Character Tables of Some Selected Groups of Extension Type Using Fischer-Clifford Matrices

by

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Declaration

I declare that *Character Tables of Some Selected Groups of Extension Type Using Fischer-Clifford Matrices* is my own work, that it has not been submitted before for any degree or assessment to any other university, and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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Abstract

The aim of this dissertation is to calculate character tables of group extensions. There are several well–developed methods for calculating the character tables of some selected group extensions. The method we study in this dissertation, is a standard application of Clifford theory, made efficient by the use of Fischer-Clifford matrices, as introduced by Fischer. We consider only extensions $\overline{G}$ of the normal subgroup N by the subgroup G with the property that every irreducible character of N can be extended to an irreducible character of its inertia group in $\overline{G}$, if N is abelian. This is indeed the case if $\overline{G}$ is a split extension, by a well known theorem of Mackey.

A brief outline of the classical theory of characters pertinent to this study, is followed by a discussion on the calculation of the conjugacy classes of extension groups by the method of coset analysis. The Clifford theory which provide the basis for the theory of Fischer-Clifford matrices is discussed in detail. Some of the properties of these Fischer-Clifford matrices which make their calculation much easier, are also given.

We restrict ourselves to split extension groups $\overline{G} = N{:}G$ in which N is always an elementary abelian 2-group. In this thesis we are concerned with the construction of the character tables (by means of the technique of Fischer-Clifford matrices) of certain extension groups which are associated with the orthogonal group $O_{10}^{+}(2)$, the automorphism groups $U_6(2){:}2$, $U_6(2){:}3$ of the unitary group $U_6(2)$ and the smallest Fischer sporadic sim-

ple group Fi_{22}. These groups are of the type $2^8{:}(U_4(2){:}2)$, $(2^9{:}L_3(4)){:}2$, $(2^9{:}L_3(4)){:}3$ and $2^6{:}(2^5{:}S_6)$.

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Notation and conventions

Throughout the thesis all groups will be assumed to be finite, unless otherwise stated. We will use the notation and terminology from the ATLAS [22] and [37].

$\mathbb{N}$	natural numbers
$\mathbb{Z}$	integers
$\mathbb{Q}$	rational numbers
$\mathbb{R}$	real numbers
$\mathbb{C}$	complex numbers
$\overline{G}, N, G, H$	groups
1_G	the identity element of G.
F	a field
F^*	$F - 0$
$H \leq G$	H is a subgroup of G
$H \trianglelefteq G$	H is a normal subgroup of G
$H \cong G$	H is isomorphic to G
$< x, y >$	the subgroup generated by x and y
$N{\cdot}G$	an extension of N by G
$N{:}G$	a split extension of N by G
$o(g)$	order of $g \in G$
$[g]$	a conjugacy class of G with representative g
$N_G(H)$	the normalizer of the subgroup H in G
Hg	the right coset of H in G
X, Y, Ω	sets

$\|\Omega\|$	the cardinality of the set Ω
G_θ	stabilizer of θ in G
$I_G(\theta)$	inertia group of θ in G
n^g	conjugation of n by g
$g_1 \sim g_2$	g_1 is conjugate to g_2
$\psi \downarrow H$	the restriction of the character ψ of G to the subgroup H
ϕ^G	the induction of the character ϕ of subgroup H to G
$C_G(g)$	centralizer of g in G
$Irr(G)$	set of irreducible characters of G
I_G	the identity character of G
$\chi(G/H)$	the permutation character of G on H
$dim(V)$	the dimension of a vector space V
S_n	the symmetric group on n symbols
$GF(q)$	the Galois field of q elements
$V(n,q)$	a vector space of dimension n over $GF(q)$
$Sp(2n,q)$	symplectic group of dimension $2n$ over $GF(q)$
$U_n(q)$	unitary group

Contents

Chapter 1

Introduction

The Classification of the Finite Simple Groups (CFSG) emphasizes the importance of the finite simple groups in Group Theory. It is one of the most impressive achievements in the history of Mathematics. The reader is referred to [69] for more literature on the CFSG theorem. Classification of Finite Simple Groups states that each finite simple group is isomorphic to exactly one of the following

- A cyclic group of prime order,
- An Alternating group A_n of degree at least 5,
- A group of Lie type,
- One of twenty–six sporadic simple groups.

Since the CFSG, more recent work in group theory has involved methods for the computation of character tables of finite groups, and in particular, the character tables of the maximal subgroups of the sporadic simple groups and their automorphism groups. The character tables of most of these maximal subgroups are known but there are still some of the character tables of the maximal subgroups of the Baby Monster group B and the Monster group M which are not yet computed. Many of these maximal subgroups are extensions of elementary abelian groups, so methods have been developed for the calculation of the character table of extensions of elementary abelian groups. A knowledge of the character table of a group provides considerable information about the group, and hence it is of importance in the physical sciences as well as in pure mathematics. Bernd Fischer developed a powerful and interesting technique for calculating the character tables of group extensions although there exist techniques such as the Schreier-Sims algorithm, Todd-Coxeter coset enumeration method, the Burnside-Dixon algorithm, etc, to construct character tables of finite groups. This technique, known as the technique of the Fischer–Clifford matrices [26], derives its fundamentals from the Clifford theory. If $\overline{G} = N.G$ is an appropriate extension of N by G, the method involves the construction of a nonsingular matrix for each conjugacy class of $\overline{G}/N$. In this thesis, we apply the Fischer-Clifford theory only to split extensions. This technique has also been discussed and used by many other researchers, but applied only to split extensions or to the case when every irreducible character of N can be extended to an irreducible character of its inertia group in $\overline{G}$ (see for example [1], [4], [5], [6], [8], [10], [24], [27], [41], [42], [48], [49], [51], [50], [57], [60], [61] and [64]).

However the same method cannot be used to construct character tables of certain non-split group extensions. For example, it cannot be applied to the non-split extensions of the forms $3^{7\cdot}O_7(3)$ and $3^{7\cdot}(O_7(3){:}2)$ which are maximal subgroups of Fischer's largest sporadic simple group Fi_{24}' and its automorphism group Fi_{24} [22], respectively. In an attempt to generalize these methods to such type of non-split group extensions, Ali[1] considered the projective representations and characters and showed how the technique of Fischer-Clifford matrices can be applied to any such type of non-split extension. However in order to apply this technique, the projective characters of the inertia factors must be known and these can be difficult to determine for some groups. Ali (see [2] and [7]), successfully applied the technique of Fischer-Clifford matrices and determined the Fischer-Clifford matrices and hence the character table of the non-split extensions $3^{7\cdot}O_7(3)$ and $3^{7\cdot}(O_7(3){:}2)$. More recent publications in this regards can be found in [9], [51] and [62].

In Chapter 2, we present some results on group characters which are used in the later chapters. We mostly concentrate on those results which are applied in Chapters 4 and 5 to describe the Fischer-Clifford matrix methods. We start by discussing the general theory of representations and characters, and go on to discuss the restricted, induced and permutation characters, which will be used in the later chapters for constructing the character tables of the groups that are studied in this thesis. The characters being studied are ordinary complex characters.

In Chapter 3 we give some preliminary results on group extensions that will be needed in subsequent chapters. In Section 3.1 we define group extensions and then we describe the method of *coset analysis* to compute the conjugacy classes of group extensions, where we first restrict ourselves to split extensions of abelian groups (Section 3.2) and then the case where the extension is not necessarily split (Section 3.3). In the remainder of Section 3.3 we prove and discuss techniques that are useful in the determination of the orders of the elements of $\overline{G} = N{:}G$. The technique of coset analysis was developed and first used by Moori in [44], [45] and has since been widely used for computing the conjugacy classes of group extensions.

Chapter 4 is devoted to the study of Clifford theory for ordinary representations of a group $\overline{G}$ and its related results which will be required to describe the Fischer-Clifford matrices. In Section 4.1 we study the relationship between characters of a group $\overline{G}$ and its normal subgroup N. We present various sufficient conditions for the extendability of an irreducible character θ of N to its inertia group $\overline{H}$ in $\overline{G}$.

In Chapter 5 we describe the theory of the Fischer-Clifford matrices. If $\overline{G} = N{\cdot}G$ is an appropriate extension of N by G, the method involves the construction of a Fischer matrix for each conjugacy class of $\overline{G}/N \cong G$. Then by using these matrices together with the fusion maps and character tables of some subgroups of G which are inertia factors of the inertia groups in $\overline{G}$, we are able to construct the complete character table of $\overline{G}$. Section 5.1 gives definitions and preliminaries, Section 5.2 deals with the properties of the Fischer-Clifford matrices, in particular we study a special case of Fischer-Clifford matrices of an extension $\overline{G} = N \cdot G$ with the property that every irreducible character of N can be extended to an irreducible character of its inertia group in $\overline{G}$. In the last part of Section 5.2 we discuss additional properties of the Fischer-Clifford matrix $M(g)$ for a split extension $\overline{G} = N{:}G$, where N is an elementary abelian 2-group.

In Chapters 6 and 7 we study the maximal subgroups $(2^9{:}L_3(4)){:}2$ and $(2^9{:}L_3(4)){:}3$ of the automorphism groups $U_6(2){:}2$ and $U_6(2){:}3$ of the unitary group $U_6(2)$, respectively. We will show with the aid of the computer algebra systems GAP [67] and MAGMA [15] that the groups $(2^9{:}L_3(4)){:}2$ and $(2^9{:}L_3(4)){:}3$ are isomorphic to the split extension groups of the type $2^9{:}(L_3(4){:}2)$ and $2^9{:}(L_3(4){:}3)$, respectively. The conjugacy classes of both of these split extension are determined by the technique of coset analysis. For this purpose of finding all the classes of $2^9{:}(L_3(4){:}2)$ and $2^9{:}(L_3(4){:}3)$, we are assisted by the Programme A and Progamme B which are based on the technique of coset analysis. These MAGMA programmes are found in [1]. Having the classes of these groups in the format allowed by coset-analysis, we can proceed to compute the Fischer-Clifford matrices of these groups. Hence we can construct the associated character tables which are partitioned into blocks according to the inertia groups of $2^9{:}(L_3(4){:}2)$ and $2^9{:}(L_3(4){:}3)$. Lastly, the complete fusion of each of these groups into their respective main groups $U_6(2){:}2$ and $U_6(2){:}3$ will be fully determined using the technique of set intersections (see [44],[45] and [48]).

In the ATLAS [22] we see that one of the 9 classes of maximal subgroups of the orthogonal simple group $O_{10}^{+}(2)$ has the form $2^8{:}O_8^{+}(2)$. Ali in [4] study a group of the form $2^8{:}Sp_6(2)$ which sits maximally in $2^8{:}O_8^{+}(2)$. Also, in the ATLAS we observed that the group $U_4(2){:}2$ is one of the maximal subgroups of the symplectic group $Sp_6(2)$. Now, the pre-image of $U_4(2){:}2$ in $2^8{:}Sp_6(2)$ is a group of the form $2^8{:}(U_4(2){:}2)$. In Chapter 8 we study the group $2^8{:}(U_4(2){:}2)$ and apply Fischer-Clifford theory to determine its character table. The complete fusion of the classes of $2^8{:}(U_4(2){:}2)$ into the classes of $2^8{:}Sp_6(2)$ will also be determined.

In the paper [48] the authors study the maximal subgroup $2^6{:}Sp_6(2)$ of the smallest Fischer sporadic simple group Fi_{22} of index 694980. They constructed the character table of $2^6{:}Sp_6(2)$ by using the technique of Fisher-Clifford matrices. It was found that one of the two inertia groups of $2^6{:}Sp_6(2)$ has the form $2^6{:}(2^5{:}S_6)$, where $2^5{:}S_6$ is maximal and affine in $Sp_6(2)$ of index 63. In general it is more complicated to construct the character tables of the inertia groups $\overline{H_i} = N \cdot H_i$ of an extension group $\overline{G} = N \cdot G$ by using Fischer-Clifford theory. In the final chapter we use the same methodology as in Chapters 6,7 and 8 to construct the character table of the split extension $2^6{:}(2^5{:}S_6)$ and as well as the fusion of $2^6{:}(2^5{:}S_6)$ into the group $2^6{:}Sp_6(2)$.

Most of our computations are carried out with the assistance of the computer algebra systems MAGMA and GAP. Our notation is standard and the reader may refer to the ATLAS and the ATLAS of Brauer Characters [37]. All our groups and sets are finite unless otherwise specified. Programmes A and B that have been used to compute the conjugacy classes of our groups are given in Appendix A. Consistency and accuracy checks for the character tables of our 4 split extension groups were implemented using GAP codes labelled as Programme C in Appendix A.

Chapter 2

Group Characters

Two ways of approaching representation and character theory are through the use of modules on the one hand (for instance, the approach used by James and Liebeck [37]), and through the classical approach used by Feit [25] for example, on the other hand. Our discussion is along the classical approach and for this purpose we follow the lecture notes of Moori [47], the works of Mpono [53] and Whitley [68].

In this chapter we give preliminary results on group characters that will be needed in later chapters to construct the character tables of the groups that are studied in this thesis. We start by presenting in section 2.1 the general theory on representations and characters of groups. In section 2.2 we discuss the role of normal subgroups in finding some characters of a group. In sections 2.3 to 2.5 restricted, induced and permutation characters are discussed to establish the relationship between characters of groups and the characters of their subgroups. For further reading on representations and characters, readers are referred to [11], [13], [16], [18], [19], [25] , [34], [35], [36], [40], [43], [55] and many other relevant sources.

2.1 Representations and Characters

In this section we give some preliminary results on representations and characters of groups .

Definition 2.1.1. Let G be a finite group and F a field. A homomorphism $\rho : G \to GL_n(F)$ is called a representation of G over F or simply an F-representation. The general linear group, $GL_n(F)$, is the multiplicative group of all non-singular $n \times n$ matrices over F for some integer n. The representation ρ is said to have degree n.

Two F-representations ρ_1 and ρ_2 are said to be *equivalent* if there exists $P \in GL_n(F)$ such that $\rho_1(g)=P\rho_2(g)P^{-1}$ for all $g \in G$. An F-representation ρ of G is said to be *reducible* if it is equivalent to a representation α which is given by

$$\alpha(g) = \begin{pmatrix} \beta(g) & \gamma(g) \\ 0 & \delta(g) \end{pmatrix}$$

for all $g \in G$, where β, γ, δ are F-representations of G. If ρ is not reducible, then it is said to be *irreducible*. ρ is defined to be fully reducible if it is equivalent to a representation α where

$$\alpha(g) = \begin{pmatrix} \beta(g) & 0 \\ 0 & \delta(g) \end{pmatrix}$$

for all $g \in G$. ρ is completely reducible if it is equivalent to a representation α given by

$$\alpha(g) = \begin{pmatrix} \beta_1(g) & 0 & \cdots 0 \\ 0 & \beta_2(g) & \cdots 0 \\ \vdots & \ddots & \cdots 0 \\ 0 & 0 \cdots & \beta_r(g) \end{pmatrix}$$

for all $g \in G$ and where each β_i is an irreducible F-representation of G. Then $\beta_1, \beta_2, .., \beta_r$ are called constituents of ρ.

Theorem 2.1.2. *(**Mashke's Theorem**) Let G be a finite group. If F is a field of characteristic zero, or whose characteristic does not divide the order of G, then every F-representation of G is completely reducible.*

Proof:See[[25],(1.1)].

Theorem 2.1.3. ***(Schur's Lemma)*** *Let ρ_1 and ρ_2 be two irreducible F-representations of a group G over a field F. Suppose P is a non-zero matrix over F such that $\rho_1(g)P = P\rho_2(g)$ for all $g \in G$. Then P is nonsingular and ρ_1 is equivalent to ρ_2.*

Proof:See[[25],(1.2)]

Corollary 2.1.4. *[45] Let F be an algebraically closed field, and ρ an irreducible F-representation of a group G. Then the only matrices that commute with every matrix $\rho(g)$, $g \in G$ are the scalar matrices aI_n, where $a \in F$ and I_n is the $n \times n$ identity matrix.*

Proof: Let P be an $n \times n$ matrix such that $P\rho(g) = \rho(g)P$ for all $g \in G$. Then for any $a \in F$ we have that

$$(aI_n - P).\rho(g) = \rho(g).(aI_n - P), \text{ for all } g \in G. \quad (1)$$

Let $m(x) = det(xI_n - P)$ be the characteristic polynomial of P. Since $m(x)$ is a polynomial over F and F is algebraically closed, then there exists $a_1 \in F$ such that $m(a_1) = 0_F$. Hence $det(a_1 I_n - P) = 0_F$ and thus $a_1 I_n - P$ is singular. Then from relation (1) and Schur's Lemma, we obtain that $a_1 I_n - P = 0$ and thus $a_1 I_n = P$. □

Definition 2.1.5. The function $\chi_\rho : G \to F$ defined by $\chi_\rho(g) = trace(\rho(g))$ is called the $F-character$ of G afforded by the $F-representation$ ρ. The degree of χ_ρ is the degree of ρ. The *trivial character* is the character 1_G defined by $1_G(g) = 1_F$ for every $g \in G$. An *irreducible character* is a character afforded by an irreducible representation.

Lemma 2.1.6. [*68*] *The following properties hold for a group G:*

1. *A character of G is constant on the conjugacy classes of G.*
2. *Equivalent representations afford the same character.*
3. *$\chi(1)$ is the degree for any character χ.*
4. *The sum of any two characters of G is again a character of G.*

Proof: Parts 1 and 2 follow from the fact that for matrices A and P, $trace(P^{-1}AP) = trace(A)$.

3. Let χ have degree n. Then $\chi(1) = trace(I_n) = n$.

4. Let χ_{ρ_1} and χ_{ρ_2} be characters of G, afforded by the representations ρ_1 and ρ_2, respectively. Define the function ψ on G by $\psi(g)=\begin{pmatrix} \rho_1(g) & 0 \\ 0 & \rho_2(g) \end{pmatrix}$. Then ψ is a representation of G with $\chi_\psi = \chi_{\rho_1} + \chi_{\rho_2}$. □

Definition 2.1.7. Let G be a group, F a field and $\phi : G \to F$ be a function which is *constant* on conjugacy classes. Then ϕ is called a *class function* of G.

From the above definition, we observe that every character is a class function. We shall use the notation $Irr(G)$ to denote the set of all irreducible characters of the group G. From now on, we will consider representations and characters of a finite group G over the complex field $\mathbb{C}$.

If ϕ is any class function on G, then ϕ can be uniquely expressed in the form $\phi = \sum_{i=1}^{r} a_i\chi_i$ where $a_i \in \mathbb{C}$ and $Irr(G) = \{\chi_1, ..., \chi_r\}$. Moreover ϕ is a character if and only if all $a_i \in \mathbb{N} \cup 0$.(See[[32],(2.8)]).

Theorem 2.1.8. *The following properties hold:*

1. *Two representations of G have the same character if and only if they are equivalent.*
2. *The number of irreducible characters of G is equal to the number of conjugacy classes of G.*
3. *Any character can be written as a sum of irreducible characters.*

Proof:

1. See [[25], (2.6)]
2. See [[25], (2.16)]
3. This follows from Mashke's Theorem. □

Lemma 2.1.9. *Let χ be a character of G afforded by a representation ρ of degree n. Then*

1. *for* $g \in G$, $\rho(g)$ *is similar to a diagonal matrix* $diag(\epsilon_1, ..., \epsilon_n)$ *where each* ϵ_i *is a complex root of unity.*

2. $\chi(g) = \epsilon_1 + ... + \epsilon_n$ *and* $\chi(g^{-1}) = \overline{\chi(g)}$*, where* $\overline{\chi(g)}$ *denotes the complex conjugate of* $\chi(g)$*.*

Proof See[[33], (2.15)]. □

$Irr(G)$ are presented in a table, called the *character table* of G. In this character table of G, the rows correspond to the irreducible characters of G and the columns to the conjugacy classes of G. The entry a_{ij} in this table is the value of the i-th irreducible character on an element of the j-th conjugacy class. This character table of G satisfies certain orthogonality relations, which are given in the next few theorems following the definition.

Definition 2.1.10. Let χ_1 and χ_2 be two characters of a group G. Then the *inner product* of χ_1 and χ_2 is defined by

$$< \chi_1, \chi_2 >= \frac{1}{|G|} \sum_{g \in G} \chi_1(g)\overline{\chi_2(g)}.$$

Theorem 2.1.11. *(**Generalized Orthogonality Relation**) Let G be a group and $Irr(G)= \{\chi_1, \chi_2, ..., \chi_r\}$. Then the following holds for every $h \in G$.*

$$\frac{1}{|G|} \sum_{g \in G} \chi_i(gh)\chi_j(g^{-1}) = \delta_{ij} \frac{\chi_i(h)}{\chi_i(1_G)}.$$

Proof: See Theorem 2.13 of [33]. □

Theorem 2.1.12. [*33*] *(**First Orthogonality Relation**) Let G be a group and $Irr(G)= \{\chi_1, \chi_2, ..., \chi_r\}$. Then*

$$\frac{1}{|G|} \sum_{g \in G} \chi_i(g)\overline{\chi_j(g)} = \delta_{ij} =< \chi_i, \chi_j > .$$

Proof: Let $h = 1_G$ in the generalized orthogonality relation to obtain the desired result. $\square$

Theorem 2.1.13. [*33*], [*53*]*(**Second Orthogonality Relation**) Let G be a group and $Irr(G)= \{\chi_1, \chi_2, ..., \chi_r\}$ and $\{g_1, g_2, ..., g_r\}$ be a set of representatives of the conjugacy classes of elements of G. Then*

$$\sum_{\chi \in Irr(G)} \chi(g_i)\overline{\chi(g_j)} = \delta_{ij}|C_G(g_i)|.$$

Proof Let X be the character table of G. Then viewed as a matrix, X is an $r \times r$ matrix whose (i, j)-th entry is given by $\chi_i(g_j)$. Let C_i be the conjugacy class which contains g_i and D be the diagonal matrix with entries $\delta_{ij}|C_i|$. Then by the first orthogonality relation, we obtain that

$$|G|\delta_{ij} = \sum_{g \in G} \chi(g_i)\overline{\chi(g_j)} = \sum_{t=1}^{r} |C_t|\chi_i(g_t)\overline{\chi_j(g_t)}.$$

Then we obtain a system of r^2 equations which can be written as a single matrix equation as follows

$$|G|I = XD\overline{X}^T,$$

where I is the identity matrix and $\bar{X}^T$ is the transpose of $\bar{X}$. Since X is a nonsingular matrix, then we obtain that

$$|G|I = D\overline{X}^T X.$$

Rewriting the above matrix system as a system of equations yields

$$|G|\delta_{ij} = \sum_{t=1}^{r} |C_i|\overline{\chi_t(g_i)}\chi_t(g_j).$$

Hence we obtain that

$$\sum_{\chi \in Irr(G)} \chi(g_i)\overline{\chi(g_j)} = |C_G(g_i)|\delta_{ij}. \quad \square$$

Theorem 2.1.14. [*68*] *Let $Irr(G) = \{\chi_1, \chi_2, ..., \chi_r\}$ and χ be any character of G. Then*

1. *χ can be expressed uniquely as $\chi = \sum_{i=1}^{r} a_i\chi_i$ where $a_i \in \mathbb{N} \cup \{0\}$.*

2. *If $\chi = \sum_{i=1}^{r} a_i\chi_i$ then $\langle \chi, \chi \rangle = \sum_{i=1}^{r} a_i^2$.*

3. *χ is irreducible if and only if $\langle \chi, \chi \rangle = 1$.*

Proof:

1. By Theorem 2.1.8(3), $\chi = \sum_{i=1}^{r} a_i\chi_i$ for $a_i \in \mathbb{N} \cup \{0\}$. For each i, $\langle \chi, \chi_i \rangle = \langle \sum_{i=1}^{r} a_i\chi_i, \chi \rangle = a_i \langle \chi_i, \chi_i \rangle = a_i$ by the First Orthogonality Relation, so the a_i are unique.
2. Follows from First Orthogonality Relation.
3. Follows from parts 1 and 2 .

Definition 2.1.15. Let G be a group, χ be a character of G and $Irr(G) = \{\chi_1, \chi_2, ..., \chi_r\}$ such that $\chi = \sum_{i=1}^{r} a_i\chi_i$, where $a_i \in \mathbb{N} \cup \{0\}$. Then those χ_i with $a_i \in \mathbb{N}$ are called the *irreducible constituents* of χ. We also say that χ contains a_i *copies* of the *irreducible character* χ_i. In general, if ϕ is a character of G such that $\chi - \phi$ is a character or is zero, then ϕ is a constituent of χ.

2.2 Normal Subgroups

Lemma 2.2.1. [68] *Let χ be a character of a group G afforded by a representation ρ. Then $g \in ker(\rho)$ if and only if $\chi(g) = \chi(1)$.*

Proof: Let $n = \chi(1)$, so n is the degree of ρ. If $g \in ker(\rho)$ then $\rho(g) = I_n = \rho(1)$, where I_n is the $n \times n$ identity matrix, so $\chi(g) = n = \chi(1)$. Conversely, assume $\chi(g) = \chi(1) = n$. By Lemma 2.1.9, $\chi(g) = \epsilon_1 + ... + \epsilon_n$ where each ϵ_i is a complex root of unity. Therefore, $\epsilon_1 + \epsilon_2 + ... + \epsilon_n = n$. But $|\epsilon_i| = 1$ for each i, so we must have $\epsilon_i = 1$ for each i. Hence $\rho(g)$ is similar to diag$(\epsilon_1, \epsilon_2, ..., \epsilon_n) = I_n$, so $g \in ker(\rho)$. $\square$

Definition 2.2.2. Let χ be a character of a group G. We define

$$ker(\chi) = \{g \in G : \chi(g) \ = \chi(1)\}.$$

From Lemma 2.2.1, it follows that $ker(\chi) = ker(\rho)$ and hence $ker(\chi)$ is a normal subgroup of G. If $Irr(G) = \{\chi_1, \chi_2, ..., \chi_r\}$, then every normal subgroup is the intersection of some of the $ker(\chi_i)$.

Theorem 2.2.3. *Let N be a normal subgroup of a group G. Then*

1. *If χ is a character of G and $N \subseteq ker(\chi)$, then χ is constant on the cosets of N in G and the function $\hat{\chi}$ defined on G/N by $\hat{\chi}(Ng) = \chi(g)$ is a character of G/N.*

2. *If $\hat{\chi}$ is a character of G/N, then the function χ defined by $\chi(g) = \hat{\chi}(Ng)$ is a character of G and the χ has the same degree as $\hat{\chi}$.*

3. *In both of the statements above , $\chi \in Irr(G)$ if and only if $\hat{\chi} \in Irr(G/N)$.*

Proof: See Theorem 2.2.2 of [68]

If N is a normal subgroup of G and ρ is a representation of G such that $N \subseteq ker(\rho)$, then there exists a unique representation $\hat{\rho}$ of G/N defined by $\hat{\rho}(Ng) = \rho(g)$. Thus knowing ρ, we can obtain $\hat{\rho}$ and vice versa. We also obtain that ρ is irreducible if and only if $\hat{\rho}$ is irreducible. Hence ρ and $\hat{\rho}$ can be identified. If ρ affords a character χ of G, then $\hat{\rho}$ affords a character $\hat{\chi}$ of G/N and hence χ and $\hat{\chi}$ can be identified. Under this identification, we obtain that

$$Irr(G/N) = \{\chi \in Irr(G) | N \subseteq ker(\chi)\}.$$

Therefore the irreducible characters of G/N are precisely those irreducible characters of G which contain N in their kernels.

Definition 2.2.4. Let N be a normal subgroup of G and let $\hat{\chi}$ be a character of G/N, then the character χ which is defined by

$$\chi(g) = \hat{\chi}(Ng) \ \text{ for } \ g \in G$$

is called the *lift* of $\hat{\chi}$ to G.

The process of obtaining characters of a group from the characters of any of its quotient groups using theorem 2.2.3 is called the *lifting process*.

2.3 Restriction of Characters

Definition 2.3.1. Let G be a group and H be a subgroup of G. If $\rho : G \longrightarrow GL_n(\mathbb{C})$ is a representation of G, then $(\rho \downarrow H)$:$H \longrightarrow GL_n(\mathbb{C})$ given by

$$(\rho \downarrow H)(h) = \rho(h), \quad \forall\, h \in H,$$

is a representation of H. We say that $\rho \downarrow H$ is the restriction of ρ to H. If χ is a character of G afforded by ρ, then we say $\chi \downarrow H$ is the restriction of χ to H and is a character of H. $\chi \downarrow H$ is a character of H afforded by the representation $\rho \downarrow H$ such that

$$\chi \downarrow H = \sum_{\psi \in Irr(H)} \delta_\psi \psi, \quad where \ \ \delta_\psi \in \mathbb{N} \cup \{0\}\,.$$

The characters $\chi \downarrow H$ and χ take on the same values on the elements of H. If $\chi \downarrow H$ is irreducible, then χ is irreducible in G but the converse is not true in general. Karpilovsky in [39] proves a theorem (Theorem 23.1.4) due to Gallagher(1966) that if $H \leq G$, $\chi \in Irr(G)$ such that $\chi(g) \neq 0$ for each $g \in G \setminus H$, then $\chi \downarrow H$ is irreducible and for any $g \in G \setminus H$, $\chi(g)$ is a root of unity. We also observe that (see [36]) every irreducible character of H is a constituent of some irreducible character of G restricted to H.

Theorem 2.3.2. [36] *Let* G *be a group,* $H \leq G$, $\chi \in Irr(G)$ *and* $Irr(H) = \{\psi_1, \psi_2, ..., \psi_r\}$. *Then*

$$\chi \downarrow H \;= \sum_{i=1}^{r} \delta_i \psi_i,$$

where $\delta_i \in \mathbb{N} \cup \{0\}$ *satisfy the following inequality*

$$\sum_{i=1}^{r} \delta_i^2 \;\leq [G{:}H] \qquad (**)$$

Moreover, we have equality in $(**)$ *if and only if* $\chi(g) = 0$*, for all* $g \in (G \setminus H)$.

Proof: We obtain that

$$\begin{aligned} \sum_{i=1}^{r} \delta_i^2 &= \frac{1}{|H|} \sum_{h \in H} \chi(h).\overline{\chi(h)} \quad \textit{so that} \\ |H| \sum_{i=1}^{r} \delta_i^2 &= \sum_{h \in H} \chi(h).\overline{\chi(h)} \qquad (***) \end{aligned}$$

Since χ is irreducible, then we have that

$$\begin{aligned} 1 = \langle \chi, \chi \rangle_G &= \frac{1}{|G|} \sum_{g \in G} \chi(g).\overline{\chi(g)} \\ &= \frac{1}{|G|} \sum_{g \in H} \chi(g).\overline{\chi(g)} + \frac{1}{|G|} \sum_{g \in (G \setminus H)} \chi(g).\overline{\chi(g)} \\ &= \frac{|H|}{|G|} \sum_{i=1}^{r} \delta_i^2 + \frac{1}{|G|} \sum_{g \in (G \setminus H)} \chi(g).\overline{\chi(g)} \quad \textit{by} \qquad (***) \\ &= \frac{|H|}{|G|} \sum_{i=1}^{r} \delta_i^2 + \frac{1}{|G|} \sum_{g \in (G \setminus H)} |\chi(g)|^2 \end{aligned}$$

and therefore

$$\frac{|H|}{|G|} \sum_{i=1}^{r} \delta_i^2 \;=\; 1 - \frac{1}{|G|} \sum_{g \in (G \setminus H)} |\chi(g)|^2 \;\leq\; 1$$

Hence

$$\sum_{i=1}^{r} \delta_i^2 \;\leq\; \frac{|G|}{|H|} = [G{:}H]$$

Equality in $(**)$ will be obtained if

$$\begin{aligned}\frac{1}{|G|}\sum_{g\in(G\setminus H)}|\chi(g)|^2 &= 0\\ \text{if and only if } |\chi(g)|^2 &= 0 \quad \forall\, g\in(G\setminus H)\\ \text{if and only if } \chi(g) &= 0 \quad \forall\, g\in(G\setminus H)\end{aligned}$$

Thus the equality in $(**)$ holds. □

Theorem 2.3.3. *Let G be a group, H be a normal subgroup of G and $\chi \in Irr(G)$. Then all the constituents of $\chi \downarrow H$ have the same degree.*

Proof: See Proposition 20.7 of [36]. □

We have seen how the irreducible characters of G can be used to find characters of a subgroup H and can now look at a technique of finding the characters of G from the irreducible characters of any subgroup H.

2.4 Induced Characters

Definition 2.4.1. Let H be a subgroup of G. The right transversal, $\{x_1, x_2, ..., x_r\}$, of H in G is a set of representatives for the right cosets of H in G.

Let H be a subgroup of a group G such that the set $\{x_1, x_2, ..., x_r\}$ is a transversal for H in G. Let ϕ be a representation of H of degree n. Then we define ϕ^* on G as follows :

$$\phi^*(g) = \begin{pmatrix} \phi(x_1gx_1^{-1}) & \phi(x_1gx_2^{-1}) & ... & ... & ... & \phi(x_1gx_r^{-1}) \\ \phi(x_2gx_1^{-1}) & \phi(x_2gx_2^{-1}) & ... & ... & ... & \phi(x_2gx_r^{-1}) \\ ... & ... & ... & ... & ... & ... \\ ... & ... & ... & ... & ... & ... \\ \phi(x_ngx_1^{-1}) & \phi(x_ngx_2^{-1}) & ... & ... & ... & \phi(x_ngx_r^{-1}) \end{pmatrix}$$

where $\phi(x_igx_j^{-1})$ are $n \times n$ submatrices of $\phi^*(g)$ satisfying the property that

$$\phi(x_igx_j^{-1}) = 0_{n\times n} \ \forall x_igx_j^{-1} \notin H$$

Then we can show $\phi^*(g)$ is a representation of G of degree nr.

Definition 2.4.2. Let G, H, ϕ and ϕ^* be as above. Then the representation ϕ^* is called the representation of G induced from the representation ϕ of H and we denote it by ϕ^G.

If ψ is a representation of H which is equivalent to ϕ, then it can be shown that ψ^G is equivalent to ϕ^G. Thus the induction process preserves equivalence between representations.

Definition 2.4.3. Let G be a group and $H \leq G$. Let χ be a class function of H. Then we define χ^G as follows:

$$\chi^G(g) = \frac{1}{|H|}\sum_{x\in G}\chi^o(xgx^{-1}), \ g \in G,$$

$$\text{where} \quad \chi^o(h) = \begin{cases} \chi(h) & \text{if } \ x \in H \\ 0 & \text{otherwise} \end{cases}$$

Then χ^G is a class function of G, called the induced class function of G induced from χ. Also we have that $deg(\chi^G) = [G{:}H]deg(\chi)$.

Theorem 2.4.4. *[33] (**Frobenius Reciprocity**) Let $H \leq G$, χ be a class function on H and ϕ a class fuction on G. Then*

$$\langle \ \chi \ , \ \phi \downarrow H\rangle_H = \langle \ \chi^G, \ \phi\rangle_G$$

Proof:

$$\begin{aligned}
\langle \ \chi^G, \ \phi\rangle_G &= \frac{1}{|G|}\sum_{g\in G}\chi^G(g).\overline{\phi(g)} \\
&= \frac{1}{|G|}\sum_{g\in G}\Big(\frac{1}{|H|}\sum_{x\in G}\chi^0(xgx^{-1})\Big).\overline{\phi(g)} \\
&= \frac{1}{|G|.|H|}\sum_{g\in G}\sum_{x\in G}\chi^0(xgx^{-1}).\overline{\phi(g)} \qquad (****)
\end{aligned}$$

Let $y = xgx^{-1}$. Then as g runs over G, xgx^{-1} runs through G. Also since ϕ is a class function on $G, \phi(y) = \phi(xgx^{-1}) = \phi(g)$. Thus by $(****)$ we have

$$\begin{aligned} \langle\, \chi^G,\ \phi\rangle_G &= \frac{1}{|G|.|H|} \sum_{y\in G} \sum_{x\in G} \chi^0(y).\overline{\phi(y)} \\ &= \frac{1}{|G|.|H|} \sum_{x\in G} \Big(\sum_{y\in G} \chi^0(y).\overline{\phi(y)} \Big) \\ &= \frac{1}{|G|.|H|}\, |G| \sum_{y\in G} \chi^0(y).\overline{\phi(y)} \\ &= \frac{1}{|H|} \sum_{y\in H} \chi(y).\overline{\phi(y)} \end{aligned}$$

$$= \langle\, \chi, \phi \downarrow H\rangle_H \quad \square$$

Let $H \leq G$ and ϕ be a representation of H that affords a character χ of H. Then χ^G is a character of G afforded by the induced representation ϕ^G of G. The character χ^G is called the *induced character* of G. The induction and restriction processes do not necessarily preserve irreducibility of characters.

Theorem 2.4.5. *Let G be a group and $H \leq G$. Let χ be a character of H, $g \in G$ and $\{x_1, x_2, ..., x_m\}$ be a set of representatives of conjugacy classes of elements of H which fuse into the conjugacy class $[g]$ in G. Then we have*

1. $\chi^G(g) = 0$ *if* $H \cap [g] = \emptyset$.
2. $\chi^G(g) = |C_G(g)| \sum_{i=1}^{m} \frac{\chi(x_i)}{|C_H(x_i)|}$ *if* $H \cap [g] \neq \emptyset$.

Proof: We obtain that

$$\chi^G(g) = \frac{1}{|H|} \sum_{x\in G} \chi^0(xgx^{-1}).$$

If $H \cap [g] = \emptyset$, then $xgx^{-1} \notin H$ and thus $\chi^0(xgx^{-1}) = 0 \quad \forall\, x \in G$ and hence $\chi^G(g) = 0$. Now if $H \cap [g] \neq \emptyset$, then as x runs over G, $xgx^{-1} = y$ covers $[g]$ exactly $|C_G(g)|$ times, thus we have

$$
\begin{aligned}
\chi^G(g) &= \frac{1}{|H|} \times |C_G(g)| \sum_{y \in [g]} \chi^0(y) \\
&= \frac{1}{|H|} \times |C_G(g)| \sum_{y \in ([g] \cap H)} \chi(y) \\
&= \frac{|C_G(g)|}{|H|} \times \sum_{i=1}^{m} [H : C_H(x_i)].\chi(x_i) \\
&= |C_G(g)| \sum_{i=1}^{m} \frac{\chi(x_i)}{|C_H(x_i)|}.
\end{aligned}
$$

Hence the result. $\square$

Theorem 2.4.6. *Let G be a group, K and H subgroups of G such that $K \leq H \leq G$ and χ be a character of K. Then for all $g \in G$ we have*

1. $(\chi^H)^g = (\chi^g)^{g^{-1}Hg}$
2. $(\chi^g)^G = (\chi^G)$

Proof: See[39]

Readers are referred to [12], [14] [35]and [54] for further reading on induced characters.

2.5 Permutation Characters

Let G be a finite group throughout.

Definition 2.5.1. G acts on a finite set Ω if for each $g \in G$ and $\alpha \in \Omega$, there is an element α^g in Ω such that $\alpha^1 = \alpha$ and $(\alpha^g)^h = \alpha^{gh}$ for all $\alpha \in \Omega$ and $g, h \in G$.

Proposition 2.5.2. *G acts on Ω $\Leftrightarrow$ there is a homomorphism $\rho : G \to S_\Omega$, where S_Ω is the symmetric group on Ω.*

Proof: Suppose G acts on Ω. Define a mapping

$\rho : G \to S_\Omega$ by $\rho(g) : \alpha \mapsto \alpha^g \;\; \forall \alpha \in \Omega, g \in G$. If $\alpha, \beta \in \Omega$ then $(\alpha)\rho(g) = (\beta)\rho(g)$ implies that $\alpha^g = \beta^g$ and $\alpha = \alpha^1 = \alpha^{gg^{-1}} = (\alpha^g)^{g^{-1}} = (\beta^g)^{g^{-1}} = \beta^{gg^{-1}} = \beta^1 = \beta$. Also if $\beta \in \Omega$ then $(\beta)^{g^{-1}} \in \Omega$ and $(\beta)^{g^{-1}}\rho(g) = (\beta^{g^{-1}})^g = \beta^{g^{-1}g} = \beta^1 = \beta$.

Therefore $\rho(g) \in S_\Omega$.

Furthermore,

$(\alpha)\rho(gh) = \alpha^{gh} = (\alpha^g)^h = (\alpha^g)\rho(h) = (\alpha\rho(g))\rho(h) = (\alpha)(\rho(g)\rho(h))$. This proves that $\forall \alpha \in \Omega : \rho(gh) = \rho(g)\rho(h)$; so ρ is the required homomorphism from G to S_Ω.

Conversely, suppose $\rho : G \to S_\Omega$ is a homomorphism. Then $\rho(1) = 1_{S_\Omega}$ and $\rho(gh) = \rho(g)\rho(h)$; so

1. $(\alpha)\rho(1) = \alpha 1_{S_\Omega} = \alpha$ and

2. $(\alpha)\rho(gh) = (\alpha)(\rho(g)\rho(h)) = ((\alpha)\rho(g))\rho(h) \ \ \forall \alpha \in \Omega$.

Define an action of G on Ω by $\alpha^g = \alpha(\rho(g)) \ \ \forall \alpha \in \Omega, g \in G$.

Then $\alpha = \alpha^1$ and $\alpha^{gh} = (\alpha^g)^h$ by (1) and (2). This shows that G acts on Ω. $\square$

If G acts on Ω then this defines an equivalence relation on Ω as follows: If $\alpha, \beta \in \Omega$ then $\alpha \sim \beta \Leftrightarrow \exists\ g \in G$ such that $\alpha = \beta^g$. The relation $\sim$ on Ω is

(i) reflexive: For $\alpha = \alpha^1$ implies $\alpha \sim \alpha$

(ii) symmetric: If $\alpha \sim \beta$ then $\exists\ g \in G$ such that $\alpha = \beta^g$. Then $\beta = \beta^1 = \beta^{gg^{-1}} = (\beta^g)^{g^{-1}} = \alpha^{g^{-1}}$ so $\beta \sim \alpha$.

(iii) transitive: If $\alpha \sim \beta$ and $\beta \sim \gamma$ then $\alpha = \beta^g$ and $\beta = \gamma^h$ for some $g, h \in G$. Therefore, $\alpha = (\gamma^h)^g = \gamma^{hg}$ hence $\alpha \sim \gamma$.

Denote by $[\alpha]$ the equivalence class containing $\alpha \in \Omega$. Then

$$\begin{aligned} [\alpha] &= \{\beta \in \Omega | \beta \sim \alpha\} \\ &= \{\beta \in \Omega | \beta = \alpha^g \text{ for some g} \in \text{G}\} \\ &= \{\alpha^g | g \in G\} \\ &= \alpha^G \end{aligned}$$

α^G is called the *orbit of* G *on* Ω containing α. The equivalence relation $\sim$ determines a partition of Ω by these equivalence classes, that is, $\Omega = \cup_{\alpha \in \Omega}[\alpha]$ and $[\alpha] \cap [\beta] = \emptyset$ if $\alpha \neq \beta$.

Let $G_\alpha = \{g \in G | \alpha^g = \alpha\}$. Then $G_\alpha \leq G$. Since $\alpha = \alpha^1$ we have $1 \in G_\alpha$. Also if $g, h \in G_\alpha$, then $\alpha^{gh} = (\alpha^g)^h = \alpha^h = \alpha$ and $\alpha = \alpha^1 = \alpha^{gg^{-1}} = (\alpha^g)^{g^{-1}} = \alpha^{g^{-1}}$, which shows that $gh \in G_\alpha, g^{-1} \in G_\alpha$.
Hence, G_α is a subgroup of G, called the stabilizer of α in G. Let $T = \{G_\alpha g | g \in G\}$ be the set of right cosets of G_α in G. We show next that there is a one-to-one correspondence between T and the orbit α^G of G on Ω containing α.

Lemma 2.5.3. $G_\alpha g = G_\alpha h \Leftrightarrow \alpha^g = \alpha^h$.

Proof:

$$\begin{aligned} G_\alpha g = G_\alpha h &\Leftrightarrow gh^{-1} \in G_\alpha \\ &\Leftrightarrow \alpha^{gh^{-1}} = \alpha \\ &\Leftrightarrow (\alpha^g)^{h^{-1}} = \alpha \\ &\Leftrightarrow \alpha^g = \alpha^h \quad \square \end{aligned}$$

Define a mapping $\Phi : T \to \alpha^G$ by $G_\alpha g \mapsto \alpha^g$. If $G_\alpha g = G_\alpha h$ then $\alpha^g = \alpha^h$ and so $\Phi(G_\alpha g) = \Phi(G_\alpha h)$, which shows that Φ is well-defined, by lemma 2.5.3. If $G_\alpha g, G_\alpha h \in T$ with $\Phi(G_\alpha g) = \Phi(G_\alpha h)$ then $\alpha^g = \alpha^h$ and so $G_\alpha g = G_\alpha h$, by lemma 2.5.3. Hence Φ is injective.

If $x \in \alpha^G$ then $x = \alpha^h$ for some $h \in G$. So $\Phi(G_\alpha h) = \alpha^h = x$. This shows that Φ is also surjective. Thus Φ is the required one-to-one correspondence between T and α^G.
Therefore $|\alpha^G| = |T| = [G{:}G_\alpha] = \frac{|G|}{|G_\alpha|}$ or $|\alpha^G||G_\alpha| = |G|$.

Corollary 2.5.4. *The length of any orbit of G on Ω divides the order of G.*

Proof: $|\alpha^G||G_\alpha| = |G|$ and therefore $|\alpha^G|$ divides $|G|$. $\square$

If G acts on Ω, this action defines a representation of G as follows:

Let $\Omega = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ and for each $g \in G$ define the $n \times n$ matrix π_g by $\pi_g = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1, & \text{if } \alpha_i^g = \alpha_j \\ 0, & \text{if otherwise} \end{cases}$$

Remark 2.5.5. *$a_{ii} = 1$ if $\alpha_i^g = \alpha_i$ and so a point $\alpha_i \in \Omega$ is fixed by $g \Leftrightarrow$ the entry in the (i, i) position on the main diagonal of the matrix (a_{ij}) is 1.*

Then $\pi : G \to \mathrm{GL}(n, \mathbb{C})$ defined by $\pi : g \mapsto \pi_g = (a_{ij})_{n \times n}$ is a representation of G.

For $g, h \in G$, let $\pi_g = (a_{ij})$ and $\pi_h = (b_{ij})$. Then $\pi_g\pi_h = (a_{ij})(b_{ij}) = (c_{ij})$ and so

$$\begin{aligned} c_{ij} = 1 \quad &\Leftrightarrow \quad a_{ik}b_{kj} = 1 \ \text{ for some } \ 1 \leq \mathrm{k} \leq \mathrm{n} \\ &\Leftrightarrow \quad a_{ik} = 1 \ \text{ and } \ \mathrm{b_{kj}} = 1 \\ &\Leftrightarrow \quad \alpha_i^g = \alpha_k \ \text{ and } \ \alpha_{\mathrm{k}}^{\mathrm{h}} = \alpha_{\mathrm{j}} \\ &\Leftrightarrow \quad \alpha_i^{gh} = (\alpha_i^g)^h = \alpha_k^h = \alpha_j \end{aligned}$$

Hence $\pi_{gh} = (c_{ij}) = \pi_g\pi_h$ and so $\pi(gh) = \pi(g)\pi(h)$. Therefore $\pi(g) = \pi_g$ is a representation of G.

Denote by χ_π the character, called the *permutation character*, afforded by the representation $\pi{:}G \to \mathrm{GL}(n, \mathbb{C})$ then $\chi_\pi(g) = tr(\pi(g)) = tr(\pi_g) = |\{\alpha \in \Omega | \alpha^g = \alpha\}| =$ number of points of Ω fixed by g, by the above remark. Therefore, $\chi_\pi(1) =$ degree of the permutation character $= |\Omega|$.

Definition 2.5.6. The action of G on Ω is said to be *transitive* if G has only one orbit on Ω i.e. if $\alpha \in \Omega$ then $\Omega = \alpha^G$

Example 2.5.7. Let H be a subgroup of G ($H \leq G$) and let $\Omega = \{Ha | a \in G\}=$ set of right cosets of H in G. Define $(Ha)^g := Hag$. Then $(Ha)^1 = Ha.1 = Ha$ and

$(Ha)^{gh} = Ha(gh) = H(ag)h = (Hag)^h = ((Ha)^g)^h$
Therefore $(Ha)^g = Hag$ defines an action of G on Ω.
Since $(Ha)^G = \{(Ha)^g | g \in G\} = \{Hag | g \in G\} = \Omega$, it follows that the action of G on Ω has only one orbit and so the action is transitive i.e. G acts *transitively* on Ω.

Let $\Omega = \{Ha_1, Ha_2, \cdots, Ha_r\}$ where $\{a_1, a_2, \cdots, a_r\}$ is a transversal for H in G.
This action of G on Ω gives rise to a permutation character χ_π of degree $|\Omega| = [G{:}H] =$ index of H in G. In fact $\chi_\pi = (1_H)^G =$ trivial character 1_H of H induced to G. $\chi_\pi(g) =$ number of points of $\Omega = \{Ha_1, Ha_2, \cdots, Ha_r\}$ fixed by g. Now $(Ha_i)^g = Ha_i \Leftrightarrow Ha_ig = Ha_i \Leftrightarrow a_iga_i^{-1} \in H$. In other words, Ha_i is fixed by $g \Leftrightarrow a_iga_i^{-1} \in H$. Therefore

$$\chi_\pi(g) = \sum_{i=1}^{r} \Phi^0(a_iga_i^{-1}) \ ,$$

where

$$\Phi^0(y) = \begin{cases} 1, & \text{if } y \in H \\ 0, & \text{if } y \notin H, \end{cases}$$

Thus $\chi_\pi = (1_H)^G$
Conversely, if G acts transitively on any set, then the associated permutation character is induced from the trivial character of some subgroup of G, according to the following theorem.

Theorem 2.5.8. *Let G act transitively on Ω. Let $\alpha \in \Omega$ and let $H = G_\alpha$. Then $(1_H)^G$ is the permutation character of the action, where 1_H is the trivial character of H.*

Proof: $\Omega = \alpha^G$ since G acts transitively on Ω. There is a one-to-one correspondence between Ω and $T = \{G_\alpha g | g \in G\}$ given by $\alpha^g \mapsto G_\alpha g$ for $g \in G$.
Let $g \in G$. Then $(\alpha^k)^g = \alpha^k \Leftrightarrow \alpha^{kg} = \alpha^k \Leftrightarrow \alpha^{kgk^{-1}} = \alpha \Leftrightarrow kgk^{-1} \in G_\alpha = H \Leftrightarrow kg \in Hk \Leftrightarrow Hkg = Hk \Leftrightarrow (Hk)^g = Hk$, where G acts on the right cosets of H as in the example 2.5.7.
Therefore the permutation character of the action of G on Ω is the same as the permutation character of the action of G on the right cosets of H in G, which is $(1_H)^G$. $\square$

Corollary 2.5.9. *If G acts on Ω with permutation character χ and has k orbits on Ω, then $< \chi, 1_G >= k$.*

Proof: Write $\Omega = \bigcup_{i=1}^{k} \theta_i$, where θ_i are the orbits of G on Ω. Let χ_i be the permutation character of G on θ_i, so $\chi = \sum_{i=1}^{k} \chi_i$ (this is because $\chi_i(g)$ = number of points in θ_i fixed by g and so $\chi(g) = \sum_{i=1}^{k} \chi_i(g)$ = number of points in Ω fixed by g).

For $\alpha_i \in \theta_i, \chi_i = (1_{G_{\alpha_i}})^G$, by theorem 2.5.8, so

$$\begin{aligned} < \chi_i, 1_G > &= < (1_{G_{\alpha_i}})^G, 1_G >_G \\ &= < 1_{G_{\alpha_i}}, 1_G \downarrow G_{\alpha_i} >_{G_{\alpha_i}}, \text{ by Frobenius Reciprocity} \\ &= < 1_{G_{\alpha_i}}, 1_{G_{\alpha_i}} >_{G_{\alpha_i}} = 1 \end{aligned}$$

Therefore,

$$\begin{aligned} < \chi, 1_G > &= < \sum_{i=1}^{k} \chi_i, 1_G > \\ &= \sum_{i=1}^{k} < \chi_i, 1_G > = k \quad \square \end{aligned}$$

Every subgroup of G gives rise to a permutation character, as shown by the previous results.
Conversely, we can show the existence of a subgroup H if we can identify the character $(1_H)^G$. Because this character is a transitive permutation character, it must satisfy certain necessary conditions. We give these conditions in theorem 2.5.11 but first prove a lemma.

Lemma 2.5.10. *If G acts transitively on Ω, then all subgroups G_α of G (for $\alpha \in \Omega$) are conjugate in G.*

Proof: Let $\alpha, \beta \in \Omega$. Then $\beta = \alpha^h$ for some $h \in G$.

Now

$$\begin{aligned} g \in G_\alpha &\Leftrightarrow \alpha^g = \alpha \\ &\Leftrightarrow \beta^{h^{-1}g} = \beta^{h^{-1}} \\ &\Leftrightarrow \beta^{h^{-1}gh} = \beta \\ &\Leftrightarrow h^{-1}gh \in G_\beta \\ &\Leftrightarrow g \in hG_\beta h^{-1}\ , \\ \text{so} \quad G_\alpha &= hG_\beta h^{-1} \text{ as required} \quad \square \end{aligned}$$

Theorem 2.5.11. *Let $H \leq G$ and $\chi = (1_H)^G$ then*

1. $\chi(1)$ divides $|G|$
2. $< \chi, \psi > \leq \psi(1)$ for all $\psi \in Irr(G)$
3. $< \chi, 1_G >= 1$
4. $\chi(g)$ is a nonnegative integer for all $g \in G$
5. $\chi(g) \leq \chi(g^m)$ for all $g \in G$ and m a nonnegative integer
6. $\chi(g) = 0$ if order of g does not divide $\frac{|G|}{\chi(1)}$
7. $\chi(g)\frac{|[g]|}{\chi(1)}$ is an integer for all $g \in G$

Proof:

1. $\chi(1) = [G{:}H]$ and $[G{:}H]||G|$
2. $< \chi, \psi > \; = \; < (1_H)^G, \psi >_G \; = \; < 1_H, \psi \downarrow H >_H \; \leq \psi(1)$, by Frobenius Reciprocity Theorem.
3. G acts transitively on the set of all right cosets of H in G, by $(Ha)^g = Hag$. This action gives rise to the permutation character $\chi = (1_H)^G$ with $deg\chi = [G{:}H]$. Therefore $< \chi, 1_G >= 1$ by corollary 2.5.9.

4. $\chi(g)$ is the number of points of Ω fixed by g, so must be a nonnegative integer.

5. For any $\alpha_i \in \Omega, \alpha_i^g = \alpha_i \Rightarrow \alpha_i^{g^m} = \alpha_i$ and so $\chi(g) \leq \chi(g^m)$ i.e the number of points of Ω fixed by g cannot exceed the number of points fixed by g^m.

6. If $ord(g)$ does not divide $\frac{|G|}{\chi(1)} = |H| \quad (\chi(1) = [G{:}H] = \frac{|G|}{|H|})$ then no conjugate of g lies in H).

