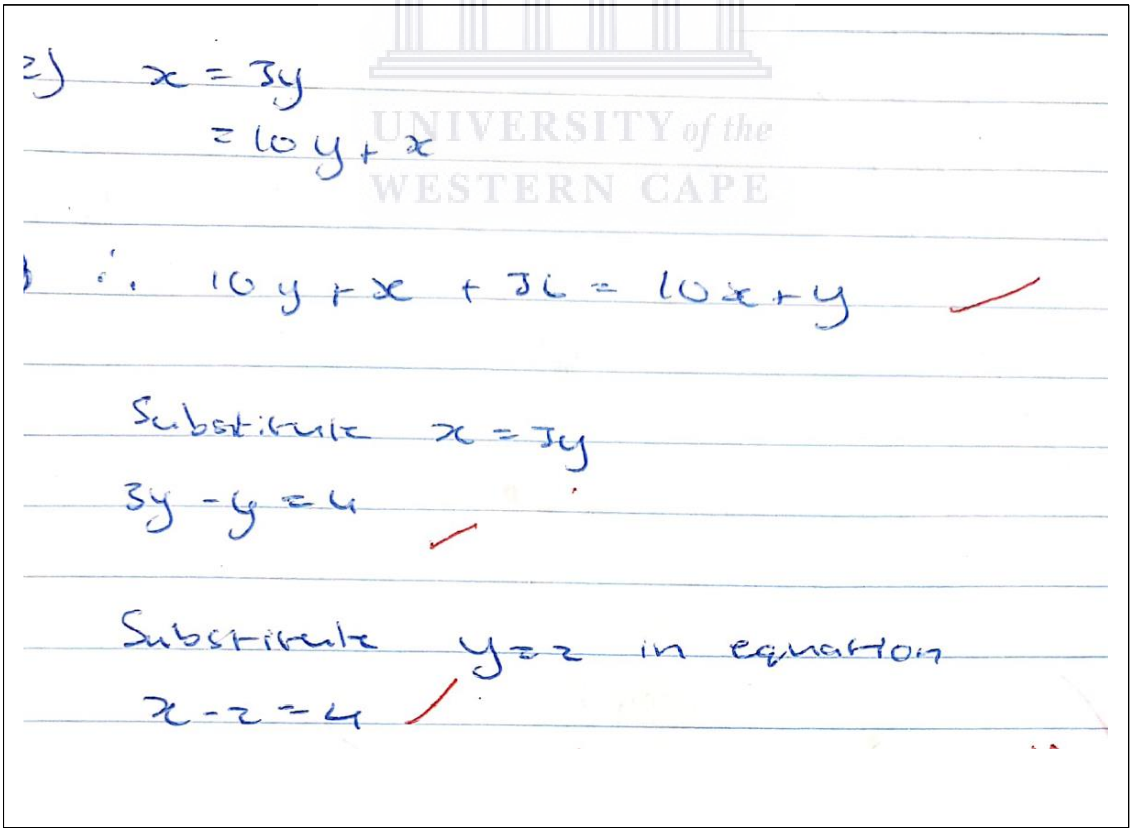


Figure 4.21: Learner 13 Responses to Problem 4

In line 3, the learner wrote: '$\therefore 10y + x + 36 = 10x + y$'. The construction of this algebraic linear equation in terms of the assigned variables x and y, suggests that the learner comprehended the problem to the extent of recognising pertinent quantities associated with the problem and constructing meaningful relationships between the variables. The successful building of a mathematical model is a reflection of the learner's profound sense of understanding of the salient aspects of the problems. As evident in lines 4-8, the learner has a sense that this model can be solved by solving the following system of linear equations simultaneously:

$$x = 3y \qquad \ldots\ldots\ldots\ldots..(1)$$

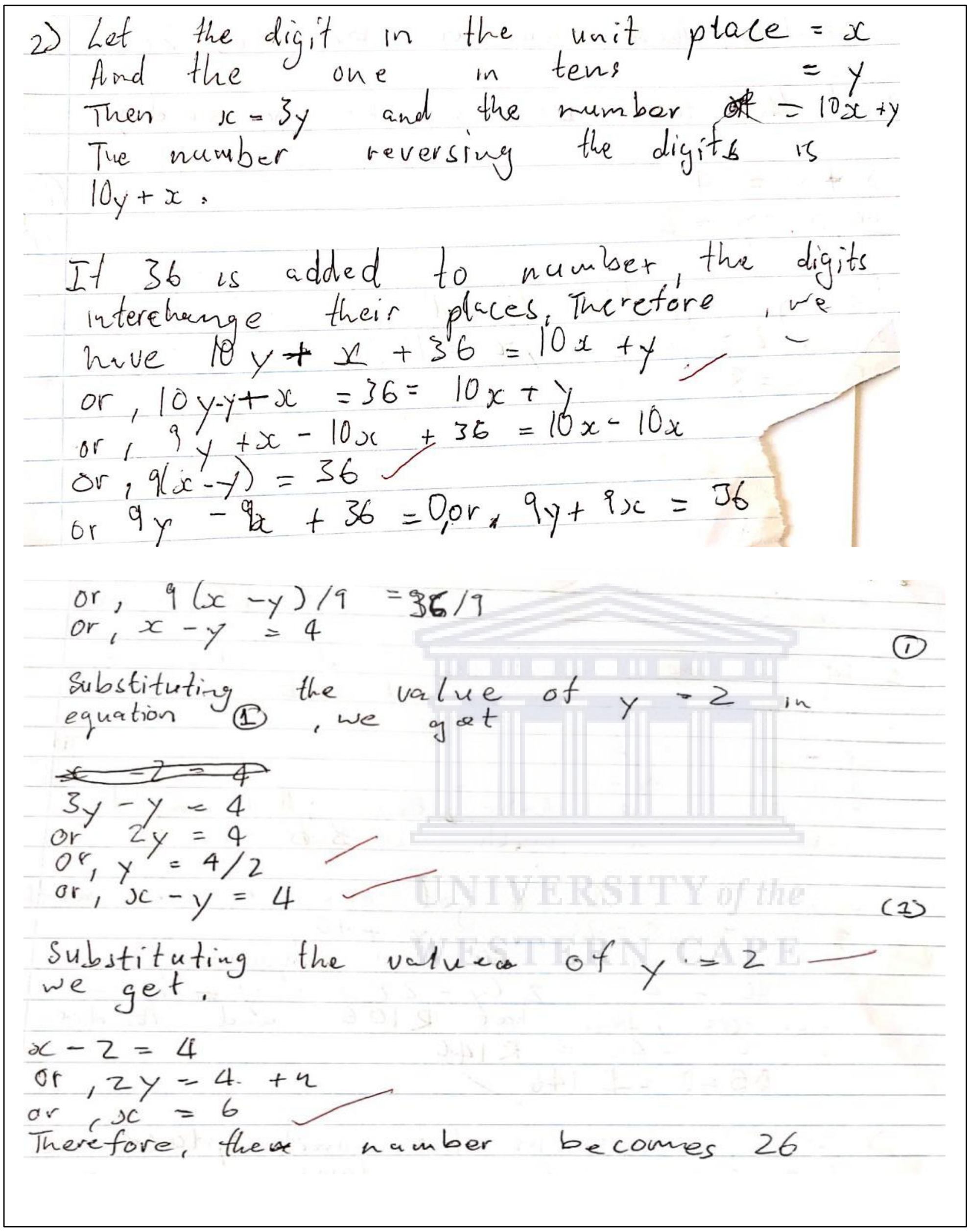
$$10y + x + 36 = 10x + y \qquad \ldots\ldots\ldots\ldots(2)$$

In line 4, the learner says he substitutes $x = 3y$ but does not say into which equation nor shows any details of the substitution and simplification resulting in getting correctly to $3y - y = 4$ in line 5. The learner, as evident in line 6, seemed to have simplified this further to get $y = 2$. Although the learner has not explicitly stated into which equation he substituted $y = 2$, it is highly likely that he substituted into (2) to get $-2 = 4$. Unfortunately, the learner did not proceed any further, and hence his solving of the mathematical model is incomplete even though he shows capabilities of mathematical reasoning.