   Therefore,

$$\begin{aligned}\chi(g) &= (1_H)^G(g) \\ &= \sum_{i=1}^{r}(1_H)^0(x_i g x_i^{-1}) \\ &= 0\end{aligned}$$

7. Let $\mathcal{S} = \{(\alpha, x)|\alpha \in \Omega, x \in [g], \alpha^x = \alpha\}$. If $x \in [g]$ then $\chi(x) =$ number of points in Ω fixed by x. Therefore $|\mathcal{S}| = \sum_{x\in[g]} \chi(x)$.

   But $\chi(x) = \chi(y)$ if x $\sim$ y (i.e. if $x, y \in [g]$) and so $|\mathcal{S}| = |[g]|\chi(g)$. However $|\mathcal{S}| = \sum_{\alpha\in\Omega} |[g] \cap G_\alpha|$ and for any $\alpha, \beta \in \Omega$, define $\Phi_h : [g] \cap G_\alpha \to [g] \cap G_\beta$ by $x \mapsto hxh^{-1}$ where $G_\beta = hG_\alpha h^{-1}$. Note: since $x \in [g]$, $x = kgk^{-1}$ for some $k \in \overline{G}$ and so $hxh^{-1} = hkgk^{-1}h^{-1} = hkg(hk)^{-1} \in [g] \cap (hG_\alpha h^{-1})$. Then Φ_h is a bijection and so for any $\alpha, \beta \in \Omega$, $|[g] \cap G_\alpha| = |[g] \cap G_\beta| = m$, say. Hence $|\mathcal{S}| = |\Omega|m = |[g]|\chi(g)$. Therefore, $m = \frac{\chi(g)|[g]|}{|\Omega|} = \frac{\chi(g)|[g]|}{\chi(1)}$ □

Theorem 2.5.12. *Let $H \leq G$ with $\chi = (1_H)^G$. Let $g \in G$ and let $x_1, x_2, ..., x_m$ be representatives of the conjugacy classes of H that fuse to $[g]$. Then*

$$\chi(g) = \sum_{i=1}^{m} \frac{|C_G(g)|}{|C_H(x_i)|} \ .$$

(If $H \cap [g] = \emptyset$, then $\chi(g) = 0$)

Proof: This follows from Theorem 2.4.5. □

Chapter 3

Conjugacy Classes of Group Extensions

In this chapter we describe a method that can be used to determine the conjugacy classes of group extensions. We first restrict ourselves to split extensions of abelian groups. These methods were used by Moori ([44],[45]), Salleh [64], Whitley [68], Mpono [53] and Ali [1]to determine the conjugacy classes of extensions of elementary abelian groups.

3.1 Extensions of Groups

Definition 3.1.1. If N and G are groups, an *extension* of N by G is a group $\overline{G}$ such that

(i) $N \trianglelefteq \overline{G}$

(ii) $\overline{G}/N \cong G$

We denote the fact that $\overline{G}$ is an extension of N by G by $\overline{G} = N.G$.

Definition 3.1.2. If $\overline{G}$ is an extension of N by G such that there exists a subgroup G_1 of $\overline{G}$ satisfying

(i) $G_1 \cong G$

(ii) $NG_1 = \overline{G}$

(iii) $N \cap G_1 = \{1\}$,

then we say that $\overline{G}$ is a split extension and we denote this fact by $\overline{G} = N{:}G$. If an extension is not split , then it is called non-split and is denoted by $N \cdot G$.

Remark 3.1.3. If $\overline{G}$ is a split extension of N by G , then $\overline{G}$ is also called a semi-direct product of N by G and we identify G and G_1.

3.2 Conjugacy classes of semi-direct products.

Let $\overline{G}$ be a *semi-direct product* of N by G where N is abelian . Then every $y \in \overline{G}$ has a unique expression of the form $y = ng$, $n \in N$, $g \in G$. For suppose $y = n_1\, g_1 = n_2\, g_2$, then $n_2^{-1}\, n_1 = g_2\, g_1^{-1}$; hence $g_2\, g_1^{-1}$, $n_2^{-1}\, n_1 \in N \cap G = \{1\}$ and so $n_1 = n_2$ and $g_1 = g_2$. It is clear that $\bigcup_{g \in G} Ng \subseteq \overline{G}$. For any $\overline{g} \in \overline{G}$, $\overline{g} = ng$ for some $n \in N$, $g \in G$.

Hence $\overline{g} \in Ng$ for some $g \in G$ and so $\overline{G} \subseteq \bigcup_{g \in G} Ng$. Therefore $\overline{G} = \bigcup_{g \in G} Ng$ and furthermore, suppose $z \in Ng \cap Ny$ for any two elements $g, y \in G$. Then $z = n_1\, g = n_2\, y$ and so $n_1 = n_2$ and $g = y$, that is, $g \neq y$ implies that $Ng \cap Ny = \emptyset$ and so G can be regarded as a *right transversal* for N in $\overline{G}$. To determine the conjugacy classes of $\overline{G}$, we analyze the cosets Ng for each conjugacy class of G with representative g, and corresponding classes of $\overline{G}$ are determined by the action (by conjugation) of C_g, the set stabilizer in $\overline{G}$ of Ng. For $g \in G$, define $C_g = \{y \in \overline{G} \,|\, (Ng)^y = Ng\} = \{y \in \overline{G} \,|\, y(Ng) = (Ng)y\} = C_{\overline{G}}(Ng)$. Now C_g is a subgroup of $\overline{G}$ because $C_{\overline{G}}(H)$ is a subgroup of $\overline{G}$ for any subset H of $\overline{G}$.

We now prove (i) $N \trianglelefteq C_g$ **and** (ii) $C_g = N{:}C_G(g)$

(i) $\forall\, n \in N$, $n_1 g \in Ng$,

$(n_1 g)^n = n(n_1 g)n^{-1} = nn_1 gn^{-1}g^{-1}g \in Ng$, which implies that $(Ng)^n = Ng$ and thus $N \subseteq C_g$ and so $N \leq C_g$. Since $N \trianglelefteq \overline{G}$, it follows that $N \trianglelefteq C_g$.

(ii) If $y \in C_G(g)$ then $y(Ng) = yNy^{-1}yg = Nyg = Ngy = (Ng)y$; so $C_G(g) \leq C_g$ and since $N \cap G = \{1\}$, it follows that $N \cap C_G(g) = \{1\}$.

Since $N \trianglelefteq C_g$ and $C_G(g) \leq C_g$, it follows that $NC_G(g) \leq C_g$. For any $y \in C_g$, $(Ng)^y = Ng$

$\Leftrightarrow Ng^y = Ng \Leftrightarrow g^y g^{-1} \in N$. Now $y = n'q'$ for some $n' \in N$, $q' \in G$ and so

$$\begin{aligned} g^y g^{-1} &= n'q'g(n'q')^{-1}g^{-1} \\ &= n'q'gq'^{-1}n'^{-1}g^{-1} \in N \Leftrightarrow q' \in C_G(g) \end{aligned}$$

For if $q' \in C_G(g)$ then $n'q'gq'^{-1}n'^{-1}g^{-1} = n'gq'q'^{-1}n'^{-1}g^{-1} = n'gn'^{-1}g^{-1} \in N$. Conversely if $n'q'gq'^{-1}n'^{-1}g^{-1} \in N$ then $q'gq'^{-1}g^{-1}gn'^{-1}g^{-1} \in N$. Therefore $q'gq'^{-1}g^{-1} \in N \cap G = \{1\}$ and so $q'g = gq'$ which implies $q' \in C_G(g)$. This proves that $y = n'q' \in C_g \Leftrightarrow q' \in C_G(g)$. Therefore $C_g \leq NC_G(g)$ and so $C_g = NC_G(g)$; that is, $C_g = N{:}C_G(g)$, a semi-direct product of N by $C_G(g)$. $\square$

Now $C_g = \{y \in \overline{G} \,|\, (Ng)^y = Ng\}$ and so C_g acts on Ng by conjugation. It follows that N and $C_G(g)$ act on Ng. We consider the *action of N and $C_G(g)$ on Ng* separately.

1. **Action of N on Ng by conjugation**

   Let $z \in Ng$, then under the action of N on Ng, $N_z = \{n \in N \,|\, nz = zn\} = C_N(z)$, the stabilizer of z in N. Now $C_N(g) = C_N(z)$, for if $n \in C_N(g)$ then $nz = n(n'g)$ for some $n' \in N$; hence $nz = n'(ng)$, since N is abelian. But then $nz = n'(gn) = (n'g)n = zn$, because $n \in C_N(g)$, and this proves that $n \in C_N(z)$, so $C_N(g) \subseteq C_N(z)$. Conversely, if $n \in C_N(z)$, then $ng = nn'^{-1}n'g = nn'^{-1}z = n'^{-1}nz$, since N is abelian, so $ng = n'^{-1}zn$, because $n \in C_N(z)$. This shows that $ng = n'^{-1}n'gn = gn$ and $n \in C_N(g)$. Therefore $C_N(z) \subseteq C_N(g)$; so we have equality $C_N(z) = C_N(g)$ and $C_N(g) \leq N$.

   Now $z^N = \{z^n \,|\, n \in N\} = \{nzn^{-1} \,|\, n \in N\}$ and under the action of N on Ng, Ng

splits into orbits $Q_{z_1}, \cdots, Q_{z_k}$ where $|Q_{z_i}| = \dfrac{|N|}{|C_N(z_i)|} = \dfrac{|N|}{|C_N(g)|} = [N{:}C_N(g)]$. Let $|C_N(g)| = k$. Then as N acts on Ng, Ng splits into k orbits $Q_{z_1}, Q_{z_2}, \cdots, Q_{z_k}$, that is, $Ng = \dot{\bigcup\limits_{i=1}^{k}} Q_{z_i}$ and for each i, $1 \leq i \leq l$, $|Q_{z_i}| = [N{:}C_N(g)] = \dfrac{|Ng|}{k}$, $1 \leq i \leq k$. Equivalently, $|Ng| = |N| = k\,|Q_{z_i}|$ for $1 \leq i \leq k$.

2. **Action of $C_G(g)$ on Ng**

From (1) above, the elements of Ng are in the orbits $Q_{z_1}, \cdots, Q_{z_k}$; thus we act $C_G(g)$ on these orbits. Suppose under this action that f of the orbits $Q_{z_1}, \cdots, Q_{z_k}$ fuse together to form an orbit $\triangle_f$ of $C_G(g)$.

Then $|\triangle_f| = f\frac{|N|}{k}$. Let $y \in \triangle_f$, then the stabilizer in C_g of y is

$$(C_g)_y = \{\bar{g} \in C_g \,|\, \bar{g}y = y\bar{g}\} = C_{\bar{G}}(y).$$

Therefore $|\triangle_f| = \dfrac{|C_g|}{|C_{\overline{G}}(y)|} = \dfrac{|N|\,|C_G(x)|}{|C_{\overline{G}}(y)|}$ and hence

$$\begin{aligned}
|C_{\overline{G}}(y)| &= \frac{|N|\,|C_G(g)|}{|\triangle_f|} \\
&= \frac{k\,|N|\,|C_G(g)|}{k|\triangle_f|} \\
&= \frac{k|N||C_G(g)|}{f|N|} \\
&= \frac{k}{f}|C_G(g)|
\end{aligned}$$

Thus to compute the conjugacy classes of $\overline{G} = N{:}G$ we just need to find the values of k and f for each conjugacy class representative g of G.

3.3 Conjugacy classes of group extensions (not necessarily split)

Proposition 3.3.1. *Suppose that $\overline{G}$ is any extension of N by G, not necessarily split. There is an onto homomorphism $\lambda : \overline{G} \longrightarrow G$ with kernel N.*

Proof: Define $\lambda : \overline{G} \longrightarrow G$ by $\lambda = \phi \circ \gamma$, where $\lambda : \overline{G} \xrightarrow{\gamma} \overline{G}/N \xrightarrow{\phi} G$ where $\overline{G}/N \overset{\phi}{\cong} G$. $\forall \overline{g_1}, \overline{g_2} \in \overline{G}$

$\lambda(\overline{g_1}\,\overline{g_2}) = (\phi \circ \gamma)(\overline{g_1}\,\overline{g_2}) = \phi(\gamma)(\overline{g_1}\,\overline{g_2}) = \phi(N\overline{g_1}\,\overline{g_2}) = \phi(N\overline{g_1}N\overline{g_2}) = \phi(N\overline{g_1})\phi(N\overline{g_2})$.

On the other hand,

$\lambda(\overline{g_1})\lambda(\overline{g_2}) = (\phi\circ\gamma)(\overline{g_1})(\phi\circ\gamma)(\overline{g_2}) = \phi(\gamma(\overline{g_1}))\phi(\gamma(\overline{g_2})) = \phi(N\overline{g_1})\phi(N\overline{g_2})$. Therefore $\lambda(\overline{g_1}\,\overline{g_2}) = \lambda(\overline{g_1})\lambda(\overline{g_2})$; hence λ is a homomorphism.

Let $x \in G$. Then $\phi(N\overline{g}) = x$ for some $\overline{g} \in \overline{G}$, since ϕ is surjective. But then $\gamma(\overline{g}) = N\overline{g}$.

Hence $\lambda(\overline{g}) = (\phi \circ \gamma)(\overline{g}) = \phi(\gamma(\overline{g})) = \phi(N\overline{g}) = x$; therefore λ is surjective.

Furthermore,

$$\begin{aligned}
\overline{x} \in Ker\lambda &\iff \lambda(\overline{x}) = 1_G \\
&\iff (\phi \circ \gamma)(\overline{x}) = 1_G \\
&\iff \phi(\gamma(\overline{x})) = 1_G \\
&\iff \phi(N\overline{x}) = 1_G \\
&\iff N\overline{x} \in Ker\phi = N \\
&\iff N\overline{x} = N \\
&\iff \overline{x} \in N
\end{aligned}$$

Therefore $Ker\lambda = N$

For $g \in G$ define a *lifting of* g to be an element $\bar{g} \in \overline{G}$ such that $\lambda(\bar{g}) = g$. Choosing a lifting of each element of G, we get the set $\{\bar{g}{:}g \in G\}$ which is a transversal for N in $\overline{G}$ (that is, a complete set of right coset representatives of N in $\overline{G}$).

For, let $\overline{x}$ be an arbitrary element of $\overline{G}$. Then $\lambda(\overline{x}) = y$ for some $y \in G$ consider the lifting of $\overline{y}$ of y in $\overline{G}$ then $\lambda(\overline{y}) = y = \lambda(\overline{x})$. Hence $\lambda(\overline{x})\lambda(\overline{y})^{-1} = 1_G$ so .
$\lambda(\overline{x}\,\overline{y}^{-1}) = \lambda(\overline{x})\lambda(\overline{y}^{-1}) = \lambda(\overline{x})\lambda(\overline{y})^{-1} = 1_G$

Therefore $\overline{x}\ \overline{y}^{-1} \in Ker\lambda = N$. Hence $\overline{x} \in N\overline{y}$. This shows $\overline{G} = \bigcup_{g\in G} N\overline{g}$. If $\overline{x} \in N\overline{g} \bigcap N\overline{g'}$ then $\overline{x} = n_1\overline{g} = n_2\overline{g'}$ for some $n_1, n_2 \in N$. Hence $\overline{g}\ \overline{g}^{-1} = n_1^{-1}n_2 \in N$. Thus $\overline{g} \in N\overline{g'}$ and so $N\overline{g} = N\overline{g'}$. This proves that $\{\bar{g}{:}g \in G\}$ is a traversal for N in $\overline{G}$.

Lemma 3.3.2. *For any set of liftings $\{\bar{g} : g \in G\}$, the map $\phi : G \longrightarrow \overline{G}/N$ defined by $\phi(g) = N\bar{g}\ \forall\, g \in G$ is an isomorphism and ϕ is independent of the choice of liftings.*

Proof: If $g = g'$ then $\lambda(\overline{g}) = g = g' = \lambda(\overline{g'})$. Hence $\lambda(\overline{g}\overline{g'}^{-1}) = 1_G$ and so $\overline{g}\overline{g'} \in N$ and so $\overline{g} \in N\overline{g'}$. Therefore $N\overline{g} = N\overline{g'}$ but $\phi(g) = N\overline{g} = n\overline{g'} = \phi(g')$. This shows that ϕ is well defined. Conversely, suppose $\phi(g) = \phi(g')$ for any $g, g' \in G$. Then $N\overline{g} = N\overline{g'}$, thus $\overline{g}\overline{g'}^{-1} \in N$. Hence $\lambda(\overline{g}\overline{g'}^{-1}) = 1_G$. Therefore $g = \lambda(\overline{g}) = \lambda(\overline{g'}) = g'$ and so λ is injective. If $X \in \overline{G}/N$ then $X = N\overline{y}$ for some $\overline{y} \in \overline{G}$. But $\lambda(\overline{y}) = x$ for some $x \in G$. Hence $\lambda(\overline{y}) = x = \lambda(\overline{x})$, so $\overline{y}\ \overline{x}^{-1} \in N$. Therefore $\overline{y} \in N\overline{x}$ and so $N\overline{y} = N\overline{x}$ for some $\overline{x} \in \overline{G}$. Hence $\phi(x) = N\overline{x} = N\overline{y} = X$, which shows that ϕ is surjective.

For any $g, g' \in G, \lambda(\overline{g}) = g$ and $\lambda(\overline{g'}) = g'$. Then $\lambda(\overline{gg'}) = gg' = \lambda(\overline{g})\lambda(\overline{g'}) = \lambda(\overline{g}\ \overline{g'}) \Rightarrow \overline{g}\overline{g'} \in N\overline{gg'} \Rightarrow N\overline{g}\ \overline{g'} = N\overline{gg'}$. Therefore $\phi(gg') = N\overline{gg'} = N\overline{g}N\overline{g'} = \phi(g)\phi(g')$, thus ϕ is a homomorphism.

If $\{\bar{g}:g \in G\}$ is another choice of liftings then for any $x \in G$, $x = \lambda(\overline{g})$ and $x = \lambda(\overline{g'})$ for some $\overline{g} \in \overline{G}$, $\overline{g'} \in \overline{G}$. Therefore $\lambda(\overline{g}) = \lambda(\overline{g'})$ implies $\lambda(\overline{g}\overline{g'}^{-1}) = 1_G$, so $\overline{g}\overline{g'}^{-1} \in N$. Therefore $N\overline{g} = N\overline{g'}$ thus ϕ is independent of the choice of liftings.

We now show that even for a non-split extension of N by G , if N is abelian, G acts on N ,that is, there $\exists$ a homomorphism $\theta{:}G \longrightarrow Aut(N)$.

Lemma 3.3.3. *Let $\overline{G}$ be an extension of N by G, with N abelian. Then there is a homomorphism $\theta{:}G \to Aut(N)$ such that $\theta_g(n) = \bar{g}n\bar{g}^{-1}$ for all $n \in N$, and θ , and θ is independent of the choice of liftings $\{\bar{g} : g \in G\}$.*

Proof: For $a \in \overline{G}$, define $\gamma_a : \overline{G} \longrightarrow \overline{G}$ by $\gamma_a(\overline{g}) = a\overline{g}a^{-1}$. Since $N \trianglelefteq \overline{G}$, $\gamma_a|_N \in Aut(N)$, the restriction of γ_a to N, and the function $\mu : \overline{G} \longrightarrow Aut(N)$ defined by $\mu(a) = \gamma_a|_N$ $\forall\, a \in \overline{G}$ is a homomorphism. This is because $\gamma_{ab}|_N(n) = abn(ab)^{-1} = abna^{-1}b^{-1} =$

$(\gamma_a|_N \circ \gamma_b|_N)(n)$ and therefore $\mu(ab) = \gamma_{ab}|_N = \gamma_a|_N \circ \gamma_b|_N = \mu(a) \circ \mu(b)$. If $a \in N$, then $\mu(a) = \gamma_a|_N$ where $\gamma_a|_N : n \longrightarrow ana^{-1} = naa^{-1} = n \quad \forall n \in N$, so $\mu(a) = \gamma_a|_N = 1_N$, since N is abelian. Define $\mu^* : \overline{G}/N \longrightarrow Aut(N)$ by $\mu^*(Na) = \mu(a)$. If $Na = Nb$ then $Nab^{-1} = N$, so $ab^{-1} \in N$. But then $\mu(ab^{-1}) = 1_N$, as above, which implies $\mu(a)\mu(b^{-1}) = \mu(a)(\mu(b))^{-1} = 1_N$ and $\mu(a) = \mu(b)$. But then $\mu^*(Na) = \mu^*(Nb)$ and so μ^* is well-defined.

Furthermore, $\mu^*(NaNb) = \mu^*(Nab) = \mu(ab) = \mu(a)\mu(b) = \mu^*(Na)\mu^*(Nb)$, therefore μ^* is a homomorphism.

Now let $\theta : G \longrightarrow Aut(N)$ be the composite map $\mu^* \circ \phi$, where ϕ is the map defined in Lemma 3.3.2 . If $g \in G$ and $\bar{g}$ is a lifting, then $\theta(g) = (\mu^* \circ \phi)(g) = \mu^*(\phi(g)) = \mu^*(N\bar{g}) = \mu(\bar{g}) \in Aut(N)$, so for $n \in N$, $\theta_g(n) = \mu(\bar{g})(n) = \bar{g}n\bar{g}^{-1}$, as required.

If $\{\bar{g'} : g' \in G\}$ is another choice of liftings then as in Lemma 3.3.2 $N\bar{g} = N\overline{g'}$ but then $\theta_g = \theta(g) = (\mu^* \circ \phi)(g) = \mu^*(\phi(g)) = \mu^*(N\bar{g}) = \mu^*(N\bar{g'}) = \mu(\bar{g'})$, and so for $\theta_g(n) = \mu(\bar{g'})(n) = \bar{g'}n\bar{g'}^{-1} = \bar{g}n\bar{g}^{-1}$ for any $n \in N$. This shows that θ is independent of the choice of liftings.

Conjugacy Classes of $\overline{G} = N \cdot G$ (N abelian).

To determine the conjugacy classes of $\overline{G}$, we analyse the cosets $N\bar{g}$, where $\overline{G} = \bigcup_{g \in G} N\bar{g}$ and $\bar{g}$ is a lifting of g in G. It is only necessary to consider one coset $N\bar{g}$ for each conjugacy class of G with representative g, and the corresponding classes of $\overline{G}$ are determined by the action (by conjugation) of $C_{\bar{g}}$, the *set stabilizer in* $\overline{G}$ *of* $N\bar{g}$, that is, $C_{\bar{g}} = \{g \in \overline{G} | g(N\bar{g}) = (N\bar{g})g\} = \{g \in \overline{G} | gn\bar{g}g^{-1} \in N\bar{g}, \quad \forall n \in N\}$. Now $N \subseteq C_{\bar{g}}$, since for $n \in N$ and $n_1\bar{g} \in N\bar{g}$, $n(n_1\bar{g})n^{-1} = nn_1\bar{g}n^{-1}\bar{g}^{-1}\bar{g} = nn_1(n^{-1})^{\bar{g}}\bar{g} \in N\bar{g}$. Therefore

$N \trianglelefteq C_{\bar{g}}$ and we have $C_{\bar{g}}/N = C_{\overline{G}/N}(N\bar{g})$ because

$$\begin{aligned}
Nh \in C_{\overline{G}/N}(N\bar{g}) &\iff NhN\bar{g}(Nh)^{-1} = N\bar{g} \\
&\iff NhN\bar{g}Nh^{-1} = N\bar{g} \\
&\iff NhN\bar{g}N\bar{g}^{-1}\bar{g}h^{-1} = N\bar{g} \\
&\iff NhN\bar{g}h^{-1} = N\bar{g} \\
&\iff NhNn\bar{g}h^{-1} = N\bar{g} \quad \forall\ n \in n \\
&\iff NhNh^{-1}hn\bar{g}h^{-1} = N\bar{g} \quad \forall\ n \in N \\
&\iff Nhn\bar{g}h^{-1} = N\bar{g} \\
&\iff hn\bar{g}h^{-1} \in N\bar{g} \quad \forall\ n \in N \\
&\iff h \in C_{\bar{g}} \\
&\iff Nh \in C_{\bar{g}}/N
\end{aligned}$$

Therefore, $C_{\bar{g}}/N = C_{\overline{G}/N}(N\bar{g}) = C_G(g)$, identifying $C_{\overline{G}/N}(N\bar{g})$ and $C_G(g)$. This shows that $C_{\bar{g}}$ is an extension of N by $C_G(g)$, that is, $C_{\bar{g}} = N.C_G(g)$.

Remark 3.3.4. If $\overline{G} = N : G$ we can identify $C_{\bar{g}} = N \cdot C_G(g)$ with $C_g = \{y \in G | y(Ng) = (Ng)y\}$, where the lifting of g in $\overline{G}$ is g itself since $G \leq \overline{G}$ in the case of a split extension.

Next we determine the *orbits* of $C_{\bar{g}}$ on $N\bar{g}$. Let $z \in N\bar{g}$. Then under the action of N on $N\bar{g}$, $N_z = \{n \in N | z^n = z\} = C_N(z)$, is the stabilizer in N of z. Then for any $nz \in N\bar{g}$ $(n \in N)$, $(nz)^y = n^y z^y = yny^{-1}yzy^{-1} = nyy^{-1}yzy^{-1} = nyzy^{-1} = nzyy^{-1} = nz$, for $y \in C_N(z)$, since N is abelian. Therefore $C_N(z)$ fixes each element in $N\bar{g}$.
Let $k = |C_N(z)|$. Then $|[z]| = \frac{|N|}{|C_N(z)|} = \frac{|N|}{k}$, that is, under conjugation by N, each element of $N\bar{g}$ is conjugate to $\frac{|N|}{k}$ elements of $N\bar{g}$, so $N\bar{g}$ splits into k blocks with $\frac{|N|}{k}$ elements in each block. Denote these blocks by $Q_1, \cdots, Q_k$. The orbits of $C_{\bar{g}}$ (that is, the conjugacy classes of $N\bar{g}$) are unions of these blocks which fuse together by the action of $C_{\bar{g}}$. Since $C_{\bar{g}} = N.C_G(g)$, this fusion is completely determined by the action of $\{\bar{y} | y \in C_G(g)\}$ (there is a homomorphism $\lambda : C_{\bar{g}} \to C_G(g)$ with kernel N; $\bar{y} \mapsto y$). For suppose Q_i and Q_j fuse $(i \neq j)$. Then there exist $n_1\bar{g} \in Q_i,\ n_2\bar{g} \in Q_j$ such that $(n_1\bar{g})^l = n_2\bar{g}$ for some $l \in C_{\bar{g}}$.
But $l \in C_{\bar{g}}$ implies that $l = n\bar{y}$ for some $n \in N,\ y \in C_G(g)$. So $(n_1\bar{g})^{n\bar{y}} = n_2\bar{g}$ implies that

$((n_1\bar{g})^n)^{\bar{y}} = n_2\bar{g}$. Now $(n_1\bar{g})^n \in Q_i$, so by the action of $\bar{y}$, Q_i and Q_j have fused. Suppose f blocks fuse to form an orbit Ω of $C_{\bar{g}}$. Then $|\Omega| = \frac{f.|N|}{k}$. Let $y \in \Omega$. Then the stabilizer in $C_{\bar{g}}$ of y is $C_G(y)$, so $|\Omega| = \frac{|C_{\bar{g}}|}{|C_G(y)|} = \frac{|N||C_G(g)|}{|C_G(y)|}$.
Therefore $|C_{\overline{G}}(y)| = \frac{|N||C_G(g)|}{|\Omega|} = \frac{k|N||C_G(g)|}{f|N|} = \frac{k}{f}|C_G(g)|$. So to calculate the conjugacy classes of $\overline{G}$ we need to find the values of k and f for each conjugacy class of G. Note that the values of k can be determined from the action of G on N given in lemma 3.3.3. Consider a class representative g of G. For this class, k is the number of elements of N that fix z for $z \in N\bar{g}$. Take $z = \bar{g}$. Now for $n \in N$, n fixes $\bar{g} \Leftrightarrow \bar{g}^n = \bar{g} \Leftrightarrow n\bar{g}n^{-1} = \bar{g} \Leftrightarrow n\bar{g} = \bar{g}n \Leftrightarrow n = \bar{g}n\bar{g}^{-1} \Leftrightarrow n = n^{\bar{g}} \Leftrightarrow n = n^g$. Therefore k is the number of elements of N fixed by g, which equals $\chi(g)$ where χ is the permutation character of the action of G on N.

Theorem 3.3.5. *[53] Let $\overline{G} = N{:}G$ and $dg \in \overline{G}$ where $d \in N$ and $g \in G$ such that $o(g) = m$ and $o(dg) = k$. Then m divides k.*

Proof: We have that

$$1_G = (dg)^k = dd^g d^{g^2} d^{g^3} ... d^{g^{k-1}} g^k \quad .$$

Since G acts on N and $d \in N$, we have $d, d^g, d^{g^2}, d^{g^3}, ..., d^{g^{k-1}} \in N$. Hence $dd^g d^{g^2} d^{g^3} ... d^{g^{k-1}} \in N$. Thus we have that $dd^g d^{g^2} d^{g^3} ... d^{g^{k-1}} = 1_K$ and $g^k = 1_G$. Hence m divides k. $\square$

Theorem 3.3.6. *Let $\overline{G} = N{:}G$ such that N is an elementary abelian p-group, where p is prime. Let $dg \in \overline{G}$ where $d \in N$ and $g \in G$ such that $o(g) = m$ and $o(dg) = k$. Then either $k = m$ or $k = pm$.*

Proof: See [53], Theorem 2.3.10.

Remark 3.3.7. Let $\overline{G} = N{:}G$ where N is an elementary abelian p-group, where p is prime. Let $dg \in \overline{G}$ with $d \in N$, $g \in G$ such that $o(g) = m$ and $o(dg) = k$, then we observe that

$$1_{\overline{G}} = (dg)^m = dd^g d^{g^2} d^{g^3} ... d^{g^{m-1}} g^m \quad .$$

Since $g^m = 1_G$, we obtain that $(dg)^m = w$, where $w \in N$ and is given by

$$w = dd^g d^{g^2} d^{g^3} ... d^{g^{m-1}} \quad .$$

By Theorem 3.2.4 above , we have that if $w = 1_N$ then $k = m$ and if $w \neq 1_N$ then $k = pm$.

By using the method of coset analysis(discussed earlier) together with Theorems 3.3.5 and 3.3.6 and Remark 3.3.7,[1] developed Programmes A and B (see Appendix A) in MAGMA [15] to compute the conjucacy classes and the orders of the class representatives of the extension $\overline{G} = N{:}G$ where N is an elementary abelian p - group for prime p on which a linear group G acts.

Chapter 4

Clifford Theory

If $\overline{G}$ is an extension of N by G, then Fischer showed how the character table of $\overline{G}$ can be determined by constructing a matrix corresponding to each conjugacy class of G. The character table of $\overline{G}$ can then be determined from these matrices (the so-called Clifford matrices) and the character tables of certain subgroups of $\overline{G}$ called the *inertia factors*. In this thesis we describe how the method of the Fischer–Clifford matrices is applied in the case of extensions of elementary abelian groups. However this method also applies to extensions of any normal subgroup N with the property that each character of N can be extended to its inertia group. The theoretical foundation for this method is the Clifford theory and the theory of the Fischer matrices.

In this chapter we discuss the Clifford theory which will be applied to describe the Fischer matrices method in the next chapter. We refer the reader to Moori [47] for the definitions and results not given here.

4.1 Clifford's Theorem

Let $\overline{G} = N \cdot G$ be the extension of N by G. Here N is any group, not necessarily abelian. Let $\theta \in Irr(N)$ be an irreducible character of N. Define θ^g by $\theta^g(n) = \theta(gng^{-1})$ for $g \in \bar{G}$, $n \in N$.

Let P be a representation of N affording θ, that is, $P{:}N \longrightarrow GL(n, \mathbb{F})$ is a homomorphism such that $\chi_p = \theta$. Then

$$\chi_{p^g}(n) = tr(P^g(n)) = tr(P(gng^{-1})) = \chi_p(gng^{-1}) = \theta(gng^{-1}) = \theta^g(n)$$

for all $n \in N$. Therefore, $\chi_{p^g} = \theta^g$ and furthermore $\theta \in Irr(N)$ implies $\theta^g \in Irr(N)$ because $< \theta^g, \theta^g >_N = < \theta, \theta >_N = 1$ (Prop. v.5(iii), [47]).

Moreover, $\theta^{n'}(n) = \theta(n'nn'^{-1}) = \theta(n)$ (since $n \sim n'nn'^{-1}$ in N, $\theta(n) = \theta(n'nn'^{-1})$) and therefore $\theta^{n'} = \theta$ for all $n' \in N$, that is, N *acts trivially on* $Irr(N)$.

Remark 4.1.1. $P^g : N \longrightarrow GL(n, \mathbb{F})$ defined by $P^g(n) = P(gng^{-1})$ is a representation of N affording θ^g. This is because $\forall n_1, n_2 \in N$, $P^g(n_1 n_2) = P(gn_1n_2g^{-1}) = P(gn_1g^{-1}gn_2g^{-1}) = P(gn_1g^{-1})\,P(gn_2g^{-1}) = P^g(n_1)\,P^g(n_2)$.

Since $\theta^1(n) = \theta(1n1) = \theta(n)$, $\forall n \in N$, $\theta^1 = \theta$. Moreover we have $\theta^{gg'} = (\theta^g)^{g'}$ (Prop.v.5(ii), [47]). It follows that G acts on $Irr(N)$ with $G_\theta = \{g \in G | \theta^g = \theta\}$, the stabilizer of θ in G. Denote G_θ by $I_G(\theta)$ and call it the *inertia group of* θ *in* G.

Remark 4.1.2. The action of G on $Irr(N)$ determines a partition of $Irr(N)$ by the orbits θ^G for $\theta \in Irr(N)$, that is,

$$Irr(N) = \bigcup_{\theta \in Irr(N)} \theta^G$$

and $\theta^G \cap \phi^G \neq \emptyset \Rightarrow \theta^G = \phi^G$ for any θ, $\phi \in Irr(N)$.

Now $I_G(\theta)$ is a subgroup of G and $N \leqslant I_G(\theta)$ since N acts trivially on $Irr(N)$, that is, $\theta^n = \theta$ $\forall n \in N$. Also $|\theta^G| = |\{\theta^g \,|\, g \in G\}| = [G{:}I_G(\theta)]$, that is, the size of the orbit $\theta^G = \{\theta^g \,|\, g \in G\}$ containing θ is $[G{:}I_G(\theta)]$, the index of the subgroup $I_G(\theta)$ in G. We say that θ *extends to an irreducible character of* $\overline{H} = I_{\overline{G}}(\theta)$ if there exists $\phi \in Irr(\overline{H})$ such that $\theta = \phi \downarrow N$.

Theorem 4.1.3. ([[18], Theorem 3.3.1],[33]) (**Clifford's Theorem**) *Let* $N \trianglelefteq \overline{G}$ *and* $\chi \in Irr(\overline{G})$*. Let* θ *be an irreducible constituent of* $\chi \downarrow N$*, that is,* $\theta \in Irr(N)$ *and* $< \theta, \chi \downarrow N >_N \neq 0$*.*

Then

$$\chi \downarrow N = c\sum_{i=1}^{t} \theta_i$$

where $\theta = \theta_1, \theta_2, \cdots, \theta_t$ *are the distinct conjugates of* θ *in* $\overline{G}$ *and* c *is a constant such that* $c =< \chi \downarrow N, \theta >_N$

Proof: We compute $\theta^{\bar{G}} \downarrow N$. Define θ° on $\overline{G}$ by

$$\theta^{\circ}(x) = \begin{cases} \theta(x) & if \quad x \in N \\ 0 & if \quad x \notin N \end{cases}$$

Let $n \in N$. Then

$$\theta^{\bar{G}}(n) = |N|^{-1} \sum_{x \in \bar{G}} \theta^{\circ}(xnx^{-1}) = |N|^{-1} \sum_{x \in \bar{G}} \theta(xnx^{-1}),$$

since $xnx^{-1} \in N$ for all $x \in \bar{G}$ (Prop.iv.2.3, [47]). Therefore,

$$\theta^{\bar{G}}(n) = |N|^{-1} \sum_{x \in \bar{G}} \theta^{x}(n) \text{ and hence } \theta^{\bar{G}} \downarrow N = |N|^{-1} \sum_{x \in \bar{G}} \theta^{x}$$

Now if $\phi \in Irr(N)$ and $\phi \notin \{\theta = \theta_1, \theta_2, \cdots, \theta_t\}$, then $< \phi, \theta^x >_N = 0 \ \forall x \in \bar{G}$. Therefore

$$\left\langle \sum_{x \in \bar{G}} \theta^{x}, \phi \right\rangle_N = 0,$$

whence $< \theta^{\bar{G}} \downarrow N, \phi >_N = 0$. Using the Frobenius Reciprocity Theorem (Theorem 2.4.4) we get

$$0 =< \theta^{\bar{G}} \downarrow N, \phi >_N =< \theta^{\bar{G}}, \phi^{\bar{G}} >_{\bar{G}} \text{ and } 0 \neq < \theta, \chi \downarrow N >_N =< \theta^{\bar{G}}, \chi >_{\bar{G}} .$$

Then $< \chi, \phi^{\bar{G}} >_{\bar{G}} = 0$, for if $< \chi, \phi^{\bar{G}} > \neq 0$ then χ is a constituent of $\phi^{\bar{G}}$, but then $< \theta^{\bar{G}}, \phi^{\bar{G}} >_{\bar{G}} \neq 0$ because χ is a constituent of $\theta^{\bar{G}}$, contradicting $< \theta^{\bar{G}}, \phi^{\bar{G}} >_{\bar{G}} = 0$.

Therefore $< \chi \downarrow N, \phi >_N = < \phi^{\bar{G}}, \chi >_{\bar{G}} = 0$ for all $\phi \in Irr(N) \backslash \{\theta = \theta_1, \cdots, \theta_t\}$, again by the Frobenius Reciprocity Theorem. Thus all the irreducible constituents of $\chi \downarrow N$ are among the θ_i, so

$$\chi \downarrow N = \sum_{i=1}^{t} < \chi \downarrow N, \theta_i >_N \theta_i$$

But $< \chi \downarrow N, \theta_i >_N = < \chi \downarrow N, \theta >_N = c$, for all $i = 1, 2, \cdots, t$ because θ_i and θ are conjugate (Prop.v.5(iv), [47]). Thus

$$\chi \downarrow N = \sum_{i=1}^{t} c\,\theta_i = c \sum_{i=1}^{t} \theta_i$$

□

Definition 4.1.4. Let $N \trianglelefteq \overline{G}$ and $\theta \in Irr(N)$. Then $I_{\bar{G}}(\theta) = \left\{ g \in \overline{G} : \theta^g = g \right\}$ is the inertia group of θ in $\overline{G}$.

Remark 4.1.5. Since $I_{\overline{G}}(\theta)$ is the stabilizer of θ in the action of $\overline{G}$ on $Irr(N)$, we have that $I_{\overline{G}}(\theta)$ is a subgroup of $\overline{G}$ and $N \subseteq I_{\overline{G}}(\theta)$. Also, $[\overline{G}:I_{\overline{G}}(\theta)]$ is the size of the orbit containing θ, so in the above formula $\chi \downarrow N = c\sum_{i=1}^{t} \theta_i$, we have $t = [\overline{G}:I_{\overline{G}}(\theta)]$. As a consequence of Clifford's Theorem we have the following result.

Theorem 4.1.6. [68] *Let $N \trianglelefteq \overline{G}$, $\theta \in Irr(N)$ and $\overline{H} = I_{\overline{G}}(\theta)$. Then induction to $\overline{G}$ maps the irreducible characters of $\overline{H}$ that contain θ in their restriction to N faithfully onto the irreducible characters of $\overline{G}$ which contain θ in their restriction to N.*

Proof: Let $\mathcal{A} = \{\psi \in Irr(\overline{H}) \mid < \psi \downarrow N, \theta > \neq 0\}$, $\mathcal{B} = \{\chi \in Irr(\overline{G}) \mid < \chi \downarrow N, \theta > \neq 0\}$. Define the mapping $\Phi : \mathcal{A} \longrightarrow \mathcal{B}$ by $\psi \mapsto \psi^{\bar{G}}$ where $\psi^{\bar{G}}$ is the character of G induced by ψ.

For $\psi \in \mathcal{A}$ we first show that

(i) $\psi^{\bar{G}} \in Irr(\overline{G})$

(ii) $< \psi^{\bar{G}} \downarrow N, \theta > \neq 0$

(iii) $< \psi \downarrow N, \theta > = < \psi^{\bar{G}} \downarrow N, \theta >$.

(i) Let χ be an irreducible constituent of $\psi^{\bar{G}}$, that is, $\chi \in Irr(\overline{G})$ and $< \chi, \psi^{\bar{G}} >_{\bar{G}} \neq 0$. Now $0 \neq < \chi, \psi^{\bar{G}} >_{\bar{G}} \;=\; < \psi, \chi \downarrow \bar{H} >_{\bar{H}}$, by Frobenius Reciprocity Theorem and so ψ is an irreducible constituent of $\chi \downarrow \bar{H}$. Thus
$\chi \downarrow \bar{H} = \cdots + \lambda\psi + \cdots$, $\lambda \geq 1$, and therefore

$$\chi \downarrow N = \cdots \lambda\psi \downarrow N + \cdots \;=\; \cdots \lambda(\cdots + \delta\theta + \cdots) \;=\; \cdots + \lambda\delta\theta + \cdots, \; \delta \geq 1$$

which shows that $< \chi \downarrow N, \theta > \neq 0$. Since $\chi \in Irr(\overline{G})$, it follows that $\chi \in \mathcal{B}$. We show that $\chi = \psi^{\bar{G}}$.
By Clifford's theorem

$$\chi \downarrow N = e \sum_{i=1}^{t} \theta_i$$

where $\theta = \theta_1, \theta_2, \cdots, \theta_t$ are the distinct conjugates of θ in $\overline{G}$ and e is a constant such that $e =< \chi \downarrow N, \theta >_N$ and $t = [\bar{G}{:}I_{\bar{G}}(\theta)] = |\theta^{\bar{G}}|$.
Therefore

$$\chi(1) = (\chi \downarrow N)(1) = e \sum_{i=1}^{t} \theta_i(1) = e \cdot t \cdot \theta(1) \tag{4.1}$$

because $\theta^g(1) = \theta(g1g^{-1}) = \theta(1) \; \forall g \in \bar{G}$. Now $N \trianglelefteq \overline{H}$, $\psi \in Irr(\bar{H})$ and $< \psi \downarrow N, \theta > \neq 0$, that is, θ is an irreducible constituent of $\psi \downarrow N$. Therefore, again by Clifford's Theorem

$$\psi \downarrow N = f\theta \tag{4.2}$$

for some constant f since $\{\theta^h \,|\, h \in \bar{H} = I_{\bar{G}}(\theta)\} = \{\theta\}$, and thus $< \psi \downarrow N, \theta >= f$. As before $\chi \downarrow \bar{H} = \cdots + \lambda\psi + \cdots$, $\lambda \geq 1$, therefore

$$\chi \downarrow N \;=\; \cdots + \lambda(\psi \downarrow N) + \cdots \;=\; \cdots + \lambda(\cdots + f\,\theta + \cdots) \;=\; \cdots + \lambda f\,\theta + \cdots$$

and so

$$e =< \chi \downarrow N, \theta >_N = \lambda f \geq f \;\text{ since } \lambda \geq 1 \tag{4.3}$$

Also $0 \neq < \psi^{\bar{G}}, \chi >_{\bar{G}}$ implies

$$\psi^{\bar{G}} = \cdots + \delta\chi + \cdots, \; (\delta \geq 1) \tag{4.4}$$

and hence $\psi^{\bar{G}}(1) = \cdots + \delta\chi(1) + \cdots \geq \chi(1) = e \cdot t \cdot \theta(1)$ by (4.1.1). But $\psi^{\bar{G}}(1) = [\bar{G}{:}\bar{H}]\psi(1) = t\,f\,\theta(1)$ by (4.1.2) and Defn.iv.2.2 [47]. Therefore $t\,f\,\theta(1) \leq t\,e\,\theta(1) = \chi(1)$, since $f \leq e$ by 4.3.

Since $e\,t\,\theta(1) = \chi(1) \leq \psi^{\bar{G}}(1) = t\,f\,\theta(1) \leq t\,e\,\theta(1) = \chi(1)$, it follows that $\chi(1) = \psi^{\bar{G}}(1)$ and $e = f$. That is,

$$e = < \chi \downarrow N, \theta > = < \psi \downarrow N, \theta > = f \tag{4.5}$$

Also $0 \neq < \psi^{\bar{G}}, \chi >_{\bar{G}}$ implies that $\psi^{\bar{G}} = \lambda_1\chi + \lambda_2\chi_2 + \cdots$ and so $\chi(1) = \psi^{\bar{G}}(1) = \lambda_1\chi(1) + \lambda_2\chi_2(1) + \cdots$ Hence $\lambda_1 = 1, \lambda_2 = \lambda_3 = \cdots = 0$, which proves that $\psi^{\bar{G}} = \chi$.
(ii) Restating (4.1.5) we get

$$< \psi^{G} \downarrow N, \theta > = < \psi \downarrow N, \theta > \neq 0 \tag{4.6}$$

(iii) The map $\psi \mapsto \psi^{\bar{G}}$ is onto. Let $\chi \in \mathcal{B}$. Then $\chi \in Irr(\bar{G})$ and $< \chi \downarrow N, \theta > \neq 0$. If $\chi \downarrow \bar{H} = \lambda_1\psi_1 + \lambda_2\psi_2 + \cdots + \lambda_r\psi_r$, $\psi_i \in Irr(\bar{H})$ for $i = 1, 2, \cdots, r$, then $\chi \downarrow N = \lambda_1(\psi_1 \downarrow N) + \cdots + \lambda_r(\psi_r \downarrow N)$. So $\exists\, i$, $1 \leq i \leq r$ such that $< \psi_i \downarrow N, \theta > \neq 0$, since $< \chi \downarrow N, \theta > \neq 0$.
Let $\psi = \psi_i$, then $\psi \in Irr(\bar{H})$ and $< \psi \downarrow N, \theta > \neq 0$. Hence $\psi \in \mathcal{A}$ and $< \psi, \chi \downarrow \bar{H} >_{\bar{H}} \neq 0$ and so $< \psi^{\bar{G}}, \chi > =^{F-R} < \psi, \chi \downarrow \bar{H} >_{\bar{H}} \neq 0$. This shows that χ is an irreducible constituent of $\psi^{\bar{G}}$. The fact that $\chi = \psi^{\bar{G}}$ can be proved by repeating the above steps. To show that the map $\psi \mapsto \psi^{\bar{G}}$ is one-to-one, we need to show that for $\psi \in \mathcal{A}$, ψ is the unique irreducible constituent of $\psi^{\bar{G}} \downarrow \bar{H}$ which lies in $\mathcal{A}$.
Let $\chi = \psi^{\bar{G}}$. Suppose $\psi_1 \in \mathcal{A}$ such that ψ_1 is a constituent of $\chi \downarrow \bar{H}$ and $\psi_1 \neq \psi$. That is, $\chi \downarrow \bar{H} = \lambda_1\psi + \lambda_2\psi_1 + \cdots$ and so $\chi \downarrow N = \lambda_1(\psi \downarrow N) + \lambda_2(\psi_1 \downarrow N) + \cdots$
Hence

$$\begin{aligned} < \chi \downarrow N, \theta > &= \lambda_1 < \psi \downarrow N, \theta > + \lambda_2 < \psi_1 \downarrow N, \theta > + \cdots \\ &\geq < \psi \downarrow N, \theta > + < \psi_1 \downarrow N, \theta > \\ &> < \psi \downarrow N, \theta > \end{aligned}$$

contradicting $< \chi \downarrow N, \theta > = < \psi \downarrow N, \theta >$ by (4.1.6).
This completes the proof. $\square$

Theorem 4.1.6 implies that, to obtain the irreducible characters of $\overline{G}$ that contain θ in their restriction to N, *it is sufficient to find the irreducible characters of* $\overline{H}$ *that contain* θ *in their restriction to* N . If θ can be extended to an irreducible character of $\overline{H} = I_{\overline{G}}(\theta)$ (that is, $\exists\, \psi \in Irr(\bar{H})$ and $\theta = \psi \downarrow N$), then the relevant characters of $\overline{H}$ can be obtained by using the following theorem.

Theorem 4.1.7. *[28] Let* $N \trianglelefteq \overline{G}$, $\theta \in Irr(N)$ *and* $\bar{H} = I_{\bar{G}}(\theta)$. *If* θ *extends to an irreducible character* φ *of* $\overline{H}$, *then*
$\mathcal{A} = \{\psi | \psi \in Irr(\bar{H})\,,\, < \psi \downarrow N, \theta > \neq 0\} = \{\beta\varphi \,|\, \beta \in Irr(\bar{H}), N \subseteq ker\beta\} =$
$\{\beta\varphi \,|\, \beta \in Irr(\overline{H}/N)\}$

Proof: If $\{x_1, \cdots, x_f\}$ is a right transversal for N in $\overline{H}$, it follows that for every $n \in N$,

$$\theta^{\bar{H}}(n) = \sum_{i=1}^{f} \theta^{\circ}(x_i\, n\, x_i{}^{-1}) = \sum_{i=1}^{f} \theta(x_i\, n\, x_i{}^{-1}) = \sum_{i=1}^{f} \theta^{x_i}(n) = \sum_{i=1}^{f} \theta(n) = f\,\theta(n)$$

since $x_i\, n\, x_i{}^{-1} \in N$ for each $i = 1, 2, \cdots, f$ and $\bar{H} = I_{\bar{G}}(\theta)$ implies that θ is the only $\overline{H}$-conjugate (that is, $\theta^{x_i} = \theta$ for each i, $1 \leq i \leq f$). Hence $\theta^{\bar{H}} \downarrow N = f\theta$ for some integer f.
Now $f\,\theta(1) = (\theta^{\bar{H}} \downarrow N)(1) = \theta^{\bar{H}}(1) = [\bar{H}{:}N]\,\theta(1)$ by Definition iv.2.2 [47] , and so $f = [\bar{H}{:}N]$.
Therefore $\theta^{\bar{H}} \downarrow N = [\bar{H}{:}N]\,\theta$. Hence

$< \theta^{\bar{H}}, \theta^{\bar{H}} >_{\bar{H}} =^{F-R} < \theta, \theta^{\bar{H}} \downarrow N >_N = < \theta, [\bar{H}{:}N]\,\theta >_N = [\bar{H}{:}N] < \theta, \theta >_N = [\bar{H}{:}N]$,

since $\theta \in Irr(N)$.

Claim :

$$\theta^{\bar{H}} = \sum_{\beta} \beta(1)\,\beta\,\varphi$$

where β runs over all irreducible characters of $\overline{H}$ that contain N in their kernel, or equivalently over all irreducible characters of $\bar{H}/N$.
For $g \notin N$, $xgx^{-1} \notin N$ for all $x \in \bar{N}$, so $\theta^{\bar{H}}(g) = 0$ and

$$\sum_{\beta} \beta(1)\,(\beta\,\varphi)(g) = \left(\sum_{\beta} (\beta(1)\,\beta(g))\right) \varphi(g) = 0$$

(column orthogonality for the character table of $\bar{H}/N$, since $g \notin N$).

By Note 7 [47] and the fact that $\beta(g) = \beta(1)$ since $N \subseteq ker\beta$, then for $g \in N$

$$\sum_{\beta} \beta(1)\,(\beta\,\varphi)(g) \quad = \sum_{\beta} \beta(1)\,\beta(g)\,\varphi(g) = \left(\sum_{\beta} (\beta(1))^2\right)\varphi(g) = |\bar{H}/N|\,\varphi(g),$$

$$= [\bar{H}{:}N]\,\varphi(g) = [\bar{H}{:}N]\,\theta(g)$$

since for $g \in N$, $\theta(g) = (\varphi \downarrow N)(g) = \varphi(g)$. Therefore,

$$\theta^{\bar{H}} \downarrow N = [\bar{H}{:}N]\theta = \left(\sum_{\beta} \beta(1)\,\beta\,\varphi\right) \downarrow N.$$

This proves that

$$\theta^{\bar{H}} = \sum_{\beta} \beta(1)\,\beta\,\varphi$$

as claimed.

Now

$$\begin{aligned}
[\bar{H}{:}N] &= <\theta^{\bar{H}}, \theta^{\bar{H}}> \\
&= \left\langle \sum_{\beta} \beta(1)\,\beta\,\varphi, \sum_{\gamma} \gamma(1)\,\gamma\,\varphi \right\rangle \\
&= \sum_{\beta,\gamma} \beta(1)\,\gamma(1) < \beta\,\varphi, \gamma\,\varphi > \\
&= \sum_{\beta} (\beta(1))^2 < \beta\,\varphi, \beta\,\varphi > + \sum_{\beta \neq \gamma} \beta(1)\,\gamma(1) < \beta\,\varphi, \gamma\,\varphi > \\
&\geq \sum_{\beta} (\beta(1))^2 + \sum_{\gamma \neq \beta} \gamma(1)\,\beta(1) < \beta\,\varphi, \gamma\,\varphi > \\
&\geq [\bar{H}{:}N] + \sum_{\gamma \neq \beta} \gamma(1)\,\beta(1) < \beta\,\varphi, \gamma\,\varphi >
\end{aligned}$$

Therefore

$$\sum_{\gamma \neq \beta} \gamma(1)\,\beta(1) < \beta\,\varphi, \gamma\,\varphi >= 0$$

and hence $< \beta\,\varphi, \gamma\,\varphi >= 0 \ \forall \gamma, \beta\ ,\ \gamma \neq \beta$, that is, the $\beta\,\varphi$ are distinct. But then

$$[\bar{H}{:}N] = \sum_{\beta} (\beta(1))^2 < \beta\,\varphi, \beta\,\varphi >$$

and so $< \beta\varphi, \beta\varphi >= 1$. This shows that $\beta\varphi$ is irreducible, where β runs over all irreducible characters of $\bar{H}/N$. Since

$$\theta^{\bar{H}} = \sum_{\beta} \beta(1)(\beta\varphi)$$

these $\beta\varphi$ are all the irreducible constituents of $\theta^{\bar{H}}$ $(< \beta\varphi \downarrow N, \theta >=^{F-R} < \beta\varphi, \theta^{\bar{H}} >_{\bar{H}} \neq 0)$, *so are all the irreducible characters of* $\overline{H}$ *that contain* θ *in their restriction*, since for $\gamma \in Irr(\bar{H})$ with $0 \neq < \gamma \downarrow N, \theta >_N$ we have
$0 \neq < \gamma \downarrow N, \theta >_N =^{F-R} < \gamma, \theta^{\bar{H}} >$ and so γ is an irreducible constituent of $\theta^{\bar{H}}$. Thus $\gamma = \beta\varphi$ for some $\beta \in Irr(\bar{H}/N)$ and this completes the proof of the theorem. □

In Theorem 4.1.6 the set $\mathcal{B} = \{\chi \in Irr(\bar{G}) \,|\, < \chi \downarrow N, \theta > \neq 0\} = \{\chi \in Irr(\bar{G}) \,|\, \chi = \psi^{\bar{G}}$ where $\psi \in Irr(\bar{H})$ and $< \psi \downarrow N, \theta > \neq 0\}$. If $\gamma \in Irr(\bar{H})$ with $< \gamma \downarrow N, \theta >_N \neq 0$ then $\gamma = \beta\varphi$ for some $\beta \in Irr(\bar{H}/N)$, where $\varphi \in Irr(\bar{H})$ and $\varphi \downarrow N = \theta$, by Theorem 4.1.3. Hence

$$\begin{aligned} \mathcal{B} &= \{\chi \in Irr(\bar{G}) \,|\, \chi = (\beta\varphi)^{\bar{G}} \text{ for some } \beta \in Irr(\bar{H}/N)\} \\ &= \{\chi \in Irr(\bar{G}) \,|\, \chi = (\beta\varphi)^{\bar{G}}, \beta \in Irr(\bar{H}), N \subseteq ker\beta\} \end{aligned}$$

Now suppose that $\overline{G}$ is an extension of N by G and suppose further that every irreducible character of N can be extended to its inertia group in $\overline{G}$. Let $\theta_1, \cdots, \theta_t$ be representatives of the distinct orbits of $\overline{G}$ on $Irr(N)$.