4.5.2 Exemplification of Level 3 Modelling Competence

As illustrated in Figure 4.22, learner 1 recognised quantities such as the units and ten's digit in the problem and appropriately assigned variables $x \ and \ y$ to them. This has helped the learner's first construction of an algebraic representation of the unknown 2-digit number in the form '$10x + y'$, which is pivotal for moving forward in building a complete mathematical model to find the digits that constitute the unknown number. Furthermore, the learner meaningfully translated the meaning of the statement '*the unit digit is thrice the tens digit*' using mathematical symbols and variables to establish an algebraic relationship, namely: '$x = 3y'$ in line 3. In addition, the learner seemed to have made sense of the conditional statement: '*If 36 is added to the number, the digits interchange their place*', by expressing resultant 2-digit number if the digits interchange correcting in the algebraic form '$10y + x'$. All of the aforementioned established algebraic relationships, provided the necessary warrants for the learner to express his profound understanding of the conditional statement in line 7, as '$10y + x + 36 = 10x + y'$.

The latter algebraic representation is a critical ‘relationship’ that forms part of the system of linear equations that characterized the mathematical model required to solve this problem.


2) Let the digit in the unit place = x
And the one in tens = y
Then x = 3y and the number = 10x + y
The number reversing the digits is
10y + x.

If 36 is added to number, the digits interchange their places, Therefore, we have 10y + x + 36 = 10x + y
or, 10y - y + x = 36 = 10x + y
or, 9y + x - 10x + 36 = 10x - 10x
or, 9(x - y) = 36
or 9y - 9x + 36 = 0, or, 9y + 9x = 36

or, 9(x - y)/9 = 36/9
or, x - y = 4 (1)

Substituting the value of y = 2 in equation (1), we get

~~x - 2 = 4~~
3y - y = 4
or 2y = 4
or, y = 4/2
or, x - y = 4 (2)

Substituting the values of y = 2 we get,

x - 2 = 4
or, 2y = 4 + 2
or, x = 6
Therefore, the number becomes 26

Figure 4.22: Learner 1 Response to Problem 4

As evident in Figure 4.22, the learner proceeds efficiently to solve the following system of linear equations,

$$x = 3y \qquad \text{………….(1)}$$

$$10y + x + 36 = 10x + y \qquad \text{…………(2)}$$

In doing so, the learner successfully obtains the correct values for $x\ and\ y$, namely $y = 2$ and $x = 6$. In effect, this means that the learner was able to achieve the correct mathematical solution by solving the correctly constructed mathematical model.

Importantly, the learner has correctly interpreted the value $y = 2$ and $x = 6$ within the context of the problem in relation to the unknown number expressed as $10x + y$ to realize that the number is 26. However, the learner did not make any effort to check or verify whether his/her answer was correct or made mathematical sense.

4.6 Analysis of modelling competencies: Problem 5 in the worksheet

Problem 5 reads as follows:

If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82. Find the ages of father and son.

An expected solution for Problem5 is as follows:

Let father's age be x years and the son's age be y years

Then $x + 2y = 56$ ………… (1)

and $2x + y = 82$ ………… (2)

Multiplying equation (1) by 2, to get:

$4y + 2x = 112$ ……… (3)

Substrate (2) from (3) :

$4y + 2x = 11$ ……… (3)

$\underline{y + 2x = 82}$ ……. (2)

$3y = 30$

$\Rightarrow y = 10$

Substituting $y = 10$ in (2) ;

$x + 2\,(10) = 56$

$\Rightarrow x + 20 = 56$

$\Rightarrow x = 36$

∴ the father is 36 years and son is 10 years.

Check: $x + 2y$

$= 36 + 2(10)$

$= 56$

OR

$= 2x + y$

$= 2(36) + 10$

$= 82$

As illustrated in Figure 4.23, 11 out of 20 learners (55% of the sample) demonstrated level 1 modelling competence, 7 out of 20 learners (35%) demonstrated level 2 competence and 2 out of 20 learners (10%) demonstrated level 3 competence.

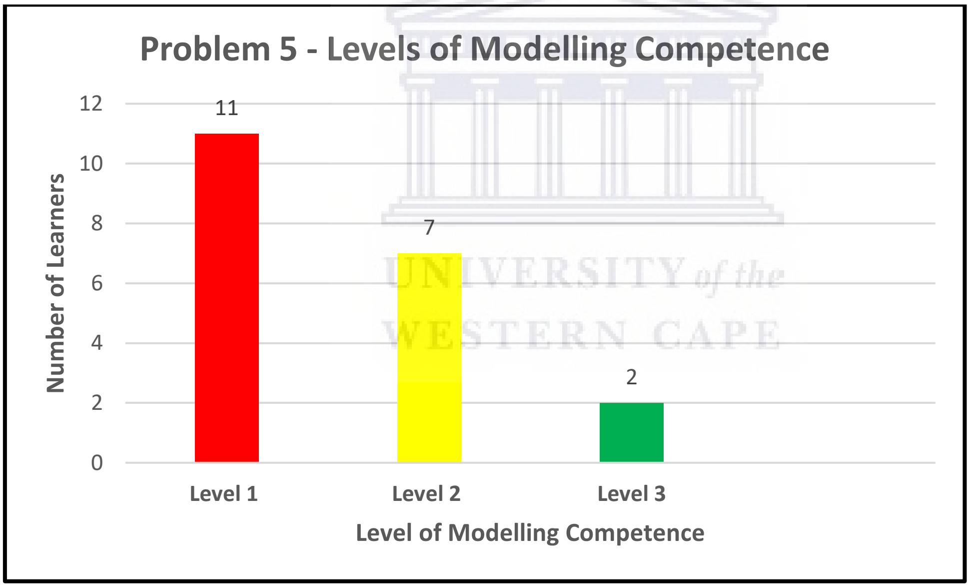


Figure 4.23: Problem 5- Levels of Modelling Competence

4.6.1 Exemplification of Level 1 Modelling Competence

As illustrated in Figure 4.24, learner 9, seemed to not to have made sense of the given quantities 82 and 56 within the context of the problem. The learner literally took 56, which is mentioned in the conclusion of the first conditional statement (*If twice the age of son is added to age of father, the sum is 56*), and literally subtracted it from 82 which is mentioned in the conclusion of the second conditional statement (*But if twice the age of the father is added to the age of son, the* *sum is 82*). This implies the learner has no comprehension of the given information, does not recognize the meaning of quantities given in the problem, and writes number sentences that show that his/her thinking is muddled up. The learner shows no evidence of mathematizing and building a mathematical model.