For each i, $1 \leq i \leq t$, let $\psi_i \in Irr(\bar{H}_i)$ where $\bar{H}_i = I_{\bar{G}}(\theta_i)$ such that $\psi_i \downarrow N = \theta_i$.
If $\mathcal{B}_i = \{\chi \in Irr(\bar{G}) \,|\, \chi = (\beta\psi_i)^{\bar{G}}, \beta \in Irr(\bar{H}_i), N \subseteq ker\beta\}$ then it is clear that

$$\bigcup_{i=1}^{t} \mathcal{B}_i \subseteq Irr(\bar{G})$$

Conversely, let $\chi \in Irr(\bar{G})$. If $< \gamma, \chi \downarrow N >_N \neq 0$ where $\gamma \in Irr(N)$, then $\gamma \in \theta_j^{\bar{G}}$ for some j, $1 \leq j \leq t$. Then by Clifford's Theorem,

$$\chi \downarrow N = c \sum_{i=1}^{r} \delta_i \tag{4.7}$$

where $\delta_1, \delta_2, \cdots, \delta_r$ are the distinct conjugates of γ in $\overline{G}$ and $c = <\chi \downarrow N, \gamma>_N$. It is clear that $\gamma^{\bar{G}} = \{\gamma^x \mid x \in \bar{G}\} = \theta_j^{\bar{G}}$. Hence it follows from 4.7 that $<\chi \downarrow N, \theta_j> \neq 0$ and so $\chi = \psi^{\bar{G}}$ where $\psi \in Irr(\bar{H}_j)$ such that $<\chi \downarrow N, \theta_j> \neq 0$. Therefore $\psi = \beta\,\psi_j$ for some $\beta \in Irr(\bar{H}_j)$ where $\psi_j \downarrow N = \theta_j$ and so $\chi = (\beta\,\psi_j)^{\bar{G}}$, $\beta \in Irr(\bar{H}_j)$, $N \subseteq ker\beta$. Hence

$$Irr(\overline{G}) = \bigcup_{i=1}^{t} \{(\beta\,\psi_i)^G \mid \beta \in Irr(\overline{H_i}) \text{ and } N \subseteq ker\beta\}\,. \qquad \square$$

Hence the character table of $\overline{G}$ is partitioned into t blocks $\triangle_1, \triangle_2, \cdots, \triangle_t$ where $\triangle_i$ is produced from the subgroup $\overline{H_i}$. We now give some results which give sufficient conditions for the irreducible characters of N to be extendible to their respective inertia groups, so that the above method can be used to calculate the characters of $\overline{G}$.

The first of these results is Mackey's theorem. The proof given here is obtained from Curtis and Reiner [53], page 353.

Theorem 4.1.8. *([23],[68]) Suppose that N is a normal subgroup of $\overline{G}$ such that N is abelian and $\overline{G}$ is a semi-direct product of N by G for some $G \leq \overline{G}$. If $\theta \in Irr(N)$ is invariant in $\overline{G}$, that is, $\theta^g = \theta\ \forall g \in \bar{G}$, then θ can be extended to a linear character of $\overline{G}$.*

Proof: Since $\overline{G}$ is a semi-direct product, any $g \in \bar{G}$ can be written uniquely as $g = nx$, $n \in N$, $x \in G$. Define χ on $\bar{G}$ by $\chi(nx) = \theta(n)$. Since N is abelian, θ has degree 1 , so is linear, and the fact that $\theta = \theta^g$ for all $g \in G$ implies that $\theta(n) = \theta(gng^{-1})$ for all $g \in \bar{G}$. Then if $g_1 = n_1x_1$, $g_2 = n_2x_2$, we have

$$\begin{aligned}\chi(g_1g_2) \quad &= \chi(n_1x_1\,n_2x_2) = \chi(n_1n_2^{x_1}\,x_1x_2) = \theta(n_1n_2^{x_1}) = \theta(n_1)\,\theta(n_2^{x_1}) = \theta(n_1)\,\theta(x_1n_2\,{n_1}^{-1}) \\ &= \theta(n_1)\,\theta^{x_1}(n_2) = \theta(n_1)\,\theta(n_2) = \chi(g_1)\,\chi(g_2), \text{ since } \theta \text{ is linear.}\end{aligned}$$

Therefore χ is a linear character of $\overline{G}$ and $\chi \downarrow N = \theta \qquad \square$

Mackey's theorem is a corollary of a more general result by Karpilovsky [38] which we state without proof.

Theorem 4.1.9. [*38*] *Let the group $\overline{G}$ contain a subgroup G of order n such that $\overline{G} = N{:}G$ for some N normal in $\overline{G}$ and let $\chi \in Irr(N)$ be invariant in G. Then χ extends to an irreducible character of $\overline{G}$ if the following conditions hold:*

(i) $gcd(m, n) = 1$ where $m = \chi(1)$;

(ii) $N \cap G \leq N'$ where N' is the derived subgroup of N. $\square$

Another extension theorem is the following.

Theorem 4.1.10. [*53*] *Suppose $\overline{G}$ is a split extension of N by G, then any linear character $\theta \in Irr(N)$ can be extended to its inertia group $I_{\overline{G}}(\theta)$.*

Proof: Let $\overline{G} = N{:}G$ and $\theta \in Irr(N)$ which is linear. Let $\overline{H} = I_{\overline{G}}(\theta)$ and let $H = I_G(\theta)$. Then $\overline{H} = N{:}H$ and so $\overline{H}/N \cong H$ can be regarded as the inertia group of θ in the factor group $\overline{G}/N \cong G$. We have $N \cap H = \{1\} \leq N'$ and since θ is linear, $deg(\theta) = 1$. Furthermore θ is clearly H–invariant, and $< deg(\theta), |H| > = < 1, |H| > = 1$. Therefore θ can be extended to $\overline{H} = I_{\overline{G}}(\theta)$, by Theorem 4.1.8. $\square$

Remark 4.1.11. We give a different proof of Mackey's theorem using Theorem 4.1.8. Let $\overline{G} = N{:}G$, a split extension of N by G. Since N is abelian, $N' = \{1\}$ and $deg(\theta) = 1$. Also $N \cap G = \{1\}$ and so $N \cap G \leq N'$. Since $\gcd(deg(\theta), |G|) = 1$, it follows from Theorem 4.1.8 that θ can be extended to $\overline{G}$.

Let ϕ be a representation of $\overline{G}$ and α an automorphism of $\overline{G}$. Define ϕ^α by $\phi^\alpha(x) = \phi(\alpha(x))$ $\forall x \in \bar{G}$. It follows that for $x, y \in \bar{G}$, $\phi^\alpha(xy) = \phi(\alpha(xy)) = \phi(\alpha(x)\alpha(y)) = \phi(\alpha(x))\ \phi(\alpha(y)) = \phi^\alpha(x)\ \phi^\alpha(y)$ and hence ϕ^α is a representation of $\overline{G}$.
Let the representation ϕ afford the character χ_ϕ. Define χ_ϕ^α by $\chi_\phi^\alpha(x) = \chi_\phi(x^\alpha)$ $\forall x \in \bar{G}$. Then $\chi_{\phi^\alpha}(x) = tr(\phi^\alpha(x)) = tr(\phi(x^\alpha)) = \chi_\phi(x^\alpha) = \chi_\phi^\alpha(x)$ and so $\chi_{\phi^\alpha} = \chi_\phi^\alpha$, that is, ϕ^α affords the character χ_ϕ^α.

The representation ϕ^α and the character χ_ϕ^α are called the *algebraic conjugates* of ϕ and χ_ϕ, respectively, induced by the automorphism α. Furthermore, if χ_ϕ is irreducible then

$$
\begin{aligned}
< \chi_\phi^\alpha, \chi_\phi^\alpha > &= \frac{1}{|\bar{G}|}\sum_{g\in\bar{G}} \chi_\phi^\alpha(g)\, \chi_\phi^\alpha(g^{-1}) \\
&= \frac{1}{|\bar{G}|}\sum_{g\in\bar{G}} \chi_\phi(g^\alpha)\, \chi_\phi((g^\alpha)^{-1}) \\
&= < \chi_\phi, \chi_\phi >= 1 \text{ because } \alpha(\bar{G}) = \bar{G}
\end{aligned}
$$

and thus χ_ϕ irreducible implies χ_ϕ^α irreducible. Also if $a \in [x]$, where x is a representative of a conjugacy class of $\overline{G}$ then $a = gxg^{-1}$ for some $x \in \bar{G}$ and so $a^\alpha = \alpha(a) = \alpha(gxg^{-1}) = \alpha(g)\,\alpha(x)\,\alpha(g^{-1}) = \alpha(g)\,\alpha(x)\,(\alpha(g))^{-1} = g^\alpha\, x^\alpha\, (g^\alpha)^{-1} \in [x^\alpha]$.Thus $([x])^\alpha \subseteq [x^\alpha]$.

Conversely, for any $a \in [x^\alpha]$, $a = g\, x^\alpha\, g^{-1} = (h\, x\, h^{-1})^\alpha \in ([x])^\alpha$. Thus $([x])^\alpha = [x^\alpha]$ and therefore $\alpha \in Aut(\bar{G})$ induces a permutation on the conjugacy classes of $\overline{G}$.
Let $X = (\chi_i(x_j))$ be the character table of $\overline{G}$, where $\chi_i \in Irr(G)$, $1 \leq i \leq n$ and x_j, $1 \leq j \leq n$ are representatives of the conjugacy classes of elements of $\overline{G}$. The automorphism α induces a permutation on the columns of X. Also for each $\chi_i \in Irr(\bar{G})$, we know that $\chi_i^\alpha \in Irr(\bar{G})$.
Hence α induces a permutation on the irreducible characters χ_i of $\overline{G}$ and thus induces a permutation on the rows of X. Moreover, since $\chi_i^\alpha(x_j) = \chi_i(x_j^\alpha)$, the matrices obtained from X by these two operations are identical.

Theorem 4.1.12. ***(Brauer's Theorem)*** *[29] Let G be a group and N be a group of automorphisms of G. Then the number of orbits of N as a group of permutations on the irreducible characters of G is the same as the number of orbits of N as a group of permutations on the conjugacy classes of G.*

Proof: Let X be the character table of G. Then as a matrix, X is square and nonsingular. Let α be an automorphism of G such that $\alpha \in N$. Then α induces a permutation on the conjugacy classes of G and thus a permutation on the columns of X.

Hence N acts on the conjugacy classes of G. Since $\alpha \in N$, then to each character χ of G, we obtain a character χ^α of G such that $\chi^\alpha \in Irr(G)$ whenever $\chi \in Irr(G)$. For $y \in G$

we obtain that $\chi^{\alpha}(y) = \chi(y^{\alpha})$, thus α induces a permutation on the rows of X. Hence N acts on the irreducible characters of G.

Let X^{α} denote the image of X under α. Then we obtain that $P(\alpha)\, X = X^{\alpha} = X\, Q(\alpha)$ where $P(\alpha)$, $Q(\alpha)$ are appropriate permutation matrices which are uniquely determined by $\alpha \in N$. Suppose that α, $\beta \in N$, then $X^{\alpha\,\beta} = (X^{\alpha})^{\beta}$ and hence

$$P(\alpha\,\beta)\, X = X^{\alpha\,\beta} = (X^{\alpha})^{\beta} = (P(\alpha)\, X)^{\beta} = P(\beta)\, P(\alpha)\, X \tag{4.8}$$

Also

$$X\, Q(\alpha\,\beta) = X^{\alpha\,\beta} = (X^{\alpha})^{\beta} = (X\, Q(\alpha))^{\beta} = X\, Q(\alpha)\, Q(\beta) \tag{4.9}$$

Since X is nonsingular, it follows from 4.8 and 4.9 that $P(\alpha\,\beta) = P(\beta)\, P(\alpha)$ and $G(\alpha\,\beta) = G(\alpha)\, G(\beta)$. Define mappings π_1 and π_2 on N by $\pi_1(\alpha) = (P(\alpha))^t$ and $\pi_2(\alpha) = G(\alpha)$, where t denotes the transpose operation on matrices.

Since

$$\pi_1(\alpha\,\beta) = (P(\alpha\,\beta))^t = (P(\beta)\, P(\alpha))^t = (P(\alpha))^t\, (P(\beta))^t = \pi_1(\alpha)\, \pi_1(\beta)$$

and

$$\pi_2(\alpha\,\beta) = G(\alpha\,\beta) = G(\alpha)\, G(\beta) = \pi_2(\alpha)\, \pi_2(\beta),$$

it follows that π_1 and π_2 are permutation representations of N. Let θ_1 and θ_2 be the permutation characters afforded by π_1 and π_2, respectively.

Since $X^{-1}\, P(\alpha)\, X = G(\alpha)$, $P(\alpha)$ and $G(\alpha)$ are similar and thus have the same trace. Therefore $trace(P(\alpha))^t = trace(P(\alpha)) = trace(G(\alpha))$ and so $\theta_1(\alpha) = \theta_{\pi_1}(\alpha) = tr(\pi_1(\alpha)) = tr((P(\alpha))^t) = tr(P(\alpha)) = tr(G(\alpha)) = tr(\pi_2(\alpha)) = \theta_{\pi_2}(\alpha) = \theta_2(\alpha)$, $\forall \alpha \in N$.

Hence $\theta_1 = \theta_2$. Let d_1, d_2 be the number of orbits of N on the irreducible characters and on the conjugacy classes of G, respectively. Thus we observe that d_1 is the number of orbits of $\pi_1(N)$ in its action as a group of permutations. Also d_2 is the number of orbits of $\pi_2(N)$ in its action as a group of permutations. Since θ_1 and θ_2 are the permutation characters of N acting on the irreducible characters of G and on the conjugacy classes of G, respectively, we have $d_1 =< \theta_1, 1_N >=< \theta_2, 1_N >= d_2$ by Corollary 2.5.9. This completes the proof. $\square$

Chapter 5

The Fischer–Clifford matrices

Character tables of finite groups can be constructed using various techniques. We are particularly interested in the method known as the technique of the Fischer–Clifford matrices. The technique derives its fundamentals from the Clifford theory and provides very powerful information for constructing character tables of group extensions. Given a group extension $\overline{G} = N \cdot G$ such that every irreducible character of N can be extended to its inertia group then for each conjugacy class representative $g \in G$, we are able to construct a matrix $M(g)$ called the Fischer-Clifford matrix. By using these matrices together with the fusion maps and character tables of some subgroups of G which are inertia factors of the inertia groups in $\overline{G}$, the complete character table of $\overline{G}$ can be constructed. These constructions of Fischer-Clifford matrices have been discussed and used by Salleh [64], List [41] , List and Mahmoud [42] , Fischer [26] , Darafshesh [24], Pahlings [56], Moori and Mpono [48], Whitley [68], Ali [1] and Zimba [71].

We first discuss the theory of the Fischer–Clifford matrices and follow closely the work of Whitley [68].

Let $\overline{G} = N \cdot G$ such that every irreducible character of N is extendible to its inertia group.

We have that $\overline{G}$ permutes $Irr(N)$ by $x : \theta \mapsto \theta^x$, where $x \in \overline{G}$ and $\theta \in Irr(N)$. Now let $\theta_1 = 1_N, \theta_2, \cdots, \theta_t$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$, $\overline{H_i} = I_{\overline{G}}(\theta_i)$, $1 \leq i \leq t$, $\psi_i \in Irr(\overline{H_i})$ be an extension of θ_i to $\overline{H_i}$ and $\beta \in Irr(\overline{H_i})$ such that $N \subseteq ker(\beta)$. It was shown in Chapter 4 that

$$Irr(\bar{G}) = \bigcup_{i=1}^{t} \{(\beta\,\psi_i)^{\overline{G}} \mid \beta \in Irr(\overline{H_i}), N \subseteq ker(\beta)\}$$

Hence the irreducible characters of $\overline{G}$ will be divided into blocks, where each block corresponds to an inertia group $\overline{H_i}$.

5.1 Definitions and Preliminaries

Remark 5.1.1. If $\overline{G} = N{:}G$, with N abelian, is a semi-direct product then by theorem 4.1.7 (Mackey's theorem) every irreducible character of N can be extended to its inertia group in $\overline{G}$. Hence the above result can be used to calculate the full character table of $\overline{G}$.

Let $\overline{G} = N \cdot G$ with the property that every irreducible character of N can be extended to its inertia group. Let $\bar{g} \in \overline{G}$ be a lifting of $g \in G$ under the natural homomorphism $\overline{G} \longrightarrow G$ and $[g]$ be a conjugacy class of elements of G with representative g.
Let $X(g) = \{x_1, x_2, \cdots, x_{c(g)}\}$ be a set of representatives of the conjugacy classes of $\overline{G}$ from the coset $N\bar{g}$ whose images under the natural homomorphism $\overline{G} \longrightarrow G$ are in $[g]$ and we take $x_1 = \bar{g}$. Let $\{\theta_1, \theta_2, \cdots, \theta_t\}$ be representatives of the orbits of $\overline{G}$ on $Irr(N)$ such that for $1 \leq i \leq t$, we have $N \leq \overline{H_i} = I_{\overline{G}}(\theta_i)$ with $H_i = \overline{H_i}/N \leq G$ and that $\psi_i \in Irr(\overline{H_i})$ is an extension of θ_i to H_i. Then without loss of generality suppose that $\theta_1 = I_N$ is the identity character of N. Then $\overline{H_1} = \overline{G}$ and $H_1 = G$. Now choose $y_1, y_2, \cdots, y_r$ to be the representatives of the conjugacy classes of elements of H_i which fuse to $[g]$ in G. Since $y_k \in H_i$ for $1 \leq k \leq r$, then we define $y_{l_k} \in \overline{H_i}$ such that y_{l_k} ranges over all the representatives of the conjugacy classes of elements of H_i which map to y_k under the homomorphism $\overline{H_i} \longrightarrow H_i$ whose kernel is N. Let $\beta \in Irr(H_i)$ such that $N \subseteq ker(\beta)$. Then β is a lifting of $\hat{\beta} \in Irr(\overline{H_i})$ such that $\beta(y_{l_k}) = \hat{\beta}(y_k)$ for any lifting $y_{l_k} \in \overline{H_i}$ of

$y_k \in H_i$. Then we obtain that $(\psi_i\,\beta)^{\bar{G}}(x_j) = \sum_{1\le k\le r} \sum_k' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{l_k})|} (\psi_i\,\beta)(y_{l_k})$
$= \sum_{1\le k\le r} \sum_k' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{l_k})|} \psi_i(y_{l_k})\,\beta(y_{l_k})$
$= \sum_{1\le k\le r} (\sum_k' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{l_k})|} \psi_i(y_{l_k}))\,\hat{\beta}(y_k)$
where $\sum_k'$ is the summation over all k for which $y_{l_k} \sim x_j$ in $\overline{G}$. Now we define a matrix $M_i(g)$ by $M_i(g) = (a_{uv})$, where $1 \le u \le r$ and $1 \le v \le c(g)$, and
$a_{uv} = \sum_k' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{l_k})|} \psi_i(y_{l_k})$. Then we obtain that

$$(\psi_i\,\beta)^{\bar{G}}(x_j) = \sum_{1\le k\le r} a_{uv}\hat{\beta}(y_k)\,.$$

By doing this for all $1 \le i \le t$ such that H_i contains an element in $[g]$ we obtain the matrix $M(g)$ given by

$$M(g) = \begin{bmatrix} M_1(g) \\ M_2(g) \\ \vdots \\ M_t(g) \end{bmatrix},$$

where $M_i(g)$ is the submatrix corresponding to the inertia group $\overline{H_i}$ and its inertia factor H_i. If $H_i \cap [g] = \emptyset$, then $M_i(g)$ will not exist and $M(g)$ does not contain $M_i(g)$. The size of the matrix $M(g)$ is $p \times c(g)$ where p is the number of conjugacy classes of elements of the inertia factors H_i's for $1 \le i \le t$ which fuse into $[g]$ in G and $c(g)$ is the number of conjugacy classes of elements of G which correspond to the coset $N\bar{g}$. Then $M(g)$ is the *Fischer–Clifford matrix* of G corresponding to the coset $N\bar{g}$. We will see later that $M(g)$ is a $c(g) \times c(g)$ nonsingular matrix.

Let $R(g) = \{(i, y_k)\,|\,1 \le i \le t, H_i \cap [g] \ne \emptyset, 1 \le k \le r\}$ and we note that y_k runs over representatives of the conjugacy classes of elements of H_i which fuse into $[g]$ in G. Following the notation used in Whitley [68] we denote $M(g)$ by writing $M(g) = (a^j_{(i,y_k)})$, where

$a^j_{(i,y_k)} = \sum_k' \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_i}(y_{l_k})|} \psi_i(y_{l_k})$, with columns indexed by $X(g)$ and rows indexed by $R(g)$.

Then the partial character table of $\bar{G}$ on the classes $\{x_1, x_2, \cdots, x_{c(g)}\}$ is given by

$$\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$$

where the Fischer-Clifford matrix $M(g)$ is divided into blocks $M_i(g)$ with each block corresponding to an inertia group $\overline{H_i}$ and $C_i(g)$ is the partial character table of H_i consisting of the columns corresponding to the classes that fuse into $[g]$ in G. We can also observe that the number of irreducible characters of $\bar{G}$ is the sum of the numbers of irreducible characters of the inertia factors H_i's.

5.2 Properties of Fischer-Clifford matrices

We shall discuss the properties which are useful in the computation of the Fischer–Clifford matrices. We only provide a selection of proofs of these properties. At the end of the section the properties of the Fischer–Clifford matrices will be summarized. Let K be a group and $A \leq Aut(K)$. Then by Brauer's theorem A acts on the conjugacy classes of elements of K and on the irreducible characters of K resulting in the same number of orbits.

Lemma 5.2.1. Suppose we have the following matrix describing the above actions :

$$\begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_i \\ \vdots \\ s_t \end{array} \begin{array}{c} \begin{array}{cccccc} 1 = l_1 & l_2 & \dots & l_j & \cdots & l_t \end{array} \\ \begin{pmatrix} 1 & 1 & \cdots & 1 & \cdots & 1 \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2t} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{it} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{tj} & \cdots & a_{tt} \end{pmatrix} \end{array}$$

where $a_{1j} = 1$ for $j \in \{1, 2, \cdots, t\}$, l_j's are lengths of orbits of A on the conjugacy classes of K, s_i's are the lengths of orbits of A on $Irr(K)$ and a_{ij} is the sum of s_i irreducible characters of K on the element x_j, where x_j is an element of the orbit of length l_j. Then the following relation holds for $i,\, i' \in \{1, 2, \cdots, t\}$: $\sum_{j=1}^{t} a_{ij}\, \overline{a_{i'j}}\, l_j = |K| s_i\, \delta_{ii'}$.

Proof: Let $\underline{s_i}$ denote the sum of s_i irreducible characters of K,
so $\underline{s_i}(x_j) = a_{ij}$. Then $< \underline{s}_i, \underline{s}_{i'} >= \frac{1}{|K|} \sum_{j=1}^{t} l_j \underline{s_i}(x_j) \overline{\underline{s_{i'}}(x_j)} = |K|^{-1} \sum_{j=1}^{t} l_j a_{ij} \overline{a_{i'j}}$

By the orthogonality of irreducible characters,

$< \underline{s}_i, \underline{s}_{i'} >= s_i \delta_{ii'}$, and hence $\sum_{j=1}^{t} l_j a_{ij} \overline{a_{i'j}} = |K| s_i\, \delta_{ii'}$. $\square$

Let $x_j \in X(g)$ and define $m_j = [C_{\bar{g}} : C_{\bar{G}}(x_j)]$. The Fischer-Clifford matrix $M(g)$ is partitioned row–wise into blocks, where each block corresponds to an inertia group. The columns of $M(g)$ are indexed by $X(g)$ and for each $x_j \in X(g)$, at the top of the columns of $M(g)$, we write $|C_{\bar{G}}(x_j)|$ and at the bottom we write m_j. The rows of $M(g)$ are indexed by $R(g)$ and on the left of each row we write $|C_{\bar{H}_i}(y_k)|$, where y_k fuses into $[g]$ in G. Then in general we can write $M(g)$ with corresponding weights for rows and columns as follows, where blocks corresponding to the inertia groups are separated by horizontal lines.

$$
\begin{array}{c|cccc|}
 & |C_{\bar{G}}(x_1)| & |C_{\bar{G}}(x_2)| & \cdots & |C_{\bar{G}}(x_{c(g)})| \\
\hline
|C_G(g)| & a^1_{(1,g)} & a^2_{(1,g)} & \cdots & a^{c(g)}_{(1,g)} \\
\hline
|C_{H_2}(y_1)| & a^1_{(2,y_1)} & a^2_{(2,y_1)} & \cdots & a^{c(g)}_{(2,y_1)} \\
|C_{H_2}(y_2)| & a^1_{(2,y_2)} & a^2_{(2,y_2)} & \cdots & a^{c(g)}_{(2,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_i}(y_1)| & a^1_{(i,y_1)} & a^2_{(i,y_1)} & \cdots & a^{c(g)}_{(i,y_1)} \\
|C_{H_i}(y_2)| & a^1_{(i,y_2)} & a^2_{(i,y_2)} & \cdots & a^{c(g)}_{(i,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
|C_{H_t}(y_1)| & a^1_{(t,y_1)} & a^2_{(t,y_1)} & \cdots & a^{c(g)}_{(t,y_1)} \\
|C_{H_t}(y_2)| & a^1_{(t,y_2)} & a^2_{(t,y_2)} & \cdots & a^{c(g)}_{(t,y_2)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
 & m_1 & m_2 & \cdots & m_{c(g)}
\end{array}
$$

From the theory of coset analysis for computing the conjugacy classes of elements of $\overline{G} = N \cdot G$ where N is abelian, we observe that

$$m_j = [C_{\bar{g}} : C_{\bar{G}}(x_j)] = \frac{f \cdot |N|}{k}.$$

The following result gives the orthogonality relation for $M(g)$.

Proposition 5.2.2. ***(Column orthogonality)*** *Let $\overline{G} = N \cdot G$, then*

$$\sum_{(i,y_k)\in R(g)} |C_{\bar{H}_i(y_k)}| \, a^j_{(i,y_k)} \, \overline{a^{j'}_{(i,y_k)}} = \delta_{jj'} \, |C_{\bar{G}}(x_j)| \, .$$

Proof: The partial character table of $\overline{G}$ at classes $x_1, \cdots, x_{c(g)}$ is given by

$$\begin{bmatrix} C_1(g)\, M_1(g) \\ C_2(g)\, M_2(g) \\ \vdots \\ C_t(g)\, M_t(g) \end{bmatrix}$$

By column orthogonality of the character table of $\overline{G}$, we have

$$\begin{aligned}
|C_{\bar{G}}(x_j)|\,\delta_{jj'} &= \sum_{i=1}^{t} \sum_{\beta_i \in Irr(\bar{H}_i)} \Big(\sum_{y_k:(i,y_k)\in R(g)} a^{j}_{(i,y_k)}\, \beta_i(y_k) \Big) \overline{\Big(\sum_{y'_k:(i,y'_k)\in R(g)} a^{j'}_{(i,y'_k)}\, \beta_i(y'_k) \Big)} \\
&= \sum_{i=1}^{t} \sum_{\beta_i \in Irr(\bar{H}_i)} \Big(\sum_{y_k} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y'_k)}}\, \beta_i(y_k)\, \overline{\beta_i(y_k)} + \\
&\quad \sum_{y_k} \sum_{y'_k \neq y_k} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y'_k)}}\, \beta_i(y_k)\, \overline{\beta_i(y'_k)} \Big) \\
&= \sum_{i=1}^{t} \Big(\sum_{y_k} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y_k)}} \sum_{\beta_i \in Irr(\bar{H}_i)} \beta_i(y_k)\, \overline{\beta_i(y_k)} + \\
&\quad \sum_{y_k} \sum_{y'_k \neq y_k} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y'_k)}} \sum_{\beta_i \in Irr(\bar{H}_i)} \beta_i(y_k)\, \overline{\beta_i(y'_k)} \Big) \\
&= \sum_{i=1}^{t} \Big(\sum_{y_k} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y_k)}}\, |C_{\bar{H}_i}(y_k)| + 0 \Big) \\
&= \sum_{(i,y_k)\in R(g)} a^{j}_{(i,y_k)}\, \overline{a^{j'}_{(i,y_k)}}\, |C_{\bar{H}_i}(y_k)|\,. \quad \square
\end{aligned}$$

Theorem 5.2.3. $a^{j}_{(1,g)} = 1$ *for all* $j = \{1, \cdots, c(g)\}$

Proof: For $y_{l_k} \sim x_j$ in $\bar{G}$, we have $|C_{\bar{G}}(x_j)| = |C_{\bar{H}_i}(y_{l_k})|$. Thus we obtain that $a^{j}_{(1,g)} = \sum'_l \frac{|C_{\bar{G}}(x_j)|}{|C_{\bar{H}_1}(y_{l_k})|} \psi_1(y_{l_k}) = \sum'_l 1 = 1$. Hence the result. $\square$

The properties of the Fischer–Clifford matrix $M(g)$ can be summarized as follows :

(a) $|X(g)| = |R(g)|$

(b) $\sum_{j=1}^{c(g)} m_j\, a^{j}_{(i,y_k)}\, \overline{a^{j}_{(i',y'_k)}} = \delta_{(i,y_k),(i',y'_k)} \frac{|C_G(g)|}{|C_{\bar{H}_i}(y_k)|}\, |N|$

(c) $\sum_{(i,y_k)\in R(g)} a^j_{(i,y_k)} \overline{a^{j'}_{(i,y_k)}} \, |C_{\bar{H}_i}(y_k)| = \delta_{jj'} \, |C_G(x_j)|$

(d) $M(g)$ is a square and nonsingular.

If N is elementary abelian, then we obtain the following additional properties of $M(g)$.

(e) $a^1_{(i,y_k)} = \frac{|C_G(g)|}{|C_{\bar{H}_i}(y_k)|}$

(f) $|a^1_{(i,y_k)}| \geq |a^j_{(i,y_k)}|$

Remark 5.2.4. Let $\bar{G} = N{:}G$ be a split extension, where N is an elementary abelian 2-group. It was shown in section 5.2.2 [53] that the Fischer-Clifford matrix $M(g)$ satisfies the following properties:

1. $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (mod\ 2)$ for all $j \geq 2$
2. $a^1_{(1,y_k)} \in \mathbb{N}$
3. For any j-th column of $M(g)$ for which $j \geq 2$, we obtain $\sum_i a^j_{(i,y_k)} = 0$

Since $a^j_{(i,y_k)} \in \mathbb{Z}$, we deduce that the Fischer-Clifford matrix $M(g)$ in Remark 5.2.4 will have integer entries $a^j_{(i,y_k)}$ such that $a^1_{(i,y_k)} \geq |a^j_{(i,y_k)}|$ and $a^j_{(i,y_k)} \equiv a^1_{(i,y_k)} (mod 2)$. If $a^1_{(i,y_k)} = n$ for some $n \in \mathbb{N}$, then for $j \geq 2$ we have $a^j_{(i,y_k)} \in \{\pm 1, \pm 3, ..., \pm n\}$ if n is odd and $a^j_{(i,y_k)} \in \{0, \pm 2, \pm 4, ..., \pm n\}$ if n is even. It is easy to see that for a fixed n there are $n+1$ possible values for each $a^j_{(i,y_k)}$ with $j \geq 2$.

The following additional information obtained from [53] is sometimes useful in the computations of the entries of $M(g)$:

1. For χ a character of any group H and $h \in H$, we have $|\chi(h)| \leq \chi(1_H)$, where 1_H is the identity element of H.
2. For χ a character of any group H and h a $p-$singular element of H, where p is a prime, then we have $\chi(h) \equiv \chi(h^p) mod p$.

3. For any irreducible character χ of a group H and for $h_i \in C_i$ then $d_i = \frac{b_i\chi(h_i)}{\chi(1_H)}$ is an algebraic integer, where C_i is the ith conjugacy class of H and $b_i = |C_i| = [H:C_H(h_i)]$. It is clear if $d_i \in \mathbb{Q}$, then $d_i \in \mathbb{Z}$.

Chapter 6

On a maximal subgroup of the automorphism group $U_6(2){:}2$ of $U_6(2)$

The unitary simple group $U = U_6(2)$ has outer automorphisms of order 2, 3 and 6 and hence automorphism groups of the form $U_1 = U_6(2){:}2$, $U_2 = U_6(2){:}3$ and $U_3 = U_6(2){:}S_3$ exist for U (see the ATLAS [22]). The reader is referred to [66] for more information about the construction of matrix representations for the covering and automorphism groups of $U_6(2)$. Recently in [58], a 3-local identification is given for the group $PSU_6 \cong U_6(2)$ and its automorphism groups $PSU_6(2){:}3$, $PSU_6(2){:}2$ and $PSU_6(2){:}S_3$. Also, we found in the ATLAS that one of the 16 maximal subgroups of U is a split extension group, say A, of the type $2^9{:}L_3(4)$ of index 891 and has order 10321920. The group A has automorphism groups of the form $A_1 = (2^9{:}L_3(4)){:}2$, $A_2 = (2^9{:}L_3(4)){:}3$ and $A_3 = (2^9{:}L_3(4)){:}S_3$ which sit maximally inside the groups U_1, U_2 and U_3, respectively. The character table of A is stored in the GAP Library [67], where as the character table of A_1 and A_2 are not yet uploaded in GAP. Also, since the Fischer-Clifford matrices of A_1 and A_2 are still not known, we will calculate the character tables of A_1 and A_2 by using the method of Fischer-Clifford matrices. In this chapter the Fischer-Clifford matrices of A_1 and its associated character table will be constructed and similar computations will be carried out for the group A_2 in Chapter 7. The method of coset-analysis as discussed in Chapter 3 will be used in the computation of the conjugacy classes of elements in both of the groups A_1 and A_2. The Fischer-Clifford matrices and character table of A_3 was determined in [59].

6.1 The group $2^9{:}(L_3(4){:}2)$

In this section, using a suitable permutation representation of $U_6(2){:}2$, we identify our group $A_1 = (2^9{:}L_3(4)){:}2$ as the split extension 2^9 by $L_3(4){:}2$ with the aid of GAP [67] and MAGMA [15]. Then with the help of MAGMA we represent $L_3(4){:}2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. We found that G acts absolutely irreducibly on its natural module 2^9 and hence the existence of a split extension $S = 2^9{:}(L_3(4){:}2)$. Then we create S as a subgroup of $SL_{10}(2)$ and show with the help of MAGMA that A_1 is indeed an isomorphic copy of S.

We construct $U_1 = U_6(2){:}2$ within GAP, using its smallest permutation representation of degree 672 found in Wilson's online ATLAS [70]. Next, we use the GAP commands "MS:= ConjugacyClassesMaximalSubgroups (U_1)", "A1:=MS[4]" and "Size(A1)" to represent $A_1 = (2^9{:}(L_3(4)){:}2$ as a permutation group on 672 points and then use this permutation representation to construct A_1 within MAGMA. Using the sequence of MAGMA commands "a,b:=ChiefSeries(A1)", "N:= a[3]", "NormalSubgroups(A1)", "IsNormal(A1,a[3])", "IsElemenataryAbelian(N)", "C:= Complements(A1,N)", "Order(C[1])", "C[1] meet N" and "IsIsomorphic(C[1], $L_3(4){:}2$)", we verified that $A_1 = (2^9{:}L_3(4)){:}2 \cong 2^9{:}(L_3(4){:}2)$.

Having A_1 as a permutation group on 672 points, we use the MAGMA commands "M:=GModule(A1,N)" and "'M:Maximal" to represent $L_3(4){:}2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. Thus we obtain the matrix group $G = < g_1, g_2 >$, where $o(g_1) = 2$ and $o(g_2) = 12$. The generators g_1 and g_2 of G are as follows:

$$g_1 = \begin{pmatrix} 1&0&0&0&0&1&1&1&0 \\ 0&1&0&1&0&1&0&0&0 \\ 1&1&0&1&0&0&0&0&0 \\ 1&0&0&1&1&1&0&0&1 \\ 0&0&1&1&1&1&1&1&1 \\ 0&1&1&1&1&0&0&0&1 \\ 0&1&0&0&1&0&0&0&0 \\ 1&0&1&0&1&1&1&0&0 \\ 0&1&0&0&1&0&1&0&1 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1&0&0&1&1&0&0&1&1 \\ 1&1&0&0&0&1&1&1&1 \\ 0&0&0&0&0&1&1&1&0 \\ 0&1&0&1&0&1&1&1&0 \\ 1&0&1&0&1&1&0&0&0 \\ 1&1&1&1&1&0&1&1&1 \\ 0&1&1&1&0&1&1&1&1 \\ 1&0&1&1&0&0&1&0&1 \\ 0&1&0&1&0&1&1&0&0 \end{pmatrix}.$$

The class representatives of each class $[g]_G$ of $G = L_3(4){:}2$ are given in terms of 9×9 matrices over $GF(2)$ and in total there are 14 conjugacy classes of elements and are listed in Table 6.1. The computation to determine the classes of G is carried out in MAGMA.

Table 6.1: The conjugacy classes of $L_3(4){:}2$

$[g]_{L_3(4):2}$	Class representative	$\lvert[g]_{L_3(4):2}\rvert$	$[g]_{L_3(4):2}$	Class representative	$\lvert[g]_{L_3(4):2}\rvert$
$1A$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0&0 \\ 1&1&1&1&0&0&0&0&0 \\ 1&0&0&0&1&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&1&0&0 \\ 1&1&1&0&0&0&0&0&1 \end{pmatrix}$	120
$2B$	$\begin{pmatrix} 1&1&0&0&0&0&0&0&0 \\ 0&1&0&1&1&1&1&0&1 \\ 1&1&0&1&1&0&1&1&1 \\ 1&1&1&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&1&0 \\ 1&1&1&1&0&1&0&0&1 \\ 0&0&0&1&1&1&0&0&1 \\ 0&1&0&1&0&1&1&0&1 \\ 0&1&0&1&0&1&1&1&0 \end{pmatrix}$	315	$3A$	$\begin{pmatrix} 0&0&1&0&1&0&0&1&1 \\ 1&1&0&0&1&1&1&1&0 \\ 1&0&1&1&1&0&0&0&1 \\ 1&0&1&0&0&0&1&0&0 \\ 1&1&0&0&0&1&0&1&1 \\ 0&0&1&0&0&1&1&1&1 \\ 0&0&0&1&1&0&0&1&0 \\ 0&1&1&0&0&0&0&0&0 \\ 1&1&0&1&0&0&1&1&0 \end{pmatrix}$	2240
$4A$	$\begin{pmatrix} 0&1&0&0&1&1&1&1&1 \\ 1&1&0&0&0&1&1&1&1 \\ 1&0&1&1&1&0&0&0&1 \\ 1&0&1&0&0&0&1&0&0 \\ 1&1&0&0&0&1&0&1&1 \\ 0&0&1&0&1&1&0&1&1 \\ 0&0&0&1&1&1&1&0&0 \\ 1&1&1&1&1&0&0&0&0 \\ 0&0&0&1&1&1&0&0&1 \end{pmatrix}$	1260	$4B$	$\begin{pmatrix} 1&0&0&1&0&1&0&0&0 \\ 0&1&0&1&1&1&0&1&1 \\ 0&1&1&0&1&0&1&1&1 \\ 0&0&0&1&0&0&0&0&1 \\ 0&0&0&0&1&1&0&1&0 \\ 0&0&0&0&1&1&0&1&1 \\ 1&1&0&1&0&1&1&0&1 \\ 0&0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&0&0&0 \end{pmatrix}$	2520

Table 6.1 (continue)

$[g]_{L_3(4):2}$	Class representative	$\lvert[g]_{L_3(4):2}\rvert$	$[g]_{L_3(4):2}$	Class representative	$\lvert[g]_{L_3(4):2}\rvert$
$4C$	$\begin{pmatrix} 1&0&1&1&0&1&0&1&1\\ 1&1&0&0&1&1&0&0&1\\ 1&0&0&1&1&0&0&1&1\\ 1&1&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&1&1&1\\ 1&0&1&0&0&0&1&1&1\\ 0&1&1&1&0&1&0&1&0\\ 0&0&0&0&1&0&1&0&0\\ 1&0&1&0&0&1&1&1&1 \end{pmatrix}$	2520	$5A$	$\begin{pmatrix} 1&0&0&0&0&0&1&0&0\\ 0&0&1&1&1&1&0&0&1\\ 1&0&0&1&0&0&1&0&1\\ 1&1&0&0&1&0&0&1&0\\ 1&1&1&1&0&1&0&1&0\\ 1&0&1&1&0&1&0&0&0\\ 1&1&1&1&0&1&1&0&0\\ 0&1&0&1&0&1&1&1&0\\ 1&0&1&1&0&1&0&1&1 \end{pmatrix}$	8064
$6A$	$\begin{pmatrix} 0&1&0&0&0&0&1&0&0\\ 1&1&0&1&1&0&0&1&0\\ 1&0&0&0&1&1&1&0&0\\ 1&0&0&1&1&0&1&0&0\\ 0&1&1&0&1&0&0&1&0\\ 1&1&1&1&1&0&1&1&1\\ 1&1&1&1&0&0&0&0&1\\ 1&0&1&0&0&1&1&1&0\\ 1&0&0&1&0&1&1&0&0 \end{pmatrix}$	6720	$7A$	$\begin{pmatrix} 0&1&1&0&0&0&1&1&0\\ 0&1&0&0&0&0&1&0&0\\ 1&1&0&0&0&0&1&0&0\\ 0&0&1&0&0&1&1&0&0\\ 1&1&0&1&0&0&1&1&1\\ 0&1&1&1&0&0&1&1&1\\ 0&0&0&1&1&1&1&1&1\\ 1&0&0&1&1&1&0&0&1\\ 1&0&0&1&0&0&0&1&0 \end{pmatrix}$	2880
$7B$	$\begin{pmatrix} 1&0&0&0&0&0&1&1&0\\ 1&1&0&0&1&0&0&1&1\\ 1&1&0&0&1&0&1&0&1\\ 0&0&0&1&1&1&1&0&1\\ 1&1&0&0&1&0&0&1&0\\ 1&1&1&1&0&1&0&0&1\\ 0&0&1&1&0&1&1&0&1\\ 1&1&0&1&0&1&0&1&1\\ 1&1&0&1&1&0&0&1&0 \end{pmatrix}$	2880	$8A$	$\begin{pmatrix} 0&0&1&0&1&0&0&1&1\\ 1&1&0&0&1&0&0&1&0\\ 0&1&0&1&0&1&0&1&0\\ 1&0&1&1&1&0&0&1&1\\ 0&1&1&0&1&1&1&0&0\\ 0&0&1&0&0&1&1&1&1\\ 1&1&1&0&0&0&0&0&0\\ 0&0&0&1&0&1&1&1&0\\ 0&0&1&0&0&1&1&0&0 \end{pmatrix}$	5040
$14A$	$\begin{pmatrix} 1&1&1&0&0&0&1&1&0\\ 1&1&1&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&1&0\\ 0&1&0&0&0&0&0&1&1\\ 0&1&0&1&1&1&0&0&1\\ 1&0&1&0&0&0&1&1&1\\ 1&1&1&0&1&0&0&1&0\\ 0&1&1&0&1&0&0&1&0\\ 1&0&0&1&0&0&1&0&1 \end{pmatrix}$	2880	$14B$	$\begin{pmatrix} 1&1&1&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&0\\ 0&0&0&1&1&1&1&1&0\\ 0&1&1&0&1&0&1&1&1\\ 1&1&1&0&1&1&1&0&0\\ 0&0&1&0&0&1&1&1&1\\ 0&1&1&1&1&0&0&1&0\\ 0&0&0&1&1&0&0&1&0\\ 0&0&1&0&1&0&1&1&0 \end{pmatrix}$	2880

The MAGMA command "IsAbsolutelyIrreducible(G)" tells us that the action of the matrix group G on its natural module 2^9 is absolutely irreducible. Thus a split extension of the type $2^9{:}(L_3(4){:}2)$ does exist. Hence we can construct $2^9{:}(L_3(4){:}2)$ as a subgroup S of $SL_{10}(2)$ such that $S =< s_1, s_2, s_3 >$ and $L_3(4){:}2 =< s_1, s_2 >$, where $o(s_1) = 4$, $o(s_2) = 14$ and $o(s_3) = 2$. The generators of the matrix group S of degree 10 over $GF(2)$ are as follows:

$$s_1=\begin{pmatrix} 1&0&0&0&0&1&1&1&0&0\\ 0&1&0&1&0&1&0&0&0&0\\ 1&1&0&1&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1&0\\ 0&0&1&1&1&1&1&1&1&0\\ 0&1&1&1&1&0&0&0&1&0\\ 0&1&0&0&1&0&0&0&0&0\\ 1&0&1&0&1&1&1&0&0&0\\ 0&1&0&0&1&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix} \quad s_2=\begin{pmatrix} 1&0&0&1&1&0&0&1&1&0\\ 1&1&0&0&0&1&1&1&1&0\\ 0&0&0&0&0&1&1&1&0&0\\ 0&1&0&1&0&1&1&1&0&0\\ 1&0&1&0&1&1&0&0&0&0\\ 1&1&1&1&1&0&1&1&1&0\\ 0&1&1&1&0&1&1&1&1&0\\ 1&0&1&1&0&0&1&0&1&0\\ 0&1&0&1&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$s_3=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

We also generate $2^9 =< n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9 >$ as a matrix group of degree 10 over $GF(2)$ within S and the generators are listed in Table 6.2.

Table 6.2: The generators of 2^9

$n_1=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0&0&1 \end{pmatrix}$	$n_2=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&1&0&1&0&1&0&0&0&1 \end{pmatrix}$
$n_3=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&1&1&0&0&1&1&0&0&1 \end{pmatrix}$	$n_4=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&1&0&1&1&1&1&0&1&1 \end{pmatrix}$
$n_5=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&1&1&1&0&0&0&1 \end{pmatrix}$	$n_6=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&1&1&0&0&1&0&1 \end{pmatrix}$

Table 6.2 (continue)

$n_7=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&1&1&1&1&1&1&0&1&1 \end{pmatrix}$	$n_8=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&1&1&1&1&0&1 \end{pmatrix}$
$n_9=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&1&0&0&0&1&0&1 \end{pmatrix}$	

Since we can represent S and A_1 as a matrix and permutation group, respectively, we use the MAGMA command "IsIsomorphic (S, A_1) to confirm that $S \cong A_1$. Hence we can regard $S = (2^9{:}L_3(4)){:}2$ as the split extension $S = 2^9{:}(L_3(4){:}2)$.

6.2 The conjugacy classes of $\overline{G} = 2^9{:}(L_3(4){:}2)$

In this section, the conjugacy classes of $\overline{G}$ are computed using the technique of coset analysis as discussed in Chapter 3.

Throughout the remainder of this chapter, let $\overline{G} = 2^9{:}(L_3(4){:}2)$ be a split extension of $N = 2^9$ by $L_3(4){:}2$, where N is the vector space $V_9(2)$ of dimension 9 over $GF(2)$ on which the linear group G acts. Since $G = < g_1, g_2 >$ is represented as a matrix group, we used the MAGMA commands "O:= Orbits(G)", "#O", "#O[1]", "#O[2]",#O[3]" and "#O[4]" to compute the orbit lengths of the action of G on N. We obtain 4 orbits of lengths 1, 21, 210 and 280 and using the MAGMA commands "P1:= Stabilizer(G,O[1,1])", "P2:= Stabilizer(G,O[2,1])", "P3:= Stabilizer(G,O[3,1])" and "P4:= Stabilizer(G,O[4,1])", we are able to compute the corresponding point stabilizers P_i, $i = 1, 2, 3, 4$, which are subgroups of G. With the aid of MAGMA and also checking the indices of the maximal subgroups of G in the ATLAS, the structures of the stabilizers P_i are identified as $P_1 = L_3(4){:}2$, $P_2 = 2^4{:}S_5$, $P_3 = 2^4{:}(2 \times S_3)$ and $P_4 = 3^2{:}(Q_8.2)$, where P_2 and P_4 are maximal subgroups of P_1 . We should note here that the group $L_3(4){:}2$ has two non-conjugate isomorphic maximal subgroups $L_1 = P_2$ and L_2, having the same structure $2^4{:}S_5$. The stabilizer P_3 sits maximally in L_2. Alternatively, we can use [21] to identify the structures of the groups P_i. Since the action of G on N does not fix any nontrivial subspace of 2^9, we have that 2^9 is an irreducible module for G. We can readily verify this fact by using the MAGMA command "IsIrreducible(G)".

Let $\chi(L_3(4){:}2|2^9)$ be the permutation character of $L_3(4){:}2$ on the classes of 2^9. Then, from methods that were developed by Mpono [53], we obtain that $\chi(L_3(4){:}2|2^9) = I_{P_1}^{P_1} + I_{P_2}^{P_1} + I_{P_3}^{P_1} + I_{P_4}^{P_1} = 4 \times 1a + 4 \times 20a + 2 \times 35a + 45a + 45b + 2 \times 64a + 2 \times 70a$, where $I_{P_1}^{P_1}$, $I_{P_2}^{P_1}$, $I_{P_3}^{P_1}$ and $I_{P_4}^{P_1}$ are the identity characters of the point stabilizers P_i, $i = 1, 2, 3, 4$, induced to G. Note that the identity characters $I_{P_i}^{P_1}$ are identified with the permutation characters $\chi(L_3(4){:}2|P_i)$ of $L_3(4){:}2$ acting on the classes of the point stabilizers P_i. We found that $I_{P_1}^{P_1} = \chi(L_3(4){:}3|P_1) = 1a$, $I_{P_1}^{P_2} = \chi(L_3(4){:}2|P_2) = 1a + 20a$, $I_{P_1}^{P_3} = \chi(L_3(4){:}2|P_3) = 1a + 2 \times 20a + 35a + 64a + 70a$ and $I_{P_1}^{P_4} = \chi(L_3(4){:}3|P_4) = 1a + 20a + 35a + 45a + 45b + 64a + 70a$. The permutation characters $\chi(L_3(4){:}2|P_i)$ are written in terms of the ordinary irreducible characters of G. Since we have the generators g_1 and g_2 for G, we compute the character tables of G and the P_i's directly in MAGMA and use these tables together with the fusion maps of the stabilizers into G , to compute $\chi(L_3(4){:}2|P_i)$ and $\chi(L_3(4){:}2|2^9)$. The values of $\chi(L_3(4){:}2|2^9)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^9. The values of k are listed in Table 6.3.

Table 6.3: The values of $\chi(L_3(4){:}2|2^9)$ on the different classes of $L_3(4){:}2$

$[h]_{L_3(4):2}$	$1A$	$2A$	$2B$	$3A$	$4A$	$4B$	$4C$
$\chi(L_3(4){:}3\|P_1)$	1	1	1	1	1	1	1
$\chi(L_3(4){:}3\|P_2)$	21	7	5	3	1	1	3
$\chi(L_3(4){:}3\|P_3)$	210	28	18	3	2	2	4
$\chi(L_3(4){:}3\|P_4)$	280	28	8	1	4	4	0
k	512	64	32	8	8	8	8

$[h]_{L_3(4):3}$	$5A$	$6A$	$7A$	$7B$	$8A$	$14A$	$14B$
$\chi(L_3(4){:}3\|P_1)$	1	1	1	1	1	1	1
$\chi(L_3(4){:}3\|P_2)$	1	1	0	0	1	0	0
$\chi(L_3(4){:}3\|P_3)$	0	1	0	0	0	0	0
$\chi(L_3(4){:}3\|P_4)$	0	1	0	0	2	0	0
k	2	4	1	1	4	1	1

The values of k enabled us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$, which have fused together under the action of $C_G(g)$, for each class representative $g \in G$, to form one orbit $\triangle_f$. Mpono in [53] used the technique of coset analysis to develop Programmes A and B (see Appendix A) in CAYLEY [20] for the computation of the conjucacy classes of a split extension $\overline{G} = N{:}G$, where N is an elementary abelian p - group for a prime p on which a linear group G acts. Ali [1] adapted Programmes A and B to be used in MAGMA. Programme A computes the values of the $f_j's$, whereas Programme B determines the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. We obtain that $\overline{G}$ has exactly 49 conjugacy classes. See Section 3.3 of Chapter 3 and Programmes A and B for information about the parameters d_j and w. The formula $|C_{\overline{G}}(x)| = \frac{k}{f}|C_G(g)|$ obtained from Chapter 3 is used to calculate the centralizer order of each class $[x]$ of $\overline{G}$. All the information involving the conjugacy classes of $\overline{G}$ are listed in Table 6.4.

Table 6.4: The conjugacy classes of elements of $\overline{G} = 2^9{:}(L_3(4){:}2)$

$[g]_G$	k	f_j	d_j	w	$[x]_{\overline{G}}$	$\lvert[x]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(x)\rvert$
$1A$	512	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$1A$	1	20643840
		$f_2 = 21$	$(0,0,0,0,1,1,0,1,1)$	$(0,0,0,0,1,1,0,1,1)$	$2A$	21	983040
		$f_3 = 210$	$(1,0,0,0,0,0,0,0,0)$	$(1,0,0,0,0,0,0,0,0)$	$2B$	210	98304
		$f_4 = 280$	$(1,0,1,0,1,1,1,0,1)$	$(1,0,1,0,1,1,1,0,1)$	$2C$	280	73728
$2A$	64	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2D$	960	21504
		$f_2 = 7$	$(0,0,0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2E$	6720	3072
		$f_3 = 7$	$(0,0,0,0,0,0,0,1,1)$	$(0,1,1,1,1,0,0,1,0)$	$4A$	6720	3072
		$f_4 = 21$	$(1,0,0,0,1,1,1,0,0)$	$(0,0,0,0,1,1,1,0,0)$	$4B$	20160	1024
		$f_5 = 28$	$(0,0,0,0,0,0,0,0,1)$	$(0,1,1,1,0,1,1,1,0)$	$4C$	26880	768
$2B$	32	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2F$	5040	4096
		$f_2 = 1$	$(0,0,0,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$2G$	5040	4096
		$f_3 = 1$	$(1,1,1,1,1,1,1,1,1)$	$(0,1,1,1,1,0,0,0,1)$	$4D$	5040	4096
		$f_4 = 1$	$(1,1,0,0,0,0,0,1,1)$	$(0,1,1,1,1,0,0,0,1)$	$4E$	5040	4096
		$f_5 = 4$	$(1,1,1,0,1,0,0,1,1)$	$(0,0,1,0,0,1,1,0,0)$	$4F$	20160	1024
		$f_6 = 8$	$(0,1,1,1,0,1,0,1,0)$	$(0,0,1,1,1,1,1,1,1)$	$4G$	40320	512
		$f_7 = 8$	$(1,1,1,1,1,0,1,1,1)$	$(0,0,0,0,1,1,1,1,0)$	$4H$	40320	512
		$f_8 = 8$	$(1,0,0,0,0,0,0,0,0)$	$(0,0,1,1,0,0,0,0,1)$	$4I$	40320	512
$3A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3A$	143360	144
		$f_2 = 1$	$(1,0,0,1,0,1,1,1,1)$	$(1,0,0,1,0,1,1,1,1)$	$6A$	143360	144
		$f_3 = 3$	$(1,1,1,1,0,1,0,0,1)$	$(1,1,1,1,0,1,0,0,1)$	$6B$	430080	48
		$f_4 = 3$	$(0,0,0,0,0,0,0,1,0)$	$(1,0,0,1,1,0,1,1,0)$	$6C$	430080	48
$4A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4J$	80640	256
		$f_2 = 1$	$(0,1,0,0,1,0,0,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$4K$	80640	256
		$f_3 = 2$	$(0,0,1,0,0,0,0,0,0)$	$(0,1,1,1,1,0,0,0,1)$	$8A$	161280	128
		$f_4 = 4$	$(1,0,1,0,1,0,1,1,1)$	$(0,1,0,1,1,1,1,0,1)$	$8B$	322560	64
$4B$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4L$	161280	128
		$f_2 = 1$	$(1,0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0,0)$	$4M$	161280	128
		$f_3 = 2$	$(1,0,0,0,1,1,1,1,1)$	$(0,0,1,1,0,0,0,1,0)$	$8C$	322560	64
		$f_4 = 4$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,0,1,1,0,0,1,0)$	$8D$	645120	32
$4C$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4N$	161280	128
		$f_2 = 1$	$(1,0,1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$4O$	161280	128
		$f_3 = 1$	$(1,0,0,0,1,1,0,1,0)$	$(1,0,1,0,1,1,1,1,0)$	$8E$	161280	128
		$f_4 = 1$	$(0,1,1,1,1,1,1,1,0)$	$(1,0,1,0,1,1,1,1,0)$	$8F$	161280	128
		$f_5 = 2$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,0,1,1,1,0,0,1)$	$8G$	322560	64
		$f_6 = 2$	$(1,0,1,0,0,1,0,0,1)$	$(0,1,1,1,0,0,1,1,1)$	$8H$	322560	64
$5A$	2	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$5A$	2064384	10
		$f_2 = 1$	$(1,0,0,0,0,0,0,0,0)$	$(0,0,0,0,1,1,0,0,1)$	$10A$	2064384	10
$6A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$6D$	860160	24
		$f_2 = 1$	$(1,0,0,0,1,0,1,1,0)$	$(0,0,0,0,0,0,0,0,0)$	$6E$	860160	24
		$f_3 = 1$	$(1,0,0,1,1,0,1,1,1)$	$(1,0,0,1,0,1,1,1,0)$	$12A$	860160	24
		$f_4 = 1$	$(1,1,1,0,1,0,1,1,1)$	$(1,0,0,1,0,1,1,1,0)$	$12B$	860160	24
$7A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$7A$	1474560	14
$7B$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$7B$	1474560	14
$8A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$8H$	645120	32
		$f_2 = 1$	$(1,0,0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0,0,0)$	8	645120	32
		$f_3 = 1$	$(1,1,1,0,1,0,1,1,1)$	$(0,0,0,0,0,0,0,1,1)$	$16A$	645120	32
		$f_4 = 1$	$(1,1,1,1,1,1,0,0,0)$	$(0,0,0,0,0,0,0,1,1)$	$16B$	645120	32
$14A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$14A$	1474560	14
$14B$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$14B$	1474560	14

6.3 The inertia groups of $\overline{G} = 2^9{:}(L_3(4){:}2)$

Since G has four orbits on N, then by Brauer's Theorem [29] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be 1 ,r, s and t where $r + s + t = 511$, with corresponding point stabilizers H_1, H_2, H_3 and H_4 as subgroups of G such that $[G : H_1] =1$, $[G : H_2] =r$, $[G : H_3]= s$ and $[G : H_4] = t$. We generate G as a permutation group on a set of cardinality 672 within MAGMA. Then the maximal and submaximal subgroups of G are computed. Now, considering the indices of these subgroups in G, the number of the classes of these subgroups, and also the fact that $\overline{G}$ has 49 conjugacy classes, we deduce that the action of G on $Irr(N)$ has orbits of lengths 1, $r = 21$, $s = 210$ and $t = 280$ with respective point stabilizers $H_1 = L_3(4){:}2$, $H_2 = 2^4{:}S_5$, $H_3 = 2^4{:}(2 \times S_3)$ and $H_4 = 3^2{:}Q_8{:}2$. Thus we obtain four inertia groups $\overline{H}_i = 2^9{:}H_i$, $i \in \{1, 2, 3, 4\}$, in $2^9{:}(L_3(4){:}2)$. Alternatively, we can also determine the inertia factor groups if we let T be the matrix group of dimension 9 over $GF(2)$ formed by the transpose of the generators of G. Then the action of T on the classes of $N = 2^9$ is the equivalent of G acting on $Irr(N)$. Then with the help of MAGMA or GAP, we can easily verify that the action of T on N has orbits of lengths 1, 21, 210 and 280 with corresponding point stabilizers T, $2^4{:}S_5$, $2^4{:}(2 \times S_3)$ and $3^2{:}Q_8{:}2$. The structures of H_2 and H_4 have been identified by checking the indices of the maximal subgroups of $L_3(4){:}2 \cong L_3(4).2_2$ in the ATLAS. The structure of H_3 was determined by direct computations in MAGMA. The groups H_2, H_3 and H_4 are constructed from elements within G and the generators are as follows:

- $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 3A$, $\alpha_2 \in 6A$ where

$$\alpha_1 = \begin{pmatrix} 0&0&0&0&1&0&1&0&0 \\ 0&1&0&0&1&0&1&0&0 \\ 1&0&0&1&0&0&1&1&1 \\ 1&1&1&0&1&1&1&0&0 \\ 0&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&1&0&0&0 \\ 1&0&0&0&1&0&0&1&0 \\ 1&0&0&0&0&1&1&0&0 \\ 0&0&1&1&0&1&0&1&0 \end{pmatrix} , \quad \alpha_2 = \begin{pmatrix} 1&0&0&0&1&1&0&0&0 \\ 0&1&1&1&1&0&0&1&0 \\ 1&1&0&0&1&0&1&1&1 \\ 1&0&1&0&0&0&0&1&0 \\ 1&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&1&0 \\ 0&0&0&0&0&1&1&1&0 \\ 0&0&1&1&0&1&1&0&0 \end{pmatrix}$$

- $H_3 = \langle \beta_1, \beta_2 \rangle$, $\beta_1 \in 4C$, $\beta_2 \in 6A$ where

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&1&0&1&0&1&1&0&0\\ 0&0&0&1&1&1&1&1&1\\ 0&0&1&1&1&1&0&1&1\\ 1&0&0&1&1&1&0&0&0\\ 1&0&1&0&0&0&1&1&1\\ 1&0&0&1&0&1&1&0&0\\ 1&1&0&1&0&1&1&0&1\\ 1&1&0&1&1&0&1&1&0 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1\\ 1&0&0&1&1&1&1&1&1\\ 0&1&0&0&1&0&0&0&1\\ 1&0&0&1&1&1&0&0&0\\ 1&0&1&0&0&0&1&1&1\\ 0&0&1&1&0&1&0&1&1\\ 1&1&0&1&0&1&1&0&1\\ 1&1&0&1&1&0&1&1&0 \end{pmatrix}$$

- $H_4 = \langle \gamma_1, \gamma_2 \rangle$, $\gamma_1 \in 2A$, $\gamma_2 \in 8A$ where

$$\gamma_1 = \begin{pmatrix} 0&0&0&0&1&0&1&0&0\\ 0&1&0&0&0&0&0&0&0\\ 1&0&0&1&1&0&0&1&1\\ 1&0&0&1&1&0&1&0&0\\ 1&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1&1&0\\ 0&0&1&1&0&1&0&1&0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 1&1&0&1&0&0&0&0&0\\ 1&1&0&0&0&1&1&1&1\\ 0&1&1&1&1&1&1&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&1&1\\ 0&0&0&0&1&1&0&0&1\\ 0&0&0&1&1&0&0&1&1\\ 0&0&0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0&0&1 \end{pmatrix}$$

For the purpose of constructing the character table of $\overline{G}$, we use the above generators of the H_i's to compute their character tables and these tables are available in the Appendix A.

6.4 The fusion of H_2, H_3 and H_4 into G

We obtain the fusions of the inertia factors H_2, H_3 and H_4 into G by using direct matrix conjugation in G and their permutation characters in G of degrees 21, 210 and 280, respectively. MAGMA was used for the various computations. The fusion maps of H_2, H_3 and H_4 into G are shown in Tables 6.5, 6.6 and 6.7.

Table 6.5: **The fusion of H_2 into $L_3(4){:}2$**

$[h]_{H_2}$ $\rightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$ $\rightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$ $\rightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$ $\rightarrow$	$[g]_{L_3(4):2}$
$1A$	$1A$	$2C$	$2B$	$4B$	$4B$	$5A$	$5A$
$2A$	$2B$	$3A$	$3A$	$4C$	$4C$	$6A$	$6A$
$2B$	$2A$	$4A$	$4A$	$4D$	$4C$	$8A$	$8A$

Table 6.6: **The fusion of H_3 into $L_3(4){:}2$**

$[h]_{H_3}$ →	$[g]_{L_3(4):2}$	$[h]_{H_3}$ →	$[g]_{L_3(4):2}$	$[h]_{H_3}$ →	$[g]_{L_3(4):2}$	$[h]_{H_3}$ →	$[g]_{L_3(4):2}$
$1A$	$1A$	$2D$	$2B$	$4A$	$4C$	$4E$	$4B$
$2A$	$2B$	$2E$	$2B$	$4B$	$4C$	$6A$	$6A$
$2B$	$2A$	$2F$	$2A$	$4C$	$4A$		
$2C$	$2B$	$3A$	$3A$	$4D$	$4C$		

Table 6.7: **The fusion of H_4 into $L_3(4){:}2$**

$[h]_{H_4}$ →	$[g]_{L_3(4):2}$	$[h]_{H_4}$ →	$[g]_{L_3(4):2}$	$[h]_{H_4}$ →	$[g]_{L_3(4):2}$	$[h]_{H_4}$ →	$[g]_{L_3(4):2}$
$1A$	$1A$	$2B$	$2A$	$4B$	$4B$	$6B$	$6B$
$2A$	$2B$	$3A$	$3A$	$4A$	$4A$		

6.5 The Fischer-Clifford Matrices of $2^9{:}(L_3(4){:}2)$

Having obtained the fusions of the inertia factors into $L_3(4){:}2$ and the conjugacy classes of $L_3(4){:}2$ displayed in the format of Table 6.4, we can proceed to use the theory and properties discussed in Chapter 5 to help us in the construction of the Fischer-Clifford matrices of $2^9{:}(L_3(4){:}2)$. Note that all the relations hold since 2^9 is an elementary abelian group.

For example, consider the conjugacy class $2A$ of $L_3(4){:}2$. Then we obtain that $M(2A)$ has the following form with corresponding weights attached to the rows and columns:

$$
M(2A) = \begin{array}{c} |C_G(2A)| \\ |C_{H_2}(2B)| \\ |C_{H_3}(2B)| \\ |C_{H_3}(2F)| \\ |C_{H_4}(2B)| \end{array}
\overset{\begin{array}{ccccc} |C_{\overline{G}}(2D)| & |C_{\overline{G}}(2E)| & |C_{\overline{G}}(4A)| & |C_{\overline{G}}(4B)| & |C_{\overline{G}}(4C)| \end{array}}{\underset{\begin{array}{ccccc} m_1 & m_2 & m_3 & m_4 & m_5 \end{array}}{\left(\begin{array}{ccccc} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{array}\right)}}.
$$

$$
M(2A) = \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array}
\begin{array}{c}
\begin{array}{ccccc} 21504 & 3072 & 3072 & 1024 & 768 \end{array} \\
\left(\begin{array}{ccccc} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{array} \right) \\
\begin{array}{ccccc} 8 & 56 & 56 & 168 & 224 \end{array}
\end{array}.
$$

By Theorem 5.2.3 and property (e) of the Fischer-Clifford matrix $M(g)$ (see Chapter 5), we have $a = f = k = p = u = 1$, $b = c = 7$, $d = 21$ and $e = 28$. Thus we get the following form

$$
M(2A) = \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array}
\begin{array}{c}
\begin{array}{ccccc} 21504 & 3072 & 3072 & 1024 & 768 \end{array} \\
\left(\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 7 & g & l & q & v \\ 7 & h & m & r & w \\ 21 & i & n & s & x \\ 28 & j & o & t & y \end{array} \right) \\
\begin{array}{ccccc} 8 & 56 & 56 & 168 & 224 \end{array}
\end{array}.
$$

By the orthogonality relations for columns and rows (see properties (b) and (c) of Fischer-Clifford matrices in Chapter 5), we obtain 28 equations and are listed as follows:

1. $12g^2 + 12h^2 + 4i^2 + 3j^2 = 684$.
2. $12l^2 + 12m^2 + 4n^2 + 3o^2 = 684$.
3. $12q^2 + 12r^2 + 4s^2 + 3t^2 = 172$.
4. $12v^2 + 12w^2 + 4x^2 + 3y^2 = 108$.
5. $12gl + 12hm + 4ni + 3jo = -84$.
6. $12gq + 12hr + 4si + 3jt = -84$.
7. $12gv + 12hw + 4xi + 3jy = -84$.
8. $12lq + 12mr + 4ns + 3ot = -84$.
9. $12lv + 12mw + 4nx + 3oy = -84$.
10. $12qv + 12rw + 4sx + 3yt = -84$.
11. $g + h + i + j = -1$.

12. $l + m + n + o = -1$.

13. $q + r + s + t = -1$.

14. $v + w + x + y = -1$.

15. $hi + mn + 3sr + 4xw = -21$.

16. $gi + ln + 3sq + 4xv = -21$.

17. $i + n + 3s + 4x = -3$.

18. $gh + ml + 3qr + 4wv = -7$.

19. $h + m + 3r + 4w = -1$.

20. $g + l + 3q + 4v = -1$.

21. $ij + on + 3st + 4xy = -84$.

22. $jh + om + 3rt + 4yw = -28$.

23. $jg + ol + 3qt + 4yv = -28$.

24. $j + o + 3t + 4y = -4$.

25. $j^2 + o^2 + 3t^2 + 4y^2 = 144$.

26. $i^2 + n^2 + 3s^2 + 4x^2 = 129$.

27. $h^2 + m^2 + 3r^2 + 4w^2 = 57$.

28. $g^2 + l^2 + 3q^2 + 4r^2 = 57$.

Solving these equations and with the help of the remaining properties discussed in Chapter 5, we obtain the desired Fischer-Clifford matrix $M(2A)$ of $\overline{G}$ given below:

$$M(2A)=\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}.$$

For each class representative $g \in L_3(4){:}2$, we construct a Fischer-Clifford matrix $M(g)$. These are listed in Table 6.8 .

Table 6.8: The Fischer-Clifford Matrices of $2^9{:}(L_3(4){:}2)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 21 & -11 & 5 & -3 \\ 210 & 50 & 2 & -6 \\ 280 & -40 & -8 & 8 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & 0 & -2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & -2 \\ 4 & -4 & 4 & -4 & 0 & 2 & -2 & 0 \\ 4 & 4 & 4 & 4 & -4 & 0 & 0 & 0 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(7A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(7B) = \begin{pmatrix} 1 \end{pmatrix}$	$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(14A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(14B) = \begin{pmatrix} 1 \end{pmatrix}$

6.6 Character Table of $2^9{:}(L_3(4){:}2)$

Having obtained the Fischer-Clifford matrices, the fusion maps of the H_i's into $L_3(4){:}2$, and the character tables of the inertia factors H_i, we construct the character table of $2^9{:}(L_3(4){:}2)$ following the methodology discussed in Chapter 5. For example, we calculate the partial character table of $2^9{:}(L_3(4){:}2)$ corresponding to the coset of $2A \in L_3(4){:}2$. From the Fischer-Clifford matrix $M(2A)$ we obtain that

$M_1(2A) = (1 \quad 1 \quad 1 \quad 1 \quad 1)$, $\qquad M_2(2A)=\begin{pmatrix} 7 & -1 & -5 & 3 & -1 \end{pmatrix}$,

$M_3(2A)=\begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix}$ and $M_4(2A) = (28 \ -4 \ -4 \ -4 \ 4)$.

Let $C_1(2A)$, $C_2(2A)$, $C_3(2A)$ and $C_4(2A)$ be the partial character tables of the inertia factors for the classes which fuse to $2A \in L_3(4){:}2$. Then the partial character table of $2^9{:}(L_3(4){:}2)$ on the classes $\{2D, 2E, 4A, 4B, 4C\}$ is given by:

$$C_1(2A)M_1(2A)=\begin{pmatrix} 1 \\ -1 \\ -6 \\ 6 \\ -7 \\ 7 \\ -3 \\ -3 \\ 3 \\ 3 \\ 8 \\ -8 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -6 & -6 & -6 & -6 & -6 \\ 6 & 6 & 6 & 6 & 6 \\ -7 & -7 & -7 & -7 & -7 \\ 7 & 7 & 7 & 7 & 7 \\ -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 8 & 8 & 8 & 8 & 8 \\ -8 & -8 & -8 & -8 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_2(2A)M_2(2A)=\begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \\ -1 \\ 1 \\ 0 \\ -3 \\ -3 \\ 3 \\ 3 \\ 0 \end{pmatrix}\begin{pmatrix} 7 & -1 & -5 & 3 & -1 \end{pmatrix} = \begin{pmatrix} 7 & -1 & -5 & 3 & -1 \\ -7 & 1 & 5 & -3 & 1 \\ 14 & -2 & -10 & 6 & -2 \\ -14 & 2 & 10 & -6 & 2 \\ -7 & 1 & 5 & -3 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ -21 & 3 & 15 & -9 & 3 \\ -21 & 3 & 15 & -9 & 3 \\ 21 & -3 & -15 & 9 & -3 \\ 21 & -3 & -15 & 9 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_3(2A)M_3(2A)=\begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ 3 & -1 \\ -3 & -1 \\ 3 & 1 \\ -3 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 28 & 4 & 8 & 0 & -4 \\ -28 & -4 & -8 & 0 & 4 \\ -14 & 10 & -10 & -2 & 2 \\ 14 & -10 & 10 & 2 & -2 \\ -14 & -14 & 2 & 2 & 2 \\ 14 & 14 & -2 & -2 & -2 \\ 0 & 24 & -12 & -4 & 0 \\ -42 & -18 & -6 & 2 & 6 \\ 42 & 18 & 6 & -2 & -6 \\ 0 & -24 & 12 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -42 & 6 & -18 & -2 & 6 \\ 42 & -6 & 18 & 2 & -6 \end{pmatrix}$$

$$C_4(2A)M_4(2A)=\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \end{pmatrix}\begin{pmatrix} 28 & -4 & -4 & -4 & 4 \end{pmatrix} = \begin{pmatrix} 28 & -4 & -4 & -4 & 4 \\ -28 & 4 & 4 & 4 & -4 \\ -28 & 4 & 4 & 4 & -4 \\ 28 & -4 & -4 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -56 & 8 & 8 & 8 & -8 \\ 56 & -8 & -8 & -8 & -8 \end{pmatrix}$$

Similarly, the partial character table associated with each coset Ng is computed. If necessary, we will restrict some characters of $Irr(U_6(2){:}2)$ to $\overline{G}$, to ensure that each partial character table corresponding to a coset Ng, will give rise to the desired set $Irr(\overline{G})$.

The character table of $\overline{G}$ will be partitioned row-wise into 4 blocks Δ_1, Δ_2,Δ_3 and Δ_4 where each block corresponds to an inertia group $\overline{H}_i = 2^9{:}H_i$. Therefore $Irr(2^9{:}(L_3(4){:}2)) = \bigcup_{i=1}^{4} \Delta_i$, where $\Delta_1 = \{\chi_j | 1 \leq j \leq 14\}$, $\Delta_2 = \{\chi_j | 15 \leq j \leq 26\}$, $\Delta_3 = \{\chi_j | 27 \leq j \leq 40\}$ and $\Delta_4 = \{\chi_j | 41 \leq j \leq 49\}$. The character table of $2^9{:}(L_3(4){:}2)$ is shown in Table 6.9. The consistency and accuracy of the character table of $2^9{:}(L_3(4){:}2)$ have been tested by using the GAP codes labelled as Programme C (see Appendix A).

Table 6.9: The Character table of $2^9{:}(L_3(4){:}2)$

	1A				2A					2B		
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$4B$	$4C$	$2F$	$2G$	$4D$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	1	1	1
χ_3	20	20	20	20	-6	-6	-6	-6	-6	4	4	4
χ_4	20	20	20	20	6	6	6	6	6	4	4	4
χ_5	35	35	35	35	-7	-7	-7	-7	-7	3	3	3
χ_6	35	35	35	35	7	7	7	7	7	3	3	3
χ_7	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_8	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_9	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{10}	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{11}	64	64	64	64	8	8	8	8	8	0	0	0
χ_{12}	64	64	64	64	-8	-8	-8	-8	-8	0	0	0
χ_{13}	70	70	70	70	0	0	0	0	0	6	6	6
χ_{14}	126	126	126	126	0	0	0	0	0	-2	-2	-2
χ_{15}	21	-11	5	-3	7	-1	-5	3	-1	5	-3	5
χ_{16}	21	-11	5	-3	-7	1	5	-3	1	5	-3	5
χ_{17}	84	-44	20	-12	14	-2	-10	6	-2	4	4	4
χ_{18}	84	-44	20	-12	-14	2	10	-6	2	4	4	4
χ_{19}	105	-55	25	-15	-7	1	5	-3	1	9	1	9
χ_{20}	105	-55	25	-15	7	-1	-5	3	-1	9	1	9
χ_{21}	126	-66	30	-18	0	0	0	0	0	-2	14	-2
χ_{22}	315	-165	75	-45	-21	3	15	-9	3	11	-13	11
χ_{23}	315	-165	75	-45	-21	3	15	-9	3	-5	3	-5
χ_{24}	315	-165	75	-45	21	-3	-15	9	-3	11	-13	11
χ_{25}	315	-165	75	-45	21	-3	-15	9	-3	-5	3	-5
χ_{26}	630	-330	150	-90	0	0	0	0	0	-10	6	-10
χ_{27}	210	50	2	-6	28	4	8	0	-4	18	10	2
χ_{28}	210	50	2	-6	-28	-4	-8	0	4	18	10	2
χ_{29}	210	50	2	-6	-14	10	-10	-2	2	2	-6	18
χ_{30}	210	50	2	-6	14	-10	10	2	-2	2	-6	18
χ_{31}	420	100	4	-12	-14	-14	2	2	2	20	4	20
χ_{32}	420	100	4	-12	14	14	-2	-2	-2	20	4	20
χ_{33}	630	150	6	-18	0	24	-12	-4	0	-10	-2	6
χ_{34}	630	150	6	-18	-42	-18	-6	2	6	6	14	-10
χ_{35}	630	150	6	-18	42	18	6	-2	-6	6	14	-10
χ_{36}	630	150	6	-18	0	-24	12	4	0	-10	-2	6
χ_{37}	1260	300	12	-36	0	0	0	0	0	-20	-4	12
χ_{38}	1260	300	12	-36	0	0	0	0	0	12	28	-20
χ_{39}	1260	300	12	-36	-42	6	-18	-2	6	-4	-20	-4
χ_{40}	1260	300	12	-36	42	-6	18	2	-6	-4	-20	-4
χ_{41}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{42}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{43}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{44}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{45}	560	-80	-16	16	0	0	0	0	0	16	-16	-16
χ_{46}	560	-80	-16	16	0	0	0	0	0	-16	16	16
χ_{47}	560	-80	-16	16	0	0	0	0	0	-16	16	16
χ_{48}	2240	-320	-64	64	-56	8	8	8	-8	0	0	0
χ_{49}	2240	-320	-64	64	56	-8	-8	-8	8	0	0	0

Table 6.9 (continue)

	$2B$					$3A$				$4A$			
	$4E$	$4F$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$6C$	$4J$	$4K$	$8A$	$8B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	4	4	4	4	4	2	2	2	2	0	0	0	0
χ_4	4	4	4	4	4	2	2	2	2	0	0	0	0
χ_5	3	3	3	3	3	-1	-1	-1	-1	3	3	3	3
χ_6	3	3	3	3	3	-1	-1	-1	-1	3	3	3	3
χ_7	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_8	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_9	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_{10}	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_{11}	0	0	0	0	0	1	1	1	1	0	0	0	0
χ_{12}	0	0	0	0	0	1	1	1	1	0	0	0	0
χ_{13}	6	6	6	6	6	-2	-2	-2	-2	-2	-2	-2	-2
χ_{14}	-2	-2	-2	-2	-2	0	0	0	0	-2	-2	-2	-2
χ_{15}	-3	1	-3	1	1	3	-3	1	-1	1	1	1	-1
χ_{16}	-3	1	-3	1	1	3	-3	1	-1	1	1	1	-1
χ_{17}	4	4	-4	-4	4	3	-3	1	-1	0	0	0	0
χ_{18}	4	4	-4	-4	4	3	-3	1	-1	0	0	0	0
χ_{19}	1	5	-7	-3	5	-3	3	-1	1	1	1	1	-1
χ_{20}	1	5	-7	-3	5	-3	3	-1	1	1	1	1	-1
χ_{21}	14	6	-2	-10	6	0	0	0	0	-2	-2	-2	2
χ_{22}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	-1	1
χ_{23}	3	-1	3	-1	-1	0	0	0	0	3	3	3	-3
χ_{24}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	-1	1
χ_{25}	3	-1	3	-1	-1	0	0	0	0	3	3	3	-3
χ_{26}	6	-2	6	-2	-2	0	0	0	0	-2	-2	-2	2
χ_{27}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	-2	0
χ_{28}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	-2	0
χ_{29}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	2	0
χ_{30}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	2	0
χ_{31}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0	0
χ_{32}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0	0
χ_{33}	14	10	-2	2	-6	0	0	0	0	-2	-2	2	0
χ_{34}	-2	10	-2	2	-6	0	0	0	0	2	2	-2	0
χ_{35}	-2	10	-2	2	-6	0	0	0	0	2	2	-2	0
χ_{36}	14	10	-2	2	-6	0	0	0	0	-2	-2	2	0
χ_{37}	28	-12	-4	4	4	0	0	0	0	4	4	-4	0
χ_{38}	-4	-12	-4	4	4	0	0	0	0	-4	-4	4	0
χ_{39}	-20	4	4	-4	4	0	0	0	0	0	0	0	0
χ_{40}	-20	4	4	-4	4	0	0	0	0	0	0	0	0
χ_{41}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{42}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{43}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{44}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{45}	16	0	0	0	0	2	-2	-2	2	-8	8	0	0
χ_{46}	-16	0	0	0	0	2	-2	-2	2	0	0	0	0
χ_{47}	-16	0	0	0	0	2	-2	-2	2	0	0	0	0
χ_{48}	0	0	0	0	0	-1	1	1	-1	0	0	0	0
χ_{49}	0	0	0	0	0	-1	1	1	-1	0	0	0	0

Table 6.9 (continue)

	4B				4C						5A	
	$4L$	$4M$	$8C$	$8D$	$4N$	$4E$	$8F$	$4O$	$8G$	$8H$	$5A$	$10A$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
χ_3	0	0	0	0	-2	-2	-2	-2	-2	-2	0	0
χ_4	0	0	0	0	2	2	2	2	2	2	0	0
χ_5	-1	-1	-1	-1	1	1	1	1	1	1	0	0
χ_6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
χ_7	1	1	1	1	1	1	1	1	1	1	0	0
χ_8	1	1	1	1	1	1	1	1	1	1	0	0
χ_9	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0
χ_{10}	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_{12}	0	0	0	0	0	0	0	0	0	0	-1	-1
χ_{13}	2	2	2	2	0	0	0	0	0	0	0	0
χ_{14}	-2	-2	-2	-2	0	0	0	0	0	0	1	1
χ_{15}	1	1	1	-1	3	-1	1	-3	-1	1	1	-1
χ_{16}	1	1	1	-1	-3	1	-1	3	1	-1	1	-1
χ_{17}	0	0	0	0	2	2	-2	-2	-2	2	-1	1
χ_{18}	0	0	0	0	-2	-2	2	2	2	-2	-1	1
χ_{19}	1	1	1	-1	1	-3	3	-1	1	-1	0	0
χ_{20}	1	1	1	-1	-1	3	-3	1	-1	1	0	0
χ_{21}	-2	-2	-2	2	0	0	0	0	0	0	1	-1
χ_{22}	-1	-1	-1	1	-1	3	-3	1	-1	1	0	0
χ_{23}	-1	-1	-1	1	3	-1	1	-3	-1	1	0	0
χ_{24}	-1	-1	-1	1	1	-3	3	-1	1	-1	0	0
χ_{25}	-1	-1	-1	1	-3	1	-1	3	1	-1	0	0
χ_{26}	2	2	2	-2	0	0	0	0	0	0	0	0
χ_{27}	2	2	-2	0	4	0	-2	2	0	-2	0	0
χ_{28}	2	2	-2	0	-4	0	2	-2	0	2	0	0
χ_{29}	-2	-2	2	0	-2	2	0	-4	2	0	0	0
χ_{30}	-2	-2	2	0	2	-2	0	4	-2	0	0	0
χ_{31}	0	0	0	0	-2	-2	2	2	-2	2	0	0
χ_{32}	0	0	0	0	2	2	-2	-2	2	-2	0	0
χ_{33}	2	2	-2	0	0	-4	-2	2	0	2	0	0
χ_{34}	-2	-2	2	0	2	-2	-4	0	2	0	0	0
χ_{35}	-2	-2	2	0	-2	2	4	0	-2	0	0	0
χ_{36}	2	2	-2	0	0	4	2	-2	0	-2	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	2	2	2	2	-2	-2	0	0
χ_{40}	0	0	0	0	-2	-2	-2	-2	2	2	0	0
χ_{41}	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{42}	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{43}	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{44}	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.9 (continue)

	$6A$				$7A$	$7B$	$8A$				$14A$	$14B$
	$6D$	$6E$	$12A$	$12B$	$7A$	$7B$	$8I$	$8J$	$16A$	$16B$	$14A$	$14B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
χ_3	0	0	0	0	-1	-1	0	0	0	0	1	1
χ_4	0	0	0	0	-1	-1	0	0	0	0	-1	-1
χ_5	-1	-1	-1	-1	0	0	1	1	1	1	0	0
χ_6	1	1	1	1	0	0	-1	-1	-1	-1	0	0
χ_7	0	0	0	0	A	B	-1	-1	-1	-1	$-A$	$-B$
χ_8	0	0	0	0	B	A	-1	-1	-1	-1	$-B$	$-A$
χ_9	0	0	0	0	A	B	1	1	1	1	A	B
χ_{10}	0	0	0	0	B	A	1	1	1	1	B	A
χ_{11}	-1	-1	-1	-1	1	1	0	0	0	0	1	1
χ_{12}	1	1	1	1	1	1	0	0	0	0	-1	-1
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	-1	-1	1	0	0	1	1	-1	-1	0	0
χ_{16}	-1	1	1	-1	0	0	-1	-1	1	1	0	0
χ_{17}	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{18}	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{19}	-1	1	1	-1	0	0	1	1	-1	-1	0	0
χ_{20}	1	-1	-1	1	0	0	-1	-1	1	1	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{23}	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{24}	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{25}	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{28}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{29}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{30}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{31}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{32}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	1	-1	1	-1	0	0	2	-2	0	0	0	0
χ_{42}	-1	1	-1	1	0	0	2	-2	0	0	0	0
χ_{43}	-1	1	-1	1	0	0	-2	2	0	0	0	0
χ_{44}	1	-1	1	-1	0	0	-2	2	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	C	$-C$	0	0
χ_{47}	0	0	0	0	0	0	0	0	$-C$	C	0	0
χ_{48}	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{49}	-1	1	-1	1	0	0	0	0	0	0	0	0

where $A = E(7)^3 + E(7)^5 + E(7)^6$, $B = E(7) + E(7)^2 + E(7)^4$ and $C = 2 * E(8) + 2 * E(8)^3$.

The information about the conjugacy classes found in Table 6.4 can be used to compute the power maps for the elements of $\overline{G}$ and then with the aid of Programme C (see Appendix A) we can verify that we obtained the unique p-power maps listed in Table 6.10 for our Table 6.9.

Table 6.10: The power maps of the elements of 2^9:($L_3(4)$:2)

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7
$1A$	$1A$					$2A$	$2D$	$1A$			
	$2A$	$1A$					$2E$	$1A$			
	$2B$	$1A$					$4A$	$2B$			
	$2C$	$1A$					$4B$	$2B$			
							$4C$	$2B$			
$2B$	$2F$	$1A$				$3A$	$3A$		$1A$		
	$2G$	$1A$					$6A$	$3A$	$2C$		
	$4D$	$2A$					$6B$	$3A$	$2A$		
	$4E$	$2A$					$6C$	$3A$	$2B$		
	$4F$	$2B$									
	$4G$	$2B$									
	$4H$	$2B$									
	$4I$	$2B$									
$4A$	$4J$	$2F$				$4B$	$4L$	$2F$			
	$4K$	$2F$					$4M$	$2F$			
	$8A$	$4E$					$8C$	$4E$			
	$8B$	$4F$					$8D$	$4I$			
$4C$	$4N$	$2F$				$5A$	$5A$			$1A$	
	$4O$	$2F$					$10A$	$5A$		$2A$	
	$8E$	$4F$									
	$8F$	$4F$									
	$8G$	$4D$									
	$8H$	$4F$									
$6A$	$6D$	$3A$	$2D$			$7A$	$7A$				$1A$
	$6E$	$3A$	$2E$								
	$12A$	$6C$	$4C$								
	$12B$	$6C$	$4A$								
$7B$	$7B$				$1A$	$8A$	$8I$	$4J$			
							$8J$	$4J$			
							$16A$	$8A$			
							$16B$	$8A$			
$14A$	$14A$	$7A$			$2D$	$14B$	$14B$	$7B$			$2D$

6.7 The Fusion of 2^9:($L_3(4)$:2) into $U_6(2)$:2

Since $\overline{G}$ is a maximal subgroup of $U_6(2)$:2 of index 891, then the action of $U_6(2)$:2 on the cosets of $\overline{G}$ gives rise to a permutation character $\chi(U_6(2):2|\overline{G})$ of degree 891. We deduce from the character table of $U_6(2)$:2 found in GAP that $\chi(U_6(2):2|\overline{G})$ =

$1a + 22a + 252a + 616a$, where $1a$, $22a$, $252a$ and $616a$ are irreducible characters of $U_6(2){:}2$ of degrees 1, 22, 252 and 616, respectively.

We are able to obtain the partial fusion of $\overline{G}$ into $U_6(2){:}2$, using the information provided by the values of $\chi(U_6(2){:}2|\overline{G})$ on the classes of $\overline{G}$ and the power maps of $\overline{G}$ and $U_6(2){:}2$. Then, the technique of set intersections for characters (see [45],[46],[48] and [53]) is applied to restrict some ordinary irreducible characters of $U_6(2){:}2$ of small degrees to $\overline{G}$, to determine fully the fusion of the classes of $\overline{G}$ into $U_6(2){:}2$.

Let ζ be the character afforded by the regular representation of $L_3(4){:}2$. We obtain that $\zeta = \Sigma_{i=1}^{14}\alpha_i\Phi_i$, where $\Phi_i \in Irr(L_3(4){:}2)$ and $\alpha_i = deg(\Phi_i)$. Then ζ can be regarded as a character of $2^9{:}(L_3(4){:}2)$ which contains 2^9 in its kernel such that

$$\zeta(x) = \begin{cases} |L_3(4){:}2| & \text{if } \ x \in 2^9 \\ 0 & \text{otherwise} \end{cases}.$$

If ϕ is a character of $U_6(2){:}2$ than we have that

$$\begin{aligned} <\zeta,\phi>_{\overline{G}} = & \frac{1}{|2^9{:}(L_3(4){:}2)|}\{\zeta(1A)\phi(1A) + 21\zeta(2A)\phi(2A) + 210\zeta(2B)\phi(2B) + 280\zeta(2C)\phi(2C)\} \\ = & \frac{1}{|2^9{:}(L_3(4){:}2|}\{|L_3(4){:}2|(\phi(1A) + 21\phi(2A) + 210\phi(2B) + 280\phi(2C))\} \\ = & \frac{1}{512}\{\phi(1A) + 21\phi(2A) + 210\phi(2B) + 280\phi(2C)\} \\ = & <\phi_{2^9}, 1_{2^9}> . \end{aligned}$$

Here 1_{2^9} is the identity character of 2^9 and ϕ_{2^9} is the restriction of ϕ to 2^9. We obtain that

$$\phi_{2^9} = a_1\theta_1 + a_2\theta_2 + a_3\theta_3 + a_4\theta_4,$$

where $a_i \in \mathbb{N}\cup\{0\}$ and θ_i are the sums of the irreducible characters of 2^9 which are in the same orbit under the action of $L_3(4){:}2$ on $Irr(2^9)$, for $i \in \{1, 2, 3, 4\}$. Let $\varphi_j \in Irr(2^9)$, where $j \in \{1, 2, 3, ..., 49\}$. Then we obtain that

$$\begin{aligned} \theta_1 &= \varphi_1 \quad , \quad deg(\theta_1) = 1 \\ \theta_2 &= \sum_{j=2}^{22} \varphi_j \ , \quad deg(\theta_2) = 21 \\ \theta_3 &= \sum_{j=23}^{232} \varphi_j \ , \quad deg(\theta_3) = 210 \\ \theta_4 &= \sum_{j=233}^{512} \varphi_j \ , \quad deg(\theta_4) = 280 \ . \end{aligned}$$

Hence

$$\phi_{2^9} = a_1\varphi_1 + a_2\sum_{j=2}^{22}\varphi_j + a_3\sum_{j=23}^{232}\varphi_j + a_4\sum_{j=233}^{512}\varphi_j,$$

and therefore

$$\begin{aligned} \langle \phi_{2^9}, \phi_{2^9} \rangle = & \quad a_1^2 + 21a_2^2 + 210a_3^2 + 280a_4^2 \\ = & \quad \frac{1}{512}\{\phi(1A)\phi(1A) + 21\phi(2A)\phi(2A) + 210\phi(2B)\phi(2B) + 280\phi(2C)\phi(2C)\} \ , \end{aligned}$$

where $a_1 = < \zeta, \phi >_{2^9:(L_3(4):2)}$.

We apply the above results to some of the irreducible characters of $U_6(2){:}2$ of small degrees, which in this case are $\phi_1 = 22a$, $\phi_2 = 22b$, $\phi_3 = 231a$, $\phi_4 = 231b$, $\phi_5 = 440a$ and $\phi_6 = 440b$. Their respective degrees are 22, 22, 231, 231, 440 and 440. For ϕ_1 we calculate that

$$< \zeta, \phi_1 >_{2^9:(L_3(4):2)} = \frac{1}{512}\{22 + 21(-10) + 210(6) + 280(-2)\} = 1 \ .$$

Now $a_1 + 21a_2 + 210a_3 + 280a_4 = 22$, since $deg\phi_1 = 22$. Since $a_1 = 1$, we must have that $a_2 = 1$, $a_3 = a_4 = 0$. Note that $2^9{:}(L_3(4){:}2)$ does not have irreducible characters of degree 22. We obtain that $(\phi_1)_{2^9:(L_3(4):2} = \chi_1 + \chi_{15}$ if the partial fusion of 2^9:($L_3(4)$:2) into $U_6(2)$:2 is taken into consideration. Similarly, for ϕ_3 and ϕ_5 we calculate that

$$< \zeta, \phi_3 >_{2^9:(L_3(4):2)} = \frac{1}{512}\{231 + 21(39) + 210(7) + 280(-9)\} = 0$$

and

$$< \zeta, \phi_5 >_{2^9:(L_3(4):2)} = \frac{1}{512}\{440 + 21(120) + 210(24) + 280(8)\} = 20\ .$$

Since the respective degrees of ϕ_3 and ϕ_5 are 231 and 440, we have to solve the equations (i) $a_1 + 21a_2 + 210a_3 + 280a_4 = 231$ and (ii) $a_1 + 21a_2 + 210a_3 + 280a_4 = 440$, separately. If we are taking into account that the set $Irr(\overline{G})$ (See Table 6.9) does not have any irreducible characters of degrees 231 and 440 and also that $< \zeta, \phi_3 >_{2^9:(L_3(4):2)} = 0$ and $< \zeta, \phi_5 >_{2^9:(L_3(4):2)} = 20$, we deduce that the two sets of values $\{a_1 = a_4 = 0, a_2 = a_3 = 1\}$ and $\{a_1 = 20, a_2 = 10, a_3 = 1, a_4 = 0\}$ are the only possibilities that satisfy equation (i) and (ii) respectively, hence we obtained that $(\phi_3)_{2^9:(L_3(4):2)} = \chi_{16} + \chi_{27}$ and $(\phi_5)_{2^9:(L_3(4):2)} = \chi_4 + \chi_{32}$. Similar computations were carried out to restrict the characters ϕ_2, ϕ_4 and ϕ_6 to $\overline{G}$ and we found that $(\phi_2)_{2^9:(L_3(4):2)} = \chi_2 + \chi_{16}$, $(\phi_4)_{2^9:(L_3(4):2)} = \chi_{15} + \chi_{28}$ and $(\phi_6)_{2^9:(L_3(4):2)} = \chi_3 + \chi_{31}$.

By making use of the values of ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5 and ϕ_6 on the classes of $U_6(2)$:2 and the values of $(\phi_1)_{2^9:(L_3(4):2)}$, $(\phi_2)_{2^9:(L_3(4):2)}$, $(\phi_3)_{2^9:(L_3(4):2)}$, $(\phi_4)_{2^9:(L_3(4):2)}$, $(\phi_5)_{2^9:(L_3(4):2)}$ and $(\phi_6)_{2^9:(L_3(4):2)}$ on the classes of 2^9:($L_3(4)$:2) together with the partial fusion, the complete fusion map of 2^9:($L_3(4)$:2) into $U_6(2)$:2 is given in the Table 6.11.

Table 6.11: The fusion of 2^9:($L_3(4)$:2) into $U_6(2)$:2

$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)}$	$\longrightarrow$	$[y]_{U_6(2):2}$	$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)}$	$\longrightarrow$	$[y]_{U_6(2):2}$
$1A$	$1A$		$1A$	$2A$	$2D$		$2D$
	$2A$		$2A$		$2E$		$2E$
	$2B$		$2B$		$4A$		$4G$
	$2C$		$2C$		$4B$		$4H$
					$4C$		$4I$
$2B$	$2F$		$2B$	$3A$	$3A$		$3C$
	$2G$		$2C$		$6A$		$6G$
	$4D$		$4A$		$6B$		$6E$
	$4E$		$4B$		$6C$		$6F$
	$4F$		$4C$				
	$4G$		$4E$				
	$4H$		$4F$				
	$4I$		$4D$				
$4A$	$4J$		$4C$	$4B$	$4L$		$4D$
	$4K$		$4F$		$4M$		$4F$
	$8A$		$8A$		$8C$		$8A$
	$8B$		$8B$		$8D$		$8C$
$4B$	$4N$		$4H$	$5A$	$5A$		$5A$
	$4O$		$4I$		$10A$		$10A$
	$8E$		$8F$				
	$8F$		$8G$				
	$8G$		$8D$				
	$8H$		$8E$				
$6A$	$6D$		$6K$	$7A$	$7A$		$7A$
	$6E$		$6L$				
	$12A$		$12K$				
	$12B$		$12J$				
$7B$	$7B$		$7A$	$8A$	$8I$		$8E$
					$8J$		$8F$
					$16A$		$16A$
					$16B$		$16B$
$14A$	$14A$		$14A$	$14B$	$14B$		$14A$

Chapter 7

On a maximal subgroup of the automorphism group $U_6(2){:}3$ of $U_6(2)$

As we mentioned in Chapter 6 that $A_2 = (2^9{:}L_3(4)){:}3$ is a maximal subgroup of the automorphism group of the unitary group $U_6(2)$. In this chapter, it will be shown with the aid of GAP and MAGMA that A_2 is a split extension of 2^9 by $L_3(4){:}3$. Firstly, the group A_2 will be constructed as a permutation group of degree 693 within $U_6(2){:}3$, and secondly we show that A_2 also exists as a subgroup of $SL_{10}(2)$. Having obtained A_2 as permutation group on 672 points, we use a similar method as in Section 6.1 to represent A_2 as a split extension $\overline{G} = 2^9{:}(L_3(4){:}3)$, where we regard 2^9 as the vector space $V_9(2)$, where upon $L_3(4){:}3$ acts irreducibly as a matrix group of dimension 9 over Galois field $GF(2)$. Then the technique of coset analysis will be used (as in the case of $2^9{:}(L_3(4){:}2)$) to compute the conjugacy classes of $\overline{G}$. The point stabilizers for the action of $L_3(4){:}3$ on $Irr(2^9)$ and as well their fusion maps into $L_3(4){:}3$ will be determined. Next, we will compute the Fischer-Clifford matrices of $\overline{G}$ and then use these matrices and all the relevant information which is needed to construct the character table of $\overline{G} \cong A_2$. Lastly, the fusion of the classes of $\overline{G}$ into $U_6(2){:}3$ will be computed. If details or explanations of computations are left out, the reader is referred to Chapter 6 for clarification.

7.1 The group $2^9{:}(L_3(4){:}3)$

In this section, we identify our group $A_2 = (2^9{:}L_3(4)){:}3$ as the split extension 2^9 by $L_3(4){:}3$ with the aid of GAP, MAGMA and the Wilson's online ATLAS. Then with the help of MAGMA we represent $L_3(4){:}3$ as a matrix group G of dimension 9 over the Galois field $GF(2)$. Since G acts absolutely irreducibly on its natural module 2^9, we construct an isomorphic copy of A_2, say S, as a subgroup of $SL_{10}(2)$.

Using the smallest permutation representation of degree 693 of $U_6(2){:}S_3$ found in Wilson's online ATLAS, we construct a copy of $U_6(2){:}3$ within $U_6(2){:}S_3$ with the help of GAP. Having obtained a permutation representation of $U_6(2){:}3$ we generate $A_2 = (2^9{:}L_3(4)){:}3$ within $U_6(2){:}3$. Using appropriate GAP and MAGMA commands, as explained in Section 6.1, it is verified that A_2 is a split extension of 2^9 by $L_3(4){:}3$ and also we manage to represent $L_3(4){:}3$ as a matrix group of dimension 9 over the Galois field $GF(2)$. The generators g_1 and g_2 obtained to represent $G \cong L_3(4){:}3$ as a matrix group of dimension 9 over the Galois field $GF(2)$, are as follows:

$$g_1 = \begin{pmatrix} 1&0&0&0&0&1&1&1&0 \\ 0&1&0&1&0&1&0&0&0 \\ 1&1&0&1&0&0&0&0&0 \\ 1&0&0&1&1&1&0&0&1 \\ 0&0&1&1&1&1&1&1&1 \\ 0&1&1&1&1&0&0&0&1 \\ 0&1&0&0&1&0&0&0&0 \\ 1&0&1&0&1&1&1&0&0 \\ 0&1&0&0&1&0&1&0&1 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1&1&0&0&0&1&1&1&1 \\ 0&1&0&1&0&1&1&0&1 \\ 1&0&1&1&1&0&0&0&1 \\ 1&1&0&1&0&1&1&0&0 \\ 0&0&0&0&1&0&0&1&1 \\ 0&0&0&0&0&1&0&0&1 \\ 1&0&0&1&1&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

where $o(g_1) = 2$ and $o(g_2) = 12$. We obtained 22 conjugacy classes for $L_3(4){:}3 = < g_1, g_2 >$ and they are listed in Table 7.1.

Table 7.1: The conjugacy classes of $L_3(4){:}3$

$[g]_{L_3(4):3}$	Class representative	$\lvert [g]_{L_3(4):3} \rvert$	$[g]_{L_3(4):3}$	Class representative	$\lvert [g]_{L_3(4):3} \rvert$
$1A$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&1&1&1&1&1 \\ 1&1&0&1&0&1&1&0&0 \\ 0&0&1&1&1&1&0&1&1 \\ 1&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 1&0&0&0&0&1&1&0&0 \\ 0&0&0&0&1&1&1&1&0 \\ 0&1&1&1&1&1&0&0&1 \end{pmatrix}$	315
$3A$	$\begin{pmatrix} 0&0&0&1&1&0&0&1&0 \\ 1&1&1&0&1&1&1&1&0 \\ 0&0&0&1&0&1&1&0&0 \\ 1&0&0&0&1&0&0&1&0 \\ 0&1&1&0&1&0&1&1&0 \\ 0&0&1&1&0&0&1&0&0 \\ 1&0&1&0&1&1&1&0&0 \\ 1&0&0&1&1&0&1&1&0 \\ 0&0&1&1&0&1&1&0&1 \end{pmatrix}$	336	$3B$	$\begin{pmatrix} 0&1&1&1&1&0&0&1&0 \\ 1&0&0&0&1&1&0&0&0 \\ 0&0&0&1&0&1&0&1&0 \\ 1&1&1&0&1&0&0&1&0 \\ 1&0&1&0&0&1&1&1&0 \\ 0&0&1&1&0&0&0&1&0 \\ 1&1&1&1&1&0&0&0&0 \\ 1&1&0&0&1&1&1&0&0 \\ 0&0&1&1&0&1&0&1&1 \end{pmatrix}$	336
$3C$	$\begin{pmatrix} 0&1&0&0&1&0&1&0&0 \\ 1&1&0&0&0&0&1&1&0 \\ 0&0&0&1&0&1&1&0&1 \\ 1&0&0&0&0&0&0&0&0 \\ 0&0&1&1&0&1&1&0&0 \\ 1&0&1&1&0&0&1&1&0 \\ 1&1&1&0&0&1&0&1&0 \\ 0&0&1&1&0&1&0&1&0 \\ 1&0&0&0&0&0&0&1&0 \end{pmatrix}$	960	$3D$	$\begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 0&1&0&1&1&0&0&1&0 \\ 0&1&0&1&0&1&1&1&0 \\ 0&1&0&0&1&0&1&0&0 \\ 1&1&0&0&0&0&0&0&1 \\ 0&0&0&0&1&1&0&0&1 \\ 0&0&0&1&1&0&0&1&1 \\ 0&0&0&1&0&0&0&0&1 \\ 0&1&1&1&1&1&1&1&0 \end{pmatrix}$	960
$3E$	$\begin{pmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 1&1&0&0&1&0&0&1&0 \\ 0&1&0&1&1&1&1&1&1 \\ 0&1&1&1&0&1&1&1&0 \\ 0&0&1&0&0&1&1&1&1 \\ 1&1&1&1&1&0&0&1&0 \\ 1&0&0&1&0&1&1&1&0 \\ 0&0&1&0&0&0&1&1&0 \end{pmatrix}$	2240	$4A$	$\begin{pmatrix} 1&0&0&0&1&1&1&0&0 \\ 1&0&0&1&1&1&1&0&1 \\ 0&0&0&1&1&1&0&0&1 \\ 1&0&1&1&1&0&0&1&1 \\ 0&1&1&0&1&1&1&0&0 \\ 0&0&1&0&0&1&1&1&1 \\ 0&1&0&0&0&1&1&1&1 \\ 0&1&0&1&1&1&1&0&1 \\ 0&0&1&0&0&0&1&1&0 \end{pmatrix}$	3780
$5A$	$\begin{pmatrix} 0&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&1&1&1&0 \\ 1&1&1&1&1&0&0&1&0 \\ 1&1&1&0&0&0&1&1&0 \\ 1&1&0&1&1&0&1&1&1 \\ 0&0&1&0&1&1&0&1&1 \\ 0&0&0&1&1&1&1&0&0 \\ 1&0&1&1&1&0&0&1&1 \\ 0&1&0&0&0&0&1&0&0 \end{pmatrix}$	4032	$5B$	$\begin{pmatrix} 1&1&0&0&1&0&0&1&1 \\ 0&0&1&1&0&1&1&0&1 \\ 0&0&1&1&1&1&1&1&1 \\ 1&0&1&1&0&1&1&1&0 \\ 0&1&0&0&0&0&1&0&1 \\ 1&1&1&0&0&0&1&0&1 \\ 0&1&1&1&1&1&0&0&0 \\ 0&0&1&1&1&1&0&0&1 \\ 1&0&0&1&0&0&0&1&0 \end{pmatrix}$	4032
$6A$	$\begin{pmatrix} 0&1&0&1&1&1&1&0&1 \\ 1&1&0&0&0&1&0&1&1 \\ 0&1&0&1&1&1&0&0&1 \\ 0&1&1&1&1&1&1&0&0 \\ 0&0&0&1&1&0&0&1&1 \\ 0&1&1&1&1&0&0&0&1 \\ 0&0&0&0&1&1&1&1&0 \\ 1&1&0&0&0&0&0&0&0 \\ 1&1&0&0&1&1&0&0&1 \end{pmatrix}$	5040	$6B$	$\begin{pmatrix} 0&0&1&1&1&1&1&0&1 \\ 0&0&1&1&1&1&1&1&1 \\ 1&1&0&0&0&1&1&0&1 \\ 0&0&0&1&1&1&1&0&0 \\ 1&0&1&1&0&1&0&1&1 \\ 1&1&1&0&0&0&1&0&1 \\ 1&0&1&0&0&0&0&0&0 \\ 1&1&1&1&0&1&0&1&0 \\ 0&1&0&1&0&1&1&0&1 \end{pmatrix}$	5040

Table 7.1 (continue)

$[g]_{L_3(4):3}$	Class representative	$\vert[g]_{L_3(4):3}\vert$	$[g]_{L_3(4):3}$	Class representative	$\vert[g]_{L_3(4):3}\vert$
$7A$	$\begin{pmatrix} 1&0&0&1&1&0&0&1&0 \\ 1&0&0&1&1&0&1&1&0 \\ 1&1&0&1&1&0&0&1&0 \\ 0&0&0&1&0&1&1&1&1 \\ 1&0&0&1&1&0&1&0&0 \\ 1&0&1&1&0&0&1&1&0 \\ 0&1&1&0&0&0&1&1&0 \\ 1&1&0&0&1&1&1&1&0 \\ 1&1&0&0&1&0&1&1&1 \end{pmatrix}$	2880	$7B$	$\begin{pmatrix} 0&1&0&1&0&1&0&0&0 \\ 0&0&0&1&0&1&0&1&1 \\ 1&0&0&0&0&1&0&0&0 \\ 1&1&1&1&1&0&0&1&0 \\ 0&1&0&1&1&1&0&0&1 \\ 0&0&0&0&0&1&0&0&1 \\ 1&0&0&1&1&1&1&1&1 \\ 1&1&0&0&1&1&1&0&0 \\ 0&0&1&1&0&1&0&1&1 \end{pmatrix}$	2880
$15A$	$\begin{pmatrix} 1&0&0&0&1&0&0&1&0 \\ 0&1&0&1&1&1&0&0&1 \\ 0&1&1&0&0&0&0&0&0 \\ 0&1&1&1&0&0&1&0&0 \\ 1&0&0&1&1&0&0&1&1 \\ 0&1&1&0&1&1&1&1&1 \\ 0&0&0&1&0&1&1&0&1 \\ 0&0&1&0&0&0&0&0&0 \\ 1&1&0&1&1&0&1&1&1 \end{pmatrix}$	4032	$15B$	$\begin{pmatrix} 0&1&1&1&0&0&1&0&0 \\ 1&1&0&1&0&0&0&0&0 \\ 0&1&1&1&1&1&1&0&0 \\ 1&0&0&0&0&0&1&1&0 \\ 1&1&0&1&0&0&1&1&0 \\ 0&1&1&0&1&1&0&1&1 \\ 1&0&0&1&0&0&1&0&1 \\ 1&1&1&1&1&0&0&1&0 \\ 0&1&0&0&1&0&1&0&0 \end{pmatrix}$	4032
$15C$	$\begin{pmatrix} 0&0&1&1&0&0&0&0&1 \\ 0&0&0&0&0&0&1&1&0 \\ 0&0&1&1&1&1&0&0&1 \\ 0&1&0&1&1&0&0&0&0 \\ 1&0&0&1&1&0&0&1&0 \\ 0&0&1&0&0&1&1&1&1 \\ 1&1&0&1&0&0&0&0&1 \\ 0&1&1&0&0&0&0&1&0 \\ 1&1&0&1&0&0&1&1&0 \end{pmatrix}$	4032	$15D$	$\begin{pmatrix} 1&1&0&0&0&0&1&1&0 \\ 1&1&0&0&0&1&0&1&0 \\ 0&0&1&0&1&0&1&0&0 \\ 0&0&1&1&0&0&0&0&1 \\ 0&1&0&0&0&0&1&0&0 \\ 1&0&1&1&0&1&0&0&0 \\ 0&0&0&1&0&1&1&0&0 \\ 0&1&1&0&0&0&1&0&0 \\ 1&0&0&1&1&0&0&1&1 \end{pmatrix}$	4032
$21A$	$\begin{pmatrix} 1&1&0&1&0&0&0&0&1 \\ 1&0&0&1&0&0&0&1&0 \\ 1&1&1&0&0&1&1&0&1 \\ 0&1&1&1&1&1&0&0&0 \\ 1&0&0&0&1&1&1&0&0 \\ 0&0&1&0&0&1&1&1&1 \\ 1&0&0&1&0&1&1&1&0 \\ 0&0&1&0&1&0&0&1&1 \\ 1&0&0&0&0&1&0&0&0 \end{pmatrix}$	2880	$21B$	$\begin{pmatrix} 1&0&1&1&1&0&0&1&1 \\ 1&0&0&0&0&1&0&1&0 \\ 1&0&0&1&0&0&1&1&1 \\ 1&0&1&0&0&0&1&0&0 \\ 1&1&1&0&1&1&1&0&0 \\ 1&1&1&1&1&0&1&1&1 \\ 0&0&1&1&0&1&0&1&1 \\ 1&1&0&0&0&1&0&1&0 \\ 1&1&1&1&0&1&1&1&0 \end{pmatrix}$	2880
$21C$	$\begin{pmatrix} 1&1&0&0&1&0&1&0&1 \\ 0&1&0&0&1&0&0&1&1 \\ 1&1&0&1&0&1&0&1&1 \\ 1&1&1&0&1&1&1&1&0 \\ 1&1&0&0&0&1&1&1&1 \\ 0&0&0&0&0&1&0&0&1 \\ 0&1&0&1&0&1&0&1&0 \\ 1&0&0&1&0&1&1&1&0 \\ 1&0&1&1&0&0&0&0&1 \end{pmatrix}$	2880	$21D$	$\begin{pmatrix} 1&0&1&0&0&0&0&1&0 \\ 1&0&1&0&1&0&0&1&1 \\ 0&1&0&0&1&1&1&1&0 \\ 0&1&0&1&0&1&1&1&1 \\ 1&1&1&1&0&1&0&1&0 \\ 1&0&1&1&0&1&0&0&0 \\ 0&0&1&1&0&1&0&1&0 \\ 1&1&1&0&0&0&1&1&0 \\ 0&1&0&0&0&0&0&0&1 \end{pmatrix}$	2880

Using the MAGMA command "IsAbsolutelyIrreducible(G)", we found that G acts absolutely irreducibly on its natural module 2^9 and hence we can construct an isomorphic copy of A_2, say S, as a subgroup of $SL_{10}(2)$. The generators for $S = 2^9{:}(L_3(4){:}3)$ as a subgroup of $SL_{10}(2)$ are given as:

$$s_1 = \begin{pmatrix} 1&0&0&0&0&1&1&1&0&0\\ 0&1&0&1&0&1&0&0&0&0\\ 1&1&0&1&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1&0\\ 0&0&1&1&1&1&1&1&1&0\\ 0&1&1&1&1&0&0&0&1&0\\ 0&1&0&0&1&0&0&0&0&0\\ 1&0&1&0&1&1&1&0&0&0\\ 0&1&0&0&1&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix} \quad s_2 = \begin{pmatrix} 1&1&0&0&0&1&1&1&1&0\\ 0&1&0&1&0&1&1&0&1&0\\ 1&0&1&1&1&0&0&0&1&0\\ 1&1&0&1&0&1&1&0&0&0\\ 0&0&0&0&1&0&0&1&1&0\\ 0&0&0&0&0&1&0&0&1&0\\ 1&0&0&1&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$s_3 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

where $o(s_1) = 4$, $o(s_2) = 6$ and $o(s_3) = 2$. We can construct the groups $L_3(4){:}3$ and 2^9 within $S =< s_1, s_2, s_3 >$, such that $L_3(4){:}2 =< s_1, s_2 >$ and $2^9 =< n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9 >$. The generators $n_i, i = 1, 2, 3, ..., 9$, for 2^9 are listed in Table 7.2.