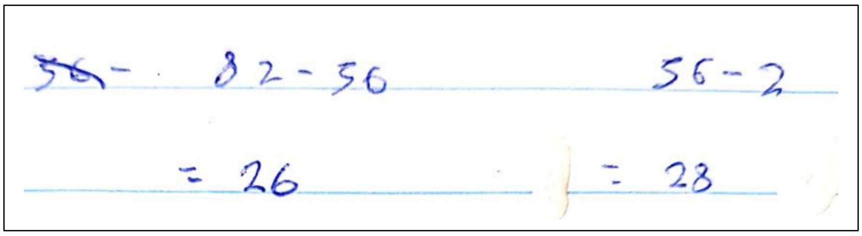


Figure 4.24: Learner 9 Responses to Problem 5

Learner 7, showed some effort to use variables, which were not explicitly assigned to build 2 sets of relationships (i.e. algebraic linear equations), but both were seriously flawed (Figure 4.25). As can be seen in line 1, the learner by writing,$2x + x = 56$, seems to suggest that the learner did not make sense the first conditional statement: '*If twice the age of son is added to age of father, the sum is 56*'. It seems that the learner is thinking that the son's age (represented by $2x$) is twice the father's age (represented by x), which is clearly not the case, hence a conceptual breakdown. As is evident in line 4, the learner makes a similar kind of mistake, as a result of the lack of comprehension of the meaning of second the conditional statement: '*But if twice the age of the father is added to the age of son, the sum is 82*'. This is further compounded by the negligent interchange of the use of the variable, x, where the son's age is represented by x and the father's age is represented by $2x$ in the wrongly expressed relationship: $2x + x = 82$. This shows that the learner includes expressions showing that he/she does not understand the problem.

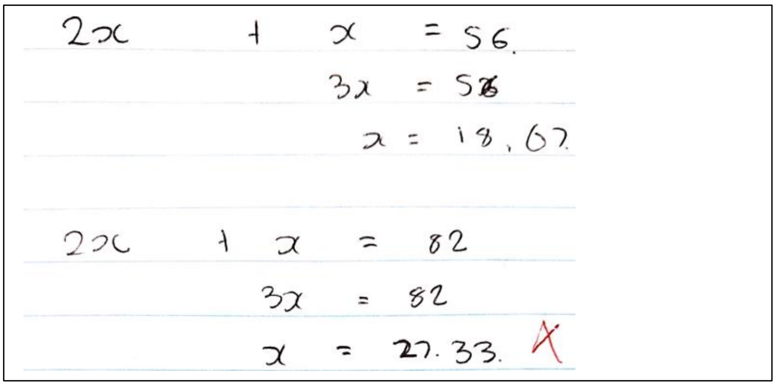


Figure 4.25: Learner 7 Response to Problem 5

4.6.2 Exemplification of Level 2 Modelling Competence

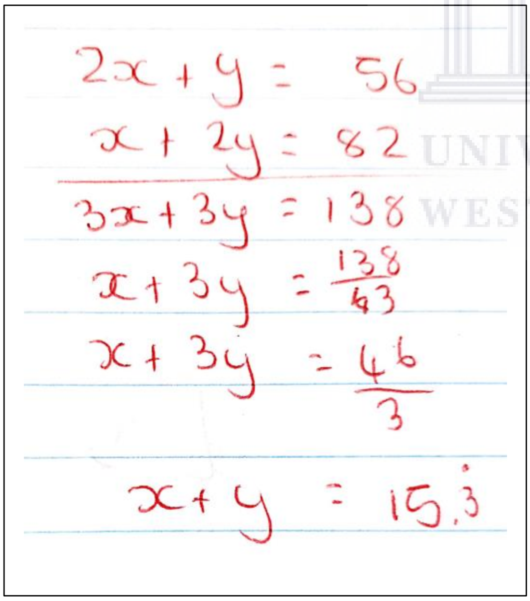
Although learners 15 and 11, did not explicitly state to what the variables are assigned, it is plausible to assume that they have each assigned the variable x to represent the son's age and the variable y to represent the father's age. Using these variables, it seems evident that both learners 15 and 11 have correctly interpreted both conditional statements in the problem to yield their following system of linear equations:

$$2x + y = 56 \ldots\ldots..(1)$$

And

$$x + 2y = 82 \ldots\ldots\ldots(2)$$

The aforementioned system of linear equations is indicative of the learners' abilities to mathematize relevant quantities and their relationships, and construct a correct mathematical model represented by a system of correct linear equations.



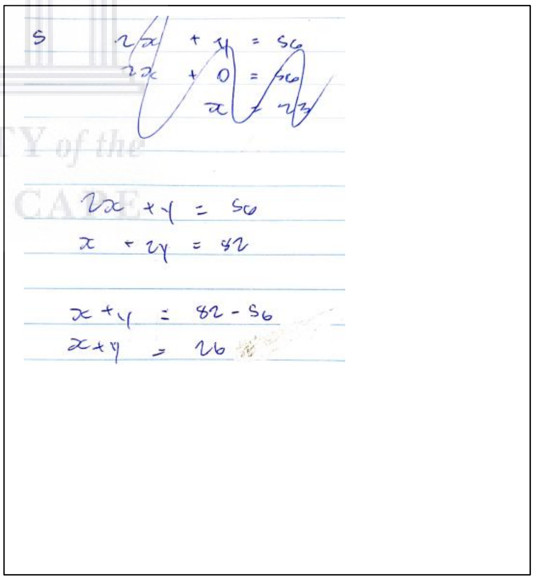


Figure 4.26: Learner 15 Response to Problem 5 **Figure 4.27: Learner 11 Response to Problem 5**

As illustrated in Figures 4.26 and Figures 27, it is evident that both learners had some inclination that they need to work with both equation systems to try solving them simultaneously. However, both only manage to get so far as to determine a value of $x + y$, which was incorrect in both cases because of computation and manipulation errors. This signals that both learners were not able to solve their mathematical models, which were correctly constructed, and were not able to complete their solution or reflect on their work to see if it made sense.

4.6.3 Exemplification of Level 3 Modelling Competence

As illustrated in Figures 4.28 and 29, both learners 12 and 5, in their responses, explicitly stated the variables x and y they assigned, respectively to the son's age and father's age. The allocation of two unique variables x and y by the learners seems to suggest that they have comprehended the pertinent aspects of the problem, and hence have recognised all quantities associated with the problem. Using their assigned variables x and y, both learners 12 and 5, correctly interpreted the respective conditional statements in the given problem to yield following pair linear equations:

$$2x + y = 56 \ldots \ldots . . (1)$$

and

$$x + 2y = 82 \ldots \ldots \ldots (2)$$

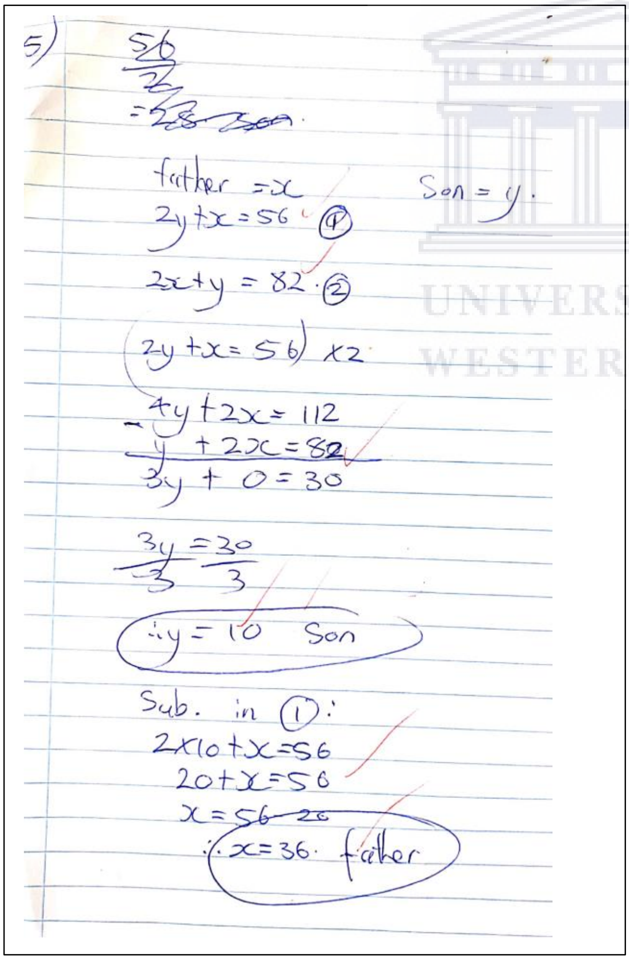


Figure 4.28: Learner 12 Response to Problem 5

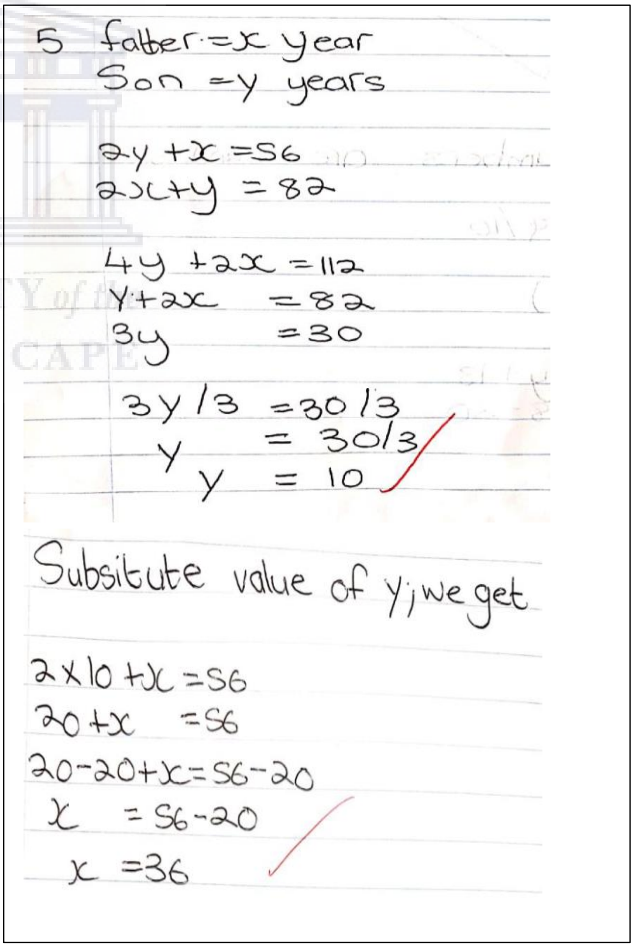


Figure 4.29: Learner 5 Response to Problem 5

The assimilated system of linear equations, sufficiently demonstrates that the learners were able to mathematize relevant quantities and their relationships and construct a correct mathematical model to represent the conditions imposed in the given problem and context. As evident in Figures 4.28 and 4.29, both learners demonstrate a clear understanding of the method to solve a system of linear equations simultaneously and with logical reasoning used correct mathematics and procedures to solve x and y correctly. Learner 12 reaffirms that the son's age is 10 years and the father's age is 36 years, while learner 5 just stops and finds the value of x and y. It is plausible to suggest that learner 12 did not move further to explicitly interpret the son's age is 10 years and the father's age is 36 years, given that he/she defined what x and y represented at the start of his/her solution. However, both learners did not demonstrate evidence of checking if their answer made sense or whether it satisfied the given conditions of the problem

4.7 Discussion of findings in relation to the research question

What levels of mathematical modelling competency do Grade 10 learners demonstrate when solving word problems using simultaneous equations?

Taking into account the core steps the modelling processes, which constitute the core modelling competency framework accompanied by their respective sub-competencies is outlined Blum and Kaiser (1997, p. 9 as cited in Maaß, 2006, pp. 116-117). The rubric, as per Table 3.2, was constructed with the prime focus on, *understanding a problem*, *building a model* and *solving a model,* to ascertain the levels (Level 1, Level 2, Level 3) of mathematical modelling competency that Grade 10 learners demonstrate in solving word problems using simultaneous equations. Beyond this, aspects pertaining to interpretation and validation were looked at and analysed to further elucidate the extent of the competency of the level 3 modellers.

As illustrated through the analysis of the learners' responses to each of the five-word problems in the preceding sections, learners demonstrated modelling competencies ranging across the following three levels: Level 1 (Not competent); Level 2 (Partially competent), and Level 3 (Competent). As summarily illustrated in Figure 4.33, results have shown that a high percentage (ranging between a low of 55% and a high of 75%) of the learners' mathematical modelling competency was found to be at level 1, with a low percentage of learners (ranging between a low of 20% and a high of 35%) performing at level 2, and very low levels (ranging between a low of 10% and a high of 25%) of level 3 across the five tackled problems. These findings agree with those of Chan, Ng, Widjaja and Seto (2012) study with primary 5 learners where they found that majority of the learners did not complete the modelling cycle or exemplified proficient modelling competence. Chan, Ng, Widjaja and Seto (2012) found that generally novice modellers have shown that their competence to make assumptions, understand a problem, mathematize a situation or build a feasible model is rather weak.