Table 7.2: The generators of 2^9

$$n_1=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0&0&1 \end{pmatrix} \quad n_2=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&1&0&0&1&1 \end{pmatrix}$$

$$n_3=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&1&0&1 \end{pmatrix} \quad n_4=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&1&1&1&0&1&0&1&1&1 \end{pmatrix}$$

$$n_5=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&1&1&1&0&1&0&1&0&1 \end{pmatrix} \quad n_6=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&1&0&1&0&0&1&0&0&1 \end{pmatrix}$$

$$n_7=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&1&1&1&1&1&1&0&0&1 \end{pmatrix} \quad n_8=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&0&1&1&0&1&1&0&1&1 \end{pmatrix}$$

$$n_9=\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 1&0&1&1&1&1&0&0&0&1 \end{pmatrix}$$

7.2 The conjugacy classes of $\overline{G} = 2^9{:}(L_3(4){:}3)$

In this section, we will determine the orbit lengths resulting from the action of $\overline{G} = 2^9{:}(L_3(4){:}3)$ on $N = 2^9$. The structure of the stabilizers corresponding to these orbit lengths, are also identified. The information obtained from the permutation character of $L_3(4){:}3$ on the conjugacy classes of 2^9, Programme A and Programme B is then used to compute the conjugacy classes of $2^9{:}(L_3(4){:}3)$.

Throughout the remainder of this chapter, let $\overline{G} = 2^9{:}(L_3(4){:}3)$ be a split extension of $N = 2^9$ by $G = L_3(4){:}3$, where N is the vector space $V_9(2)$ of dimension 9 over $GF(2)$ on which the linear group $G =< g_1, g_2 >$ acts irreducibly. Using MAGMA (as in Section 6.2), it is found that the action of G on N partitioned the classes of N into 4 orbits of lengths $1, 21, 210$ and 280. The structure of the point stabilizers P_i, $i = 1, 2, 3,\ 4$, corresponding to these orbit lengths are $P_1 = L_3(4){:}3$, $P_2 = 2^4{:}(3 \times A_5)$, $P_3 = 2^4{:}(3 \times S_3)$ and $P_4 = 3^2{:}(2A_4)$, respectively. The structures of P_2 and P_4 were obtained by checking the indices of the maximal subgroups of $L_3(4){:}3$ in the ATLAS. MAGMA is used to generate the submaximal subgroups of G and represented as a permutation group, then their indices in G are checked. We obtained that $P_3 = 2^4{:}(3 \times S_3)$ sits maximally inside one of the two maximal subgroups of G of the form $2^4{:}(3 \times A_5)$. This group, say L, is isomorphic to P_2 but they are not in the same conjugacy class.

Let $\chi(G|2^9)$, $\chi(G|P_i)$ and $I_{P_i}^{P_1}$ be the permutation character of G on the classes of 2^9, the permutation character of G on the classes of a point stabilizer P_i, and the identity character of a stabilizer P_i induced to G. We obtain that $\chi(L_3(4){:}3|2^9) = \sum_{i=1}^{4} I_{P_i}^{P_1}$ $= \sum_{i=1}^{4} \chi(L_3(4){:}3|P_i)$ $=\chi(L_3(4){:}3|P_1) + \chi(L_3(4){:}3|P_2) + \chi(L_3(4){:}3|P_3) + \chi(L_3(4){:}3|P_4)$ $=1a + (1a + 20a) + (1a + 2 \times 20a + 64a + 105a) + (1a + 20a + 45a + 45b + 64a + 105a)$ $= 4 \times 1a + 4 \times 20a + 45a + 45b + 2 \times 64a + 2 \times 105a$. The information provided by the character tables of G and the P_i's (See Appendix B) and the fusion maps of the point stabilizers P_i into G was used to calculate $\chi(L_3(4){:}3|P_i)$, $i = 1, 2, 3,\ 4$, and $\chi(L_3(4){:}3|2^9)$. The values of $\chi(L_3(4){:}3|2^9)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^9. The values of k are listed in Table 7.3. These values of k help us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$, which have fused together under the action of $C_G(g)$, for $g \in [g]_G$.

We use Programme A to compute the values f_j and Programme B to determine the class orders of the elements of $\overline{G}$. Having obtained all the relevant information we can compute the centralizer orders for each class $[x]_{\overline{G}}$ of $\overline{G}$ with the equation $|C_{\overline{G}}(x)| = \frac{k}{f}|C_G(g)|$. The information pertaining to the conjugacy classes of $\overline{G}$ is summarized in Table 7.4.

Table 7.3: The values of $\chi(L_3(4){:}3|2^9)$ on the different classes of $L_3(4){:}3$

$[h]_{L_3(4):3}$	$1A$	$2A$	$3A$	$3B$	$3C$	$3D$	$3E$	$4A$	$5A$	$5B$	$6A$
$\chi(L_3(4){:}3\|P_1)$	1	1	1	1	1	1	1	1	1	1	1
$\chi(L_3(4){:}3\|P_2)$	21	5	6	6	0	0	3	1	1	1	2
$\chi(L_3(4){:}3\|P_3)$	210	18	15	15	0	0	3	2	0	0	3
$\chi(L_3(4){:}3\|P_4)$	280	8	10	10	7	7	1	4	0	0	2
k	512	32	32	32	8	8	8	8	2	2	8
$[h]_{L_3(4):3}$	$6B$	$7A$	$7B$	$15A$	$15B$	$15C$	$15D$	$21A$	$21B$	$21C$	$21D$
$\chi(L_3(4){:}3\|P_1)$	1	1	1	1	1	1	1	1	1	1	1
$\chi(L_3(4){:}3\|P_2)$	2	0	0	1	1	1	1	0	0	0	0
$\chi(L_3(4){:}3\|P_3)$	3	0	0	0	0	0	0	0	0	0	0
$\chi(L_3(4){:}3\|P_4)$	2	0	0	0	0	0	0	0	0	0	0
k	8	1	1	2	2	2	2	1	1	1	1

Table 7.4: The conjugacy classes of elements of $G = 2^9{:}(L_3(4){:}3)$

$[g]_{L_3(4):3}$	k	f_j	d_j	w	$[x]_{2^9:(L_3(4):3)}$	$\|[x]_{2^9:(L_3(4):3)}\|$	$\|C_{2^9:(L_3(4):3)}(x)\|$
$1A$	512	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$1A$	1	30965760
		$f_2 = 21$	$(0,0,1,0,1,1,0,1,1)$	$(0,0,1,0,1,1,0,1,1)$	$2A$	21	1474560
		$f_3 = 210$	$(0,0,1,1,0,1,1,0,1)$	$(0,0,1,1,0,1,1,0,1)$	$2B$	210	147456
		$f_4 = 280$	$(0,0,1,0,1,1,1,0,1)$	$(0,0,1,0,1,1,1,0,1)$	$2C$	280	110592
$2A$	32	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2D$	5040	6144
		$f_2 = 1$	$(1,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2E$	5040	6144
		$f_3 = 1$	$(0,0,0,1,1,0,1,0,0)$	$(0,0,1,0,1,1,0,1,1)$	$4A$	5040	6144
		$f_4 = 1$	$(1,0,0,1,1,0,1,0,0)$	$(0,0,1,0,1,1,0,1,1)$	$4B$	5040	6144
		$f_5 = 8$	$(1,1,0,1,0,0,1,1,0)$	$(1,1,1,1,1,0,0,0,0)$	$4C$	40320	768
		$f_6 = 8$	$(0,0,0,1,1,1,1,1,1)$	$(0,1,0,1,1,1,1,1,1)$	$4D$	40320	768
		$f_7 = 12$	$(1,0,0,0,1,0,0,0,0)$	$(1,0,0,0,0,1,0,0,0)$	$4E$	60480	512
$3A$	32	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3A$	5376	5760
		$f_2 = 1$	$(1,0,1,1,0,0,1,1,0)$	$(1,0,1,1,0,0,1,1,0)$	$6A$	5376	5760
		$f_3 = 5$	$(1,0,0,1,0,1,0,0,1)$	$(1,0,0,1,0,1,0,0,1)$	$6B$	26880	1152
		$f_4 = 5$	$(0,1,1,1,0,0,1,1,1)$	$(0,1,1,1,0,0,1,1,1)$	$6C$	26880	1152
		$f_5 = 10$	$(0,0,0,1,1,1,1,1,1)$	$(1,1,1,0,0,1,1,1,0)$	$6D$	53760	576
		$f_6 = 10$	$(0,0,1,1,1,1,1,1,1)$	$(1,0,0,1,0,0,1,1,1)$	$6E$	53760	576
$3B$	32	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3B$	5376	5760
		$f_2 = 1$	$(1,0,1,1,0,0,1,1,0)$	$(1,0,1,1,0,0,1,1,0)$	$6F$	5376	5760
		$f_3 = 5$	$(1,0,1,1,0,1,1,1,1)$	$(1,0,1,1,0,0,1,1,1)$	$6G$	26880	1152
		$f_4 = 5$	$(1,1,1,1,1,1,1,0,0)$	$(0,0,0,0,0,1,1,1,0)$	$6H$	26880	1152
		$f_5 = 10$	$(1,1,0,0,0,1,0,0,0)$	$(1,1,0,0,0,1,0,0,0)$	$6I$	53760	576
		$f_6 = 10$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,1,0,0,0,0,0,0)$	$6J$	53760	576
$3C$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3C$	61440	504
		$f_2 = 7$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,0,1,1,0,1,0,0)$	$6K$	430080	72
$3D$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3D$	61440	504
		$f_2 = 7$	$(1,1,1,1,1,0,0,0,0)$	$(0,1,0,0,0,1,0,1,0)$	$6L$	430080	72

Table 7.4 (continue)

$[g]_{(L_3(4):3)}$	k	f_j	d_j	w	$[x]_{2^9:(L_3(4):3)}$	$\lvert[x]_{2^9:(L_3(4):3)}\rvert$	$\lvert C_{2^9:(L_3(4):3)}(x)\rvert$
$3E$	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$3E$	143360	216
		$f_2 = 1$	(1, 0, 0, 1, 1, 0, 1, 1, 1)	(1, 0, 0, 0, 0, 1, 0, 0, 1)	$6M$	143360	216
		$f_3 = 3$	(0, 0, 1, 1, 1, 1, 1, 0, 1)	(0, 0, 1, 0, 1, 1, 0, 1, 1)	$6N$	430080	72
		$f_4 = 3$	(0, 0, 0, 1, 0, 0, 0, 0, 0)	(1, 0, 1, 0, 1, 0, 0, 1, 0)	$6O$	430080	72
$4A$	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$4F$	241920	128
		$f_2 = 1$	(1, 1, 1, 1, 1, 1, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$4G$	241920	128
		$f_3 = 2$	(0, 1, 1, 0, 1, 0, 0, 1, 1)	(0, 0, 0, 0, 1, 1, 1, 0, 0)	$8A$	483840	64
		$f_4 = 4$	(0, 1, 0, 0, 0, 1, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 1, 0, 0)	$8B$	967680	32
$5A$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$5A$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$10A$	1032192	30
$5B$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$5B$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$10B$	1032192	30
$6A$	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$6P$	322560	96
		$f_2 = 1$	(0, 1, 1, 0, 1, 1, 1, 0, 0)	(0, 1, 1, 0, 1, 1, 1, 0, 0)	$6Q$	322560	96
		$f_3 = 1$	(1, 1, 0, 1, 1, 1, 0, 1, 0)	(1, 1, 0, 1, 1, 1, 0, 1, 0)	$12A$	322560	96
		$f_4 = 1$	(0, 0, 0, 0, 0, 0, 1, 1, 0)	(0, 0, 0, 0, 0, 0, 1, 1, 0)	$12B$	322560	96
		$f_5 = 2$	(0, 0, 1, 0, 0, 1, 0, 1, 1)	(0, 0, 1, 0, 0, 1, 0, 1, 1)	$12C$	645120	48
		$f_6 = 2$	(0, 1, 1, 1, 1, 1, 1, 1, 1)	(0, 1, 1, 1, 1, 1, 1, 1, 1)	$12D$	645120	48
$6B$	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$6R$	322560	96
		$f_2 = 1$	(0, 1, 0, 1, 0, 1, 0, 1, 0)	(0, 1, 0, 1, 0, 1, 0, 1, 0)	$6S$	322560	96
		$f_3 = 1$	(0, 1, 1, 0, 1, 1, 1, 0, 0)	(0, 1, 1, 0, 1, 1, 1, 0, 0)	$12E$	322560	96
		$f_4 = 1$	(0, 0, 0, 0, 0, 0, 1, 1, 0)	(0, 0, 0, 0, 0, 0, 1, 1, 0)	$12F$	322560	96
		$f_5 = 2$	(1, 1, 1, 1, 1, 1, 1, 0, 1)	(1, 1, 1, 1, 1, 1, 1, 0, 1)	$12G$	645120	48
		$f_6 = 2$	(0, 1, 1, 1, 1, 1, 1, 1, 1)	(0, 1, 1, 1, 1, 1, 1, 1, 1)	$12H$	645120	48
$7A$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$7A$	1474560	21
$7B$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$7B$	1474560	21
$15A$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$15A$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$30A$	1032192	30
$15B$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$15B$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$30B$	1032192	30
$15C$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$15C$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$30C$	1032192	30
$15D$	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$15D$	1032192	30
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	$30D$	1032192	30
$21A$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$21A$	1474560	21
$21B$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$21B$	1474560	21
$21C$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$21C$	1474560	21
$21D$	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	$21D$	1474560	21

7.3 The inertia groups of $\overline{G} = 2^9{:}(L_3(4){:}3)$

Since G has four orbits on N, then by Brauer's Theorem [29] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be 1 ,r, s and t where $r + s + t = 511$, with corresponding point stabilizers H_1, H_2, H_3 and H_4 as subgroups of G such that $[G : H_1]$ =1, $[G : H_2]$ =r, $[G : H_3]$= s and $[G : H_4]$ = t. We let T be the matrix group of dimension 9 over $GF(2)$ formed by the transpose of the generators of G. Then the action of T on the classes of $N = 2^9$ is the equivalent of G acting on $Irr(N)$. Then with the help of MAGMA it is easily verified that the action of T on N has orbits of lengths 1, 21, 210 and 280. We deduce that the action of G on $Irr(N)$ has orbits of lengths 1, $r = 21$, $s = 210$ and $t = 280$ with respective point stabilizers $H_1 = L_3(4){:}3$, $H_2 = 2^4{:}(3 \times A_5)$, $H_3 = 2^4{:}(3 \times S_3)$ and $H_4 = 3^2{:}(2A_4)$. Thus the four groups $\overline{H}_i = 2^9{:}H_i$ are the inertia groups obtained in $\overline{G}$ of the linear characters of 2^9 which are partitioned into four orbits.The reader is referred to Section 7.2 to see how the structures of the H_i were obtained. The groups H_2, H_3 and H_4 are constructed from elements within G and the generators are as follows:

- $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 3B$, $\alpha_2 \in 5A$ where

$$\alpha_1 = \begin{pmatrix} 1&0&0&0&0&0&0&1&0\\ 0&0&0&1&1&1&1&1&1\\ 0&0&1&1&1&1&1&0&1\\ 0&1&0&1&1&0&0&1&0\\ 0&0&0&0&1&0&0&1&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&1&0&0\\ 0&0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&1&1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 1&0&0&0&1&1&0&1&0\\ 0&0&0&1&1&1&0&0&0\\ 0&1&0&1&1&1&0&1&1\\ 0&1&1&1&1&0&0&1&0\\ 1&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&0&0&0\\ 1&0&0&0&0&1&1&1&0\\ 0&0&0&0&0&0&1&1&0\\ 1&0&1&1&0&0&0&0&0 \end{pmatrix}$$

- $H_3 = \langle \beta_1, \beta_2 \rangle$, $\beta_1 \in 6B$, $\beta_2 \in 6B$ where

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&1&0&1&1&0&0&1&0\\ 0&1&0&1&0&1&0&1&0\\ 0&1&1&1&0&0&1&0&0\\ 0&0&1&1&1&1&1&1&1\\ 1&0&1&0&0&0&1&1&1\\ 0&0&0&1&0&1&1&0&0\\ 0&1&0&0&1&0&1&0&1\\ 1&0&1&0&1&1&1&0&0 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&0&1&0&1&1&0&0&0\\ 1&0&1&0&0&0&0&0&0\\ 0&1&1&1&0&1&0&0&0\\ 1&0&0&1&1&0&0&1&0\\ 0&0&1&0&0&1&1&1&1\\ 0&0&1&1&0&0&0&0&1\\ 0&1&1&0&0&0&0&0&0\\ 1&1&0&1&0&0&1&1&0 \end{pmatrix}$$

- $H_4 = \langle \gamma_1, \gamma_2 \rangle$, $\gamma_1 \in 4A$, $\gamma_2 \in 6A$ where

$$\gamma_1 = \begin{pmatrix} 1&0&0&0&0&1&1&0&0 \\ 1&1&0&1&1&0&0&1&1 \\ 0&1&0&1&0&0&0&0&1 \\ 0&0&0&1&0&1&1&0&0 \\ 1&1&1&1&1&0&1&0&0 \\ 1&0&1&1&0&0&1&1&0 \\ 0&1&0&0&1&0&1&0&0 \\ 1&1&1&0&0&1&0&1&0 \\ 0&1&0&0&1&0&0&0&1 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 1&1&0&0&0&0&0&0&1 \\ 1&0&0&1&0&0&0&1&0 \\ 1&0&0&0&0&1&1&1&0 \\ 0&1&0&1&0&0&0&0&1 \\ 0&0&1&1&0&1&1&0&1 \\ 0&1&1&0&1&1&1&1&1 \\ 1&1&1&0&0&1&1&0&1 \\ 1&1&1&1&0&0&0&0&0 \\ 0&1&0&1&1&1&0&1&1 \end{pmatrix}$$

For the purpose of constructing the character table of $\overline{G}$, we use the above generators of the H_i's to compute their character tables and these tables are available in the Appendix B.

7.4 The fusion of H_2, H_3 and H_4 into G

We obtain the fusions of the inertia factors H_2, H_3 and H_4 into G by using direct matrix conjugation in G and their permutation characters in G of degrees 21, 210 and 280, respectively. MAGMA was used for the various computations. The fusion maps of H_2, H_3 and H_4 into G are shown in Tables 7.5, 7.6 and 7.7.

Table 7.5: **The fusion of H_2 into $L_3(4)$:3**

$[h]_{H_2}$ →	$[g]_{L_3(4):3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):3}$	$[h]_{H_2}$ →	$[g]_{L_3(4):3}$
$1A$	$1A$	$3C$	$3A$	$5B$	$5B$	$15A$	$15C$
$2A$	$2A$	$3D$	$3B$	$6A$	$6B$	$15B$	$15D$
$2B$	$2A$	$3E$	$3E$	$6B$	$6A$	$15C$	$15B$
$3A$	$3A$	$4A$	$4A$	$6C$	$6A$	$15D$	$15A$
$3B$	$3B$	$5A$	$5A$	$6D$	$6B$		

Table 7.6: **The fusion of H_3 into $L_3(4)$:3**

$[h]_{H_3}$ →	$[g]_{L_3(4):3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):3}$	$[h]_{H_3}$ →	$[g]_{L_3(4):3}$
$1A$	$1A$	$3A$	$3B$	$3E$	$3E$	$6C$	$6A$
$2A$	$2A$	$3B$	$3A$	$4A$	$4A$	$6D$	$6B$
$2B$	$2A$	$3C$	$3B$	$6A$	$6B$		
$2C$	$2A$	$3D$	$3A$	$6B$	$6A$		

Table 7.7: **The fusion of H_4 into $L_3(4)$:3**

$[h]_{H_4}$	$\rightarrow$	$[g]_{L_3(4):3}$	$[h]_{H_4}$	$\rightarrow$	$[g]_{L_3(4):3}$	$[h]_{H_4}$	$\rightarrow$	$[g]_{L_3(4):3}$	$[h]_{H_4}$	$\rightarrow$	$[g]_{L_3(4):3}$
$1A$		$1A$	$3B$		$3B$	$3E$		$3C$	$6B$		$6B$
$2A$		$2A$	$3C$		$3A$	$4A$		$4A$			
$3A$		$3E$	$3D$		$3D$	$6A$		$6A$			

7.5 The Fischer-Clifford Matrices of $2^9{:}(L_3(4){:}3)$

The conjugacy classes of $\overline{G}$ which are displayed in the format of Table 7.4 together with the fusion maps of the inertia factor groups H_1, H_2, H_3 and H_4 into G, enabled us to compute the Fischer-Clifford matrices for G. The properties of the Fischer-Clifford matrices, as discussed in Chapter 5, are used in the construction of these matrices for $\overline{G}$. In Section 6.5 the application of these properties are sufficiently addressed. A Fischer-Clifford matrix $M(g)$ is constructed for each class representative g in G and they are listed in Table 7.8.

7.6 Character Table of $2^9{:}(L_3(4){:}3)$

Having obtained the Fischer-Clifford matrices, the fusion maps of the H_i's into $L_3(4)$:3, and the character tables of the inertia factors H_i, we construct the character table of $2^9{:}(L_3(4){:}3)$ using the methodology discussed in Chapter 5. The reader is referred to Section 6.6 for an example on the application of this methodology. Altogether we obtain 65 irreducible characters of $\overline{G}$. The set of irreducible characters of $\overline{G}$ will be partitioned into 4 blocks Δ_1, Δ_2,Δ_3 and Δ_4 where each block corresponds to the inertia factor groups $\overline{H}_i = 2^9{:}H_i$. Therefore $Irr(2^9{:}(L_3(4){:}3)) = \bigcup_{i=1}^{4} \Delta_i$, where $\Delta_1 = \{\chi_j | 1 \leq j \leq 22\}$, $\Delta_2 = \{\chi_j | 23 \leq j \leq 41\}$, $\Delta_3 = \{\chi_j | 42 \leq j \leq 55\}$ and $\Delta_4 = \{\chi_j | 56 \leq j \leq 65\}$. The character table of $2^9{:}(L_3(4){:}3)$ is shown in Table 7.9. The consistency and accuracy of the character table of $\overline{G}$ have been tested by using Programme C (see Appendix A).

Table 7.8: The Fischer-Clifford matrices of $2^9{:}(L_3(4){:}3)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 21 & -11 & 5 & -3 \\ 210 & 50 & 2 & -6 \\ 280 & -40 & -8 & 8 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & -2 & 2 & 0 \\ 4 & -4 & 4 & -4 & 2 & -2 & 0 \\ 6 & 6 & 6 & 6 & 0 & 0 & -2 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 \end{pmatrix}$
$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 5 & 5 & -3 & -3 & 1 & 1 \\ 5 & -5 & -3 & 3 & -1 & 1 \\ 10 & 10 & 2 & 2 & -2 & -2 \\ 10 & -10 & 2 & -2 & 2 & -2 \end{pmatrix}$	$M(3B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 5 & 5 & -3 & -3 & 1 & 1 \\ 5 & -5 & -3 & 3 & -1 & 1 \\ 10 & 10 & 2 & 2 & -2 & -2 \\ 10 & -10 & 2 & -2 & 2 & -2 \end{pmatrix}$
$M(3C) = \begin{pmatrix} 1 & 1 \\ 7 & -1 \end{pmatrix}$	$M(3D) = \begin{pmatrix} 1 & 1 \\ 7 & -1 \end{pmatrix}$
$M(3E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -3 & 1 & -1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$
$M(5A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(5B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 \\ 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$
$M(7A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(7B) = \begin{pmatrix} 1 \end{pmatrix}$
$M(15A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(15B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(15C) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(15D) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(21A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(21B) = \begin{pmatrix} 1 \end{pmatrix}$
$M(21C) = \begin{pmatrix} 1 \end{pmatrix}$	$M(21D) = \begin{pmatrix} 1 \end{pmatrix}$

Table 7.9: The Character table of 2^9:($L_3(4)$:3)

	1A				2A							3A					
	1A	2A	2B	2C	2D	2E	4A	4B	4C	4D	4E	3A	6A	6B	6C	6D	6E
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	A	A	A	A	A	A
χ_3	1	1	1	1	1	1	1	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_4	20	20	20	20	4	4	4	4	4	4	4	5	5	5	5	5	5
χ_5	20	20	20	20	4	4	4	4	4	4	4	B	B	B	B	B	B
χ_6	20	20	20	20	4	4	4	4	4	4	4	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$
χ_7	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_8	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_9	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{10}	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{11}	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{12}	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0
χ_{13}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3
χ_{14}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3
χ_{15}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	C	C	C	C	C	C
χ_{16}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	C	C	C	C	C	C
χ_{17}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$
χ_{18}	63	63	63	63	-1	-1	-1	-1	-1	-1	-1	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$
χ_{19}	64	64	64	64	0	0	0	0	0	0	0	4	4	4	4	4	4
χ_{20}	64	64	64	64	0	0	0	0	0	0	0	D	D	D	D	D	D
χ_{21}	64	64	64	64	0	0	0	0	0	0	0	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$
χ_{22}	105	105	105	105	9	9	9	9	9	9	9	0	0	0	0	0	0
χ_{23}	21	-11	5	-3	5	-3	5	-3	-3	1	1	6	4	-2	-4	0	2
χ_{24}	21	-11	5	-3	5	-3	5	-3	-3	1	1	E	D	K	$-D$	0	$-K$
χ_{25}	21	-11	5	-3	5	-3	5	-3	-3	1	1	$\overline{E}$	$\overline{D}$	$\overline{K}$	$-\overline{D}$	0	$-\overline{K}$
χ_{26}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	3	-3	3	-3	-3	3
χ_{27}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	3	-3	3	-3	-3	3
χ_{28}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	C	$-C$	C	$-C$	$-C$	C
χ_{29}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	C	$-C$	C	$-C$	$-C$	C
χ_{30}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	$\overline{C}$	$-\overline{C}$	$\overline{C}$	$-\overline{C}$	$-\overline{C}$	$\overline{C}$
χ_{31}	63	-33	15	-9	-1	7	-1	7	-1	-5	3	$\overline{C}$	$-\overline{C}$	$\overline{C}$	$-\overline{C}$	$-\overline{C}$	$\overline{C}$
χ_{32}	84	-44	20	-12	4	4	4	4	-4	-4	4	9	1	1	-7	-3	5
χ_{33}	84	-44	20	-12	4	4	4	4	-4	-4	4	F	$\overline{A}$	$\overline{A}$	$-M$	$-C$	B
χ_{34}	84	-44	20	-12	4	4	4	4	-4	-4	4	$\overline{F}$	A	A	$-\overline{M}$	$-\overline{C}$	$\overline{B}$
χ_{35}	105	-55	25	-15	9	1	9	1	-7	-3	5	0	-10	8	-2	-6	4
χ_{36}	105	-55	25	-15	9	1	9	1	-7	-3	5	0	$-H$	L	K	$-E$	D
χ_{37}	105	-55	25	-15	9	1	9	1	-7	-3	5	0	$-\overline{H}$	$\overline{L}$	$\overline{K}$	$-\overline{E}$	$\overline{D}$
χ_{38}	315	-165	75	-45	11	-13	11	-13	-5	7	-1	15	15	-9	-9	3	3
χ_{39}	315	-165	75	-45	11	-13	11	-13	-5	7	-1	G	G	$-F$	$-F$	C	C
χ_{40}	315	-165	75	-45	11	-13	11	-13	-5	7	-1	$\overline{G}$	$\overline{G}$	$-\overline{F}$	$-\overline{F}$	$\overline{C}$	$\overline{C}$
χ_{41}	945	-495	225	-135	-15	9	-15	9	9	-3	-3	0	0	0	0	0	0
χ_{42}	210	50	2	-6	18	10	2	-6	2	-2	-2	15	5	-1	5	-3	-1
χ_{43}	210	50	2	-6	2	-6	18	10	2	-2	-2	15	5	-1	5	-3	-1
χ_{44}	210	50	2	-6	2	-6	18	10	2	-2	-2	G	B	$-\overline{A}$	B	$-C$	$-\overline{A}$
χ_{45}	210	50	2	-6	2	-6	18	10	2	-2	-2	$\overline{G}$	$\overline{B}$	$-A$	$\overline{B}$	$-\overline{C}$	$-A$
χ_{46}	210	50	2	-6	18	10	2	-6	2	-2	-2	G	B	$-\overline{A}$	B	$-C$	$-\overline{A}$
χ_{47}	210	50	2	-6	18	10	2	-6	2	-2	-2	$\overline{G}$	$\overline{B}$	$-A$	$\overline{B}$	$-\overline{C}$	$-A$
χ_{48}	420	100	4	-12	20	4	20	4	4	-4	-4	15	25	7	1	-3	-5
χ_{49}	420	100	4	-12	20	4	20	4	4	-4	-4	G	J	M	$\overline{A}$	$-C$	$-B$
χ_{50}	420	100	4	-12	20	4	20	4	4	-4	-4	$\overline{G}$	$\overline{J}$	$\overline{M}$	A	$-\overline{C}$	$-\overline{B}$
χ_{51}	1260	300	12	-36	-4	-20	-4	-20	4	-4	4	15	-15	-9	9	-3	3
χ_{52}	1260	300	12	-36	-4	-20	-4	-20	4	-4	4	G	$-G$	$-F$	F	$-C$	C
χ_{53}	1260	300	12	-36	-4	-20	-4	-20	4	-4	4	$\overline{G}$	$-\overline{G}$	$-\overline{F}$	$\overline{F}$	$-\overline{C}$	$\overline{C}$
χ_{54}	1890	450	18	-54	18	42	-30	-6	-6	6	-2	0	0	0	0	0	0
χ_{55}	1890	450	18	-54	-30	-6	18	42	-6	6	-2	0	0	0	0	0	0
χ_{56}	280	-40	-8	8	8	-8	-8	8	0	0	0	10	-10	2	-2	2	-2
χ_{57}	280	-40	-8	8	8	-8	-8	8	0	0	0	H	$-H$	$-K$	K	$-K$	K
χ_{58}	280	-40	-8	8	8	-8	-8	8	0	0	0	$\overline{H}$	$-\overline{H}$	$-\overline{K}$	$\overline{K}$	$-\overline{K}$	$\overline{K}$
χ_{59}	560	-80	-16	16	-16	16	16	-16	0	0	0	-10	10	-2	2	-2	2
χ_{60}	560	-80	-16	16	-16	16	16	-16	0	0	0	$-H$	H	K	$-K$	K	$-K$
χ_{61}	560	-80	-16	16	-16	16	16	-16	0	0	0	$-\overline{H}$	$\overline{H}$	$\overline{K}$	$-\overline{K}$	$\overline{K}$	$-\overline{K}$
χ_{62}	840	-120	-24	24	24	-24	-24	24	0	0	0	0	0	0	0	0	0
χ_{63}	2240	-320	-64	64	0	0	0	0	0	0	0	20	-20	4	-4	4	-4
χ_{64}	2240	-320	-64	64	0	0	0	0	0	0	0	I	$-I$	$\overline{D}$	$-\overline{D}$	$\overline{D}$	$-\overline{D}$
χ_{65}	2240	-320	-64	64	0	0	0	0	0	0	0	$\overline{I}$	$-\overline{I}$	D	$-D$	D	$-D$

Table 7.9 (continue)

	3B						3C		3D		3E				4A			
	$3B$	$6F$	$6G$	$6H$	$6I$	$6J$	$3C$	$6K$	$3D$	$6L$	$3E$	$6M$	$6N$	$6O$	$4F$	$4G$	8A	8B
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	1	1	1	1	1	1	1	1
χ_3	A	A	A	A	A	A	A	A	$\overline{A}$	$\overline{A}$	1	1	1	1	1	1	1	1
χ_4	5	5	5	5	5	5	-1	-1	-1	-1	2	2	2	2	0	0	0	0
χ_5	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	2	2	2	2	0	0	0	0
χ_6	B	B	B	B	B	B	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	2	2	2	2	0	0	0	0
χ_7	0	0	0	0	0	0	3	3	3	3	0	0	0	0	1	1	1	1
χ_8	0	0	0	0	0	0	3	3	3	3	0	0	0	0	1	1	1	1
χ_9	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$	0	0	0	0	1	1	1	1
χ_{10}	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$	0	0	0	0	1	1	1	1
χ_{11}	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C	0	0	0	0	1	1	1	1
χ_{12}	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C	0	0	0	0	1	1	1	1
χ_{13}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{14}	3	3	3	3	3	3	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{15}	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{16}	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{17}	C	C	C	C	C	C	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{18}	C	C	C	C	C	C	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{19}	4	4	4	4	4	4	1	1	1	1	1	1	1	1	0	0	0	0
χ_{20}	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$	$\overline{D}$	A	A	$\overline{A}$	$\overline{A}$	1	1	1	1	0	0	0	0
χ_{21}	D	D	D	D	D	D	$\overline{A}$	$\overline{A}$	A	A	1	1	1	1	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	-3	-3	-3	-3	1	1	1	1
χ_{23}	6	4	-2	-4	0	2	0	0	0	0	3	-3	1	-1	1	1	1	-1
χ_{24}	$\overline{E}$	$\overline{D}$	$\overline{K}$	$-\overline{D}$	0	$-\overline{K}$	0	0	0	0	3	-3	1	-1	1	1	1	-1
χ_{25}	E	D	K	$-D$	0	$-K$	0	0	0	0	3	-3	1	-1	1	1	1	-1
χ_{26}	3	-3	3	-3	-3	3	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{27}	3	-3	3	-3	-3	3	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{28}	$\overline{C}$	$-\overline{C}$	$\overline{C}$	$-\overline{C}$	$-\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{29}	$\overline{C}$	$-\overline{C}$	$\overline{C}$	$-\overline{C}$	$-\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{30}	C	$-C$	C	$-C$	$-C$	C	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{31}	C	$-C$	C	$-C$	$-C$	C	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{32}	9	1	1	-7	-3	5	0	0	0	0	3	-3	1	-1	0	0	0	0
χ_{33}	$\overline{F}$	A	A	$-\overline{M}$	$-\overline{C}$	$\overline{B}$	0	0	0	0	3	-3	1	-1	0	0	0	0
χ_{34}	F	$\overline{A}$	$\overline{A}$	$-M$	$-C$	B	0	0	0	0	3	-3	1	-1	0	0	0	0
χ_{35}	0	-10	8	-2	-6	4	0	0	0	0	-3	3	-1	1	1	1	1	-1
χ_{36}	0	$-\overline{H}$	$\overline{L}$	$\overline{K}$	$-\overline{E}$	$\overline{D}$	0	0	0	0	-3	3	-1	1	1	1	1	-1
χ_{37}	0	$-H$	L	K	$-E$	D	0	0	0	0	-3	3	-1	1	1	1	1	-1
χ_{38}	15	15	-9	-9	3	3	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{39}	$\overline{G}$	$\overline{G}$	$-\overline{F}$	$-\overline{F}$	$\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{40}	G	G	$-F$	$-F$	C	C	0	0	0	0	0	0	0	0	-1	-1	-1	1
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1
χ_{42}	15	5	-1	5	-3	-1	0	0	0	0	3	3	-1	-1	2	2	-2	0
χ_{43}	15	5	-1	5	-3	-1	0	0	0	0	3	3	-1	-1	-2	-2	2	0
χ_{44}	$\overline{G}$	$\overline{B}$	$-A$	$\overline{B}$	$-\overline{C}$	$-A$	0	0	0	0	3	3	-1	-1	-2	-2	2	0
χ_{45}	G	B	$-\overline{A}$	B	$-C$	$-\overline{A}$	0	0	0	0	3	3	-1	-1	-2	-2	2	0
χ_{46}	$\overline{G}$	$\overline{B}$	$-A$	$\overline{B}$	$-\overline{C}$	$-A$	0	0	0	0	3	3	-1	-1	2	2	-2	0
χ_{47}	G	B	$-\overline{A}$	B	$-C$	$-\overline{A}$	0	0	0	0	3	3	-1	-1	2	2	-2	0
χ_{48}	15	25	7	1	-3	-5	0	0	0	0	-3	-3	1	1	0	0	0	0
χ_{49}	$\overline{G}$	$\overline{J}$	$\overline{M}$	A	$-\overline{C}$	$-\overline{B}$	0	0	0	0	-3	-3	1	1	0	0	0	0
χ_{50}	G	J	M	$\overline{A}$	$-C$	$-B$	0	0	0	0	-3	-3	1	1	0	0	0	0
χ_{51}	15	-15	-9	9	-3	3	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	$\overline{G}$	$-\overline{G}$	$-\overline{F}$	$\overline{F}$	$-\overline{C}$	$\overline{C}$	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	G	$-G$	$-F$	F	$-C$	C	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	2	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2	-2	0
χ_{56}	10	-10	2	-2	2	-2	7	-1	7	-1	1	-1	-1	1	4	-4	0	0
χ_{57}	$\overline{H}$	$-\overline{H}$	$-\overline{K}$	$\overline{K}$	$-\overline{K}$	$\overline{K}$	$\overline{M}$	$-A$	M	$-\overline{A}$	1	-1	-1	1	4	-4	0	0
χ_{58}	H	$-H$	$-K$	K	$-K$	K	M	$-\overline{A}$	$\overline{M}$	$-A$	1	-1	-1	1	4	-4	0	0
χ_{59}	-10	10	-2	2	-2	2	-7	1	-7	1	2	-2	-2	2	0	0	0	0
χ_{60}	$-\overline{H}$	$\overline{H}$	$\overline{K}$	$-\overline{K}$	$\overline{K}$	$-\overline{K}$	$-\overline{M}$	A	$-M$	$\overline{A}$	2	-2	-2	2	0	0	0	0
χ_{61}	H	$-H$	$-K$	K	$-K$	K	M	$-\overline{A}$	$\overline{M}$	$-A$	2	-2	-2	2	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	3	-3	-3	3	-4	4	0	0
χ_{63}	20	-20	4	-4	4	-4	-7	1	-7	1	-1	1	1	-1	0	0	0	0
χ_{64}	$\overline{I}$	$-\overline{I}$	D	$-D$	D	$-D$	$-M$	$\overline{A}$	$-\overline{M}$	A	-1	1	1	-1	0	0	0	0
χ_{65}	I	$-I$	$\overline{D}$	$-\overline{D}$	$\overline{D}$	$-\overline{D}$	$-\overline{M}$	A	$-M$	$\overline{A}$	-1	1	1	-1	0	0	0	0

Table 7.9 (continue)

	5A		5B		6A						6B					
	$5A$	$10A$	$5B$	$10B$	$6P$	$6Q$	$12A$	$12B$	$12C$	$12D$	$6R$	$6S$	$12E$	$12F$	$12G$	12H
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A	A	A	A
χ_3	1	1	1	1	A	A	A	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_4	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_5	0	0	0	0	A	A	A	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$
χ_6	0	0	0	0	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A	A	A	A
χ_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	N	N	$^*N$	$^*N$	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{14}	$^*N$	$^*N$	N	N	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{15}	N	N	$^*N$	$^*N$	$-A$	$-A$	$-A$	$-A$	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$
χ_{16}	$^*N$	$^*N$	N	N	$-A$	$-A$	$-A$	$-A$	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$
χ_{17}	N	N	$^*N$	$^*N$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	$-A$	$-A$	$-A$	$-A$
χ_{18}	$^*N$	$^*N$	N	N	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	$-A$	$-A$	$-A$	$-A$
χ_{19}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{20}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{21}	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	1	-1	1	-1	2	0	2	0	0	-2	2	0	2	0	0	-2
χ_{24}	1	-1	1	-1	$-\overline{K}$	0	$-\overline{K}$	0	0	$-\overline{K}$	$-K$	0	$-K$	0	0	K
χ_{25}	1	-1	1	-1	$-K$	0	$-K$	0	0	K	$-\overline{K}$	0	$-\overline{K}$	0	0	K
χ_{26}	N	$-N$	$^*N$	$-{}^*N$	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
χ_{27}	$^*N$	$-{}^*N$	N	$-N$	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1
χ_{28}	N	$-N$	$^*N$	$-{}^*N$	$-A$	A	$-A$	A	$-A$	A	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$
χ_{29}	$^*N$	$-{}^*N$	N	$-N$	$-A$	A	$-A$	A	$-A$	A	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$
χ_{30}	$^*N$	$-{}^*N$	N	$-N$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-A$	A	$-A$	A	$-A$	A
χ_{31}	N	$-N$	$^*N$	$-{}^*N$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-A$	A	$-A$	A	$-A$	A
χ_{32}	-1	1	-1	1	1	1	1	1	-1	-1	1	1	1	1	-1	-1
χ_{33}	-1	1	-1	1	A	A	A	A	$-A$	$-A$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-\overline{A}$
χ_{34}	-1	1	-1	1	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	A	A	A	A	$-A$	$-A$
χ_{35}	0	0	0	0	0	-2	0	-2	2	0	0	-2	0	-2	2	0
χ_{36}	0	0	0	0	0	$\overline{K}$	0	$\overline{K}$	$-\overline{K}$	0	0	K	0	K	$-K$	0
χ_{37}	0	0	0	0	0	K	0	K	$-K$	0	0	$\overline{K}$	0	$\overline{K}$	$-\overline{K}$	0
χ_{38}	0	0	0	0	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1
χ_{39}	0	0	0	0	$-A$	$-A$	$-A$	$-A$	A	A	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$
χ_{40}	0	0	0	0	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$	$-A$	$-A$	$-A$	$-A$	A	A
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	3	1	-1	-3	-1	1	3	1	-1	-3	-1	1
χ_{43}	0	0	0	0	-1	-3	3	1	-1	1	-1	-3	3	1	-1	1
χ_{44}	0	0	0	0	$-A$	$-\overline{C}$	$\overline{C}$	A	$-A$	A	$-\overline{A}$	$-C$	C	$\overline{A}$	$-\overline{A}$	$\overline{A}$
χ_{45}	0	0	0	0	$-\overline{A}$	$-C$	C	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$-A$	$-\overline{C}$	$-\overline{C}$	A	$-A$	A
χ_{46}	0	0	0	0	$-\overline{C}$	A	$-A$	$-\overline{C}$	$-A$	A	C	$\overline{A}$	$\overline{A}$	$-C$	$-\overline{A}$	$\overline{A}$
χ_{47}	0	0	0	0	C	$\overline{A}$	$-\overline{A}$	$-C$	$-\overline{A}$	$\overline{A}$	$\overline{C}$	A	$-A$	$-\overline{C}$	$-A$	A
χ_{48}	0	0	0	0	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1
χ_{49}	0	0	0	0	$-A$	A	$-A$	A	A	$-A$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$
χ_{50}	0	0	0	0	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-A$	A	$-A$	A	A	$-A$
χ_{51}	0	0	0	0	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1
χ_{52}	0	0	0	0	$-A$	A	$-A$	A	A	$-A$	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$
χ_{53}	0	0	0	0	$-\overline{A}$	$\overline{A}$	$-\overline{A}$	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-A$	A	$-A$	A	A	$-A$
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	2	-2	-2	2	0	0	2	-2	-2	2	0	0
χ_{57}	0	0	0	0	$-\overline{K}$	$\overline{K}$	$\overline{K}$	$-\overline{K}$	0	0	$-K$	K	K	$-K$	0	0
χ_{58}	0	0	0	0	$-K$	K	K	$-K$	0	0	$-\overline{K}$	$\overline{K}$	$\overline{K}$	$-\overline{K}$	0	0
χ_{59}	0	0	0	0	2	-2	-2	2	0	0	2	-2	-2	2	0	0
χ_{60}	0	0	0	0	$-\overline{K}$	$\overline{K}$	$\overline{K}$	$-\overline{K}$	0	0	$-K$	K	K	$-K$	0	0
χ_{61}	0	0	0	0	$-K$	K	K	$-K$	0	0	$-\overline{K}$	$\overline{K}$	$\overline{K}$	$-\overline{K}$	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 7.9 (continue)

	7A	$7B$	$15A$		15B		$15C$		15D		21A	21B	21C	$21D$
	$7A$	$7B$	$15A$	$30A$	$15B$	$30B$	$15C$	$30C$	$15D$	$30D$	$21A$	$21B$	$21C$	$21D$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	$\overline{A}$	$\overline{A}$	A	A	$\overline{A}$	$\overline{A}$	A	A	$\overline{A}$	A	$\overline{A}$	A
χ_3	1	1	A	A	$\overline{A}$	$\overline{A}$	A	A	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	A	$\overline{A}$
χ_4	-1	-1	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_5	-1	-1	0	0	0	0	0	0	0	0	$-A$	$-\overline{A}$	$-A$	$-\overline{A}$
χ_6	-1	-1	0	0	0	0	0	0	0	0	$-\overline{A}$	$-A$	$-\overline{A}$	$-A$
χ_7	O	$\overline{O}$	0	0	0	0	0	0	0	0	$\overline{O}$	$\overline{O}$	O	O
χ_8	$\overline{O}$	O	0	0	0	0	0	0	0	0	O	O	$\overline{O}$	$\overline{O}$
χ_9	O	$\overline{O}$	0	0	0	0	0	0	0	0	R	$\overline{S}$	S	$\overline{R}$
χ_{10}	$\overline{O}$	O	0	0	0	0	0	0	0	0	S	$\overline{R}$	R	$\overline{S}$
χ_{11}	O	$\overline{O}$	0	0	0	0	0	0	0	0	$\overline{S}$	R	$\overline{R}$	S
χ_{12}	$\overline{O}$	O	0	0	0	0	0	0	0	0	$\overline{R}$	S	$\overline{S}$	R
χ_{13}	0	0	$^*N$	$^*N$	$^*N$	$^*N$	N	N	N	N	0	0	0	0
χ_{14}	0	0	N	N	N	N	$^*N$	$^*N$	$^*N$	$^*N$	0	0	0	0
χ_{15}	0	0	P	P	$\overline{P}$	$\overline{P}$	Q	Q	$\overline{Q}$	$\overline{Q}$	0	0	0	0
χ_{16}	0	0	Q	Q	$\overline{Q}$	$\overline{Q}$	P	P	$\overline{P}$	$\overline{P}$	0	0	0	0
χ_{17}	0	0	$\overline{P}$	$\overline{P}$	P	P	$\overline{Q}$	$\overline{Q}$	Q	Q	0	0	0	0
χ_{18}	0	0	$\overline{Q}$	$\overline{Q}$	Q	Q	$\overline{P}$	$\overline{P}$	P	P	0	0	0	0
χ_{19}	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{20}	1	1	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	A	$\overline{A}$	A	$\overline{A}$
χ_{21}	1	1	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	$-\overline{A}$	$-\overline{A}$	$-A$	$-A$	$\overline{A}$	A	$\overline{A}$	A
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	1	-1	1	-1	1	-1	1	-1	0	0	0	0
χ_{24}	0	0	A	$-A$	$\overline{A}$	$-\overline{A}$	A	$-A$	$\overline{A}$	$-\overline{A}$	0	0	0	0
χ_{25}	0	0	$\overline{A}$	$-\overline{A}$	A	$-A$	$\overline{A}$	$-\overline{A}$	A	$-A$	0	0	0	0
χ_{26}	0	0	$^*N$	$-^*N$	$^*N$	$-^*N$	N	$-N$	N	$-N$	0	0	0	0
χ_{27}	0	0	N	$-N$	N	$-N$	$^*N$	$-^*N$	$^*N$	$-^*N$	0	0	0	0
χ_{28}	0	0	P	$-P$	$\overline{P}$	$-\overline{P}$	$-Q$	$-Q$	$\overline{Q}$	$-\overline{A}$	0	0	0	0
χ_{29}	0	0	Q	$-Q$	$\overline{Q}$	$-\overline{Q}$	P	$-P$	$\overline{P}$	$-\overline{P}$	0	0	0	0
χ_{30}	0	0	$\overline{Q}$	$-\overline{Q}$	Q	$-Q$	$\overline{P}$	$-\overline{P}$	P	$-P$	0	0	0	0
χ_{31}	0	0	$\overline{B}$	$-\overline{P}$	P	$-P$	$\overline{Q}$	$-\overline{Q}$	Q	$-Q$	0	0	0	0
χ_{32}	0	0	-1	1	-1	1	-1	1	-1	1	0	0	0	0
χ_{33}	0	0	$-A$	A	$-\overline{A}$	$\overline{A}$	$-A$	A	$-\overline{A}$	$\overline{A}$	0	0	0	0
χ_{34}	0	0	$-\overline{A}$	$\overline{A}$	$-A$	A	$-\overline{A}$	$\overline{A}$	$-A$	A	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where $A = \frac{-1+i\sqrt{3}}{2}$=b3, $B = \frac{-5-5i\sqrt{3}}{2}$=-5-5b3, $C = \frac{-3-3i\sqrt{3}}{2}$=-3-3b3, $D = -2-2i\sqrt{3}$=-2-2i3, $E = -3-3i\sqrt{3}$=-3-3i3, $F = \frac{-9-9i\sqrt{3}}{2}$=-9-9b3, $G = \frac{-15-15i\sqrt{3}}{2}$=-15-15b3, $H = -5-5i\sqrt{3}$=-5-5i3, $I = -10+10i\sqrt{3}$=20b3, $J = \frac{-25-25i\sqrt{3}}{2}$=-25-25b3, $K = 1+i\sqrt{3}$=1+i3, $L = -4-4i\sqrt{3}$=-4-4i3, $M = \frac{-7-7i\sqrt{3}}{2}$=-7-7b3, $N = \frac{1-\sqrt{5}}{2}$=-b5, $O = \frac{-1-i\sqrt{-7}}{2}$=-1-b7, $P = -E(15)^{11} - E(15)^{14}$, $Q = -E(15)^2 - E(15)^8$, $R = E(21)^5 + E(21)^{17} + E(21)^{20}$ and $S = E(21)^2 + E(21)^8 + E(21)^{11}$.