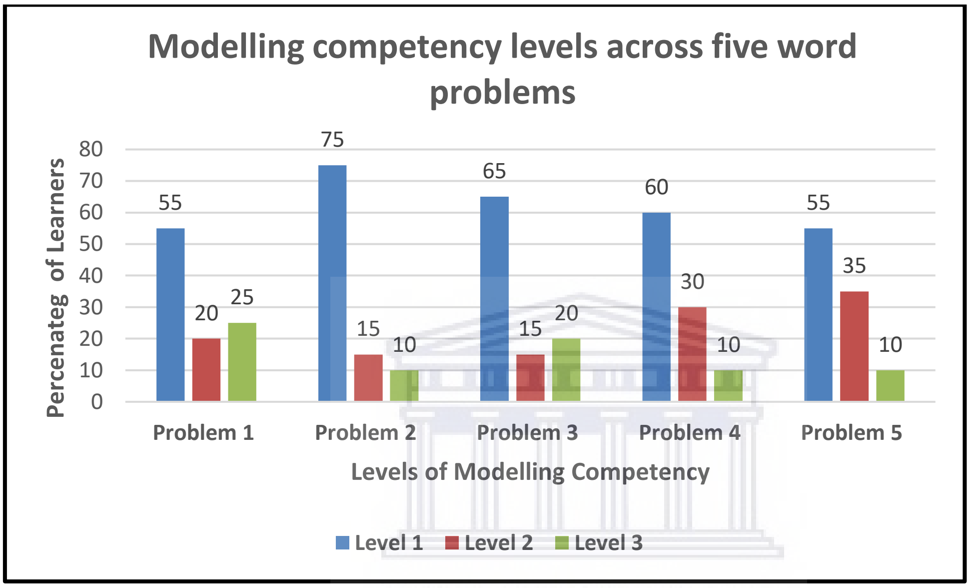


Figure 4.30: Summary of learner modelling competency levels across five word problems

According to Pólya (1973), reading and understanding a problem is the gateway to success in solving a problem. Thus learners should invest essential and sufficient time in reading the problem and unpacking the meaning of included mathematical concepts, definitions, words and phrases so that they can establish what is given and what needs to be done. Added to this basket of understanding, includes the competencies to differentiate between relevant and irrelevant information (Blum and Kaiser, 1997; Maaß Maaß, 2006), and where necessary the problem may be reformulated in learners own words (Van de Walle, 1998). According to MaaB (2006), understanding also encompasses the sub-competencies such as making assumptions relevant to the model; recognising quantities associated with the problem, naming them and assigning variables to them; and constructing relationships between them.

Reflecting on learners attempts to solve a problem, have shown that their inability to read and understand a problem was the biggest obstacle in most of their endeavour to solve a given problem. Certainly, this was evident across the learners who exemplified L1 modelling competency across the 5 sets of problems. For example, in the case of problem 1, five learners as illustrated in Figure 4.2, muddled up connections between 27 and 6 and the concepts of sum and difference and wrote down mathematical statements that made no meaningful sense and showed disrespect for the equality sign. In particular, learner X did not realize that 27 is actually the answer to the sum of the two given numbers but instead found half of 27. Furthermore, learner X could do not fathom out the quantities between which a difference of 6 exists, and hence wrote down a completely wrong number sentence, namely ‘ $= 6 - 13\frac{1}{2}$.’

In much the same way Eu and You (2015) analysed the mathematical competencies of identifying variables, making assumptions, mathematics reasoning and interpreting solutions. Their learners were assessed based on the three modelling competencies of making assumptions, computing and interpreting solution and mathematical reasoning via an analytical rubric. The assessment was carried out by investigating learners ‘written works’ and obtaining their responses through interviews. Eu and You (2015) concluded that the learners showed weaknesses in making assumptions and mathematical reasoning, and asserted that all this goes back to the lack of understanding of the problem. However, Eu and You (2015) assert that when learners understand the problem, they can use different strategies to solve it.

A lack of understanding the problem was highly manifested in most of the learners' inability to make sense of a hypothesis relating to a conclusion in a given conditional statement across the problems. For example, in the case of problem 2, most of the 15 learners who demonstrated L1 modelling competency, had some literal conception of a fraction and did not comprehend the meaning of the ratio $\frac{2}{3}$ within the context of the problem. Reflecting on (2016) study on Grade 6 learners’ mathematical word problem skills, it was that second language learners found it difficult to attach meaning to English mathematical word problems. Reflecting on Raoano's (2016) study on Grade 6 learners’ mathematical word problem skills in South Africa, it was noticed that second language English learners found it difficult to attach meaning to English written mathematical word problems. Therefore, it is plausible to conjecture that the nature of English used in the word problems could have acted as a barrier to learners in this study, making sense of what they read. This invariably has blocked learners from developing a plausible relationship between the pronounced quantities in the problem or even think of assigning variables to quantities associated with the problem. For example, for problem 2, the conclusion in the conditional statement, '*If 1 is added to the numerator and 2 to the denominator the ratio of the numerator to the new denominator is 2:3*' has literally been taken to mean the resultant fraction is $\frac{2}{3}$ rather than realizing that the ratio of the numerator to the denominator in the new fraction after adding 1 to the numerator and 2 to the denominator is $\frac{2}{3}$. Furthermore, this group of learners were possibly not conversant with the concept of equivalent fractions, like $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$, and hence did not see that the ratio, $\frac{2}{3}$ could mean that values of the numerator and denominator is not 2 and 3 respectively but could also be 4 and 6 or 6 and 9. All of these misunderstandings and misconceptions could have inhibited learners from proceeding to mathematize pertinent, relevant quantities or even mathematize identified quantities and their relations appropriately.

This challenge of most of the learners not understanding the conditional statements manifested in problem 3 as well, where for example learner 14 only chose to work the second condition and wrongly deduced from the hypothesis '*Bob gave Andre R22'* that Bob just only had R22. This learner 14, also erroneously interpreted the conclusion of the second condition, namely '*Andre would then have twice as much as Bob'* to literally mean that Andre has $2\ xR22\ =\ R44$. All this confirms that the learner did not comprehend the given information in the problem, and hence mistakenly formed relationships between them which were also devoid of the assignment and use of variables.

Similarly, in problem 4, most of the learners in the sample (12 out of 20), as shown in Figure 4.19, seemed not to have comprehended the given information in the given problem. For example, as illustrated in Figure 4.20, learner 3, incorrectly considered 2 as the '2-digit' number and subtracted it from 36 to give 34. Furthermore, it seems that the learner on reading the statement 'the unit digit is thrice the ten's digit' decided to wrongfully subtract $3x10$ from 36 (as indicated in line 2 in Figure 4.20) and even got that computation incorrect by writing '$36 = 3x10 = 43'$.

The inability to make sense of the hypothesis and conclusion embodying conditional statements was also demonstrated by 55% of the learners, who attempted problem 5. For example, as illustrated in Figure 4.24, the learner literally took 56 mentioned in the conclusion of the first conditional statement (*If twice the age of son is added to age of father, the sum is 56),* and literally subtracted it from 82 mentioned in the conclusion of the second conditional statement *(But if twice the age of the father is added to the age of son, the sum is 82).* These repetitive kinds of inconsistencies suggest reading without understanding or pure lack of comprehension of the meaning of the pertinent information presented in the given conditional statements has been exemplified by other learners as well. For example, as illustrated in Figure 4.234, learner 7 by writing $2x + x = 56$ seems not to have made sense of the first conditional statement: '*If twice the age of son is added to age of father, the sum is 56*'. It seems if learner 7 wrongly understood that the son's age is twice the father's age, and hence constructed a relationship in terms of the variables x, which is muddled up and incorrect.

The low number of learners, who exemplified Level 2 and Level 3 modelling competencies for each of problems 1-5, seemed to have comprehended pertinent aspects furnished in each of the problems to the extent that they were able to mathematize relevant quantities and their relations for each problem, and correspondingly build meaningful mathematical models represented by a system of linear equations. However, learners at Level 2 seemed to lack the necessary skills to solve the system of linear equations they constructed. For example, in the case of problem 5, learner 15 manipulated his correct system of linear equations $2x + y = 56$ *and* $x + 2y = 82$ *to* get $3\ x + 3y = 138$ through addition, but then incorrectly simplified it *to* $x + y = 15$ and stopped (see Figure 4.26 for details). Similarly, Learner 11 incorrectly reduced the very same system of linear equations $2x + y = 56$ *and* $x + 2y = 82$ to $x + y = 26$ instead of $-x + y = 26$ through the process of subtraction and stopped (see Figure 4.26 for details).
These moves signal a breakdown in the process of solving a system of linear equations correctly and completely. The abrupt stopping signals that these Level 2 learners made no effort to interpret their answers or see if they made sense. To the contrary, those few learners who demonstrated Level 3 modelling competency efficiently and amicably solved their system of linear equations to obtain the correct answers (see Figures 4.28 and 4.29).

Retrospectively, the few learners, who demonstrated level 3 competency in solving problems 1-5, figured out what the problem was, recognised the important and relevant quantities, and assigned variables in alignment to the 'simultaneous equation' strategy, which they realized could help solve each of the problems. These group of learners constructed meaningful relationships between the variables, in the form of a system of algebraic linear equations for each problem, and solved them with a great sense of accuracy. However, while in most cases, these learners stopped dead in their tracks when they ascertained the numerical values of x and y, a very limited number interpreted their respective values of x and y by reaffirming what they meant within the context of the problem. For example, learner 12 in the case of problem 5 as illustrated in Figure 4.28, clearly affirmed that $x = 36$ is associated with the age of the father and that $y = 10$ is associated with the age of the son. In a much more affirmative manner, learner 1, as illustrated in Figure 4.2.1, reflected on his initial assumption that the 2-digit number is in the formxy, and appropriately assimilated values $y = 2$ and $x = 6$ derived through solving his system of linear equations simultaneously to configure that the 2-digit number equals 26. This latter case demonstrates insight, understanding and an acceptable level of interpretation of the solution. However, like most learners in the group, Learner 1 did not make any attempt to check if the values of x and y to satisfy the conditional statements governing problem 4.

4.8 Conclusion

In this chapter, the researcher presented the analysis culminating in the findings and the discussion of these. From the analysis, the researcher gained an understanding of the levels of modelling competency exhibited by the sample of selected learners across the five-word problems. The analysis showed that across the five-word problems, a high number of learners demonstrated level 1 modelling competency, a low number level 2 modelling competency, and an extremely low number level 3 modelling competency. Understanding a given problem was found to be the biggest obstacle in the path of learners demonstrating Learner 1 competence in trying to solve the problem. Deficiencies included were not being able to recognise important quantities, or assigning variables enabling the building of relationships that could culminate in a model and be used as a vehicle to solve a problem.. This group showed that Learner1 competence did not seem to know that the respective problems could be solved by using the system of simultaneous equations.

However, those limited number of learners, who demonstrated L2 modelling competence, seemed to have shown an understanding of the problem by building a system of related linear equations in terms of using two variables (like x and y), which were appropriate models to be used in solving the problem. However, they seemed to know the heuristics about the actual solving of linear equations simultaneously. Hence, they were derailed from completing the problem and or successfully finding the correct values for their assigned variables.

On the other hand, few learners demonstrated L3 modelling competencies, as a result of their profound understanding of a problem within a given context, the successful building of a model characterized by a system of linear equations, which they successfully solved. Although some of these L3 modellers interpreted and made sense of their answers relating to a given context, the others did not. Nearly all of the L3 modellers did not test their solutions to verify whether these satisfied in the imposed conditions.

In the next chapter, I will present the summary of the findings, implications, recommendations and conclusion of the study.

Chapter 5: Conclusions and Recommendations

5.1 Introduction

The purpose of this study was to investigate the modelling competencies Grade 10 learners demonstrate when solving word problems that invoke the use of simultaneous equations. This investigation provided an opportunity for a sample of these high-school learners at a school in the Western Cape Province, South Africa to solve a set of word problems as one of their class-based activity tasks. This study attempted to provide a set of localized answers to the posited research question using a case-based design aligned to a mixed-methods research approach located within an interpretative paradigm. Hence, a summary of the study findings that assisted in answering the research question is provided in this chapter, along with a discussion of implications, recommendations and directions for future research, and ultimately the conclusion.

5.2 Summary of findings in relation to the research question

As indicated earlier, the research question pursued in this study was posited as:

> *What levels of mathematical modelling competency do Grade 10 learners demonstrate when solving word problems using simultaneous equations?*

The findings associated with the research question reveal that:

i. A high number of learners, who demonstrated Level 1 modelling competency, showed a serious lack of modelling competencies and their associated sub-competencies across their responses to the respective word problems. In this respect, the inability of learners to *understand a problem* was seen to be the main factor ***inhibiting*** learners from:

   (a) comprehending the context or sketching or writing anything about the problem, or

   (b) Recognising quantities associated with the problem and assigning variables, or finding connections to any mathematical ideas, or

   (c) Mathematizing the situation to enable the building of relationships and or a mathematical model, or

   (d) seeing that the strategy of building a system of linear equations, which could be solved simultaneously could help to solve a given problem.

   This group of learners, who demonstrated Level 1 modelling competencies (i.e. not competent) did not effectively move onto the stages of building and solving a model, which are indeed core modelling competencies.

ii. A low number of learnersdemonstrated Level 2 modelling competency and were able to read and understand a problem and build a system of linear equations (i.e. build a mathematical model). However, they lacked the necessary mathematical knowledge and skills to solve their built system of linear equations. In terms of understanding the problem, this small group of learners was able to comprehend the context.

iii. A central strength of this small group of learners is their mathematizing ability. Mathematizing includes discussing mathematical ideas that eventually relate to the mathematical entity that is represented in the constructed model. By trying to mathematize a situation, these group of learners succeeded in making a connection or bridge between the problem and the mathematical structure of the model. Although their respective models were characterized by an appropriate system of linear equations, their inability to solve a system of linear equations using a simultaneous strategy, prevented these learners (level 2 modelling competency) from finding a sensible solution to a given problem. Furthermore, this group of learners did not make any attempt to verify or check their solutions.

iv. Very few learners demonstrated Level 3 modelling competency. These Level 3 modellers moved smoothly and diligently through the first three phases of the modelling process, namely understanding a problem, building a model and solving it. In showing understanding, these few learners succeeded in recognising all relevant quantities associated with a problem, assign variables appropriately, and build meaningful relationships between generated variables. Using appropriate and correct mathematical language and notation including representations (such as diagrams, number sentences) they were able able to mathematize relevant quantities and their relations efficiently. Consequently, they could correctly build necessary mathematical models represented by a system of two linear equations involving two variables (such as x and y). More importantly, these Level 3 modellers were able to apply the method of solving their system of linear equations simultaneously using the elimination method and perfectly solve the variables (such as x and y). Most of the Level 3 modellers seemed to have taken their answers that were correct for granted, as they did not attempt to verify or check the solutions. In only one or two cases, the learner interpreted their solution within the context of the problem and or checked if the solutions satisfied the initial given conditions.