The information about the conjugacy classes found in Table 7.4 can be used to compute the power maps for the elements of $\overline{G}$ and then Programme C (see Appendix A) is used to verify that we obtained the unique p-power maps listed in Table 7.10 for our Table 7.9.

Table 7.10: The power maps of the elements of 2^9:$(L_3(4)$:$3)$

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7
$1A$	$1A$					$2A$	$2D$	$1A$			
	$2A$	$1A$					$2E$	$1A$			
	$2B$	$1A$					$4A$	$2A$			
	$2C$	$1A$					4B	2A			
							$4C$	$2B$			
							$4D$	$2B$			
							$4E$	$2B$			
$3A$	$3A$		$1A$			$3B$	$3B$		$1A$		
	$6A$	$3B$	$2A$				$6F$	$3A$	$2A$		
	$6B$	$3B$	$2A$				$6G$	$3A$	$2A$		
	$6C$	$3B$	$2B$				$6H$	$3A$	$2B$		
	$6D$	$3B$	$2C$				$6I$	$3A$	$2C$		
	$6E$	$3B$	$2B$				$6J$	$3A$	$2B$		
$3C$	$3C$		1A			$3D$	$3D$		1A		
	$6K$	$3D$	2C				$6L$	$3C$	2C		
$3E$	$3E$		$1A$			$4A$	$4F$	$2D$			
	$6M$	$3E$	$2C$				4G	2D			
	$6N$	$3E$	$2A$				$8A$	$4B$			
	$6O$	$3E$	$2B$				8B	4E			
$5A$	$5A$			$1A$		$5B$	$5B$			$1A$	
	$10A$	$5B$		$2A$			$10B$	5A		$2A$	
$6A$	$6P$	$3A$	2D			$6B$	$6R$	$3B$	2D		
	$6Q$	$3A$	2E				6S	3B	2E		
	$12A$	$6B$	4A				$12E$	$6G$	4A		
	$12B$	$6B$	4B				$12F$	$6G$	4B		
	$12C$	$6E$	4C				$12G$	$6J$	4C		
	$12D$	$6E$	4D				$12H$	$6J$	4D		
$7A$	$7A$				1A	$7B$	$7B$				1A
$15A$	$15A$		5A	3A		$15B$	$15B$		5A	3B	
	$30A$	$15D$	10A	6A			30B	15C	10A	6F	
$15C$	$15C$		5B	3A		$15D$	$15D$		5B	3B	
	$30C$	$15B$	10B	6A			30D	15A	10B	6F	
$21A$	$21A$		7A		3C	$21B$	$21B$		7A		3D
$21C$	$21C$		7B		3C	$21D$	$21D$		7B		3D

7.7 The Fusion of 2^9:$(L_3(4)$:$3)$ into $U_6(2)$:3

The group $\overline{G}$ is a maximal subgroup of $U_6(2)$:3 of index 891. Hence the action of $U_6(2)$:3 on the cosets of $\overline{G}$ gives rise to a permutation character $\chi(U_6(2)$:$3|\overline{G})$ of degree 891. We deduce from the ordinary character table of $U_6(2){:}3$ uploaded in the GAP Library that $\chi(U_6(2){:}3|\overline{G}) = 1a + 22a + 252a + 616a$. The partial fusion of $\overline{G}$ into $U_6(2){:}3$ is made possible by using the values of $\chi(U_6(2){:}3|\overline{G})$ on the classes of $\overline{G}$ and the information provided by the power maps of the classes of $U_6(2){:}3$ and $\overline{G}$. Similarly, the technique of set intersections (as it was done in Section 6.7) is used to restrict the irreducible ordinary characters $\phi_1 = 22a$, $\phi_2 = 22b$, $\phi_3 = 231a$, $\phi_4 = 385a$, $\phi_5 = 440a$ and $\phi_6 = 440b$ of $U_6(2){:}3$ to $\overline{G}$. Hence based on the partial fusion of $\overline{G}$ into $U_6(2){:}3$, we obtain that $(\phi_1)_{2^9:(L_3(4):3} = \chi_1 + \chi_{23}$, $(\phi_2)_{2^9:(L_3(4):3)} = \chi_3 + \chi_{24}$, $(\phi_3)_{2^9:(L_3(4):3)} = \chi_{23} + \chi_{43}$, $(\phi_4)_{2^9:(L_3(4):3)} = \chi_{35} + \chi_{56}$, $(\phi_5)_{2^9:(L_3(4):3)} = \chi_4 + \chi_{48}$ and $(\phi_6)_{2^9:(L_3(4):3)} = \chi_6 + \chi_{50}$. Thus the complete fusion map of $\overline{G}$ into $U_6(2){:}3$ is obtained and is listed in Table 7.11.

Table 7.11: The fusion of $2^9{:}(L_3(4){:}3)$ into $U_6(2){:}3$

$[g]_{L_3(4):3}$	$[x]_{2^9:(L_3(4):3)}$	$\longrightarrow$ $[y]_{U_6(2):3}$	$[g]_{L_3(4):3}$	$[x]_{2^9:(L_3(4):3)}$	$\longrightarrow$ $[y]_{U_6(2):3}$
$1A$	$1A$	$1A$	$2A$	$2D$	$2B$
	$2A$	$2A$		$2E$	$2C$
	$2B$	$2B$		$4A$	$4A$
	$2C$	$2C$		$4B$	$4B$
				$4C$	$4D$
				$4D$	$4E$
				$4E$	$4C$
$3A$	$3A$	$3F$	$3B$	$3B$	$3G$
	$6A$	$6K$		$6F$	$6L$
	$6B$	$6O$		$6G$	$6P$
	$6C$	$6S$		$6H$	$6T$
	$6D$	$6Y$		$6I$	$6Z$
	$6E$	$6U$		$6J$	$6V$
$3C$	$3C$	$3K$	$3D$	$3D$	$3J$
	$6K$	$6AD$		$6L$	$6AC$
$3E$	$3E$	$3C$	$4A$	$4F$	$4C$
	$6M$	$6H$		$4G$	$4E$
	$6N$	$6F$		$8A$	$8A$
	$6O$	$6G$		$8B$	$8B$
$5A$	$5A$	$5A$	$5B$	$5B$	$5A$
	$10A$	$10A$		$10B$	$10A$
$6A$	$6P$	$6V$	$6B$	$6R$	$6U$
	$6Q$	$6Z$		$6S$	$6Y$
	$12A$	$12M$		$12E$	$12L$
	$12B$	$12Q$		$12F$	$12P$
	$12C$	$12U$		$12G$	$12T$
	$12D$	$12Y$		$12H$	$12X$
$7A$	$7A$	$7A$	$7B$	$7B$	$7A$
$15A$	$15A$	$15C$	$15B$	$15B$	$15B$
	$30A$	$30B$		$30B$	$30A$
$15C$	$15C$	$15C$	$15D$	$15D$	$15B$
	$30C$	$30B$		$30D$	$30A$
$21A$	$21A$	$21B$	$21B$	$21B$	$21A$
$21C$	$21C$	$21B$	$21D$	$21D$	$21A$

Chapter 8

The Fischer-Clifford matrices of $2^8{:}(U_4(2){:}2)$ as a subgroup of $O_{10}^+(2)$

In the ATLAS we see that the orthogonal group $O_{10}^+(2)$ has a maximal subgroup $L = 2^8{:}O_8^+(2)$, where the quotient group $L/2^8 \cong O_8$ has three non-conjugate maximal subgroups K_1, K_2 and K_3 of the type $Sp_6(2)$. We should mention here that the ordinary character table of L was computed in [52]. Two of these groups, say K_1 and K_2, have eight-dimensional absolutely irreducible modules over $GF(2)$ and their pre-images in L are isomorphic groups L_1 and L_2 of the type $2^8{:}Sp_6(2)$, having 70 classes of elements. This can be easily verified by first obtaining a permutation representation on 496 points of L from Wilson's online ATLAS [70] and then computing all the classes of maximal subgroups of L using the computer algebra system GAP [67]. The permutation representation of degree 496 is the smallest permutation representation $O_{10}^+(2)$ has on the cosets of all its maximal subgroups (see Wilson's online ATLAS) and hence it make our computations for the group L relatively faster. We could also use the permutation representation of degree 527 the group $O_{10}^+(2)$ has on the cosets of L. Also, with the aid of GAP we found that the pre-image $L_3 = 2^8{:}Sp_6(2)$ of K_3 in L is a group having 168 conjugacy classes of elements. It was shown in [26] that L_3 is one of the inertia factors of the factor group $F = D/2^{10} \cong 2^{16}{:}O_{10}^+(2)$, where $2^{10+16\cdot}O_{10}^+(2)$ is one of the maximal subgroups of the Fischer-Griess "Monster" group M.

Ali and Moori in [4] computed the Fischer-Clifford matrices and the associated ordinary character of a maximal subgroup $\overline{G}_1 = 2^8{:}Sp_6(2)$ of L. This group $\overline{G}_1$ is isomorphic to the groups L_1 and L_2. Since $Sp_6(2)$ has only one maximal subgroup of the type $U_4(2){:}2 \cong GO_6^-(2)$, the pre-image of this group in G_1 is a split extension group $\overline{G} = 2^8{:}(U_4(2){:}2)$. As we will see in Section 8.1 of this chapter, the action of $U_4(2){:}2$ on 2^8 has three orbits of lengths 1, 120 and 135. Therefore, we can deduce that $U_4(2){:}2$ does not fix any non-trivial subspace of 2^8 and hence 2^8 is an irreducible module for $U_4(2){:}2$. Since the Fischer-Clifford matrices of the group $\overline{G}$ are not yet known, these matrices and the associated ordinary character table of $\overline{G}$ will be calculated.

The method of coset-analysis will be used to compute the conjugacy classes of elements in $\overline{G}$. Also, the fusion map of the conjugacy classes of $\overline{G}$ into the classes of $\overline{G}_1$ will be determined. Most of our computations were carried out with the aid of the computer algebra systems MAGMA and GAP.

8.1 On the group $2^8{:}(U_4(2){:}2)$ and its conjugacy classes

In this section, we apply the method of coset analysis, as discussed in Chapter 3, to determine the conjugacy classes of elements of $2^8{:}(U_4(2){:}2)$. Let $\overline{G} = N{\cdot}G$ be an extension of N by G, where N is abelian. Then for $g \in G$, we write $\overline{g}$ for a lifting of g in $\overline{G}$ under the natural homomorphism $\overline{G} \longrightarrow G$. We consider a coset $N\overline{g}$ for each class representative g of G, writing k for the number of orbits of N acting by conjugation on the coset $N\overline{g}$, and f_j for the number of these fused by the action of $\left\{\overline{h} : h \in C_G(g)\right\}$. These values of k are determined by the values of $\chi(G|2^8)$ on the different classes of G, where $\chi(G|2^8)$ denotes the permutation character of G on the classes of 2^8. Note if $\overline{G}$ is a split extension then $\overline{g}$

becomes g since $G \leq \overline{G}$. The order of the centralizer $C_{\overline{G}}(x)$ for each element $x \in \overline{G}$ in a conjugacy class $[x]_{\overline{G}}$ is given by $|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$.

The authors in [4] constructed the group $Sp_6(2)$ as a matrix group of dimension 8 over the Galois field $GF(2)$. The two generators α and β of $Sp_6(2)$ with respective orders of 2 and 6 are given as:

$$\alpha = \begin{pmatrix} 0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 1&0&0&1&1&1&0&0 \\ 0&0&1&0&0&0&0&0 \\ 1&1&1&0&1&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix} \quad \beta = \begin{pmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&1&1&0&1&0 \\ 1&0&1&0&0&0&0&0 \\ 0&1&1&1&1&1&1&0 \\ 1&0&0&1&1&1&1&1 \end{pmatrix}.$$

Using GAP we are able to find a copy of $U_4(2)$:2 within $Sp_6(2) =< \alpha, \beta >$ and the elements g_1 and g_2, with respective orders of 2 and 9, generating $U_4(2)$:2 are as follows:

$$g_1 = \begin{pmatrix} 1&1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 1&1&1&1&0&1&1&0 \\ 1&0&1&1&1&1&0&1 \\ 1&0&0&1&1&0&0&1 \\ 1&1&1&0&1&1&1&0 \\ 0&0&0&0&0&1&0&1 \\ 1&1&1&0&1&1&0&0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 1&0&1&1&1&0&0&1 \\ 0&0&0&1&0&0&1&0 \\ 0&0&0&1&1&1&1&0 \\ 1&0&0&1&1&1&1&0 \\ 1&1&0&1&0&0&0&1 \\ 0&1&1&0&0&1&1&1 \\ 1&0&0&1&1&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix}.$$

The class representatives of each class $[g]_{U_4(2):2}$ of 2^8:($U_4(2)$:2) are given in terms of 8×8 matrices over $GF(2)$ and in total there are 25 conjugacy classes of elements and are listed in Table 8.1.

Table 8.1: The conjugacy classes of $U_4(2){:}2$

$[g]_{U_4(2):2}$	Class representative	$\lvert [g]_{U_4(2):2} \rvert$	$[g]_{U_4(2):2}$	Class representative	$\lvert [g]_{U_4(2):2} \rvert$
$1A$	$\begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 1&0&0&0&0&1&1&0 \\ 1&0&1&0&1&1&0&1 \\ 1&1&0&0&1&1&1&0 \\ 0&0&0&1&0&0&0&1 \\ 0&0&0&0&1&1&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}$	36
$2B$	$\begin{pmatrix} 0&0&0&1&0&0&1&1 \\ 0&1&0&0&0&1&1&1 \\ 1&0&1&1&0&1&0&0 \\ 1&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&1&0 \end{pmatrix}$	45	$2C$	$\begin{pmatrix} 1&1&0&1&0&1&0&1 \\ 1&0&1&0&1&1&0&0 \\ 1&1&0&1&0&1&1&0 \\ 1&0&1&0&1&1&0&1 \\ 0&1&0&0&0&1&1&1 \\ 0&0&0&0&0&1&0&0 \\ 1&1&1&1&0&0&0&0 \\ 0&1&0&1&0&0&0&0 \end{pmatrix}$	270
$2D$	$\begin{pmatrix} 0&1&1&0&1&1&0&1 \\ 0&1&0&0&0&0&0&0 \\ 0&1&0&1&1&0&1&1 \\ 1&0&1&1&1&1&0&1 \\ 0&1&1&0&0&1&0&0 \\ 0&0&0&1&0&0&1&1 \\ 0&1&0&1&0&1&0&1 \\ 1&1&1&0&1&1&0&0 \end{pmatrix}$	540	$3A$	$\begin{pmatrix} 0&1&0&0&1&0&0&0 \\ 1&0&1&1&1&1&0&0 \\ 0&0&1&0&1&1&1&0 \\ 0&0&1&0&0&1&0&0 \\ 0&1&0&0&0&0&0&1 \\ 1&1&1&1&1&1&1&1 \\ 1&0&1&0&1&0&1&1 \\ 1&1&0&0&0&0&0&0 \end{pmatrix}$	80
$3B$	$\begin{pmatrix} 1&1&1&0&1&0&1&1 \\ 0&0&1&0&1&1&0&1 \\ 0&1&1&0&1&1&1&1 \\ 1&0&1&1&0&1&1&1 \\ 1&0&0&1&1&0&0&1 \\ 1&1&0&0&1&1&1&1 \\ 0&0&0&0&1&1&1&1 \\ 0&1&0&1&0&1&0&0 \end{pmatrix}$	240	$3C$	$\begin{pmatrix} 1&0&1&1&0&1&1&1 \\ 0&0&1&1&1&0&1&1 \\ 1&0&1&0&1&0&1&1 \\ 0&1&1&0&1&0&1&1 \\ 0&0&1&1&1&1&1&1 \\ 1&0&1&1&1&0&0&0 \\ 1&0&1&1&0&0&1&0 \\ 0&0&1&1&0&1&1&0 \end{pmatrix}$	480
$4A$	$\begin{pmatrix} 0&1&0&1&1&0&0&0 \\ 0&1&0&1&0&1&0&1 \\ 1&1&0&1&0&1&1&0 \\ 1&1&1&1&0&1&1&0 \\ 1&1&0&0&1&0&1&0 \\ 0&1&0&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0 \\ 0&1&0&0&1&1&1&1 \end{pmatrix}$	540	$4B$	$\begin{pmatrix} 1&1&1&0&1&0&0&0 \\ 0&1&0&0&1&0&0&0 \\ 1&0&1&1&1&0&0&1 \\ 1&0&1&0&1&0&1&1 \\ 0&0&0&0&1&0&0&0 \\ 1&1&0&1&1&1&1&1 \\ 1&0&0&1&0&0&1&1 \\ 1&1&0&1&0&0&1&0 \end{pmatrix}$	540
$4C$	$\begin{pmatrix} 1&1&1&1&1&1&1&0 \\ 0&0&1&0&0&0&0&1 \\ 0&1&0&1&1&1&1&0 \\ 1&1&0&1&0&0&0&1 \\ 0&0&0&0&1&1&1&1 \\ 1&0&1&1&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&0&1&0&0&1&0&1 \end{pmatrix}$	1620	$4D$	$\begin{pmatrix} 0&0&1&1&0&0&0&1 \\ 1&1&1&0&1&1&1&1 \\ 1&1&0&1&0&0&1&0 \\ 1&0&0&1&0&0&1&1 \\ 0&1&0&1&1&1&0&1 \\ 1&1&1&1&0&0&1&0 \\ 1&0&0&1&1&0&1&1 \\ 1&0&0&1&0&1&0&0 \end{pmatrix}$	3240

Table 8.1 (continue)

$[g]_{U_4(2):2}$	Class representative	$\lvert[g]_{U_4(2):2}\rvert$	$[g]_{U_4(2):2}$	Class representative	$\lvert[g]_{U_4(2):2}\rvert$
$5A$	$\begin{pmatrix} 1&1&1&0&1&1&0&0\\ 1&1&1&0&1&1&0&1\\ 1&0&1&0&0&1&1&1\\ 0&1&1&1&1&1&1&0\\ 1&1&0&0&0&0&1&1\\ 1&0&1&1&0&1&1&1\\ 0&1&0&1&1&1&1&0\\ 0&0&1&1&1&0&0&0 \end{pmatrix}$	5184	$6A$	$\begin{pmatrix} 0&1&0&0&1&0&0&0\\ 1&0&1&1&1&1&0&0\\ 1&1&0&1&0&0&1&0\\ 0&0&1&0&0&1&0&0\\ 0&1&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&1\\ 1&1&1&0&1&0&1&1\\ 1&1&0&0&0&0&0&0 \end{pmatrix}$	720
$6B$	$\begin{pmatrix} 0&1&0&1&1&1&1&0\\ 1&1&0&1&0&0&0&1\\ 1&1&1&0&0&0&1&1\\ 0&0&1&0&1&1&0&1\\ 1&1&0&0&1&0&1&1\\ 0&1&1&1&1&1&1&0\\ 1&0&1&0&0&0&0&0\\ 0&1&0&0&1&0&0&0 \end{pmatrix}$	1440	$6C$	$\begin{pmatrix} 1&1&1&1&1&1&1&1\\ 1&1&1&1&1&1&1&0\\ 0&1&0&0&1&0&0&0\\ 1&0&1&0&0&0&0&0\\ 1&0&0&1&0&0&0&1\\ 0&1&0&1&1&0&0&0\\ 0&0&0&0&1&1&0&1\\ 1&1&0&0&0&1&1&0 \end{pmatrix}$	1440
$6D$	$\begin{pmatrix} 1&1&0&1&0&1&1&0\\ 1&0&0&1&0&0&0&1\\ 1&0&0&0&0&1&1&0\\ 1&0&0&1&1&1&0&0\\ 0&0&1&0&1&1&0&1\\ 1&1&1&0&1&0&0&1\\ 0&0&0&0&0&0&1&1\\ 0&0&0&0&0&0&1&0 \end{pmatrix}$	1440	$6E$	$\begin{pmatrix} 1&0&0&0&1&1&0&1\\ 0&1&0&1&1&0&1&1\\ 0&1&1&0&0&1&1&0\\ 0&1&0&1&1&1&1&0\\ 0&1&0&1&1&1&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&1&0&0&1&0\\ 1&0&1&1&1&0&0&0 \end{pmatrix}$	1440
$6F$	$\begin{pmatrix} 0&0&0&1&1&0&1&0\\ 1&0&1&0&0&0&1&1\\ 0&0&0&1&1&0&0&0\\ 1&0&1&0&0&1&0&0\\ 1&0&1&0&0&0&0&0\\ 1&1&1&0&1&1&1&0\\ 1&1&0&0&1&0&1&1\\ 1&1&0&1&1&0&0&0 \end{pmatrix}$	2160	$6G$	$\begin{pmatrix} 1&0&0&1&1&1&0&1\\ 0&1&1&1&1&0&1&1\\ 1&0&0&1&1&1&0&0\\ 1&0&0&0&0&1&1&0\\ 1&0&0&1&0&1&0&0\\ 0&0&1&0&0&1&0&0\\ 1&1&0&1&0&0&1&0\\ 0&0&0&1&1&1&0&0 \end{pmatrix}$	4320
$8A$	$\begin{pmatrix} 0&1&1&1&1&1&0&0\\ 0&0&0&0&0&1&1&1\\ 0&1&1&0&1&1&1&1\\ 0&1&0&0&0&0&0&1\\ 1&0&0&1&1&0&0&1\\ 0&1&0&1&1&0&0&0\\ 1&0&1&1&1&1&0&0\\ 1&0&1&0&1&1&0&0 \end{pmatrix}$	6480	$9A$	$\begin{pmatrix} 1&1&1&0&0&1&0&0\\ 1&0&0&1&1&1&0&1\\ 0&0&0&0&1&0&1&0\\ 1&0&0&1&1&1&0&0\\ 1&0&1&1&1&0&0&0\\ 1&1&0&1&1&0&0&0\\ 1&0&0&0&0&0&0&0\\ 1&0&0&1&1&1&1&0 \end{pmatrix}$	5760
$10A$	$\begin{pmatrix} 0&0&1&0&0&0&0&0\\ 0&0&1&1&1&0&0&0\\ 1&0&0&1&1&0&0&1\\ 1&0&0&1&0&0&1&1\\ 0&1&1&0&1&1&0&1\\ 0&0&0&0&0&0&0&1\\ 1&1&1&0&1&1&1&1\\ 0&0&0&0&0&1&1&1 \end{pmatrix}$	5184	$12A$	$\begin{pmatrix} 1&0&0&0&0&1&0&0\\ 1&0&0&1&1&0&1&0\\ 0&1&0&1&1&1&0&1\\ 1&1&0&1&0&0&1&0\\ 0&1&0&1&0&0&0&0\\ 0&1&0&0&0&1&1&1\\ 1&1&1&0&1&1&0&1\\ 1&0&1&0&1&0&1&1 \end{pmatrix}$	4320
$12B$	$\begin{pmatrix} 0&0&0&0&0&0&0&1\\ 1&1&1&1&1&1&0&0\\ 1&0&0&0&1&1&1&0\\ 0&0&1&0&1&1&0&1\\ 0&0&1&1&1&1&0&0\\ 1&1&0&1&1&1&1&0\\ 0&0&1&1&0&0&0&1\\ 0&1&0&0&1&0&0&0 \end{pmatrix}$	4320			

Throughout this chapter, let $\overline{G} = N{:}G$ be the split extension of $N = 2^8 \cong V_8(2)$ (the vector space of dimension 8 over $GF(2)$) by $G = U4(2){:}2 =< g_1, g_2 >$, where G acts irreducibly on N. As a result of the action of G on N (using MAGMA), we obtain that the elements of N are partitioned into 3 orbits with respective lengths of 1, 120 and 135.

We should add here that in the paper (see Proposition 2.12 in [58]) it is shown that according to Witt's lemma, $G = Aut(SU_4(2)) \cong U4(2){:}2$ has exactly two orbits of lengths 135 and 120 on the non-zero elements of its irreducible 8-dimensional module $2^8 \cong V_8(2)$, where these correspond to the singular and non-singular vectors. Further it shown that since $2^8 - 1$ does not divide $|SP_6(2)|$, it follows that these orbits are also orbits under the action of $Sp_6(2) =< \alpha, \beta >$. In fact the MAGMA command "IsAbsolutelyIrreducible(G)" confirms that the linear group G acts absolutely irreducibly on its 8-dimensional module. Hence $\overline{G}$ as a split extension $2^8{:}(U_4(2){:}2)$ exists as a subgroup S of $SL_9(2)$. The group S can be generated by the following elements of $SL_9(2)$:

$$s_1 = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad s_2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$s_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

where $o(s_1) = 4$, $o(s_2) = 14$ and $o(s_3) = 2$.

Since the group $\overline{G}$ can be generated as permutation group on 496 points, it is easily verified that $\overline{G} \cong S$ by using the MAGMA command, "IsIsomorphic $(\overline{G}, S)$". With the aid of MAGMA and the ATLAS, we are able to identify the structures of the stabilizers corresponding to the 3 orbits of elements of N. The point stabilizers, which are subgroups of G, are identified as $P_1 = G$, $P_2 = 3_+^{1+2}{:}(2D_8)$ and $P_3 = 2^4{:}S_4$.

Let $\chi(G|2^8)$ be the permutation character of G on the classes of 2^8. Then using the same method as in Chapters 6 and 7, we can easily write $\chi(G|2^8) = I_{U_4(2):2}^{U_4(2):2} + I_{3_+^{1+2}:(2D_8)}^{U_4(2):2} + I_{2^4:S_4}^{U_4(2):2} = \chi(U_4(2){:}2|P_1) + \chi(U_4(2){:}2|P_2) + \chi(U_4(2){:}2|P_3) = 1a + 1a + 15b + 20c + 24a + 60c + 1a + 15a + 15c + 20a + 24a + 60c = 3 \times 1a + 15abc + 20a + 2 \times 24a + 2 \times 60c$ as the sum of the permutation characters of G acting on the classes of P_1, P_2 and P_3. For the purpose of computing $\chi(G|2^8)$, we used the character table of $U_4(2){:}2$ which was computed directly in MAGMA with the generators g_1 and g_2 (see Appendix B). The permutation character $\chi(G|2^8)$ is written in terms of of the irreducible characters of $U_4(2){:}2$. The values of $\chi(G|2^8)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^8. The values of k are listed in the second column of Table 8.2. The values of the $f_j's$ (as mentioned earlier in this section) are calculated by Programme A and hence we obtain that $\overline{G}$ has exactly 59 conjugacy classes. Having obtained all the values for the parameters k and $f_j's$, the centralizer order $|C_{\overline{G}}(x)|$ for each class $[x]$ of $\overline{G}$ are determined. Programme B is used to compute the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. All the information involving the conjugacy classes of $\overline{G}$ are listed in Table 8.2.

Table 8.2: The conjugacy classes of elements of $G = 2^8{:}(U_4(2){:}2)$

$[g]_{U_4(2):2}$	k	f_j	d_j	w	$[x]_{2^8:(U_4(2):2)}$	$\lvert[x]_{2^8:(U_4(2):2)}\rvert$	$\lvert C_{2^8:(U_4(2):2)}(x)\rvert$
$1A$	256	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$1A$	1	13271040
		$f_2 = 120$	$(0,0,1,0,1,1,0,1)$	$(0,0,1,0,1,1,0,1)$	$2A$	120	110592
		$f_3 = 135$	$(0,0,1,1,0,1,0,1)$	$(0,0,1,1,0,1,0,1)$	$2B$	135	98304
$2A$	16	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2C$	596	23040
		$f_2 = 15$	$(1,0,0,0,0,0,0,0)$	$(1,0,0,0,0,1,0,0)$	$4A$	8640	1536
$2B$	64	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2D$	180	73728
		$f_2 = 6$	$(0,1,1,0,0,1,1,0)$	$(0,1,1,0,0,1,1,0)$	$2E$	1080	12288
		$f_3 = 9$	$(0,0,1,0,1,0,0,1)$	$(0,0,1,0,1,0,0,1)$	$2F$	1620	8192
		$f_4 = 48$	$(1,1,1,0,0,1,1,1)$	$(1,1,1,0,0,1,1,1)$	$4B$	8640	1536
$2C$	16	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2G$	4320	3072
		$f_2 = 3$	$(0,1,1,0,0,1,1,0)$	$(0,1,1,0,0,1,1,0)$	$6F$	12960	1024
		$f_3 = 6$	$(0,1,1,0,1,1,1,1)$	$(0,1,1,0,0,1,1,1)$	$6G$	25920	512
		$f_4 = 6$	$(1,1,1,1,1,1,0,0)$	$(0,0,0,0,1,1,1,0)$	$6H$	25920	512
$2D$	16	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$2H$	8640	1536
		$f_2 = 1$	$(0,1,1,0,0,1,1,0)$	$(0,1,1,0,0,1,1,0)$	$4F$	8640	1536
		$f_3 = 6$	$(0,1,1,0,1,1,1,1)$	$(0,1,1,0,0,1,1,1)$	$4G$	51840	256
		$f_4 = 8$	$(1,1,1,1,1,1,0,0)$	$(0,0,0,0,1,1,1,0)$	$4H$	69120	192
$3A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3A$	5120	2592
		$f_2 = 3$	$(1,1,1,1,0,0,0,0)$	$(1,0,0,0,1,0,1,0)$	$6A$	15360	864
$3B$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3B$	61440	216
$3C$	16	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$3C$	7680	1728
		$f_2 = 6$	$(1,1,1,1,1,0,0,0)$	$(0,0,1,0,0,0,0,0)$	$6B$	46080	288
		$f_3 = 9$	$(1,1,0,1,0,0,1,1)$	$(0,0,0,1,1,1,0,0)$	$6C$	69120	192
$4A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4A$	34560	384
		$f_2 = 3$	$(1,0,0,0,0,0,0,0)$	$(0,0,0,1,1,0,1,1)$	$8A$	103680	128
$4B$	16	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4J$	8640	1536
		$f_2 = 1$	$(1,1,1,1,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4K$	8640	1536
		$f_3 = 2$	$(1,1,0,1,0,0,1,1)$	$(0,0,0,0,0,0,0,0)$	$4L$	17280	768
		$f_4 = 12$	$(1,0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4M$	103680	128

Table 8.2(continue)

$[g]_{U_4(2):2}$	k	f_j	d_j	w	$[x]_{2^8:(U_4(2):2)}$	$\lvert[x]_{2^8:(U_4(2):2)}\rvert$	$\lvert C_{2^8:(U_4(2):2)}(x)\rvert$
$4C$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4N$	103680	128
		$f_2 = 1$	$(1,1,0,1,1,1,0,0)$	$(1,1,0,1,1,1,0,0)$	$8B$	103680	128
		$f_3 = 2$	$(1,0,1,1,1,0,1,0)$	$(1,0,1,1,1,0,1,0)$	$8C$	207360	64
$4D$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$4O$	207360	64
		$f_2 = 1$	$(1,0,1,0,1,0,1,0)$	$(1,0,1,0,1,0,1,0)$	$8D$	138240	96
		$f_3 = 2$	$(1,1,0,1,1,1,0,0)$	$(1,1,0,1,1,1,0,0)$	$8E$	138240	96
$5A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$5A$	1327104	10
$6A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6D$	46080	288
		$f_2 = 3$	$(1,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6E$	138240	96
$6B$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6F$	92160	144
		$f_2 = 3$	$(1,0,0,0,0,0,0,0)$	$(0,0,0,1,1,0,1,1)$	$12A$	276480	48
$6C$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6G$	368640	36
$6D$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6H$	368640	36
$6E$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6I$	92160	144
		$f_2 = 3$	$(1,0,0,0,0,0,0,0)$	$(0,0,0,1,1,0,1,1)$	$21B$	276480	48
$6F$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6J$	552960	24
$6G$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$6K$	276480	48
		$f_2 = 1$	$(1,1,0,1,1,1,0,0)$	$(1,1,0,1,1,1,0,0)$	$12C$	276480	48
		$f_3 = 2$	$(1,0,1,1,1,0,1,0)$	$(1,0,1,1,1,0,1,0)$	$12D$	552960	24
$8A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$8F$	414720	32
		$f_2 = 1$	$(1,1,1,1,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$8G$	414720	32
		$f_3 = 1$	$(1,1,0,1,0,0,1,1)$	$(0,0,0,0,0,0,0,0)$	$8H$	414720	32
		$f_4 = 1$	$(1,0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$8I$	414720	32
$9A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$9A$	1474560	9
$10A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$10A$	1327104	10
$12A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$12E$	1105920	12
$12B$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$12F$	276480	48
		$f_2 = 1$	$(1,1,1,1,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$12G$	276480	48
		$f_3 = 1$	$(1,1,0,1,0,0,1,1)$	$(0,0,0,0,0,0,0,0)$	$12H$	276480	48
		$f_4 = 1$	$(1,0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0,0)$	$12I$	276480	48

8.2 The Inertia groups of 2^8:$(U_4(2)$:$2)$

Since G has 3 orbits on N, then by Brauer's Theorem [29] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 3 orbits will be 1, r, and s where $r + s = 255$, with corresponding point stabilizers H_1, H_2 and H_3 as subgroups of G such that $[G{:}H_1]$ $=1$, $[G{:}H_2]$ $=r$ and $[G{:}H_3] = s$.

The action of $\overline{G}$ on $Irr(2^8)$ can be seen as the action of $\overline{G}$ on the dual space N^* of N, since $N = 2^8$ is regarded as the vector space $V_8(2)$ (see [56]). Hence we act the group T formed by the transpose of the generators g_1 and g_2 of G on N. The action of T on N partitioned the vectors of N into 3 orbits of lengths 1, $r = 120$ and $s = 135$. Thus the action of $\overline{G}$ on $Irr(N)$ is self-dual to the action of $\overline{G}$ on N, since the respective actions give rise to orbits of the same lengths. We deduce that the action of G on N has orbits of lengths 1, $r = 120$ and $s = 135$ with respective point stabilizers $H_1 = G$, $H_2 = 3_+^{1+2}{:}(2D_8)$ and $H_3 = 2^4{:}S_4$. Thus we obtain 3 inertia groups $\overline{H}_i = 2^8{:}H_i$, $i = 1, 2, 3$, in $2^8{:}(U_4(2){:}2)$. The groups H_2 and H_3 are constructed from elements within G and the generators are as follows:

- $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 8A$, $\alpha_2 \in 12B$ where

$$\alpha_1 = \begin{pmatrix} 0&0&0&1&1&1&0&0\\ 0&1&0&0&1&1&0&0\\ 0&1&1&1&1&0&0&1\\ 0&0&0&0&0&1&1&1\\ 1&0&0&1&1&1&0&1\\ 1&0&0&0&0&1&0&0\\ 1&0&0&1&1&1&0&0\\ 1&0&0&1&0&1&0&0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0&1&0&1&0&0&0&0\\ 1&0&1&1&1&1&0&0\\ 1&1&0&0&1&0&1&1\\ 0&0&1&0&0&1&0&1\\ 1&1&0&1&1&0&0&0\\ 1&0&0&1&1&0&1&0\\ 0&1&1&1&1&0&1&1\\ 1&1&0&1&0&0&0&1 \end{pmatrix}$$

- $H_3 = \langle \beta_1, \beta_2 \rangle$, $\beta_1 \in 2C$, $\beta_2 \in 8A$ where

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&0&1&0\\ 1&1&0&1&1&1&1&1\\ 1&0&1&0&1&1&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&1&0&1&0&0\\ 1&0&0&1&1&0&1&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 1&0&1&0&0&1&0&0\\ 1&0&0&0&1&1&0&0\\ 1&1&0&1&1&1&1&1\\ 0&0&0&1&0&0&0&1\\ 0&1&1&0&1&1&0&1\\ 1&0&0&1&1&0&0&0\\ 1&1&1&1&1&1&0&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix}$$

We obtain the fusion maps of the inertia factors H_2 and H_3 into G (see Tables 8.3 and 8.4) by using their permutation characters in G of degrees 120 and 135, and if necessary direct matrix conjugation in G. MAGMA was used for the various computations. The character tables of H_1, H_2 and H_3 are also computed using their generators obtained in this section and can be found in Appendix B.

Table 8.3: **The fusion of H_2 into $U_4(2)$:2**

$[h]_{H_2}$ →	$[g]_{U_4(2):2}$	$[h]_{H_2}$ →	$[g]_{U_4(2):2}$	$[h]_{H_2}$ →	$[g]_{U_4(2):2}$
1A	1A	4A	4B	8B	8A
2A	2B	4B	4B	12A	12B
2B	2D	6A	6A	12B	12B
3A	3A	6B	6G	12C	12B
3B	3C	8A	8A		

Table 8.4: **The fusion of H_3 into $U_4(2)$:2**

$[h]_{H_3}$ →	$[g]_{U_4(2):2}$	$[h]_{H_3}$ →	$[g]_{U_4(2):2}$	$[h]_{H_3}$ →	$[g]_{U_4(2):2}$	$[h]_{H_3}$ →	$[g]_{U_4(2):2}$
1A	1A	2E	2B	4A	4B	4F	4D
2A	2B	2F	2C	4B	4C	6A	6B
2B	2D	2G	2C	4C	4A	6B	6G
2C	2A	2H	2D	4D	4D	6C	6E
2D	2C	3A	3C	4E	4C	8A	8A

8.3 The Fischer-Clifford Matrices of 2^8:($U4(2)$:2)

We use the fusion maps of the classes of H_2 and H_3 into G, the information in Table 8.2, and the properties of Fischer-Clifford matrices found in Chapter 5 to compute the entries of the Fischer-Clifford matrices for 2^8:($U_4(2)$:2). For example, see Chapter 6 for explicit computations to construct Fischer- Clifford matrices for a split extension group. For each class representative $g \in U_4(2)$:2, we construct a Fischer-Clifford matrix $M(g)$ which are listed in Table 8.5.

Table 8.5: The Fischer-Clifford matrices of $2^8{:}(U_4(2){:}2)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 \\ 120 & 8 & -8 \\ 135 & -9 & 7 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 24 & 8 & -8 & 0 \\ 3 & 3 & 3 & -1 \\ 36 & -12 & 4 & 0 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 3 & -1 & -1 \\ 6 & -2 & -2 & 2 \\ 6 & -2 & 2 & -2 \end{pmatrix}$
$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 8 & -8 & 0 & 0 \\ 1 & 1 & 1 & -1 \\ 6 & 6 & -2 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 \end{pmatrix}$	$M(3C) = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 2 & -2 \\ 9 & -3 & 1 \end{pmatrix}$
$M(4A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 4 & -4 & 0 \\ 8 & -8 & 0 & 0 \\ 3 & 3 & 3 & -1 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(4D) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(5A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(6A) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$
$M(6B) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$	$M(6C) = \begin{pmatrix} 1 \end{pmatrix}$
$M(6D) = \begin{pmatrix} 1 \end{pmatrix}$	$M(6E) = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$
$M(6F) = \begin{pmatrix} 1 \end{pmatrix}$	$M(6G) = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 1 & 1 & -1 \end{pmatrix}$
$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(9A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(10A) = \begin{pmatrix} 1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 \end{pmatrix}$
$M(12B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	

8.4 Character Table of $2^8{:}(U_4(2){:}2)$

Following the method of Fischer-Clifford matrices as explained in Chapter 5, the character table of $2^8{:}(U_4(2){:}2)$ can be constructed from the Fischer-Clifford matrices (Table 8.5), the fusion maps of the H_i's (Tables 8.3 and 8.4), and the character tables of the inertia factors H_i (found in Appendix B).

The character table of $\overline{G}$ (see Table 8.6) is partitioned row-wise into 3 blocks Δ_1, Δ_2 and Δ_3, where each block corresponds to an inertia group $\overline{H}_i = 2^8{:}H_i$. Therefore $Irr(2^8{:}(U_4(2){:}2)) = \bigcup_{i=1}^{3} \Delta_i$, where $\Delta_1 = \{\chi_j | 1 \leq j \leq 25\}$, $\Delta_2 = \{\chi_j | 26 \leq j \leq 39\}$ and $\Delta_3 = \{\chi_j | 40 \leq j \leq 59\}$. The consistency and accuracy of the character table of $2^8{:}(U_4(2){:}2)$ are tested as Programme C (see Appendix A).

Table 8.6: The Character table of $2^8{:}(U_4(2){:}2)$

	1A			2A		2B				2C			
	$1A$	$2A$	$2B$	$2C$	$4A$	$2D$	$2E$	$2F$	$4B$	$2G$	$4C$	$4D$	$4E$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	1	1	1	1	1	1	1	1
χ_3	6	6	6	-4	-4	-2	-2	-2	-2	2	2	2	2
χ_4	6	6	6	4	4	-2	-2	-2	-2	2	2	2	2
χ_5	10	10	10	0	0	-6	-6	-6	-6	2	2	2	2
χ_6	15	15	15	-5	-5	-1	-1	-1	-1	-1	-1	-1	-1
χ_7	15	15	15	-5	-5	7	7	7	7	3	3	3	3
χ_8	15	15	15	5	5	-1	-1	-1	-1	-1	-1	-1	-1
χ_9	15	15	15	5	5	7	7	7	7	3	3	3	3
χ_{10}	20	20	20	-10	-10	4	4	4	4	4	4	4	4
χ_{11}	20	20	20	10	10	4	4	4	4	4	4	4	4
χ_{12}	20	20	20	0	0	4	4	4	4	-4	-4	-4	-4
χ_{13}	24	24	24	-4	-4	8	8	8	8	0	0	0	0
χ_{14}	24	24	24	4	4	8	8	8	8	0	0	0	0
χ_{15}	30	30	30	-10	-10	-10	-10	-10	-10	2	2	2	2
χ_{16}	30	30	30	10	10	-10	-10	-10	-10	2	2	2	2
χ_{17}	60	60	60	-10	-10	-4	-4	-4	-4	4	4	4	4
χ_{18}	60	60	60	10	10	-4	-4	-4	-4	4	4	4	4
χ_{19}	60	60	60	0	0	12	12	12	12	4	4	4	4
χ_{20}	64	64	64	-16	-16	0	0	0	0	0	0	0	0
χ_{21}	64	64	64	16	16	0	0	0	0	0	0	0	0
χ_{22}	80	80	80	0	0	-16	-16	-16	-16	0	0	0	0
χ_{23}	81	81	81	9	9	9	9	9	9	-3	-3	-3	-3
χ_{24}	81	81	81	-9	-9	9	9	9	9	-3	-3	-3	-3
χ_{25}	90	90	90	0	0	-6	-6	-6	-6	-6	-6	-6	-6
χ_{26}	120	8	-8	0	0	24	8	-8	0	0	0	0	0
χ_{27}	120	8	-8	0	0	24	8	-8	0	0	0	0	0
χ_{28}	120	8	-8	0	0	24	8	-8	0	0	0	0	0
χ_{29}	120	8	-8	0	0	24	8	-8	0	0	0	0	0
χ_{30}	240	16	-16	0	0	48	16	-16	0	0	0	0	0
χ_{31}	240	16	-16	0	0	-48	-16	16	0	0	0	0	0
χ_{32}	240	16	-16	0	0	-48	-16	16	0	0	0	0	0
χ_{33}	720	48	-48	0	0	-48	-16	16	0	0	0	0	0
χ_{34}	720	48	-48	0	0	-48	-16	16	0	0	0	0	0
χ_{35}	720	48	-48	0	0	-48	-16	16	0	0	0	0	0
χ_{36}	720	48	-48	0	0	-48	-16	16	0	0	0	0	0
χ_{37}	960	64	-64	0	0	0	0	0	0	0	0	0	0
χ_{38}	960	64	-64	0	0	0	0	0	0	0	0	0	0
χ_{39}	1440	96	-96	0	0	96	32	-32	0	0	0	0	0
χ_{40}	135	-9	7	15	-1	39	-9	7	-1	15	-1	-1	-1
χ_{41}	135	-9	7	15	-1	-33	15	-1	-1	3	3	-5	3
χ_{42}	135	-9	7	-15	1	-33	15	-1	-1	3	3	-5	3
χ_{43}	135	-9	7	-15	1	39	-9	7	-1	15	-1	-1	-1
χ_{44}	270	-18	14	-30	2	6	6	6	-2	18	2	-6	2
χ_{45}	270	-18	14	30	-2	6	6	6	-2	18	2	-6	2
χ_{46}	405	-27	21	-45	3	-27	21	5	-3	-3	13	-3	-3
χ_{47}	405	-27	21	-45	3	45	-3	13	-3	9	9	1	-7
χ_{48}	405	-27	21	45	-3	-27	21	5	-3	-3	13	-3	-3
χ_{49}	405	-27	21	45	-3	45	-3	13	-3	9	9	1	-7
χ_{50}	540	-36	28	30	-2	60	-36	-4	4	-12	4	-4	4
χ_{51}	540	-36	28	30	-2	-84	12	-20	4	12	-4	4	-4
χ_{52}	540	-36	28	-30	2	60	-36	-4	4	-12	4	-4	4
χ_{53}	540	-36	28	-30	2	-84	12	-20	4	12	-4	4	-4
χ_{54}	810	-54	42	0	0	-54	42	10	-6	-6	-6	-6	10
χ_{55}	810	-54	42	0	0	90	-6	26	-6	18	-14	2	2
χ_{56}	810	-54	42	0	0	18	18	18	-6	-18	-2	6	-2
χ_{57}	810	-54	42	0	0	18	18	18	-6	-18	-2	6	-2
χ_{58}	1080	-72	56	60	-4	-24	-24	-24	8	0	0	0	0
χ_{59}	1080	-72	56	-60	4	-24	-24	-24	8	0	0	0	0

Table 8.6 (continue)

	2D				3A		3B	3C			4A		4B			
	$2H$	$4F$	$4G$	$4H$	$3A$	$6A$	$3B$	$3C$	$6B$	$6C$	$4I$	$8A$	$4J$	$4K$	$4L$	$4M$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	1	1	1	1
χ_3	0	0	0	0	-3	-3	3	0	0	0	2	2	2	2	2	2
χ_4	0	0	0	0	-3	-3	3	0	0	0	-2	-2	2	2	2	2
χ_5	0	0	0	0	1	1	-2	4	4	4	0	0	2	2	2	2
χ_6	3	3	3	3	6	6	3	0	0	0	-1	-1	3	3	3	3
χ_7	-1	-1	-1	-1	-3	-3	0	3	3	3	-3	-3	-1	-1	-1	-1
χ_8	-3	-3	-3	-3	6	6	3	0	0	0	1	1	3	3	3	3
χ_9	1	1	1	1	-3	-3	0	3	3	3	3	3	-1	-1	-1	-1
χ_{10}	-2	-2	-2	-2	2	2	5	-1	-1	-1	-2	-2	0	0	0	0
χ_{11}	2	2	2	2	2	2	5	-1	-1	-1	2	2	0	0	0	0
χ_{12}	0	0	0	0	-7	-7	2	2	2	2	0	0	4	4	4	4
χ_{13}	-4	-4	-4	-4	6	6	0	3	3	3	0	0	0	0	0	0
χ_{14}	4	4	4	4	6	6	0	3	3	3	0	0	0	0	0	0
χ_{15}	2	2	2	2	3	3	3	3	3	3	4	4	-2	-2	-2	-2
χ_{16}	-2	-2	-2	-2	3	3	3	3	3	3	-4	-4	-2	-2	-2	-2
χ_{17}	-2	-2	-2	-2	6	6	-3	-3	-3	-3	2	2	0	0	0	0
χ_{18}	2	2	2	2	6	6	-3	-3	-3	-3	-2	-2	0	0	0	0
χ_{19}	0	0	0	0	-3	-3	-6	0	0	0	0	0	4	4	4	4
χ_{20}	0	0	0	0	-8	-8	4	-2	-2	-2	0	0	0	0	0	0
χ_{21}	0	0	0	0	-8	-8	4	-2	-2	-2	0	0	0	0	0	0
χ_{22}	0	0	0	0	-10	-10	-4	2	2	2	0	0	0	0	0	0
χ_{23}	-3	-3	-3	-3	0	0	0	0	0	0	3	3	-3	-3	-3	-3
χ_{24}	3	3	3	3	0	0	0	0	0	0	-3	-3	-3	-3	-3	-3
χ_{25}	0	0	0	0	9	9	0	0	0	0	0	0	2	2	2	2
χ_{26}	8	-8	0	0	3	-1	0	6	2	-2	0	0	12	-4	-4	0
χ_{27}	-8	8	0	0	3	-1	0	6	2	-2	0	0	-4	12	-4	0
χ_{28}	8	-8	0	0	3	-1	0	6	2	-2	0	0	-4	12	-4	0
χ_{29}	-8	8	0	0	3	-1	0	6	2	-2	0	0	12	-4	-4	0
χ_{30}	0	0	0	0	6	-2	0	12	4	-4	0	0	-8	-8	8	0
χ_{31}	0	0	0	0	6	-2	0	12	4	-4	0	0	0	0	0	0
χ_{32}	0	0	0	0	6	-2	0	12	4	-4	0	0	0	0	0	0
χ_{33}	0	0	0	0	-9	3	0	0	0	0	0	0	-8	24	-8	0
χ_{34}	0	0	0	0	-9	3	0	0	0	0	0	0	24	-8	-8	0
χ_{35}	0	0	0	0	-9	3	0	0	0	0	0	0	-8	-8	8	0
χ_{36}	0	0	0	0	-9	3	0	0	0	0	0	0	-8	-8	8	0
χ_{37}	-16	16	0	0	24	-8	0	-6	-2	2	0	0	0	0	0	0
χ_{38}	16	-16	0	0	24	-8	0	-6	-2	2	0	0	0	0	0	0
χ_{39}	0	0	0	0	-18	6	0	0	0	0	0	0	0	0	0	0
χ_{40}	7	7	-1	-1	0	0	0	9	-3	1	3	-1	3	3	3	-1
χ_{41}	-5	-5	3	-1	0	0	0	9	-3	1	-3	1	3	3	3	-1
χ_{42}	5	5	-3	1	0	0	0	9	-3	1	3	-1	3	3	3	-1
χ_{43}	-7	-7	1	1	0	0	0	9	-3	1	-3	1	3	3	3	-1
χ_{44}	-2	-2	-2	2	0	0	0	-9	3	-1	0	0	6	6	6	-2
χ_{45}	2	2	2	-2	0	0	0	-9	3	-1	0	0	6	6	6	-2
χ_{46}	3	3	-5	3	0	0	0	0	0	0	3	-1	-3	-3	-3	1
χ_{47}	-9	-9	-1	3	0	0	0	0	0	0	-3	1	-3	-3	-3	1
χ_{48}	-3	-3	5	-3	0	0	0	0	0	0	-3	1	-3	-3	-3	1
χ_{49}	9	9	1	-3	0	0	0	0	0	0	3	-1	-3	-3	-3	1
χ_{50}	-2	-2	-2	2	0	0	0	9	-3	1	6	-2	0	0	0	0
χ_{51}	-2	-2	-2	2	0	0	0	9	-3	1	-6	2	0	0	0	0
χ_{52}	2	2	2	-2	0	0	0	9	-3	1	-6	2	0	0	0	0
χ_{53}	2	2	2	-2	0	0	0	9	-3	1	6	-2	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	-6	2
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	-6	2
χ_{56}	-12	-12	4	0	0	0	0	0	0	0	6	-2	6	6	6	-2
χ_{57}	12	12	-4	0	0	0	0	0	0	0	-6	2	6	6	6	-2
χ_{58}	-4	-4	-4	4	0	0	0	-9	3	-1	0	0	0	0	0	0
χ_{59}	4	4	4	-4	0	0	0	-9	3	-1	0	0	0	0	0	0

Table 8.6 (continue)

	4C			4D			5A	6A		6B		6C	6D	6E	
	$4N$	$8B$	$8C$	$4O$	$8D$	$8E$	$5A$	$6D$	$6E$	$6F$	$12A$	$6G$	$6H$	$6I$	$12B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	1	1	1
χ_3	-2	-2	-2	0	0	0	1	1	1	2	2	-1	1	-2	-2
χ_4	2	2	2	0	0	0	1	1	1	-2	-2	1	1	-2	-2
χ_5	0	0	0	-2	-2	-2	0	-3	-3	0	0	0	0	0	0
χ_6	-1	-1	-1	-1	-1	-1	0	2	2	-2	-2	1	-1	2	2
χ_7	0	-1	-1	-1	1	1	0	1	1	1	1	-2	-2	1	1
χ_8	1	1	1	-1	-1	-1	0	2	2	2	2	-1	-1	2	2
χ_9	-1	-1	-1	1	1	1	0	1	1	-1	-1	2	-2	1	1
χ_{10}	-2	-2	-2	0	0	0	0	-2	-2	-1	-1	-1	1	1	1
χ_{11}	2	2	2	0	0	0	0	-2	-2	1	1	1	1	1	1
χ_{12}	0	0	0	0	0	0	0	1	1	0	0	0	-2	-2	-2
χ_{13}	0	0	0	0	0	0	-1	2	2	-1	-1	2	2	-1	-1
χ_{14}	0	0	0	0	0	0	-1	2	2	1	1	-2	2	-1	-1
χ_{15}	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1
χ_{16}	0	0	0	0	0	0	0	-1	-1	1	1	1	-1	-1	-1
χ_{17}	2	2	2	0	0	0	0	2	2	-1	-1	-1	-1	-1	-1
χ_{18}	-2	-2	-2	0	0	0	0	2	2	1	1	1	-1	-1	-1
χ_{19}	0	0	0	0	0	0	0	-3	-3	0	0	0	0	0	0
χ_{20}	0	0	0	0	0	0	-1	0	0	2	2	2	0	0	0
χ_{21}	0	0	0	0	0	0	-1	0	0	-2	-2	-2	0	0	0
χ_{22}	0	0	0	0	0	0	0	2	2	0	0	0	2	2	2
χ_{23}	-1	-1	-1	-1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{24}	1	1	1	-1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	2	2	2	0	-3	-3	0	0	0	0	0	0
χ_{26}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{27}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{28}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	6	-2	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	-6	2	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	-6	2	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	3	-1	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	-6	2	0	0	0	0	0	0
χ_{40}	3	-1	-1	3	-1	-1	0	0	0	3	-1	0	0	3	-1
χ_{41}	1	-3	1	-3	1	1	0	0	0	3	-1	0	0	3	-1
χ_{42}	-1	3	-1	-3	1	1	0	0	0	-3	1	0	0	3	-1
χ_{43}	-3	1	1	3	-1	-1	0	0	0	-3	1	0	0	3	-1
χ_{44}	-4	4	0	0	0	0	0	0	0	3	-1	0	0	-3	1
χ_{45}	4	-4	0	0	0	0	0	0	0	-3	1	0	0	-3	1
χ_{46}	3	-1	-1	1	-3	1	0	0	0	0	0	0	0	0	0
χ_{47}	1	-3	1	-1	3	-1	0	0	0	0	0	0	0	0	0
χ_{48}	-3	1	1	1	-3	1	0	0	0	0	0	0	0	0	0
χ_{49}	-1	3	-1	-1	3	-1	0	0	0	0	0	0	0	0	0
χ_{50}	-2	-2	2	0	0	0	0	0	0	-3	1	0	0	-3	1
χ_{51}	2	2	-2	0	0	0	0	0	0	-3	1	0	0	-3	1
χ_{52}	2	2	-2	0	0	0	0	0	0	3	-1	0	0	-3	1
χ_{53}	-2	-2	2	0	0	0	0	0	0	3	-1	0	0	-3	1
χ_{54}	0	0	0	2	2	-2	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	-2	-2	2	0	0	0	0	0	0	0	0	0
χ_{56}	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	3	-1	0	0	3	-1
χ_{59}	0	0	0	0	0	0	0	0	0	-3	1	0	0	3	-1

Table 8.6 (continue)

	6F	6G			8A				9A	10A	12A	12B			
	$6J$	$6K$	$12C$	$12D$	$8F$	$8G$	$8H$	$8I$	$9A$	$10A$	$12E$	$12F$	$12G$	$12H$	$12I$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	-1	-1	-1	-1	-1	1	-1	-1	1	1	1	1
χ_3	-1	0	0	0	0	0	0	0	0	1	-1	-1	-1	-1	-1
χ_4	-1	0	0	0	0	0	0	0	0	-1	1	-1	-1	-1	-1
χ_5	2	0	0	0	0	0	0	0	1	0	0	-1	-1	-1	-1
χ_6	-1	0	0	0	1	1	1	1	0	0	-1	0	0	0	0
χ_7	0	-1	-1	-1	1	1	1	1	0	0	0	-1	-1	-1	-1
χ_8	-1	0	0	0	-1	-1	-1	-1	0	0	1	0	0	0	0
χ_9	0	1	1	1	-1	-1	-1	-1	0	0	0	-1	-1	-1	-1
χ_{10}	1	1	1	1	0	0	0	0	-1	0	1	0	0	0	0
χ_{11}	1	-1	-1	-1	0	0	0	0	-1	0	-1	0	0	0	0
χ_{12}	2	0	0	0	0	0	0	0	-1	0	0	1	1	1	1
χ_{13}	0	-1	-1	-1	0	0	0	0	0	1	0	0	0	0	0
χ_{14}	0	1	1	1	0	0	0	0	0	-1	0	0	0	0	0
χ_{15}	-1	-1	-1	-1	0	0	0	0	0	0	1	1	1	1	1
χ_{16}	-1	1	1	1	0	0	0	0	0	0	-1	1	1	1	1
χ_{17}	1	1	1	1	0	0	0	0	0	0	-1	0	0	0	0
χ_{18}	1	-1	-1	-1	0	0	0	0	0	0	1	0	0	0	0
χ_{19}	-2	0	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_{20}	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
χ_{21}	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
χ_{23}	0	0	0	0	1	1	1	1	0	-1	0	0	0	0	0
χ_{24}	0	0	0	0	-1	-1	-1	-1	0	1	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{26}	0	2	-2	0	2	0	-2	0	0	0	0	3	-1	-1	-1
χ_{27}	0	-2	2	0	2	0	-2	0	0	0	0	-1	-1	-1	3
χ_{28}	0	2	-2	0	-2	0	2	0	0	0	0	-1	-1	-1	3
χ_{29}	0	-2	2	0	-2	0	2	0	0	0	0	3	-1	-1	-1
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	-2	2	2	-2
χ_{31}	0	0	0	0	0	A	0	$-A$	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	$-A$	0	A	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-3
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	-3	1	1	1
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	1	B	$\overline{B}$	1
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	1	$\overline{B}$	B	1
χ_{37}	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	1	1	-1	1	-1	1	-1	0	0	0	0	0	0	0
χ_{41}	0	1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0
χ_{42}	0	-1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0
χ_{43}	0	-1	-1	1	-1	1	-1	1	0	0	0	0	0	0	0
χ_{44}	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	-1	1	-1	1	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	1	-1	1	-1	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	-1	1	-1	1	0	0	0	0	0	0	0
χ_{50}	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{52}	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{55}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	1	1	-1	0	0	0	0	0	0	0	0	0	0	0

where $A = 2i\sqrt{2}$ and $B = -1 - 2i\sqrt{3}$.