5.3 Implications and Recommendations

This study found that the lack of competence of learners to understand a problem caused them to not make any headway in solving a problem and progress to the other stages of the modelling process such as building and solving a model and interpreting their solutions. To solve a problem, one needs to have a clear understanding of what the problem is. This knowledge can be developed by allowing the learners to write the problem in their own words, distil given facts and related information, and state the goal in their own words. Therefore, a clear language focus is needed, and the learner to internalize the information by developing a representation of the problem. It is through understanding that a decision can be made which strategy should be used. The better learners understand a problem, the easier the evaluation progress is towards reaching a solution. A clear understanding will enable the solver to decide whether the answer is reasonable (Naidoo, Smit, & van Heerden, 1995).

According to Govender (2018), this inability to understand the problem could have inhibited learners in this study from "*making adequate representation of the situation which could help simplify and structure the situation for mathematization to begin*". To enable learners to develop their competencies in reading and understanding a problem, Govender (2018) asserts that teachers should do the necessary lesson planning to ensure that "*reading and understanding of problems are part our daily/regular classroom activity*". In planning such lessons, we should provide scaffold opportunities to engage learners with problems that require mathematization and provide them with on the spot feedback. Teachers should be trained by curriculum advisers and in-service courses on how to design and enact scaffold activities in their classroom that aim to develop the sub-competency of mathematization. Language, concepts, and behaviour are definitely, though complexed, interrelated. Concerning the nature of this relationship, Whorf (1956) as cited in Govender (2002), asserts that the structure and vocabulary of language determine how individuals view the world and consequently, how they act. The perceptions and behaviour of persons speaking different languages will vary to the extent that particular languages differ in structure and vocabulary. Besides, the Cockroft report (1981) emphasized that language is essential for the formulation and expression of mathematical ideas, including self-understanding of information provided, and stressed the role of discussion in mathematics. In this respect, Govender (2002) pointed out that recollection of mathematics will improve if the learner understands it better, which can only happen if there is a reasonable degree of language competence.

Given that mathematical problems selected for English second language (ESL) learners were assimilated from textbooks predominantly authored by English first language speakers, this could have been a barrier in preventing most learners from accessing, reading and understanding a particular word problem. Moreover, learners who learn the subject through a second or third language often encounter difficulties in comprehending the instruction or apply English in a mathematical context (Dicker, 2015). In this respect, Swanson (2015) suggests that mathematical items can be linguistically altered to lessen the language load without changing the construct being evaluated. However, the teachers first need to have a profound understanding of the learners' level of English proficiency and a mathematics vocabulary that is essential before they can make adaptations (Haag, Heppt, Stanat, Kuhl, & Pant, 2013). Furthermore, this study recommends that learners, who are taught in their second language (English), should more extensively be engaged with activities that focus on making sense of a problem from at least Grade 3 upwards. Also, learners should be taught in English from an early age during mathematics periods to support the reading and understanding competency.

With the extensive set of technological tools and software available to facilitate teaching and learning mathematics, a classroom atmosphere should be created to enable learners to explore problem situations and develop a common understanding of the problem through group work, discussions, rebottles and teacher feedback. Furthermore, it could be of much help if the policymakers could ensure that the medium of instruction used by teachers is aligned to the language that permeates learner's textbooks and workbooks.

Although a limited number of learners assimilated a system of linear equations defined by two variables, most of them seemed not to possess the necessary knowledge and procedural skills to solve the system of equations, and some struggled with computations with fractions. Learners should be provided with regular tutorial activities and assignments as a form of spiral revision to enhance and strengthen their competencies in solving simultaneous linear equations and simplifying fractions to address these shortcomings.

Even though a few learners demonstrated sufficient competence in understanding a given problem, and then developing and successfully solving their mathematical model accurately, most of these learners did not verify or check their answers satisfied the initial given conditions. Although more attention should be given to learners' solution attempts, and less on correct answers (Lester, 1985), teachers should encourage their learners to develop a habit of checking their work and taking their calculated answers and testing it to see if it satisfies the conditions of the problem. Furthermore, Lenchner (1983) believes that answers make sense if they can be achieved when learners their complete sentences. Writing complete sentences would result in learners reviewing the problem statement or the question being asked, and in detecting a possible error. Discussing problems with learners and sharing their experiences, as well as reasons for their choices of solutions, can build confidence in them.

5.4 Suggestions for Future Research

This study focused on Grade 10 learners' modelling competencies within the context of solving word problems involving simultaneous equations. Other important findings can be gained if this study is extended to word problems associated with other Grade 10 topics, such as quadratic equations, exponential equations, inequalities or trigonometry.Although the sample size of learners (i.e. 20 learners) was acceptable for this study, it would be ideal to repeat it on a larger population sample size. Even though numerous issues have been considered and deliberated in this study, they can be investigated further with other high school grades.

Although the CAPS curriculum and research emphasize the importance of engaging our learners in mathematical modelling activities, it is quite evident from this study that the grade 10 learners were not adequately exposed to the modelling processes and sub-processes. Hence, a study could be designed to explore how learners' mathematical modelling competencies can be developed and enhanced through working with word problems in natural classroom settings. The findings will assist in intervention strategies so that most educators feel confident in implementing this approach.

Furthermore, the researcher suggests that a large scale investigation be done on the impact of the Language of Learning and Teaching (LOLT) on the mathematical modelling capabilities of second-language learners. The findings from this kind of study could help teachers to design appropriate learning support material to compensate and eradicate some (if not all) kinds of obstacles or hindrances learners experience in reading and understanding a problem.

5.6 Acknowledgement of the limitations of the study

Study constraints involved the mobility of the researcher, who was unable to reach out to more schools. The data were collected from the school in the suburb closer to where the researcher lives for convenience. A sample of only 20 learners was used for the study. Therefore, the findings of this study could not be generalised because the sample size was not an adequate representation of the cohort population of learners in Grade 10 classes in 2016. Furthermore, because of unforeseen municipal protest and taxi violence in the vicinity surrounding the school, teachers (and learners) were not comfortable and felt unsafe to keep their learners at school after hours. In particular, learners were travelling to and from school by taxi, and it was a risk to allow them to travel after school hours. However, the findings of the study are reliable and valid. Therefore, future research is recommended that could be undertaken at a larger scale to pursue the same ambitions and purpose of this study.

5.7 Conclusion

This study has shown that a localized group of Grade 10 learners, who were all ESL speakers, demonstrated varying levels of modelling competency ranging from non-competency to partial competency and sufficient competency. However, most of the learners demonstrated non-competency in modelling, mainly because of their inability to understand the problem, as evident in their failure to comprehend the context of a problem, inability to recognise important quantities associated with a problem, and confusing relationships if any. Given that all the learners in the sample were ESL speakers, this study has recognised language as a potential barrier to their understanding of the problem and establishing its goal. In South Africa, for example, learners are taught in their second language, thus it is imperative that they are assisted in understanding the question accurately. Without this coaching, they would select incorrect strategies to solve the problem, represent and mathematize the situation and invoke unnecessary and or inaccurate mathematical concepts, skills and procedures.