The information in Table 8.2 can be used to compute the power maps for the conjugacy classes of elements of $\overline{G}$ and Programme C is used to confirm that the character table of $\overline{G}$ produced the unique p-power maps listed in Table 8.7.

Table 8.7: **The power maps of the elements of** 2^8:($U_4(2)$:2)

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5
1A	1A				2A	2C	1A		
	2A	1A				4A	2B		
	2B	1A							
2B	2D	1A			2C	2G	1A		
	2E	1A				4C	2B		
	2F	1A				4D	2B		
	4B	2B				4E	2B		
2D	2H	1A			3A	3A		1A	
	4F	2B				6A	3A	2A	
	4G	2B							
	4H	2A							
3B	3B		1A		3C	3C		1A	
						6B	3C	2A	
						6C	3C	2B	
4A	4I	2G			4B	4J	2D		
	8A	4C				4K	2D		
						4L	2D		
						4M	2E		
4C	4N	2G			4D	4O	2G		
	8B	4C				8D	4C		
	8C	4C				8E	4E		
5A	5A			1A	6A	6D	3A	2D	
						6E	3A	2E	
6B	6F	3C	2D		6C	6G	3B	2D	
	12A	6C	4B						
6D	6H	3B	2C		6E	6I	3C	2C	
						12B	6C	4A	
6F	6J	3B	2G		6G	6K	3C	2H	
						12C	6C	4F	
						12D	6B	4H	
8A	8F	4J			9A	9A		3A	
	8G	4M							
	8H	4K							
	8I	4M							
10A	10A	5A		2C	12A	12E	6J	4I	
12B	12F	6D	4J						
	12G	6D	4K						
	12H	6D	4L						
	12I	6D	4L						

8.5 The fusion of $2^8{:}(U_4(2){:}2)$ into $2^8{:}Sp_6(2)$

Since $U_4(2){:}2$ is a subgroup of $Sp_6(2)$, then its fusion into $Sp_6(2)$ (see Table 8.8) will help to determine the fusion of $\overline{G}$ into $2^8{:}Sp_6(2)$ by using the proposition below which is obtained from [53].

Proposition 8.5.1. Let G, H and N be groups such that $H \leq G$ and that class kA of H fuses into class kB of G. Let $a \in kA$ and $b \in kB$. Then the classes of $N{:}H$ corresponding to the coset Na will fuse into the classes of $N{:}G$ corresponding to the coset Nb.

Remark 8.5.2. When H and G act on N, then a and b will have the same number fixed points in N. This is true since a and b are conjugate in G and thus will have the same number of fixed points in N.

Table 8.8: The fusion of $U_4(2){:}2$ into $SP(6,2)$

$[g]_{U_4(2):2}$ $\longrightarrow$	$[g_1]_{SP(6,2)}$	$[g]_{U_4(2):2}$ $\longrightarrow$	$[g_1]_{SP(6,2)}$	$[g]_{U_4(2):2}$ $\longrightarrow$	$[g_1]_{SP(6,2)}$	$[g]_{U_4(2):2}$ $\longrightarrow$	$[g_1]_{SP(6,2)}$
$1A$	$1A$	$3C$	$3C$	$6A$	$6E$	$6G$	$6B$
$2A$	$2A$	$4A$	$4B$	$6B$	$6C$	$8A$	$8A$
$2B$	$2B$	$4B$	$4C$	$6C$	$6E$	$9A$	$9A$
$2C$	$2C$	$4C$	$4D$	$6D$	$6B$	$10A$	$10A$
$2D$	$2D$	$4D$	$4B$	$6E$	$6A$	$12A$	$12A$
$3A$	$3C$	$5A$	$5A$	$6F$	$6E$	$12B$	$12C$
$3B$	$3C$						

Since $\overline{G}$ has index 28 in $\overline{G}_1$, the action of $\overline{G}_1$ on the cosets of $\overline{G}$ gives rise to a permutation character $\chi(\overline{G}_1|\overline{G})$ of degree 28. We deduce from the character table of $\overline{G}$ found in [4] (or the GAP Library) that $\chi(\overline{G}_1|\overline{G}) = 1a + 27a$, where $1a$ and $27a$ are irreducible characters of $\overline{G}_1$ of degrees 1 and 27, respectively.

Using the values of $\chi(\overline{G}_1|\overline{G})$ on the classes of $\overline{G}$, the information provided by the conjugacy classes, Proposition 8.51 and Remark 8.52 we are able to compute the partial fusion map of $\overline{G}$ into $\overline{G}_1$. In order to complete the fusion map, we use the technique of set intersections for characters to restrict the ordinary irreducible characters $120a$, $135a$, $405c$ and

810b of $\overline{G}_1$ to $\overline{G}$. We obtained that $(\phi_1)_{2^8:(U_4(2):2)} = \chi_{26}$, $(\phi_2)_{2^8:(U_4(2):2)} = \chi_{40}$, $(\phi_3)_{2^8:(U_4(2):2)}$ $= \chi_{48}$ and $(\phi_4)_{2^8:(U_4(2):2)} = \chi_{55}$ if the partial fusion of 2^8:$(U_4(2)$:2) into 2^8:$SP_6(2)$ is taken into consideration. The complete fusion map of $\overline{G}$ into $\overline{G}_1$ is listed in Table 8.9.

Table 8.9: The fusion of 2^8:$(U_4(2)$:2) into $\overline{G} = 2^8$:$Sp_6(2)$

$[g]_G$	$[x]_{2^8:(U_4(2):2)}$ $\longrightarrow$	$[y]_{2^8:Sp_6(2)}$	$[g]_G$	$[x]_{2^8:(U_4(2):2)}$ $\longrightarrow$	$[y]_{2^8:Sp_6(2)}$
$1A$	$1A$	$1A$	$2A$	$2C$	$2C$
	$2A$	$2A$		$4A$	$4A$
	$2B$	$2B$			
$2B$	$2D$	$2D$	$2C$	$2G$	$2G$
	$2E$	$2E$		$4C$	$4C$
	$2F$	$2F$		$4D$	$4D$
	$4B$	$4B$		$4E$	$4D$
$2D$	$2H$	$2H$	$3A$	$3A$	$3B$
	$4F$	$4E$		$6A$	$6A$
	$4G$	$4F$			
	$4H$	$4G$			
$3B$	$3B$	$3A$	$3C$	$3C$	$3C$
				$6B$	$6B$
				$6C$	$6C$
$4A$	$4I$	$4M$	$4B$	$4J$	$4H$
	$8A$	$8B$		$4K$	$4I$
				$4L$	$4J$
				$4M$	$4K$
$4C$	$4N$	$4L$	$4D$	$4O$	$4S$
	$8B$	$8A$		$8D$	$8C$
	$8C$	$8A$		$8E$	$8D$
$5A$	$5A$	$5A$	$6A$	$6D$	$6F$
				$6E$	$6G$
$6B$	$6F$	$6I$	$6C$	$6G$	$6E$
	$12A$	$12A$			
$6D$	$6H$	$6D$	$6E$	$6I$	$6J$
				$12B$	$12B$
$6F$	$6J$	$6H$	$6G$	$6K$	$6K$
				$12C$	$12C$
				$12D$	$12D$
$8A$	$8F$	$8E$	$9A$	$9A$	$9A$
	$8G$	$8G$			
	$8H$	$8F$			
	$8I$	$8G$			
$10A$	$10A$	$10A$	$12A$	$12E$	$12F$
$12B$	$12F$	$12G$			
	$12G$	$12H$			
	$12H$	$12J$			
	$12I$	$12I$			

Chapter 9

The Character Table of an inertia group of the maximal subgroup $2^6{:}Sp_6(2)$ of Fi_{22}

In the ATLAS we found that $N_{Fi_{22}}(2^6) \cong 2^6{:}SP_6(2)$ is a maximal subgroup of the smallest Fischer sporadic simple group (discovered by Bernd Fischer) of index 694980. Here 2^6 is a pure $2B$-group, where $2B$ denotes a class of involutions in Fi_{22}. The character table of $2^6{:}SP_6(2)$ was constructed by J. Moori and Z.E. Mpono in [48] and it was also shown in this paper that a group $\overline{G}$ of the form $2^6{:}(2^5{:}S_6)$ is the inertia group in $2^6{:}SP_6(2)$ of one of the linear characters of the normal 2^6 subgroup. This group $\overline{G}$ of index 63 in $2^6{:}SP_6(2)$ is a split extension of an elementary abelian 2-group of order 2^6 by the centralizer of a transvection in $SP_6(2)$, which has the form $2^5{:}S_6 \cong 2^5{:}SP_4(2)$. In this chapter the character table of $\overline{G} = 2^6{:}(2^5{:}S_6)$ is calculated. The methodology in the paper [48], as in the current chapter, is a standard application of Clifford theory, made efficient by the use of Fischer-Clifford matrices, as introduced by Fischer to assist in such calculations.

The character table of a submaximal subgroup $\overline{H_3} = 2^7{:}(2^5{:}S_6)$ of $Aut(Fi_{22})$ was computed in [27] by the method of Fischer-Clifford matrices. This group $\overline{H_3}$ is a direct product of the group $\overline{G} = 2^6{:}(2^5{:}S_6)$ by cyclic group group C_2 of order 2. In general it is always more difficult to construct the character table of any inertia group $N{:}H_i$ of an appropriate split extension $N{:}G$. In Section 9.1 of this chapter it is shown how we can use Wilson's online ATLAS and MAGMA to represent $\overline{G}$ as a permutation group within $2^6{:}SP_6(2)$. Appropriate MAGMA commands are then used to confirm that $\overline{G}$ sits maximally inside $2^6{:}SP_6(2)$ and that $\overline{G}$ is indeed the split extension 2^6 by $2^5{:}S_6$. The remainder of this chapter consists largely of data concerning conjugacy classes, inertia groups, and Fischer-Clifford matrices, with descriptions of the methods used for the computations, which were assisted by the use Programme A in the computer package MAGMA. Consistency checks for the actual character table of $\overline{G}$ were implemented using Programme C, written in GAP. Finally the technique of set intersections is used to determine the fusion of $\overline{G}$ into $2^6{:}SP_6(2)$.

9.1 The group $2^6{:}(2^5{:}S_6)$

Using the Wilson's online ATLAS we can obtain a permutation representation of degree 3510 of Fi_{22}. Then the MAGMA straight line program in [70] is used to generate $D = 2^6{:}SP_6(2)$ as a permutation group inside Fi_{22}. The MAGMA commands "c:=Classes(D)" and "C=:Centralizer(D,c[2,3])" are used to construct $\overline{G} = 2^6{:}(2^5{:}S_6)$ as the centralizer $C_D(2A)$ of the conjugacy class $2A$ of D. The further MAGMA commands "IsMaximal(D, C)" and "Index(D, C)" are a confirmation that the group C sits maximally inside D and has index 63 in D. We use similar commands as in Section 6.1 to confirm that $C = C_D(2A)$ is a split extension of 2^6 by $2^5{:}S_6$.

We generated $2^5{:}S_6$ as a linear group, consisting of 6×6 matrices over the field $GF(2)$, by the two elements $g_1 \in 2B$ and $g_2 \in 12A$ in $Sp_6(2)$.

$$g_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \qquad g_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The class representatives of each class $[g]_{2^5:S_6}$ of $2^5{:}S_6$ are given in terms of 6×6 matrices over $GF(2)$ and in total there are 37 conjugacy classes of elements and are listed in Table 9.1.

Table 9.1: The conjugacy classes of $2^5{:}S_6$

$[g]_{2^5:S_6}$	Class representative	$\lvert[g]_{2^5:S_6}\rvert$	$[g]_{2^5:S_6}$	Class representative	$\lvert[g]_{2^5:S_6}\rvert$
$1A$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$	1
$2B$	$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$	15	$2C$	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	15
$2D$	$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$	30	$2E$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	30
$2F$	$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	60	$2G$	$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$	60
$2H$	$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$	180	$2I$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$	180

Table 9.1 (continue)

$[g]_{2^5:S_6}$	Class representative	$\|[g]_{2^5:S_6}\|$	$[g]_{2^5:S_6}$	Class representative	$\|[g]_{2^5:S_6}\|$
$2J$	$\begin{pmatrix} 0&0&1&0&0&1 \\ 0&0&1&0&1&0 \\ 1&1&1&1&0&0 \\ 1&1&0&1&1&1 \\ 1&0&1&1&0&0 \\ 0&1&1&1&0&0 \end{pmatrix}$	180	$3A$	$\begin{pmatrix} 1&0&1&1&1&1 \\ 0&1&1&1&1&1 \\ 0&0&0&1&1&1 \\ 0&0&1&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	160
$3B$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&1&1&0&0&0 \\ 1&0&1&0&1&0 \\ 1&1&0&1&1&0 \\ 0&1&0&0&0&1 \end{pmatrix}$	640	$4A$	$\begin{pmatrix} 1&0&0&0&1&0 \\ 1&1&1&0&0&1 \\ 0&0&1&0&0&0 \\ 1&1&0&1&1&1 \\ 1&1&0&0&1&1 \\ 0&0&0&0&1&1 \end{pmatrix}$	120
$4B$	$\begin{pmatrix} 0&1&1&0&1&1 \\ 1&1&1&0&0&1 \\ 0&0&1&0&0&0 \\ 0&0&1&1&1&0 \\ 0&1&1&0&1&0 \\ 1&1&1&0&1&0 \end{pmatrix}$	120	$4C$	$\begin{pmatrix} 1&1&1&0&0&0 \\ 1&0&0&0&1&1 \\ 0&1&0&1&0&1 \\ 0&0&0&1&1&0 \\ 0&0&0&1&0&1 \\ 0&0&0&1&0&0 \end{pmatrix}$	180
$4D$	$\begin{pmatrix} 0&1&0&0&0&1 \\ 1&0&0&0&1&1 \\ 0&1&0&1&0&1 \\ 1&0&1&1&1&1 \\ 0&0&0&1&0&1 \\ 1&0&1&1&0&1 \end{pmatrix}$	180	$4E$	$\begin{pmatrix} 0&1&0&1&0&0 \\ 0&1&0&1&0&1 \\ 0&1&1&0&1&0 \\ 0&0&1&0&1&1 \\ 1&0&1&1&1&0 \\ 1&0&0&1&1&1 \end{pmatrix}$	360
$4F$	$\begin{pmatrix} 1&0&0&1&1&1 \\ 0&0&1&1&1&1 \\ 0&0&1&1&0&1 \\ 1&1&0&0&1&0 \\ 1&1&0&1&1&0 \\ 0&0&0&0&1&1 \end{pmatrix}$	720	$4G$	$\begin{pmatrix} 1&1&0&0&1&0 \\ 1&0&0&0&1&1 \\ 0&1&0&1&0&1 \\ 0&0&1&1&0&0 \\ 1&0&1&1&0&0 \\ 0&0&1&1&1&0 \end{pmatrix}$	720
$4H$	$\begin{pmatrix} 0&0&0&0&1&1 \\ 1&1&0&0&0&1 \\ 0&0&0&1&1&1 \\ 0&0&1&1&0&0 \\ 0&0&0&0&1&0 \\ 1&0&1&1&0&1 \end{pmatrix}$	720	$4I$	$\begin{pmatrix} 1&0&1&0&1&0 \\ 1&1&0&0&0&1 \\ 0&0&0&1&1&1 \\ 1&0&0&1&0&1 \\ 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \end{pmatrix}$	720
$4J$	$\begin{pmatrix} 1&1&1&1&1&1 \\ 0&0&1&1&1&1 \\ 1&0&0&1&0&0 \\ 0&0&0&0&1&1 \\ 1&1&0&1&1&0 \\ 1&1&0&0&1&0 \end{pmatrix}$	720	$5A$	$\begin{pmatrix} 1&1&1&0&0&0 \\ 0&0&1&0&0&0 \\ 1&0&1&0&1&1 \\ 1&0&0&0&0&0 \\ 1&0&1&1&1&0 \\ 1&1&1&0&1&0 \end{pmatrix}$	2304

Table 9.1 (continue)

$[g]_{2^5:S_6}$	Class representative	$\lvert[g]_{2^5:S_6}\rvert$	$[g]_{2^5:S_6}$	Class representative	$\lvert[g]_{2^5:S_6}\rvert$
$6A$	$\begin{pmatrix} 0&0&1&1&1&0\\ 1&1&0&1&1&0\\ 1&0&1&1&1&0\\ 1&0&1&1&0&1\\ 0&0&0&0&1&0\\ 0&0&1&0&0&1 \end{pmatrix}$	160	$6B$	$\begin{pmatrix} 0&1&0&1&1&0\\ 1&1&0&1&1&0\\ 1&0&1&1&1&0\\ 0&1&1&1&0&0\\ 1&0&1&0&1&1\\ 0&1&0&0&0&1 \end{pmatrix}$	480
$6C$	$\begin{pmatrix} 1&1&1&1&0&1\\ 0&1&1&1&1&1\\ 1&0&1&1&1&0\\ 1&1&0&1&1&1\\ 1&0&1&0&1&1\\ 1&1&1&0&1&0 \end{pmatrix}$	480	$6D$	$\begin{pmatrix} 0&0&1&0&0&1\\ 0&0&1&0&0&0\\ 0&1&1&0&0&0\\ 0&0&0&0&1&1\\ 1&1&0&1&1&0\\ 1&1&1&0&0&0 \end{pmatrix}$	640
$6E$	$\begin{pmatrix} 1&0&0&0&0&0\\ 1&1&1&1&1&0\\ 1&0&0&1&1&0\\ 0&1&0&1&0&0\\ 0&1&1&0&1&0\\ 1&0&1&1&1&1 \end{pmatrix}$	960	$6F$	$\begin{pmatrix} 1&1&1&0&0&0\\ 1&1&1&1&1&0\\ 1&0&0&1&1&0\\ 1&0&0&1&0&1\\ 1&1&0&0&1&1\\ 1&1&0&1&1&1 \end{pmatrix}$	960
$6G$	$\begin{pmatrix} 0&0&1&0&0&1\\ 1&0&0&1&1&0\\ 1&1&1&1&0&0\\ 0&0&1&0&1&1\\ 0&0&1&0&1&0\\ 0&1&1&1&0&0 \end{pmatrix}$	1920	$6H$	$\begin{pmatrix} 1&0&0&0&0&0\\ 1&0&0&1&1&0\\ 1&1&1&1&0&0\\ 1&0&0&0&1&0\\ 0&0&1&0&1&0\\ 1&1&0&1&0&1 \end{pmatrix}$	1920
$8A$	$\begin{pmatrix} 1&1&0&0&1&0\\ 1&1&0&0&0&1\\ 0&0&0&1&1&1\\ 1&1&1&1&0&1\\ 1&0&1&0&1&1\\ 0&1&1&1&0&0 \end{pmatrix}$	1440	$8B$	$\begin{pmatrix} 1&1&0&0&0&1\\ 1&0&0&1&1&0\\ 0&0&1&1&0&1\\ 1&1&1&1&0&1\\ 1&1&0&1&1&0\\ 0&1&0&0&0&1 \end{pmatrix}$	1440
$10A$	$\begin{pmatrix} 1&1&1&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 1&1&0&1&0&1\\ 1&0&1&0&1&1\\ 0&1&1&0&0&1 \end{pmatrix}$	2304	$12A$	$\begin{pmatrix} 0&1&1&0&1&1\\ 1&1&1&1&1&0\\ 0&0&1&1&1&1\\ 0&0&0&1&1&0\\ 0&1&1&0&1&0\\ 1&1&1&1&0&1 \end{pmatrix}$	960
$12B$	$\begin{pmatrix} 1&0&1&0&1&0\\ 1&1&1&1&1&0\\ 0&0&1&1&1&1\\ 0&1&1&1&1&0\\ 1&1&0&0&1&1\\ 0&0&1&1&0&0 \end{pmatrix}$	960			

9.2 The conjugacy classes of $2^6{:}(2^5{:}S_6)$

In this section the conjugacy classes of $\overline{G} = 2^6{:}(2^5{:}S_6)$ are determined by the method of coset analysis as it was used in the previous three chapters.

Let $\overline{G} = 2^6{:}(2^5{:}S_6)$ be the split extension of $N = 2^6$ by $G = 2^5{:}S_6$. Having obtained $G =< g_1, g_2 >$ as a matrix group, we act G on the conjugacy classes of $N \cong V_6(2)$, where $V_6(2)$ is the vector space of dimension 6 over the Galois field $GF(2)$. As a result of this action, we obtain that the elements of N are partitioned into 4 orbits with respective lengths of 1,1, 30 and 32. With the aid of MAGMA and the ATLAS, we are able to identify the structures of the stabilizers corresponding to the 4 orbits of elements of N. The point stabilizers, which are subgroups of G, are identified as $P_1 = 2^5{:}S_6$, $P_2 = 2^5{:}S_6$, $P_3 = 2^4{:}S_5$ and $P_4 = S_6$.

Let $\chi(2^5{:}S_6|2^6)$ be the permutation character of $2^5{:}S_6$ on 2^6. We obtain that $\chi(2^5{:}S_6|2^6) = I_{2^5:S_6}^{2^5:S_6} + I_{2^5:S_6}^{2^5:S_6} + I_{2^4:S_5}^{2^5:S_6} + I_{S_6}^{2^5:S_6} = \chi(L_3(4){:}3|P_1) + \chi(L_3(4){:}3|P_2) + \chi(L_3(4){:}3|P_3) + \chi(L_3(4){:}3|P_4) = 1a + (1a + 5b + 6b + 15b) + (1a + 9a + 10f + 15b) = 4 \times 1a + 5b + 6b + 9a + 10f + 2 \times 15b$, which is the sum of the identity characters of the point stabilizers induced to $2^5{:}S_6$. We observe that the identity characters of the point stabilizers induced to $2^5{:}S_6$ are the permutation characters of $2^5{:}S_6$ on the point stabilizers.

The values of $\chi(2^5{:}S_6|2^6)$ on the different classes of $2^5{:}S_6$ determine the number k of fixed points of each $g \in 2^5{:}S_6$ in 2^6 (see Table 9.2). The value of k is the number of orbits formed as N acts on a coset Ng. Then the action of $C_G(g)$ determines the fusion of f_j of these k orbits. These values of the $f_j's$ are calculated by programme A. Programme B is used to compute the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. Since we have determined all the necessary information, the centralizer order $|C_{\overline{G}}(x)| = \frac{k|C_G(g)|}{f_j}$ for each class $[x]$ of $\overline{G}$ is obtained. See Table 9.3 for all the information about the conjugacy classes of $\overline{G}$.

Table 9.2: The values of $\chi(2^5{:}S_6|2^6)$ on the different classes of $2^5{:}S_6$

$[h]_{2^5:S_6}$	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$2G$	$2H$	$2I$	$2J$	$3A$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^4{:}S_5)$	30	30	14	14	14	14	6	6	6	6	6	6
$\chi(2^5{:}S_6\|S_6)$	32	0	0	0	0	16	8	0	0	8	0	8
k	64	32	16	16	16	32	16	8	8	16	8	16

$[h]_{2^5:S_6}$	$3B$	$4A$	$4B$	$4C$	$4D$	$4E$	$4F$	$4G$	$4H$	$4I$	$4J$	$5A$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_4)$	0	6	6	2	2	2	2	2	2	2	2	0
$\chi(2^5{:}S_6\|S_6)$	2	0	0	0	0	0	0	0	0	4	4	2
k	4	8	8	4	4	4	4	4	4	8	8	4

$[h]_{2^5:S_6}$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$6H$	$8A$	$8B$	$10A$	$12A$	$12B$
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_6)$	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(2^5{:}S_6\|2^5{:}S_4)$	6	2	2	0	2	2	0	0	0	0	0	0	0
$\chi(2^5{:}S_6\|S_6)$	0	0	0	0	2	0	0	1	0	0	0	0	0
$\chi(2^5{:}S_6\|S_6)$	0	0	0	0	2	0	0	1	0	0	0	0	0
k	8	4	4	2	8	4	2	4	2	2	2	2	2

.

Table 9.3: The conjugacy classes of elements of the groups $2^6{:}(2^5{:}S_6)$

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[x]_{2^6:(2^5:S_6)}$	$[x]_{2^6:(2^5:S_6)}\|$	$\|C_{2^6:(2^5:S_6)}(x)\|$
$1A$	64	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$1A$	1	1474560
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(1,0,1,0,0,1)$	$2A$	1	1474560
		$f_3 = 30$	$(0,1,0,0,0,0)$	$(0,1,0,0,0,0)$	$2B$	30	49152
		$f_4 = 32$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$2C$	32	46080
$2A$	32	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2D$	2	737280
		$f_2 = 15$	$(0,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$2E$	30	49152
		$f_3 = 16$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,0)$	$4A$	32	46080
$2B$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2F$	60	24576
		$f_2 = 3$	$(1,1,1,1,1,0)$	$(0,0,0,0,0,0)$	$2G$	180	8192
		$f_3 = 4$	$(0,0,0,1,1,1)$	$(1,0,1,0,0,1)$	$4B$	240	6144
		$f_4 = 8$	$(1,0,0,0,0,0)$	$(0,1,0,0,1,0)$	$4C$	480	3072
$2C$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2H$	60	24576
		$f_2 = 3$	$(0,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$2I$	180	8192
		$f_3 = 4$	$(1,1,0,0,1,1)$	$(1,0,1,0,0,1)$	$4D$	240	6144
		$f_4 = 8$	$(1,0,0,0,0,0)$	$(0,1,1,0,0,0)$	$4E$	480	3072
$2D$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2J$	120	12288
		$f_2 = 3$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$2K$	360	4096
		$f_3 = 4$	$(1,0,0,0,0,0)$	$(0,1,1,0,0,0)$	$4F$	480	3072
		$f_4 = 4$	$(1,1,1,1,0,0)$	$(1,1,0,0,0,1)$	$4G$	480	3072
		$f_5 = 4$	$(1,0,1,0,1,0)$	$(1,0,1,0,0,1)$	$4H$	480	3072
$2E$	32	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2L$	60	24576
		$f_2 = 1$	$(1,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$2M$	60	24576
		$f_3 = 6$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$2N$	360	4096
		$f_4 = 8$	$(1,1,0,0,1,1)$	$(0,1,1,0,0,0)$	$4I$	480	3072
		$f_5 = 8$	$(0,1,0,0,1,1)$	$(0,1,1,0,0,0)$	$4J$	480	3072
		$f_6 = 8$	$(0,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$2O$	480	3072
$2F$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2P$	240	6144
		$f_2 = 1$	$(1,1,1,1,1,0)$	$(0,0,0,0,0,0)$	$2Q$	240	6144
		$f_3 = 2$	$(1,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$2R$	480	3072
		$f_4 = 6$	$(1,1,1,1,1,1)$	$(0,0,0,1,0,1)$	$4K$	1440	1024
		$f_5 = 6$	$(0,1,1,1,1,1)$	$(1,1,1,0,1,1)$	$4L$	1440	1024
$2G$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2S$	480	3072
		$f_2 = 1$	$(1,0,0,1,1,1)$	$(1,0,1,0,0,1)$	$4M$	480	3072
		$f_3 = 3$	$(1,1,1,1,1,1)$	$(1,1,1,1,1,0)$	$4N$	1440	1024
		$f_4 = 3$	$(1,1,1,1,0,0)$	$(1,0,1,1,0,0)$	$4O$	1440	1024
$2H$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2T$	1440	1024
		$f_2 = 1$	$(1,1,1,0,1,1)$	$(1,0,0,0,0,1)$	$4P$	1440	1024
		$f_3 = 1$	$(1,1,1,1,0,0)$	$(1,1,0,0,0,1)$	$4Q$	1440	1024
		$f_4 = 1$	$(1,1,1,1,1,0)$	$(1,0,1,0,0,1)$	$4R$	1440	1024
		$f_5 = 2$	$(1,1,1,1,1,1)$	$(1,0,0,0,0,1)$	$4S$	2880	512
		$f_6 = 2$	$(1,1,1,1,0,1)$	$(1,1,1,0,0,1)$	$4T$	2880	512

Table 9.3 (continue)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[x]_{2^6:(2^5:S_6)}$	$\lvert[x]_{2^6:(2^5:S_6)}\rvert$	$\lvert C_{2^6:(2^5:S_6)}(x)\rvert$
$2I$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2U$	720	2048
		$f_2 = 1$	$(0,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$2V$	720	2048
		$f_3 = 2$	$(0,1,1,1,1,1)$	$(1,0,1,1,1,0)$	$4U$	1440	1024
		$f_4 = 2$	$(1,1,1,1,1,1)$	$(1,0,1,1,1,0)$	$4V$	1440	1024
		$f_5 = 2$	$(1,1,1,1,0,1)$	$(0,0,0,0,0,0)$	$2W$	1440	1024
		$f_6 = 4$	$(1,1,1,1,0,0)$	$(1,1,0,1,0,0)$	$4W$	2880	512
		$f_7 = 4$	$(1,0,1,1,1,1)$	$(1,1,0,1,0,0)$	$4X$	2880	512
$2J$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$2X$	1440	1024
		$f_2 = 1$	$(1,1,1,1,0,1)$	$(1,0,1,0,0,1)$	$4Y$	1440	1024
		$f_3 = 1$	$(1,1,1,0,0,1)$	$(1,0,1,1,1,0)$	$4Z$	1440	1024
		$f_4 = 1$	$(1,1,1,1,1,1)$	$(0,0,0,1,1,1)$	$4AA$	1440	1024
		$f_5 = 2$	$(1,1,1,1,1,0)$	$(0,1,1,0,1,0)$	$4AB$	2880	512
		$f_6 = 2$	$(1,1,0,1,1,1)$	$(1,1,0,0,1,1)$	$4AC$	2880	512
$3A$	16	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$3A$	640	2304
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(1,0,1,0,0,1)$	$6A$	640	2304
		$f_3 = 6$	$(0,0,0,1,0,0)$	$(0,0,0,0,1,1)$	$6B$	3840	384
		$f_4 = 8$	$(0,0,0,0,0,1)$	$(0,0,0,0,0,1)$	$6C$	5120	288
$3B$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$3B$	10240	144
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(1,0,1,0,0,1)$	$6D$	10240	144
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$6E$	20480	72
$4A$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AD$	960	1536
		$f_2 = 3$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$4AE$	2880	512
		$f_3 = 4$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AF$	3840	384
$4B$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AG$	2880	1536
		$f_2 = 3$	$(0,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AH$	2880	512
		$f_3 = 4$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$8A$	3840	384
$4C$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AI$	2880	512
		$f_2 = 1$	$(1,0,0,1,0,0)$	$(0,0,0,0,0,0)$	$4AJ$	2880	512
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AK$	3840	256
$4D$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AL$	2880	512
		$f_2 = 1$	$(1,0,0,1,0,0)$	$(0,0,0,0,0,0)$	$4AM$	2880	512
		$f_3 = 2$	$(0,0,1,0,0,1)$	$(0,0,0,0,0,0)$	$4AN$	5760	256
$4E$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AO$	5760	256
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AP$	5760	256
		$f_3 = 1$	$(1,0,0,0,0,1)$	$(0,0,0,0,0,0)$	$4AQ$	5760	256
		$f_4 = 1$	$(1,0,0,0,1,0)$	$(0,0,0,0,0,0)$	$4AR$	5760	256

Table 9.3 (continue)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[x]_{2^6:(2^5:S_6)}$	$\lvert [x]_{2^6:(2^5:S_6)} \rvert$	$\lvert C_{2^6:(2^5:S_6)}(x) \rvert$
$4F$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AS$	11520	128
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AT$	11520	128
		$f_3 = 1$	$(0,1,0,0,0,0)$	$(1,0,1,1,1,0)$	$8B$	11520	128
		$f_4 = 1$	$(1,1,0,0,0,0)$	$(1,0,1,1,1,0)$	$8C$	11520	128
$4G$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AU$	11520	128
		$f_2 = 1$	$(0,0,0,1,1,0)$	$(1,0,1,0,0,1)$	$8D$	11520	128
		$f_3 = 1$	$(1,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$4AV$	11520	128
		$f_4 = 1$	$(1,1,0,1,1,1)$	$(1,0,1,0,0,1)$	$8E$	11520	128
$4H$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AW$	11520	128
		$f_2 = 1$	$(1,0,0,0,1,0)$	$(1,0,1,0,1,1)$	$8F$	11520	128
		$f_3 = 1$	$(0,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$4AX$	11520	128
		$f_4 = 1$	$(1,1,0,0,1,1)$	$(1,0,1,0,1,1)$	$8G$	11520	128
$4I$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4AY$	5760	256
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(0,0,0,0,0,0)$	$4AZ$	5760	256
		$f_3 = 2$	$(1,0,0,0,1,0)$	$(1,0,1,0,1,1)$	$8H$	11520	128
		$f_4 = 2$	$(1,1,0,0,0,0)$	$(0,0,0,0,0,0)$	$4BA$	11520	128
		$f_5 = 2$	$(1,1,0,0,1,1)$	$(0,0,1,0,0,0)$	$8I$	11520	128
$4J$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$4BB$	5760	256
		$f_2 = 1$	$(1,0,1,1,1,0)$	$(0,0,0,0,0,0)$	$4BC$	5760	256
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(0,0,0,1,1,1)$	$8J$	11520	128
		$f_4 = 2$	$(1,1,1,1,0,0,1)$	$(0,0,0,1,1,1)$	$8K$	11520	128
		$f_5 = 2$	$(1,1,0,0,1,1)$	$(0,0,0,0,0,0)$	$4BD$	11520	128
$5A$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$5A$	36864	40
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(1,0,1,0,0,1)$	$10A$	36864	40
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(1,1,0,0,0,0)$	$10B$	73728	20
$6A$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6F$	1280	1152
		$f_2 = 3$	$(1,1,0,0,0,1)$	$(0,0,0,0,0,0)$	$6G$	3840	384
		$f_3 = 4$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$12A$	5120	288
$6B$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6H$	7680	192
		$f_2 = 1$	$(1,0,1,0,1,1)$	$(1,0,1,0,0,1)$	$12B$	7680	192
		$f_3 = 2$	$(1,0,1,1,0,1)$	$(0,1,1,0,0,0)$	$12C$	15360	96
$6C$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6I$	15360	192
		$f_2 = 1$	$(1,0,1,0,1,1)$	$(1,0,1,0,0,1)$	$12D$	15360	192
		$f_3 = 2$	$(1,0,1,1,1,1)$	$(0,1,1,0,1,0)$	$12E$	15360	96

Table 9.3 (continue)

$[g]_{2^5:S_6}$	k	f_j	d_j	w	$[x]_{2^6:(2^5:S_6)}$	$\|[x]_{2^6:(2^5:S_6)}\|$	$\|C_{2^6:(2^5:S_6)}(x)\|$
$6D$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6J$	20480	72
		$f_2 = 1$	$(0,0,1,0,0,1)$	$(0,0,1,0,0,1)$	$12F$	20480	72
$6E$	8	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6K$	7680	192
		$f_2 = 1$	$(1,0,1,0,0,1)$	$(0,0,0,0,0,0)$	$6L$	7680	192
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6M$	15360	96
		$f_4 = 2$	$(1,0,1,0,1,1)$	$(0,1,1,0,0,0)$	$12G$	15360	96
		$f_5 = 2$	$(1,0,0,0,1,0)$	$(0,1,1,0,0,0)$	$12H$	15360	96
$6F$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6N$	15360	96
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(0,1,1,0,0,0)$	$12I$	15360	96
		$f_3 = 1$	$(1,0,1,0,1,1)$	$(1,1,0,0,0,1)$	$12J$	15360	96
		$f_4 = 1$	$(1,0,0,0,1,0)$	$(1,0,1,0,0,1)$	$12K$	15360	96
$6G$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6O$	61440	24
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$12L$	61440	24
$6H$	4	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6P$	30720	48
		$f_2 = 1$	$(1,1,1,0,1,1)$	$(0,0,0,0,0,0)$	$6Q$	30720	48
		$f_3 = 2$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$6R$	61440	24
$8A$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$8L$	46080	32
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$8M$	46080	32
$8B$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$8N$	46080	32
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(1,0,0,0,0,0)$	$8O$	46080	32
$10A$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$10C$	73728	20
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$20A$	73728	20
$12A$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$12M$	30720	48
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$24A$	30720	48
$12B$	2	$f_1 = 1$	$(0,0,0,0,0,0)$	$(0,0,0,0,0,0)$	$12N$	30720	48
		$f_2 = 1$	$(1,0,0,0,0,0)$	$(1,0,1,0,0,1)$	$24B$	30720	48

9.3 The Inertia groups of $2^6{:}(2^5{:}S_6)$

Since G has 4 orbits on N, then by Brauer's Theorem [29] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be 1, r, s and t, where $r + s + t = 63$, with corresponding point stabilizers H_1, H_2, H_3 and H_4 as subgroups of G such that $[G{:}H_1] = 1$, $[G{:}H_2] = r$, $[G{:}H_3] = s$ and $[G{:}H_4] = t$. Let T be the matrix group of dimension 6 over $GF(2)$ formed by the transpose of the generators of G. The action of T on the classes of $N = 2^6$ is the equivalent of G acting on $Irr(N)$. The action of T on N has orbits of lengths 1, 1, 30 and 32 with point stabilizers T, T, $(2^5{:}S_6)$ and $(2^5{:}S_4)$, respectively. We deduce that $r = 1$, s=30, and $t = 32$, thus we obtain 4 inertia groups $\overline{H}_i = 2^6{:}H_i$ in $2^6{:}(2^5{:}S_6)$, $i \in \{1, 2, 3, 4\}$, with corresponding inertia factor groups $H_1 = G$, $H_2 = 2^5{:}S_6$, $H_3 = 2^5{:}S_4$ and $H_4 = S_6$. We use similar techniques as in the previous chapters to identify the structural information of the inertia factor groups H_i. Note that the action of G on N and $Irr(N)$ is self dual. The inertia factor groups $H_3 = 2^5{:}S_4$ and $H_4 = S_6$ are constructed from elements within $2^5{:}S_6$ and the generators are as follows:

- $\langle \zeta_1, \zeta_2, \zeta_3 \rangle = 2^5{:}S_4$, $\zeta_1 \in 4A$, $\zeta_2 \in 4J$, $\zeta_3 \in 6F$ where

$$\zeta_1 = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \zeta_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \zeta_3 = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- $\langle \eta_1, \eta_2 \rangle = S_6$, $\eta_1 \in 2I$, $\eta_2 \in 6E$ where

$$\eta_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \eta_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The above generators were used to construct the character tables of the inertia factor groups (see Appendix B).

9.4 The Fusions of the Inertia Factor Groups into $2^5{:}S_6$

We obtain the fusions of the inertia factors H_i into $2^5{:}S_6$ by using direct matrix conjugation in $2^5{:}S_6$ and the permutation characters of the inertia factor groups in $2^5{:}S_6$ of degrees 30 and 32 respectively. MAGMA was used for the various computations. The fusion maps of the inertia factor groups into $2^5{:}S_6$ are shown in the Table 9.4 and Table 9.5.

Table 9.4: The fusion of $2^5{:}S_4$ into $2^5{:}S_6$

$[h]_{2^5:S_4}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^5:S_4}$ $\longrightarrow$	$g]_{2^5:S_6}$	$[h]_{2^5:S_4}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{2^5:S_4}$ $\longrightarrow$	$[g]_{2^5:S_6}$
$1A$	$1A$	$2M$	$2J$	$2Z$	$2J$	$4K$	$4J$
$2A$	$2D$	$2N$	$2H$	$2AA$	$2E$	$4L$	$4B$
$2B$	$2D$	$2O$	$2F$	$3A$	$3A$	$4M$	$4I$
$2C$	$2A$	$2P$	$2H$	$4A$	$4C$	$4N$	$4G$
$2D$	$2B$	$2Q$	$2G$	$4B$	$4D$	$4O$	$4A$
$2E$	$2E$	$2R$	$2J$	$4C$	$4E$	$4P$	$4H$
$2F$	$2E$	$2S$	$2G$	$4D$	$4E$	$6A$	$6C$
$2G$	$2C$	$2T$	$2I$	$4E$	$4I$	$6B$	$6E$
$2H$	$2C$	$2U$	$2J$	$4F$	$4G$	$6C$	$6E$
$2I$	$2F$	$2V$	$2D$	$4G$	$4J$	$6D$	$6F$
$2J$	$2B$	$2W$	$2H$	$4H$	$4F$	$6E$	$6A$
$2K$	$2I$	$2X$	$2I$	$4I$	$4F$	$6F$	$6F$
$2L$	$2I$	$2Y$	$2H$	$4J$	$4H$	$6G$	$6B$

Table 9.5: The fusion of S_6 into $2^5{:}S_6$

$[h]_{S_6}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6}$ $\longrightarrow$	$[g]_{2^5:S_6}$	$[h]_{S_6}$ $\longrightarrow$	$[g]_{2^5:S_6}$
$1A$	$1A$	$3A$	$3A$	$5A$	$5A$
$2A$	$2E$	$3B$	$3B$	$6A$	$6E$
$2B$	$2F$	$4A$	$4J$	$6B$	$6H$
$2C$	$2I$	$4B$	$4I$		

9.5 The Fischer-Clifford Matrices of $2^6{:}(2^5{:}S_6)$

Having obtained the fusions of the inertia factor groups H_i into $2^5{:}S_6$, and the conjugacy classes of $\overline{G}$ lying above each coset Ng, we are now able to compute the Fischer-Clifford matrices of the group $2^6{:}(2^5{:}S_6)$. The properties discussed in Chapter 5 are used and applied to our groups as was done in the previous three chapters, in the construction of these matrices. Note that all the relations hold since 2^6 is an elementary abelian group. For each class representative $g \in 2^5{:}S_6$, we construct a Fischer-Clifford matrix $M(g)$ which are listed in Table 9.6.

Table 9.6: The Fischer-Clifford Matrices of $2^6{:}(2^5{:}S_6)$

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 30 & 30 & -2 & 0 \\ 32 & -32 & 0 & 0 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 30 & -2 & 0 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 12 & -4 & 0 & 0 \end{pmatrix}$	$M(2C) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 12 & -4 & 0 & 0 \end{pmatrix}$
$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 \\ 12 & -4 & 0 & 0 & 0 \end{pmatrix}$	$M(2E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 \\ 12 & 12 & -4 & 0 & 0 & 0 \\ 16 & -16 & 0 & 0 & 0 & 0 \end{pmatrix}$
$M(2F) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 3 & 3 & 3 & -1 & -1 \\ 3 & 3 & -3 & 1 & -1 \\ 8 & -8 & 0 & 0 & 0 \end{pmatrix}$	$M(2G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \end{pmatrix}$
$M(2H) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & -2 & 2 & -2 & 0 & 0 \end{pmatrix}$	$M(2I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 2 & 2 & 2 & -2 & -2 & 0 & 0 \\ 2 & 2 & -2 & 2 & -2 & 0 & 0 \\ 8 & -8 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
$M(2J) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & -2 & 2 & 0 & 0 \\ 2 & -2 & 2 & -2 & 0 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 6 & 6 & -2 & 0 \\ 8 & -8 & 0 & 0 \end{pmatrix}$
$M(3B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(4A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 6 & -2 & 0 \end{pmatrix}$
$M(4B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 6 & -2 & 0 \end{pmatrix}$	$M(4C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(4D) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(4E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$
$M(4F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(4G) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$

Table 9.6 (continue)

$M(g)$	$M(g)$
$M(4H) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(4I) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 0 & 0 & 0 \end{pmatrix}$
$M(4J) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 0 & 0 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 6 & -2 & 0 \end{pmatrix}$	$M(6B) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(6C) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$	$M(6D) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6E) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & -1 \\ 4 & -4 & 0 & 0 & 0 \end{pmatrix}$	$M(6F) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$
$M(6G) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(6H) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 0 \end{pmatrix}$
$M(8A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(8B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(10A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$M(12A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(12B) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	

9.6 Character Table of $2^6{:}(2^5{:}S_6)$

The character table of $\overline{G}$ is constructed following the method discussed in Chapter 5 and applied in Chapters 6, 7 and 8. The character table of $\overline{G}$ will be partitioned row-wise into blocks B_i, where each block corresponds to an inertia group $\overline{H}_i = 2^6{:}H_i$. Therefore $Irr(\overline{G}) = \bigcup_{i=1}^{4} B_i$, where $B_1 = \{\chi_j | 1 \leq j \leq 37\}$, $B_2 = \{\chi_j | 38 \leq j \leq 74\}$,

$B_3 = \{\chi_j | 75 \leq j \leq 126\}$, $B_4 = \{\chi_j | 127 \leq j \leq 137\}$. The character table of $2^6{:}(2^5{:}S_6)$ is shown in Table 9.7.