A very limited number of students showed partial modelling competency, as they were only able to understand the problem and build a correct model to solve it. Regrettably, these students lacked the knowledge of the heuristics for solving the system of linear equations correctly and completely and did not check or verify their answers. Thus, teachers must revise topics covered on an on-going basis through spiral revision, including assignments and tutorials.

The extremely small number of learners, who demonstrated sufficient modelling competency, demonstrated sufficient understanding of the problem, built and solved the system of simultaneous linear equations successfully without necessarily checking or testing whether their answers satisfied the conditions of the problem. The competence of checking whether solutions are reasonable and those finally accepted satisfy the conditions attached to the problem needs to develop over time, over a series of tasks, and be seen as a continuous process. This competency process should become a consistent natural habit for learners. Studying their modelling competencies requires further work to add to the repertoire of this knowledge domain.

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Appendix 1: RESEARCH INSTRUMENT: WORKSHEET

SOLVING WORD PROBLEMS

Grade10 Date: _____________

Learner No: ________________

Task 1

Use your knowledge of simultaneous to solve the following problems:

1. Half the sum of two numbers is 27, and their difference is 6. What are the numbers? (6 marks)

2. If 1 is added to the numerator and 2 to the denominator of a fraction, the ratio of the new numerator to the new denominator is 2: 3. If 1 is subtracted from the numerator and 2 from the denominator, the new numerator is equal to the1 new denominator. Find the fraction. (6 marks)

3. Andre has more money than Bob. If Andre gave Bob R20, they would have the same amount. While if Bob gave Andre R22, Andre would then have twice as much as Bob. How much does each one actually have?

4. In a two-digit number. The unit digit is thrice the tens digit. If 36 is added to the Number, the digits interchange their place. Find the number.

5. If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82. Find the ages of father and son.

Appendix 2: RESEARCH INSTRUMENT

THE OBSERVATION SCHEDULE

The purpose of the observation list is to track the kinds of strategies and learning resources that Grade 10 learners use when solving each of the simultaneous word problems listed in Appendix B.

Problem-Solving Strategy	Yes	No	Comments
Writing variables			
Understand operations			
Forming equation			
Number sentences			
Recognising Patterns			
Solving the equation			

Learning Resources	Yes	No	Comments
Calculator			
Ruler			
Mathematics textbook			
Mathematical set			
Number line			

Appendix 3: RESEARCH INSTRUMENT

THE SEMI-STRUCTURED INTERVIEW PROTOCOL

The following are questions that may be put to a sample of learners during the semi-structured interview sessions.

1. Did you do a same/similar type of problem before in class?
2. What is the problem about?
3. Circle the key words in the problem?
4. Do you understand all the words in the problem?
5. What information is given?
6. What is the question asking?
7. Can you think of variables that can be used to represent the problem and help solve the problem?
8. Do see you a pattern, which can help you solve this problem
9. Describe the plan you used to solve this problem?
10. Why did you choose to use your plan to solve this problem?
11. Which learning aid (calculator/number line) did you use to solve this problem?
12. Can you check your answer?
13. Did you check your answer?
14. How can you tell whether the answer is correct?
15. Does your answer makes sense?
16. Do you thinking there is another way to solve this problem?
    If yes, briefly describe it to me.
17. Is there a shorter method you think you can use to give the same answer?
18. What did you learn from doing/completing this problem?
19. What did you struggle with as you proceeded to read and solve this problem?
20. Did you enjoy doing this problem? If yes, why. If no, why?

Appendix 4: Interview questions and responses

1. Interview questions and responses of problems in Worksheet A.

Q: How did you arrive to your answer?

A:half of 20 is 10 and 27 take away 20 is 7, so he added 7 to 10 to get 17, and half of 10 is 5 plus 6 is 11 in the Equation. .

Q: How did you came up with the two answers 24 and 30 that are correct?

A: "I multiplied 27 by 2 and got 54 and then subtracted 24 from 54 to get 30, I also subtracted 3 from 27 and I got 24", when I asked why did she subtracted 3 from 27 she said she doesn't know. .

Q: Do you need someone to assist you with the question? What is the main problem you experienced when doing this activity?

A: Y*es I can see where I failed but even then I am not sure if this is correct*.

2. Interview questions and responses of problems in Worksheet B.

Q: Why is knowledge of variables and equations so important?

A: It is only now that I am learning Mathematics in English all along I was taught in my mother languages. I can read the instruction but not understanding it and the question.

Q: Is Mathematics a difficult subject to you?

A: *You know Miss, Mathematics in general is an interesting subject, it the way in which it is taught that makes it difficult.*

Q: What do you mean by the way of teaching it?

A: *No, look Miss if the teacher comes into the class and say today we are calculating variables and equations. To some of us we do not know what these variables are and it becomes boring to us.*

Q: What difference have you noticed in your knowledge of variables and equations in doing Worksheet B?

A: I did not like what I was doing in the worksheet because I did not understand the question. I just wrote whatever I thought and I did not care if it is right or wrong. Mathematics has been my worse subject because it is about formulas and numbers the x and the y and these does not make sense to me.

Q: How does my teaching assist you to think differently about yourself and Mathematics?

A:(Ja) meaning yes…… really if I can get a teacher who is patient and friendly like you and who make Mathematics so simple, I can love it. Learner (Y) elaborated to say: *No, Miss our problem is the way Mathematics is taught… it makes you lose interest. But the way you explain it to us, you started by telling us a story about variables and you explained them to us and also the relationships between them and equations all was clearer. I will love it, if it can be explained like this. My marks on knowledge of equations improved.*

Appendix 5: Ethical Clearance Certificate

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14 November 2016

Prof Ms D Machingura
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Ethics Reference Number: HS/16/8/5

Project Title: The development of mathematical modelling competences of grade 10 learners through an intentional teaching strategy: The case of modelling with equations.

Approval Period: 14 November 2016 – 14 November 2017

I hereby certify that the Humanities and Social Science Research Ethics Committee of the University of the Western Cape approved the methodology and ethics of the above mentioned research project.

Any amendments, extension or other modifications to the protocol must be submitted to the Ethics Committee for approval. Please remember to submit a progress report in good time for annual renewal.

The Committee must be informed of any serious adverse event and/or termination of the study.

Ms Patricia Josias
Research Ethics Committee Officer
University of the Western Cape

PROVISIONAL REC NUMBER - 130416-049

APPENDIX 6: Language Editor Letter

Language Practitioner

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TO WHOM IT MAY CONCERN

I am a language practitioner with an MPhil degree (cum laude) in Translation Studies, which I received in 2003 at Stellenbosch University. I have edited several doctoral and master's theses since qualifying.

For the present thesis, *Mathematical modelling with simultaneous equations – An analysis of Grade 10 learners modelling competencies* by Dzivaidzo Machingura, I did a comprehensive edit which included:

- Language editing
- Technical editing and formatting
- Adjusted Tables and figures and renumbered these
- Checked for plagiarism
- Pointed out duplication of text as well as information that is not in the correct order
- Checked and re-did Table of Contents including pagination
- Cross-checked references in text and list
- Formatted references according to APA style

JM Fourie (Ms)

22 October 2019