Table 9.7: The Character table of $2^6{:}(2^5{:}S_6)$

	1A				2A			2B				2C				2D				
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$2G$	$4B$	$4C$	$2H$	$2I$	$4D$	$4E$	$2J$	$2K$	$4F$	$4G$	$4H$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
χ_3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3
χ_4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	-3	-3	-3	-3	-3
χ_5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	-1	-1	-1	-1	-1
χ_6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	1	1	1	1	1
χ_7	6	6	6	6	-6	-6	-6	-2	-2	-2	-2	2	2	2	2	-4	-4	-4	-4	-4
χ_8	6	6	6	6	-6	-6	-6	-2	-2	-2	-2	2	2	2	2	4	4	4	4	4
χ_9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	-3	-3	-3	-3	-3
χ_{10}	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	3	3	3	3	3
χ_{11}	10	10	10	10	-10	-10	-10	2	2	2	2	-2	-2	-2	-2	4	4	4	4	4
χ_{12}	10	10	10	10	-10	-10	-10	2	2	2	2	-2	-2	-2	-2	4	4	4	4	4
χ_{13}	10	10	10	10	-10	-10	-10	2	2	2	2	-2	-2	-2	-2	-4	-4	-4	-4	-4
χ_{14}	10	10	10	10	-10	-10	-10	2	2	2	2	-2	-2	-2	-2	-4	-4	-4	-4	-4
χ_{15}	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	-2	-2	-2	-2	-2
χ_{16}	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	2	2	2	2	2
χ_{17}	15	15	15	15	15	15	15	-1	-1	-1	-1	-1	-1	-1	-1	-5	-5	-5	-5	-5
χ_{18}	15	15	15	15	15	15	15	-1	-1	-1	-1	-1	-1	-1	-1	-7	-7	-7	-7	-7
χ_{19}	15	15	15	15	15	15	15	-1	-1	-1	-1	-1	-1	-1	-1	5	5	5	5	5
χ_{20}	15	15	15	15	15	15	15	-1	-1	-1	-1	-1	-1	-1	-1	7	7	7	7	7
χ_{21}	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	0	0	0	0	0
χ_{22}	20	20	20	20	-20	-20	-20	4	4	4	4	-4	-4	-4	-4	0	0	0	0	0
χ_{23}	24	24	24	24	-24	-24	-24	-8	-8	-8	-8	8	8	8	8	8	8	8	8	8
χ_{24}	24	24	24	24	-24	-24	-24	-8	-8	-8	-8	8	8	8	8	-8	-8	-8	-8	-8
χ_{25}	30	30	30	30	-30	-30	-30	-10	-10	-10	-10	10	10	10	10	4	4	4	4	4
χ_{26}	30	30	30	30	-30	-30	-30	-10	-10	-10	-10	10	10	10	10	-4	-4	-4	-4	-4
χ_{27}	30	30	30	30	30	30	30	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{28}	30	30	30	30	30	30	30	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	2
χ_{29}	36	36	36	36	-36	-36	-36	-12	-12	-12	-12	12	12	12	12	0	0	0	0	0
χ_{30}	40	40	40	40	-40	-40	-40	8	8	8	8	-8	-8	-8	-8	-8	-8	-8	-8	-8
χ_{31}	40	40	40	40	-40	-40	-40	8	8	8	8	-8	-8	-8	-8	8	8	8	8	8
χ_{32}	40	40	40	40	-40	-40	-40	8	8	8	8	-8	-8	-8	-8	0	0	0	0	0
χ_{33}	40	40	40	40	-40	-40	-40	8	8	8	8	-8	-8	-8	-8	0	0	0	0	0
χ_{34}	45	45	45	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	9	9	9	9	9
χ_{35}	45	45	45	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-9	-9	-9	-9	-9
χ_{36}	45	45	45	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_{37}	45	45	45	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3	3	3	3	3	3
χ_{38}	1	1	1	-1	1	1	-1	1	1	1	-1	1	1	1	-1	1	1	-1	1	-1
χ_{39}	1	1	1	-1	1	1	-1	1	1	1	-1	1	1	1	-1	-1	-1	1	-1	1
χ_{40}	5	5	5	-5	5	5	-5	5	5	5	-5	5	5	5	-5	3	3	-3	3	-3
χ_{41}	5	5	5	-5	5	5	-5	5	5	5	-5	5	5	5	-5	-3	-3	3	-3	3
χ_{42}	5	5	5	-5	5	5	-5	5	5	5	-5	5	5	5	-5	-1	-1	1	-1	1
χ_{43}	5	5	5	-5	5	5	-5	5	5	5	-5	5	5	5	-5	1	1	-1	1	-1
χ_{44}	6	6	6	-6	-6	-6	6	-2	-2	-2	2	2	2	2	-2	-4	-4	4	-4	4
χ_{45}	6	6	6	-6	-6	-6	6	-2	-2	-2	2	2	2	2	-2	4	4	-4	4	-4

Table 9.7 (continue)

	1A				2A			2B				2C				2D				
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$2G$	$4B$	$4C$	$2H$	$2I$	$4D$	$4E$	$2J$	$2K$	$4F$	$4G$	$4H$
χ_{46}	9	9	9	-9	9	9	-9	9	9	9	-9	9	9	9	-9	-3	-3	3	-3	3
χ_{47}	9	9	9	-9	9	9	-9	9	9	9	-9	9	9	9	-9	3	3	-3	3	-3
χ_{48}	10	10	10	-10	-10	-10	10	2	2	2	-2	-2	-2	-2	2	4	4	-4	4	-4
χ_{49}	10	10	10	-10	-10	-10	10	2	2	2	-2	-2	-2	-2	2	4	4	-4	4	-4
χ_{50}	10	10	10	-10	-10	-10	10	2	2	2	-2	-2	-2	-2	2	-4	-4	4	-4	4
χ_{51}	10	10	10	-10	-10	-10	10	2	2	2	-2	-2	-2	-2	2	-4	-4	4	-4	4
χ_{52}	10	10	10	-10	10	10	-10	10	10	10	-10	10	10	10	-10	-2	-2	2	-2	2
χ_{53}	10	10	10	-10	10	10	-10	10	10	10	-10	10	10	10	-10	2	2	-2	2	-2
χ_{54}	15	15	15	-15	15	15	-15	-1	-1	-1	1	-1	-1	-1	1	-5	-5	5	-5	5
χ_{55}	15	15	15	-15	15	15	-15	-1	-1	-1	1	-1	-1	-1	1	-7	-7	7	-7	7
χ_{56}	15	15	15	-15	15	15	-15	-1	-1	-1	1	-1	-1	-1	1	5	5	-5	5	-5
χ_{57}	15	15	15	-15	15	15	-15	-1	-1	-1	1	-1	-1	-1	1	7	7	-7	7	-7
χ_{58}	16	16	16	-16	16	16	-16	16	16	16	-16	16	16	16	-16	0	0	0	0	0
χ_{59}	20	20	20	-20	-20	-20	20	4	4	4	-4	-4	-4	-4	4	0	0	0	0	0
χ_{60}	24	24	24	-24	-24	-24	24	-8	-8	-8	8	8	8	8	-8	8	8	-8	8	-8
χ_{61}	24	24	24	-24	-24	-24	24	-8	-8	-8	8	8	8	8	-8	-8	-8	8	-8	8
χ_{62}	30	30	30	-30	-30	-30	30	-10	-10	-10	10	10	10	10	-10	4	4	-4	4	-4
χ_{63}	30	30	30	-30	-30	-30	30	-10	-10	-10	10	10	10	10	-10	-4	-4	4	-4	4
χ_{64}	30	30	30	-30	30	30	-30	-2	-2	-2	2	-2	-2	-2	2	-2	-2	2	-2	2
χ_{65}	30	30	30	-30	30	30	-30	-2	-2	-2	2	-2	-2	-2	2	2	2	-2	2	-2
χ_{66}	36	36	36	-36	-36	-36	36	-12	-12	-12	12	12	12	12	-12	0	0	0	0	0
χ_{67}	40	40	40	-40	-40	-40	40	8	8	8	-8	-8	-8	-8	8	-8	-8	8	-8	8
χ_{68}	40	40	40	-40	-40	-40	40	8	8	8	-8	-8	-8	-8	8	8	8	-8	8	-8
χ_{69}	40	40	40	-40	-40	-40	40	8	8	8	-8	-8	-8	-8	8	0	0	0	0	0
χ_{70}	40	40	40	-40	-40	-40	40	8	8	8	-8	-8	-8	-8	8	0	0	0	0	0
χ_{71}	45	45	45	-45	45	45	-45	-3	-3	-3	3	-3	-3	-3	3	9	9	-9	9	-9
χ_{72}	45	45	45	-45	45	45	-45	-3	-3	-3	3	-3	-3	-3	3	-9	-9	9	-9	9
χ_{73}	45	45	45	-45	45	45	-45	-3	-3	-3	3	-3	-3	-3	3	-3	-3	3	-3	3
χ_{74}	45	45	45	-45	45	45	-45	-3	-3	-3	3	-3	-3	-3	3	3	3	-3	3	-3
χ_{75}	30	30	-2	0	30	-2	0	14	-2	-2	0	14	-2	-2	0	14	-2	0	-2	0
χ_{76}	30	30	-2	0	-30	2	0	-10	6	-2	0	10	-6	2	0	12	-4	-2	0	2
χ_{77}	30	30	-2	0	30	-2	0	14	-2	-2	0	14	-2	-2	0	-14	2	0	2	0
χ_{78}	30	30	-2	0	-30	2	0	-10	6	-2	0	10	-6	2	0	12	-4	2	0	-2
χ_{79}	30	30	-2	0	30	-2	0	14	-2	-2	0	14	-2	-2	0	-10	6	0	-2	0
χ_{80}	30	30	-2	0	-30	2	0	-10	6	-2	0	10	-6	2	0	-12	4	-2	0	2
χ_{81}	30	30	-2	0	30	-2	0	14	-2	-2	0	14	-2	-2	0	10	-6	0	2	0
χ_{82}	30	30	-2	0	-30	2	0	-10	6	-2	0	10	-6	2	0	-12	4	2	0	-2
χ_{83}	60	60	-4	0	60	-4	0	28	-4	-4	0	28	-4	-4	0	4	4	0	-4	0
χ_{84}	60	60	-4	0	-60	4	0	-20	12	-4	0	20	-12	4	0	0	0	4	0	-4
χ_{85}	60	60	-4	0	60	-4	0	28	-4	-4	0	28	-4	-4	0	-4	-4	0	4	0
χ_{86}	60	60	-4	0	-60	4	0	-20	12	-4	0	20	-12	4	0	0	0	-4	0	4
χ_{87}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	12	-4	-6	0	6
χ_{88}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	-18	-2	0	6	0
χ_{89}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	12	-4	6	0	-6
χ_{90}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	-6	10	0	-6	0
χ_{91}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	-12	4	-6	0	6

Table 9.7 (continue)

	1A				2A			2B				2C				2D				
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$2F$	$2G$	$4B$	$4C$	$2H$	$2I$	$4D$	$4E$	$2J$	$2K$	$4F$	$4G$	$4H$
χ_{92}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	6	-10	0	6	0
χ_{93}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	-12	4	6	0	-6
χ_{94}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	18	2	0	-6	0
χ_{95}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	12	-4	-6	0	6
χ_{96}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	-18	-2	0	6	0
χ_{97}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	12	-4	6	0	-6
χ_{98}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	-6	10	0	-6	0
χ_{99}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	-12	4	-6	0	6
χ_{100}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	6	-10	0	6	0
χ_{101}	90	90	-6	0	-90	6	0	18	2	-6	0	-18	-2	6	0	-12	4	6	0	-6
χ_{102}	90	90	-6	0	90	-6	0	-6	10	-6	0	-6	10	-6	0	18	2	0	-6	0
χ_{103}	90	90	-6	0	-90	6	0	-30	18	-6	0	30	-18	6	0	12	-4	-6	0	6
χ_{104}	90	90	-6	0	90	-6	0	42	-6	-6	0	42	-6	-6	0	-18	-2	0	6	0
χ_{105}	90	90	-6	0	-90	6	0	-30	18	-6	0	30	-18	6	0	12	-4	6	0	-6
χ_{106}	90	90	-6	0	90	-6	0	42	-6	-6	0	42	-6	-6	0	-6	10	0	-6	0
χ_{107}	90	90	-6	0	-90	6	0	-30	18	-6	0	30	-18	6	0	-12	4	-6	0	6
χ_{108}	90	90	-6	0	90	-6	0	42	-6	-6	0	42	-6	-6	0	6	-10	0	6	0
χ_{109}	90	90	-6	0	-90	6	0	-30	18	-6	0	30	-18	6	0	-12	4	6	0	-6
χ_{110}	90	90	-6	0	90	-6	0	42	-6	-6	0	42	-6	-6	0	18	2	0	-6	0
χ_{111}	120	120	-8	0	-120	8	0	-8	-8	8	0	8	8	-8	0	-32	0	0	8	0
χ_{112}	120	120	-8	0	-120	8	0	-8	-8	8	0	8	8	-8	0	16	-16	0	8	0
χ_{113}	120	120	-8	0	-120	8	0	-8	-8	8	0	8	8	-8	0	-16	16	0	-8	0
χ_{114}	120	120	-8	0	-120	8	0	-8	-8	8	0	8	8	-8	0	32	0	0	-8	0
χ_{115}	120	120	-8	0	120	-8	0	-8	-8	8	0	-8	-8	8	0	24	-8	-8	0	8
χ_{116}	120	120	-8	0	120	-8	0	-8	-8	8	0	-8	-8	8	0	-24	8	-8	0	8
χ_{117}	120	120	-8	0	120	-8	0	-8	-8	8	0	-8	-8	8	0	24	-8	8	0	-8
χ_{118}	120	120	-8	0	120	-8	0	-8	-8	8	0	-8	-8	8	0	-24	8	8	0	-8
χ_{119}	180	180	-12	0	180	-12	0	-12	20	-12	0	-12	20	-12	0	12	12	0	-12	0
χ_{120}	180	180	-12	0	-180	12	0	36	4	-12	0	-36	-4	12	0	0	0	12	0	-12
χ_{121}	180	180	-12	0	180	-12	0	-12	20	-12	0	-12	20	-12	0	-12	-12	0	12	0
χ_{122}	180	180	-12	0	-180	12	0	36	4	-12	0	-36	-4	12	0	0	0	-12	0	12
χ_{123}	240	240	-16	0	-240	16	0	-16	-16	16	0	16	16	-16	0	-16	-16	0	16	0
χ_{124}	240	240	-16	0	-240	16	0	-16	-16	16	0	16	16	-16	0	16	16	0	-16	0
χ_{125}	240	240	-16	0	240	-16	0	-16	-16	16	0	-16	-16	16	0	0	0	-16	0	16
χ_{126}	240	240	-16	0	240	-16	0	-16	-16	16	0	-16	-16	16	0	0	0	16	0	-16
χ_{127}	32	-32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	32	-32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	160	-160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	160	-160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	160	-160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	160	-160	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	288	-288	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	288	-288	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	320	-320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	320	-320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	512	-512	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	2E						2F					2G				2H					
	$2L$	$2M$	$2N$	$4I$	$4J$	$2O$	$2P$	$2Q$	$2R$	$4K$	$4L$	$2S$	$4M$	$4N$	$4O$	$2T$	$4P$	$4Q$	$4R$	$4S$	$4T$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3
χ_4	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1	1	1	-3	-3	-3	-3	-3	-3
χ_5	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1
χ_6	1	1	1	1	1	1	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_7	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_8	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_{10}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
χ_{11}	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	-4	4	4	4	4	0	0	0	0	0	0
χ_{12}	-4	-4	-4	-4	-4	-4	4	4	4	4	4	-4	-4	-4	-4	0	0	0	0	0	0
χ_{13}	4	4	4	4	4	4	-4	-4	-4	-4	-4	4	4	4	4	0	0	0	0	0	0
χ_{14}	4	4	4	4	4	4	4	4	4	4	4	-4	-4	-4	-4	0	0	0	0	0	0
χ_{15}	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2
χ_{16}	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2
χ_{17}	-5	-5	-5	-5	-5	-5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
χ_{18}	-7	-7	-7	-7	-7	-7	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_{19}	5	5	5	5	5	5	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
χ_{20}	7	7	7	7	7	7	3	3	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	-8	-8	-8	-8	-8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	-4	-4	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	4	4	4	4	4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	-2	-2	-2	-2	-2	-2	-6	-6	-6	-6	-6	-6	-6	-6	-6	-2	-2	-2	-2	-2	-2
χ_{28}	2	2	2	2	2	2	6	6	6	6	6	6	6	6	6	2	2	2	2	2	2
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	-8	-8	-8	-8	-8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	-8	-8	-8	-8	-8	8	8	8	8	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	8	8	8	8	8	-8	-8	-8	-8	0	0	0	0	0	0
χ_{34}	9	9	9	9	9	9	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1
χ_{35}	-9	-9	-9	-9	-9	-9	3	3	3	3	3	3	3	3	3	-1	-1	-1	-1	-1	-1
χ_{36}	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	5	5	5	5	5	5
χ_{37}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	-5	-5	-5	-5	-5	-5
χ_{38}	1	1	1	-1	-1	1	1	1	-1	-1	1	1	-1	1	-1	1	1	1	1	-1	-1
χ_{39}	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	-1	1	-1	1	-1	-1	-1	-1	1	1
χ_{40}	3	3	3	-3	-3	3	-1	-1	1	1	-1	-1	1	-1	1	3	3	3	3	-3	-3
χ_{41}	-3	-3	-3	3	3	-3	1	1	-1	-1	1	1	-1	1	-1	-3	-3	-3	-3	3	3
χ_{42}	-1	-1	-1	1	1	-1	3	3	-3	-3	3	3	-3	3	-3	-1	-1	-1	-1	1	1
χ_{43}	1	1	1	-1	-1	1	-3	-3	3	3	-3	-3	3	-3	3	1	1	1	1	-1	-1
χ_{44}	4	4	4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	-4	-4	-4	4	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	2E						2F					2G				2H					
	$2L$	$2M$	$2N$	$4I$	$4J$	$2O$	$2P$	$2Q$	$2R$	$4K$	$4L$	$2S$	$4M$	$4N$	$4O$	$2T$	$4P$	$4Q$	$4R$	$4S$	$4T$
χ_{46}	-3	-3	-3	3	3	-3	-3	-3	3	3	-3	-3	3	-3	3	-3	-3	-3	-3	3	3
χ_{47}	3	3	3	-3	-3	3	3	3	-3	-3	3	3	-3	3	-3	3	3	3	3	-3	-3
χ_{48}	-4	-4	-4	4	4	-4	-4	-4	4	4	-4	4	-4	4	-4	0	0	0	0	0	0
χ_{49}	-4	-4	-4	4	4	-4	4	4	-4	-4	4	-4	4	-4	4	0	0	0	0	0	0
χ_{50}	4	4	4	-4	-4	4	-4	-4	4	4	-4	4	-4	4	-4	0	0	0	0	0	0
χ_{51}	4	4	4	-4	-4	4	4	4	-4	-4	4	-4	4	-4	4	0	0	0	0	0	0
χ_{52}	-2	-2	-2	2	2	-2	2	2	-2	-2	2	2	-2	2	-2	-2	-2	-2	-2	2	2
χ_{53}	2	2	2	-2	-2	2	-2	-2	2	2	-2	-2	2	-2	2	2	2	2	2	-2	-2
χ_{54}	-5	-5	-5	5	5	-5	3	3	-3	-3	3	3	-3	3	-3	3	3	3	3	-3	-3
χ_{55}	-7	-7	-7	7	7	-7	-3	-3	3	3	-3	-3	3	-3	3	1	1	1	1	-1	-1
χ_{56}	5	5	5	-5	-5	5	-3	-3	3	3	-3	-3	3	-3	3	-3	-3	-3	-3	3	3
χ_{57}	7	7	7	-7	-7	7	3	3	-3	-3	3	3	-3	3	-3	-1	-1	-1	-1	1	1
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	-8	-8	-8	8	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	8	8	8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	-4	-4	-4	4	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	4	4	4	-4	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	-2	-2	-2	2	2	-2	-6	-6	6	6	-6	-6	6	-6	6	-2	-2	-2	-2	2	2
χ_{65}	2	2	2	-2	-2	2	6	6	-6	-6	6	6	-6	6	-6	2	2	2	2	-2	-2
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	8	8	8	-8	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	-8	-8	-8	8	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	-8	-8	8	8	-8	8	-8	8	-8	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	8	8	-8	-8	8	-8	8	-8	8	0	0	0	0	0	0
χ_{71}	9	9	9	-9	-9	9	-3	-3	3	3	-3	-3	3	-3	3	1	1	1	1	-1	-1
χ_{72}	-9	-9	-9	9	9	-9	3	3	-3	-3	3	3	-3	3	-3	-1	-1	-1	-1	1	1
χ_{73}	-3	-3	-3	3	3	-3	-3	-3	3	3	-3	-3	3	-3	3	5	5	5	5	-5	-5
χ_{74}	3	3	3	-3	-3	3	3	3	-3	-3	3	3	-3	3	-3	-5	-5	-5	-5	5	5
χ_{75}	14	14	-2	0	0	-2	6	6	0	0	-2	6	0	-2	0	6	-2	-2	-2	0	0
χ_{76}	-12	-12	4	2	-2	0	0	0	6	-2	0	0	-6	0	2	0	0	4	-4	-2	2
χ_{77}	-14	-14	2	0	0	2	-6	-6	0	0	2	-6	0	2	0	-6	2	2	2	0	0
χ_{78}	-12	-12	4	-2	2	0	0	0	-6	2	0	0	6	0	-2	0	0	4	-4	2	-2
χ_{79}	-10	-10	6	0	0	-2	6	6	0	0	-2	6	0	-2	0	-2	6	-2	-2	0	0
χ_{80}	12	12	-4	2	-2	0	0	0	6	-2	0	0	-6	0	2	0	0	-4	4	-2	2
χ_{81}	10	10	-6	0	0	2	-6	-6	0	0	2	-6	0	2	0	2	-6	2	2	0	0
χ_{82}	12	12	-4	-2	2	0	0	0	-6	2	0	0	6	0	-2	0	0	-4	4	2	-2
χ_{83}	4	4	4	0	0	-4	12	12	0	0	-4	12	0	-4	0	4	4	-4	-4	0	0
χ_{84}	0	0	0	-4	4	0	0	0	-12	4	0	0	12	0	-4	0	0	0	0	4	-4
χ_{85}	-4	-4	-4	0	0	4	-12	-12	0	0	4	-12	0	4	0	-4	-4	4	4	0	0
χ_{86}	0	0	0	4	-4	0	0	0	12	-4	0	0	-12	0	4	0	0	0	0	-4	4
χ_{87}	-12	-12	4	6	-6	0	12	12	6	-2	-4	-12	-6	4	2	0	0	4	-4	2	-2
χ_{88}	-18	-18	-2	0	0	6	-6	-6	-12	4	2	-6	-12	2	4	-2	6	-2	-2	0	0
χ_{89}	-12	-12	4	-6	6	0	-12	-12	-6	2	4	12	6	-4	-2	0	0	4	-4	-2	2
χ_{90}	-6	-6	10	0	0	-6	6	6	12	-4	-2	6	12	-2	-4	-6	2	2	2	0	0
χ_{91}	12	12	-4	6	-6	0	12	12	6	-2	-4	-12	-6	4	2	0	0	-4	4	2	-2

Table 9.7 (continue)

	2E						2F					2G				2H					
	$2L$	$2M$	$2N$	$4I$	$4J$	$2O$	$2P$	$2Q$	$2R$	$4K$	$4L$	$2S$	$4M$	$4N$	$4O$	$2T$	$4P$	$4Q$	$4R$	$4S$	$4T$
χ_{92}	6	6	-10	0	0	6	-6	-6	-12	4	2	-6	-12	2	4	6	-2	-2	-2	0	0
χ_{93}	12	12	-4	-6	6	0	-12	-12	-6	2	4	12	6	-4	-2	0	0	-4	4	-2	2
χ_{94}	18	18	2	0	0	-6	6	6	12	-4	-2	6	12	-2	-4	2	-6	2	2	0	0
χ_{95}	-12	-12	4	6	-6	0	-12	-12	6	-2	4	12	-6	-4	2	0	0	4	-4	2	-2
χ_{96}	-18	-18	-2	0	0	6	-6	-6	12	-4	2	-6	12	2	-4	-2	6	-2	-2	0	0
χ_{97}	-12	-12	4	-6	6	0	12	12	-6	2	-4	-12	6	4	-2	0	0	4	-4	-2	2
χ_{98}	-6	-6	10	0	0	-6	6	6	-12	4	-2	6	-12	-2	4	-6	2	2	2	0	0
χ_{99}	12	12	-4	6	-6	0	-12	-12	6	-2	4	12	-6	-4	2	0	0	-4	4	2	-2
χ_{100}	6	6	-10	0	0	6	-6	-6	12	-4	2	-6	12	2	-4	6	-2	-2	-2	0	0
χ_{101}	12	12	-4	-6	6	0	12	12	-6	2	-4	-12	6	4	-2	0	0	-4	4	-2	2
χ_{102}	18	18	2	0	0	-6	6	6	-12	4	-2	6	-12	-2	4	2	-6	2	2	0	0
χ_{103}	-12	-12	4	6	-6	0	0	0	-6	2	0	0	6	0	-2	0	0	4	-4	-6	6
χ_{104}	-18	-18	-2	0	0	6	6	6	0	0	-2	6	0	-2	0	-10	-2	6	6	0	0
χ_{105}	-12	-12	4	-6	6	0	0	0	6	-2	0	0	-6	0	2	0	0	4	-4	6	-6
χ_{106}	-6	-6	10	0	0	-6	-6	-6	0	0	2	-6	0	2	0	2	10	-6	-6	0	0
χ_{107}	12	12	-4	6	-6	0	0	0	-6	2	0	0	6	0	-2	0	0	-4	4	-6	6
χ_{108}	6	6	-10	0	0	6	6	6	0	0	-2	6	0	-2	0	-2	-10	6	6	0	0
χ_{109}	12	12	-4	-6	6	0	0	0	6	-2	0	0	-6	0	2	0	0	-4	4	6	-6
χ_{110}	18	18	2	0	0	-6	-6	-6	0	0	2	-6	0	2	0	10	2	-6	-6	0	0
χ_{111}	32	32	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0
χ_{112}	-16	-16	16	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	-8	8	0	0
χ_{113}	16	16	-16	0	0	8	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0
χ_{114}	-32	-32	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	-8	8	0	0
χ_{115}	24	24	-8	-8	8	0	0	0	0	0	0	0	0	0	0	-8	8	0	0	0	0
χ_{116}	-24	-24	8	-8	8	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	0	0
χ_{117}	24	24	-8	8	-8	0	0	0	0	0	0	0	0	0	0	-8	8	0	0	0	0
χ_{118}	-24	-24	8	8	-8	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	0	0
χ_{119}	12	12	12	0	0	-12	-12	-12	0	0	4	-12	0	4	0	-4	-4	4	4	0	0
χ_{120}	0	0	0	-12	12	0	0	0	12	-4	0	0	-12	0	4	0	0	0	0	-4	4
χ_{121}	-12	-12	-12	0	0	12	12	12	0	0	-4	12	0	-4	0	4	4	-4	-4	0	0
χ_{122}	0	0	0	12	-12	0	0	0	-12	4	0	0	12	0	-4	0	0	0	0	4	-4
χ_{123}	16	16	16	0	0	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	-16	-16	-16	0	0	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{125}	0	0	0	-16	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{126}	0	0	0	16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	16	-16	0	0	0	0	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	-16	16	0	0	0	0	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	-48	48	0	0	0	0	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	48	-48	0	0	0	0	-8	8	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	-16	16	0	0	0	0	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	16	-16	0	0	0	0	-24	24	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	-48	48	0	0	0	0	-24	24	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	48	-48	0	0	0	0	24	-24	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	-32	32	0	0	0	0	16	-16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	32	-32	0	0	0	0	-16	16	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	2I							2J						3A				3B		
	$2U$	$2V$	$4U$	$4V$	$2W$	$4W$	$4X$	$2X$	$4Y$	$4Z$	$4AA$	$4AB$	$4AC$	$3A$	$6A$	$6B$	$6C$	$3B$	$6D$	$6E$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	-1	-1	-1
χ_4	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	-1	-1	-1
χ_5	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	2	2	2
χ_6	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	2	2	2
χ_7	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	3	3	3	3	0	0	0
χ_8	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	3	3	3	3	0	0	0
χ_9	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
χ_{10}	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
χ_{11}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{12}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{13}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{14}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{15}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{16}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	1	1	1	1	1	1	1
χ_{17}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	0	0	0
χ_{18}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0	0	0
χ_{19}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	3	3	3	3	0	0	0
χ_{20}	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2	-2	-2	-2
χ_{22}	-4	-4	-4	-4	-4	-4	-4	4	4	4	4	4	4	2	2	2	2	2	2	2
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	3	0	0	0
χ_{25}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	0	0	0
χ_{26}	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-3	-3	-3	-3	0	0	0
χ_{27}	2	2	2	2	2	2	2	2	2	2	2	2	2	-3	-3	-3	-3	0	0	0
χ_{28}	2	2	2	2	2	2	2	2	2	2	2	2	2	-3	-3	-3	-3	0	0	0
χ_{29}	-4	-4	-4	-4	-4	-4	-4	4	4	4	4	4	4	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-2	-2	-2
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-2	-2	-2
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2	1	1	1
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	-2	1	1	1
χ_{34}	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
χ_{35}	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
χ_{36}	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	0
χ_{37}	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0	0	0	0	0	0	0
χ_{38}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	1	1	1	-1	1	1	-1
χ_{39}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	1	1	1	-1	1	1	-1
χ_{40}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	2	2	2	-2	-1	-1	1
χ_{41}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	2	2	2	-2	-1	-1	1
χ_{42}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	2	2	-2
χ_{43}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	-1	-1	1	2	2	-2
χ_{44}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	3	3	3	-3	0	0	0
χ_{45}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	3	3	3	-3	0	0	0

Table 9.7 (continue)

	2I							2J						3A				3B		
	$2U$	$2V$	$4U$	$4V$	$2W$	$4W$	$4X$	$2X$	$4Y$	$4Z$	$4AA$	$4AB$	$4AC$	$3A$	$6A$	$6B$	$6C$	$3B$	$6D$	$6E$
χ_{46}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{47}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{48}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{49}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{50}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{51}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{52}	-2	-2	2	2	-2	2	-2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{53}	-2	-2	2	2	-2	2	-2	-2	-2	2	2	-2	2	1	1	1	-1	1	1	-1
χ_{54}	-1	-1	1	1	-1	1	-1	-1	-1	1	1	-1	1	3	3	3	-3	0	0	0
χ_{55}	3	3	-3	-3	3	-3	3	3	3	-3	-3	3	-3	3	3	3	-3	0	0	0
χ_{56}	-1	-1	1	1	-1	1	-1	-1	-1	1	1	-1	1	3	3	3	-3	0	0	0
χ_{57}	3	3	-3	-3	3	-3	3	3	3	-3	-3	3	-3	3	3	3	-3	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	2	-2	-2	2
χ_{59}	-4	-4	4	4	-4	4	-4	4	4	-4	-4	4	-4	2	2	2	-2	2	2	-2
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	-3	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	3	-3	0	0	0
χ_{62}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	-3	-3	-3	3	0	0	0
χ_{63}	2	2	-2	-2	2	-2	2	-2	-2	2	2	-2	2	-3	-3	-3	3	0	0	0
χ_{64}	2	2	-2	-2	2	-2	2	2	2	-2	-2	2	-2	-3	-3	-3	3	0	0	0
χ_{65}	2	2	-2	-2	2	-2	2	2	2	-2	-2	2	-2	-3	-3	-3	3	0	0	0
χ_{66}	-4	-4	4	4	-4	4	-4	4	4	-4	-4	4	-4	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	-2	-2	2
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	-2	-2	2
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	2	1	1	-1
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	-2	-2	-2	2	1	1	-1
χ_{71}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{72}	1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{73}	-3	-3	3	3	-3	3	-3	-3	-3	3	3	-3	3	0	0	0	0	0	0	0
χ_{74}	-3	-3	3	3	-3	3	-3	-3	-3	3	3	-3	3	0	0	0	0	0	0	0
χ_{75}	6	6	0	0	-2	0	-2	6	-2	0	0	-2	0	6	6	-2	0	0	0	0
χ_{76}	2	2	4	-4	2	0	-2	-2	-2	-4	4	2	0	6	6	-2	0	0	0	0
χ_{77}	6	6	0	0	-2	0	-2	6	-2	0	0	-2	0	6	6	-2	0	0	0	0
χ_{78}	2	2	-4	4	2	0	-2	-2	-2	4	-4	2	0	6	6	-2	0	0	0	0
χ_{79}	-2	-2	0	0	6	0	-2	-2	6	0	0	-2	0	6	6	-2	0	0	0	0
χ_{80}	2	2	-4	4	2	0	-2	-2	-2	4	-4	2	0	6	6	-2	0	0	0	0
χ_{81}	-2	-2	0	0	6	0	-2	-2	6	0	0	-2	0	6	6	-2	0	0	0	0
χ_{82}	2	2	4	-4	2	0	-2	-2	-2	-4	4	2	0	6	6	-2	0	0	0	0
χ_{83}	4	4	0	0	4	0	-4	4	4	0	0	-4	0	-6	-6	2	0	0	0	0
χ_{84}	4	4	0	0	4	0	-4	-4	-4	0	0	4	0	-6	-6	2	0	0	0	0
χ_{85}	4	4	0	0	4	0	-4	4	4	0	0	-4	0	-6	-6	2	0	0	0	0
χ_{86}	4	4	0	0	4	0	-4	-4	-4	0	0	4	0	-6	-6	2	0	0	0	0
χ_{87}	2	2	8	0	2	-4	-2	-2	-2	-8	0	2	4	0	0	0	0	0	0	0
χ_{88}	6	6	4	4	-2	-4	-2	6	-2	4	4	-2	-4	0	0	0	0	0	0	0
χ_{89}	2	2	0	8	2	-4	-2	-2	-2	0	-8	2	4	0	0	0	0	0	0	0
χ_{90}	-2	-2	4	4	6	-4	-2	-2	6	4	4	-2	-4	0	0	0	0	0	0	0
χ_{91}	2	2	0	8	2	-4	-2	-2	-2	0	-8	2	4	0	0	0	0	0	0	0

Table 9.7 (continue)

	2I							2J						3A				3B		
	$2U$	$2V$	$4U$	$4V$	$2W$	$4W$	$4X$	$2X$	$4Y$	$4Z$	$4AA$	$4AB$	$4AC$	$3A$	$6A$	$6B$	$6C$	$3B$	$6D$	$6E$
χ_{92}	-2	-2	4	4	6	-4	-2	-2	6	4	4	-2	-4	0	0	0	0	0	0	0
χ_{93}	2	2	8	0	2	-4	-2	-2	-2	-8	0	2	4	0	0	0	0	0	0	0
χ_{94}	6	6	4	4	-2	-4	-2	6	-2	4	4	-2	-4	0	0	0	0	0	0	0
χ_{95}	2	2	0	-8	2	4	-2	-2	-2	0	8	2	-4	0	0	0	0	0	0	0
χ_{96}	6	6	-4	-4	-2	4	-2	6	-2	-4	-4	-2	4	0	0	0	0	0	0	0
χ_{97}	2	2	-8	0	2	4	-2	-2	-2	8	0	2	-4	0	0	0	0	0	0	0
χ_{98}	-2	-2	-4	-4	6	4	-2	-2	6	-4	-4	-2	4	0	0	0	0	0	0	0
χ_{99}	2	2	-8	0	2	4	-2	-2	-2	8	0	2	-4	0	0	0	0	0	0	0
χ_{100}	-2	-2	-4	-4	6	4	-2	-2	6	-4	-4	-2	4	0	0	0	0	0	0	0
χ_{101}	2	2	0	-8	2	4	-2	-2	-2	0	8	2	-4	0	0	0	0	0	0	0
χ_{102}	6	6	-4	-4	-2	4	-2	6	-2	-4	-4	-2	4	0	0	0	0	0	0	0
χ_{103}	-2	-2	4	-4	-2	0	2	2	2	-4	4	-2	0	0	0	0	0	0	0	0
χ_{104}	2	2	0	0	-6	0	2	2	-6	0	0	2	0	0	0	0	0	0	0	0
χ_{105}	-2	-2	-4	4	-2	0	2	2	2	4	-4	-2	0	0	0	0	0	0	0	0
χ_{106}	-6	-6	0	0	2	0	2	-6	2	0	0	2	0	0	0	0	0	0	0	0
χ_{107}	-2	-2	-4	4	-2	0	2	2	2	4	-4	-2	0	0	0	0	0	0	0	0
χ_{108}	-6	-6	0	0	2	0	2	-6	2	0	0	2	0	0	0	0	0	0	0	0
χ_{109}	-2	-2	4	-4	-2	0	2	2	2	-4	4	-2	0	0	0	0	0	0	0	0
χ_{110}	2	2	0	0	-6	0	2	2	-6	0	0	2	0	0	0	0	0	0	0	0
χ_{111}	8	8	0	0	-8	0	0	-8	8	0	0	0	0	6	6	-2	0	0	0	0
χ_{112}	-8	-8	0	0	8	0	0	8	-8	0	0	0	0	6	6	-2	0	0	0	0
χ_{113}	-8	-8	0	0	8	0	0	8	-8	0	0	0	0	6	6	-2	0	0	0	0
χ_{114}	8	8	0	0	-8	0	0	-8	8	0	0	0	0	6	6	-2	0	0	0	0
χ_{115}	0	0	8	-8	0	0	0	0	0	8	-8	0	0	6	6	-2	0	0	0	0
χ_{116}	0	0	-8	8	0	0	0	0	0	-8	8	0	0	6	6	-2	0	0	0	0
χ_{117}	0	0	-8	8	0	0	0	0	0	-8	8	0	0	6	6	-2	0	0	0	0
χ_{118}	0	0	8	-8	0	0	0	0	0	8	-8	0	0	6	6	-2	0	0	0	0
χ_{119}	-4	-4	0	0	-4	0	4	-4	-4	0	0	4	0	0	0	0	0	0	0	0
χ_{120}	-4	-4	0	0	-4	0	4	4	4	0	0	-4	0	0	0	0	0	0	0	0
χ_{121}	-4	-4	0	0	-4	0	4	-4	-4	0	0	4	0	0	0	0	0	0	0	0
χ_{122}	-4	-4	0	0	-4	0	4	4	4	0	0	-4	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0	0
χ_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0	0
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	-6	2	0	0	0	0
χ_{127}	8	-8	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	2	-2	0
χ_{128}	8	-8	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	2	-2	0
χ_{129}	8	-8	0	0	0	0	0	0	0	0	0	0	0	16	-16	0	0	-2	2	0
χ_{130}	8	-8	0	0	0	0	0	0	0	0	0	0	0	16	-16	0	0	-2	2	0
χ_{131}	8	-8	0	0	0	0	0	0	0	0	0	0	0	-8	8	0	0	4	-4	0
χ_{132}	8	-8	0	0	0	0	0	0	0	0	0	0	0	-8	8	0	0	4	-4	0
χ_{133}	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	8	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	-16	16	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	2	-2	0
χ_{136}	-16	16	0	0	0	0	0	0	0	0	0	0	0	8	-8	0	0	2	-2	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0	-16	16	0	0	-4	4	0

Table 9.7 (continue)

	4A			4B			4C			4D			4E			
	$4AD$	$4AE$	$8A$	$4AF$	$4AG$	$8B$	$4AH$	$4AI$	$4AJ$	$4AK$	$4AL$	$4AM$	$4AN$	$4AO$	$4AP$	$4AQ$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1
χ_3	3	3	3	3	3	3	1	1	1	1	1	1	-1	-1	-1	-1
χ_4	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1	1	1	1
χ_5	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	3	3	3	3
χ_6	1	1	1	1	1	1	1	1	1	1	1	1	-3	-3	-3	-3
χ_7	-2	-2	-2	2	2	2	-2	-2	-2	2	2	2	0	0	0	0
χ_8	2	2	2	-2	-2	-2	-2	-2	-2	2	2	2	0	0	0	0
χ_9	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	-3	-3	-3	-3
χ_{10}	3	3	3	3	3	3	1	1	1	1	1	1	3	3	3	3
χ_{11}	-2	-2	-2	2	2	2	2	2	2	-2	-2	-2	0	0	0	0
χ_{12}	-2	-2	-2	2	2	2	2	2	2	-2	-2	-2	0	0	0	0
χ_{13}	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	0	0	0	0
χ_{14}	2	2	2	-2	-2	-2	2	2	2	-2	-2	-2	0	0	0	0
χ_{15}	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	2	2	2	2
χ_{16}	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{17}	-1	-1	-1	-1	-1	-1	3	3	3	3	3	3	-1	-1	-1	-1
χ_{18}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
χ_{19}	1	1	1	1	1	1	3	3	3	3	3	3	1	1	1	1
χ_{20}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	-4	-4	-4	4	4	4	0	0	0	0
χ_{23}	4	4	4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0
χ_{24}	-4	-4	-4	4	4	4	0	0	0	0	0	0	0	0	0	0
χ_{25}	2	2	2	-2	-2	-2	-2	-2	-2	2	2	2	0	0	0	0
χ_{26}	-2	-2	-2	2	2	2	-2	-2	-2	2	2	2	0	0	0	0
χ_{27}	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_{28}	-2	-2	-2	-2	-2	-2	2	2	2	2	2	2	-2	-2	-2	-2
χ_{29}	0	0	0	0	0	0	4	4	4	-4	-4	-4	0	0	0	0
χ_{30}	4	4	4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0
χ_{31}	-4	-4	-4	4	4	4	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	1	1	1	1
χ_{35}	3	3	3	3	3	3	-3	-3	-3	-3	-3	-3	-1	-1	-1	-1
χ_{36}	-3	-3	-3	-3	-3	-3	1	1	1	1	1	1	1	1	1	1
χ_{37}	3	3	3	3	3	3	1	1	1	1	1	1	-1	-1	-1	-1
χ_{38}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	-1	-1	1
χ_{39}	-1	-1	1	-1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1
χ_{40}	3	3	-3	3	3	-3	1	1	-1	1	1	-1	-1	1	1	-1
χ_{41}	-3	-3	3	-3	-3	3	1	1	-1	1	1	-1	1	-1	-1	1
χ_{42}	-1	-1	1	-1	-1	1	1	1	-1	1	1	-1	3	-3	-3	3
χ_{43}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-3	3	3	-3
χ_{44}	-2	-2	2	2	2	-2	-2	-2	2	2	2	-2	0	0	0	0
χ_{45}	2	2	-2	-2	-2	2	-2	-2	2	2	2	-2	0	0	0	0

Table 9.7 (continue)

	4A			4B			4C			4D			4E			
	$4AD$	$4AE$	$8A$	$4AF$	$4AG$	$8B$	$4AH$	$4AI$	$4AJ$	$4AK$	$4AL$	$4AM$	$4AN$	$4AO$	$4AP$	$4AQ$
χ_{46}	-3	-3	3	-3	-3	3	1	1	-1	1	1	-1	-3	3	3	-3
χ_{47}	3	3	-3	3	3	-3	1	1	-1	1	1	-1	3	-3	-3	3
χ_{48}	-2	-2	2	2	2	-2	2	2	-2	-2	-2	2	0	0	0	0
χ_{49}	-2	-2	2	2	2	-2	2	2	-2	-2	-2	2	0	0	0	0
χ_{50}	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	0	0	0	0
χ_{51}	2	2	-2	-2	-2	2	2	2	-2	-2	-2	2	0	0	0	0
χ_{52}	-2	-2	2	-2	-2	2	-2	-2	2	-2	-2	2	2	-2	-2	2
χ_{53}	2	2	-2	2	2	-2	-2	-2	2	-2	-2	2	-2	2	2	-2
χ_{54}	-1	-1	1	-1	-1	1	3	3	-3	3	3	-3	-1	1	1	-1
χ_{55}	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	1	-1	-1	1
χ_{56}	1	1	-1	1	1	-1	3	3	-3	3	3	-3	1	-1	-1	1
χ_{57}	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1	-1
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	-4	-4	4	4	4	-4	0	0	0	0
χ_{60}	4	4	-4	-4	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{61}	-4	-4	4	4	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{62}	2	2	-2	-2	-2	2	-2	-2	2	2	2	-2	0	0	0	0
χ_{63}	-2	-2	2	2	2	-2	-2	-2	2	2	2	-2	0	0	0	0
χ_{64}	2	2	-2	2	2	-2	2	2	-2	2	2	-2	2	-2	-2	2
χ_{65}	-2	-2	2	-2	-2	2	2	2	-2	2	2	-2	-2	2	2	-2
χ_{66}	0	0	0	0	0	0	4	4	-4	-4	-4	4	0	0	0	0
χ_{67}	4	4	-4	-4	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{68}	-4	-4	4	4	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	-3	-3	3	-3	-3	3	-3	-3	3	-3	-3	3	1	-1	-1	1
χ_{72}	3	3	-3	3	3	-3	-3	-3	3	-3	-3	3	-1	1	1	-1
χ_{73}	-3	-3	3	-3	-3	3	1	1	-1	1	1	-1	1	-1	-1	1
χ_{74}	3	3	-3	3	3	-3	1	1	-1	1	1	-1	-1	1	1	-1
χ_{75}	6	-2	0	6	-2	0	2	-2	0	2	-2	0	2	0	0	-2
χ_{76}	6	-2	0	-6	2	0	-2	2	0	2	-2	0	0	2	-2	0
χ_{77}	-6	2	0	-6	2	0	2	-2	0	2	-2	0	-2	0	0	2
χ_{78}	6	-2	0	-6	2	0	-2	2	0	2	-2	0	0	-2	2	0
χ_{79}	-6	2	0	-6	2	0	2	-2	0	2	-2	0	2	0	0	-2
χ_{80}	-6	2	0	6	-2	0	-2	2	0	2	-2	0	0	2	-2	0
χ_{81}	6	-2	0	6	-2	0	2	-2	0	2	-2	0	-2	0	0	2
χ_{82}	-6	2	0	6	-2	0	-2	2	0	2	-2	0	0	-2	2	0
χ_{83}	0	0	0	0	0	0	4	-4	0	4	-4	0	4	0	0	-4
χ_{84}	0	0	0	0	0	0	-4	4	0	4	-4	0	0	-4	4	0
χ_{85}	0	0	0	0	0	0	4	-4	0	4	-4	0	-4	0	0	4
χ_{86}	0	0	0	0	0	0	-4	4	0	4	-4	0	0	4	-4	0
χ_{87}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	-2	2	0
χ_{88}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{89}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{90}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{91}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	-2	2	0

Table 9.7 (continue)

	4A			4B			4C			4D			4E			
	$4AD$	$4AE$	$8A$	$4AF$	$4AG$	$8B$	$4AH$	$4AI$	$4AJ$	$4AK$	$4AL$	$4AM$	$4AN$	$4AO$	$4AP$	$4AQ$
χ_{92}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{93}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{94}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{95}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	-2	2	0
χ_{96}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{97}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{98}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{99}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	-2	2	0
χ_{100}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{101}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{102}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{103}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	-2	2	0
χ_{104}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{105}	6	-2	0	-6	2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{106}	-6	2	0	-6	2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{107}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	-2	2	0
χ_{108}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	2	0	0	-2
χ_{109}	-6	2	0	6	-2	0	2	-2	0	-2	2	0	0	2	-2	0
χ_{110}	6	-2	0	6	-2	0	-2	2	0	-2	2	0	-2	0	0	2
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	4	-4	0	4	-4	0	4	0	0	-4
χ_{120}	0	0	0	0	0	0	-4	4	0	4	-4	0	0	-4	4	0
χ_{121}	0	0	0	0	0	0	4	-4	0	4	-4	0	-4	0	0	4
χ_{122}	0	0	0	0	0	0	-4	4	0	4	-4	0	0	4	-4	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	4F				4G				4H				4I				
	$4AR$	$4AS$	$8C$	$8D$	$4AJ$	$8E$	$4AU$	$8F$	$4AV$	$4AW$	$8G$	$8H$	$4AX$	$4AY$	$4AZ$	$8I$	$8J$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_3	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1
χ_4	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_5	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1
χ_6	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_7	0	0	0	0	0	0	0	0	-2	-2	-2	-2	0	0	0	0	0
χ_8	0	0	0	0	0	0	0	0	2	2	2	2	0	0	0	0	0
χ_9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{10}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_{11}	0	0	0	0	-2	-2	-2	-2	0	0	0	0	2	2	2	2	2
χ_{12}	0	0	0	0	2	2	2	2	0	0	0	0	-2	-2	-2	-2	-2
χ_{13}	0	0	0	0	2	2	2	2	0	0	0	0	-2	-2	-2	-2	-2
χ_{14}	0	0	0	0	-2	-2	-2	-2	0	0	0	0	2	2	2	2	2
χ_{15}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{18}	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_{19}	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1
χ_{20}	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{25}	0	0	0	0	0	0	0	0	-2	-2	-2	-2	0	0	0	0	0
χ_{26}	0	0	0	0	0	0	0	0	2	2	2	2	0	0	0	0	0
χ_{27}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_{35}	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1
χ_{36}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_{37}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1
χ_{38}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	1
χ_{39}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1
χ_{40}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1
χ_{41}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
χ_{42}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1
χ_{43}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
χ_{44}	0	0	0	0	0	0	0	0	-2	2	2	-2	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	2	-2	-2	2	0	0	0	0	0

Table 9.7 (continue)

	4F				4G				4H				4I				
	$4AR$	$4AS$	$8C$	$8D$	$4AJ$	$8E$	$4AU$	$8F$	$4AV$	$4AW$	$8G$	$8H$	$4AX$	$4AY$	$4AZ$	$8I$	$8J$
χ_{46}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	1
χ_{47}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1
χ_{48}	0	0	0	0	-2	2	2	-2	0	0	0	0	2	2	-2	-2	2
χ_{49}	0	0	0	0	2	-2	-2	2	0	0	0	0	-2	-2	2	2	-2
χ_{50}	0	0	0	0	2	-2	-2	2	0	0	0	0	-2	-2	2	2	-2
χ_{51}	0	0	0	0	-2	2	2	-2	0	0	0	0	2	2	-2	-2	2
χ_{52}	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{53}	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{54}	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
χ_{55}	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1
χ_{56}	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1
χ_{57}	-1	1	1	-1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	1
χ_{58}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	0	0	0	0	0	0	-2	2	2	-2	0	0	0	0	0
χ_{63}	0	0	0	0	0	0	0	0	2	-2	-2	2	0	0	0	0	0
χ_{64}	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	-2	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	1	-1
χ_{72}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	1	1	-1
χ_{73}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	-1	1
χ_{74}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1
χ_{75}	2	0	0	-2	2	0	0	-2	2	0	0	-2	2	2	0	0	-2
χ_{76}	0	-2	2	0	0	-2	2	0	2	0	0	-2	0	0	-2	2	0
χ_{77}	2	0	0	-2	2	0	0	-2	-2	0	0	2	2	2	0	0	-2
χ_{78}	0	2	-2	0	0	2	-2	0	2	0	0	-2	0	0	2	-2	0
χ_{79}	-2	0	0	2	-2	0	0	2	-2	0	0	2	-2	-2	0	0	2
χ_{80}	0	2	-2	0	0	2	-2	0	-2	0	0	2	0	0	2	-2	0
χ_{81}	-2	0	0	2	-2	0	0	2	2	0	0	-2	-2	-2	0	0	2
χ_{82}	0	-2	2	0	0	-2	2	0	-2	0	0	2	0	0	-2	2	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	0	2	-2	0	2	0	0	-2	0	2	-2	0	-2	-2	0	0	2
χ_{88}	-2	0	0	2	0	-2	2	0	0	-2	2	0	0	0	2	-2	0
χ_{89}	0	-2	2	0	-2	0	0	2	0	2	-2	0	2	2	0	0	-2
χ_{90}	2	0	0	-2	0	2	-2	0	0	-2	2	0	0	0	-2	2	0
χ_{91}	0	-2	2	0	-2	0	0	2	0	-2	2	0	2	2	0	0	-2

Table 9.7 (continue)

	4F				4G				4H				4I				
	$4AR$	$4AS$	$8C$	$8D$	$4AJ$	$8E$	$4AU$	$8F$	$4AV$	$4AW$	$8G$	$8H$	$4AX$	$4AY$	$4AZ$	$8I$	$8J$
χ_{92}	2	0	0	-2	0	2	-2	0	0	2	-2	0	0	0	-2	2	0
χ_{93}	0	2	-2	0	2	0	0	-2	0	-2	2	0	-2	-2	0	0	2
χ_{94}	-2	0	0	2	0	-2	2	0	0	2	-2	0	0	0	2	-2	0
χ_{95}	0	2	-2	0	-2	0	0	2	0	-2	2	0	2	2	0	0	-2
χ_{96}	-2	0	0	2	0	2	-2	0	0	2	-2	0	0	0	-2	2	0
χ_{97}	0	-2	2	0	2	0	0	-2	0	-2	2	0	-2	-2	0	0	2
χ_{98}	2	0	0	-2	0	-2	2	0	0	2	-2	0	0	0	2	-2	0
χ_{99}	0	-2	2	0	2	0	0	-2	0	2	-2	0	-2	-2	0	0	2
χ_{100}	2	0	0	-2	0	-2	2	0	0	-2	2	0	0	0	2	-2	0
χ_{101}	0	2	-2	0	-2	0	0	2	0	2	-2	0	2	2	0	0	-2
χ_{102}	-2	0	0	2	0	2	-2	0	0	-2	2	0	0	0	-2	2	0
χ_{103}	0	-2	2	0	0	2	-2	0	-2	0	0	2	0	0	2	-2	0
χ_{104}	2	0	0	-2	-2	0	0	2	2	0	0	-2	-2	-2	0	0	2
χ_{105}	0	2	-2	0	0	-2	2	0	-2	0	0	2	0	0	-2	2	0
χ_{106}	-2	0	0	2	2	0	0	-2	2	0	0	-2	2	2	0	0	-2
χ_{107}	0	2	-2	0	0	-2	2	0	2	0	0	-2	0	0	-2	2	0
χ_{108}	-2	0	0	2	2	0	0	-2	-2	0	0	2	2	2	0	0	-2
χ_{109}	0	-2	2	0	0	2	-2	0	2	0	0	-2	0	0	2	-2	0
χ_{110}	2	0	0	-2	-2	0	0	2	-2	0	0	2	-2	-2	0	0	2
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	0	0	0
χ_{128}	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	0	0	0
χ_{129}	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	0	0	0
χ_{130}	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	0	0	0
χ_{131}	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	0	0	0
χ_{132}	0	0	0	0	0	0	0	0	0	0	0	0	-4	4	0	0	0
χ_{133}	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	0	0	0
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	4	-4	0	0	0
χ_{135}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	4J					5A			6A			6B			6C		
	$4BA$	$4BB$	$4BC$	$8K$	$8L$	$5A$	$10A$	$10B$	$6F$	$6G$	$12A$	$6H$	$12B$	$12C$	$6I$	$12D$	$12E$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	1	1	1	0	0	0	2	2	2	2	2	2	2	2	2
χ_4	-1	-1	-1	-1	-1	0	0	0	2	2	2	2	2	2	2	2	2
χ_5	1	1	1	1	1	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_6	-1	-1	-1	-1	-1	0	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_7	2	2	2	2	2	1	1	1	-3	-3	-3	1	1	1	-1	-1	-1
χ_8	-2	-2	-2	-2	-2	1	1	1	-3	-3	-3	1	1	1	-1	-1	-1
χ_9	1	1	1	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0
χ_{10}	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0
χ_{11}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{12}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{13}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{14}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{15}	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
χ_{16}	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
χ_{17}	-1	-1	-1	-1	-1	0	0	0	3	3	3	-1	-1	-1	-1	-1	-1
χ_{18}	-1	-1	-1	-1	-1	0	0	0	3	3	3	-1	-1	-1	-1	-1	-1
χ_{19}	1	1	1	1	1	0	0	0	3	3	3	-1	-1	-1	-1	-1	-1
χ_{20}	1	1	1	1	1	0	0	0	3	3	3	-1	-1	-1	-1	-1	-1
χ_{21}	0	0	0	0	0	1	1	1	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{22}	0	0	0	0	0	0	0	0	-2	-2	-2	-2	-2	-2	2	2	2
χ_{23}	0	0	0	0	0	-1	-1	-1	-3	-3	-3	1	1	1	-1	-1	-1
χ_{24}	0	0	0	0	0	-1	-1	-1	-3	-3	-3	1	1	1	-1	-1	-1
χ_{25}	2	2	2	2	2	0	0	0	3	3	3	-1	-1	-1	1	1	1
χ_{26}	-2	-2	-2	-2	-2	0	0	0	3	3	3	-1	-1	-1	1	1	1
χ_{27}	0	0	0	0	0	0	0	0	-3	-3	-3	1	1	1	1	1	1
χ_{28}	0	0	0	0	0	0	0	0	-3	-3	-3	1	1	1	1	1	1
χ_{29}	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{31}	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	1	1	1
χ_{32}	0	0	0	0	0	0	0	0	2	2	2	2	2	2	-2	-2	-2
χ_{33}	0	0	0	0	0	0	0	0	2	2	2	2	2	2	-2	-2	-2
χ_{34}	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	1	1	-1	-1	1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
χ_{39}	-1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
χ_{40}	1	1	-1	-1	1	0	0	0	2	2	-2	2	2	-2	2	2	-2
χ_{41}	-1	-1	1	1	-1	0	0	0	2	2	-2	2	2	-2	2	2	-2
χ_{42}	1	1	-1	-1	1	0	0	0	-1	-1	1	-1	-1	1	-1	-1	1
χ_{43}	-1	-1	1	1	-1	0	0	0	-1	-1	1	-1	-1	1	-1	-1	1
χ_{44}	2	2	-2	-2	2	1	1	-1	-3	-3	3	1	1	-1	-1	-1	1
χ_{45}	-2	-2	2	2	-2	1	1	-1	-3	-3	3	1	1	-1	-1	-1	1

Table 9.7 (continue)

	4J					5A			6A			6B			6C		
	$4BA$	$4BB$	$4BC$	$8K$	$8L$	$5A$	$10A$	$10B$	$6F$	$6G$	$12A$	$6H$	$12B$	$12C$	$6I$	$12D$	$12E$
χ_{46}	1	1	-1	-1	1	-1	-1	1	0	0	0	0	0	0	0	0	0
χ_{47}	-1	-1	1	1	-1	-1	-1	1	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{49}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{50}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{51}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{52}	0	0	0	0	0	0	0	0	1	1	-1	1	1	-1	1	1	-1
χ_{53}	0	0	0	0	0	0	0	0	1	1	-1	1	1	-1	1	1	-1
χ_{54}	-1	-1	1	1	-1	0	0	0	3	3	-3	-1	-1	1	-1	-1	1
χ_{55}	-1	-1	1	1	-1	0	0	0	3	3	-3	-1	-1	1	-1	-1	1
χ_{56}	1	1	-1	-1	1	0	0	0	3	3	-3	-1	-1	1	-1	-1	1
χ_{57}	1	1	-1	-1	1	0	0	0	3	3	-3	-1	-1	1	-1	-1	1
χ_{58}	0	0	0	0	0	1	1	-1	-2	-2	2	-2	-2	2	-2	-2	2
χ_{59}	0	0	0	0	0	0	0	0	-2	-2	2	-2	-2	2	2	2	-2
χ_{60}	0	0	0	0	0	-1	-1	1	-3	-3	3	1	1	-1	-1	-1	1
χ_{61}	0	0	0	0	0	-1	-1	1	-3	-3	3	1	1	-1	-1	-1	1
χ_{62}	2	2	-2	-2	2	0	0	0	3	3	-3	-1	-1	1	1	1	-1
χ_{63}	-2	-2	2	2	-2	0	0	0	3	3	-3	-1	-1	1	1	1	-1
χ_{64}	0	0	0	0	0	0	0	0	-3	-3	3	1	1	-1	1	1	-1
χ_{65}	0	0	0	0	0	0	0	0	-3	-3	3	1	1	-1	1	1	-1
χ_{66}	0	0	0	0	0	1	1	-1	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{68}	0	0	0	0	0	0	0	0	-1	-1	1	-1	-1	1	1	1	-1
χ_{69}	0	0	0	0	0	0	0	0	2	2	-2	2	2	-2	-2	-2	2
χ_{70}	0	0	0	0	0	0	0	0	2	2	-2	2	2	-2	-2	-2	2
χ_{71}	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	1	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	1	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	2	2	0	0	-2	0	0	0	6	-2	0	2	-2	0	2	-2	0
χ_{76}	-2	-2	0	0	2	0	0	0	-6	2	0	2	-2	0	-2	2	0
χ_{77}	-2	-2	0	0	2	0	0	0	6	-2	0	2	-2	0	2	-2	0
χ_{78}	-2	-2	0	0	2	0	0	0	-6	2	0	2	-2	0	-2	2	0
χ_{79}	-2	-2	0	0	2	0	0	0	6	-2	0	2	-2	0	2	-2	0
χ_{80}	2	2	0	0	-2	0	0	0	-6	2	0	2	-2	0	-2	2	0
χ_{81}	2	2	0	0	-2	0	0	0	6	-2	0	2	-2	0	2	-2	0
χ_{82}	2	2	0	0	-2	0	0	0	-6	2	0	2	-2	0	-2	2	0
χ_{83}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	-2	2	0
χ_{84}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	2	-2	0
χ_{85}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	-2	2	0
χ_{86}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	2	-2	0
χ_{87}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{89}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	4J					5A			6A			6B			6C		
	$4BA$	$4BB$	$4BC$	$8K$	$8L$	$5A$	$10A$	$10B$	$6F$	$6G$	$12A$	$6H$	$12B$	$12C$	$6I$	$12D$	$12E$
χ_{92}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{93}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	0	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	2	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{104}	2	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{105}	2	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	2	2	0	0	-2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	-2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{108}	-2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	-2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	-2	-2	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	2	-2	0
χ_{112}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	2	-2	0
χ_{113}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	2	-2	0
χ_{114}	0	0	0	0	0	0	0	0	-6	2	0	-2	2	0	2	-2	0
χ_{115}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	-2	2	0
χ_{116}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	-2	2	0
χ_{117}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	-2	2	0
χ_{118}	0	0	0	0	0	0	0	0	6	-2	0	-2	2	0	-2	2	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	6	-2	0	2	-2	0	-2	2	0
χ_{124}	0	0	0	0	0	0	0	0	6	-2	0	2	-2	0	-2	2	0
χ_{125}	0	0	0	0	0	0	0	0	-6	2	0	2	-2	0	2	-2	0
χ_{126}	0	0	0	0	0	0	0	0	-6	2	0	2	-2	0	2	-2	0
χ_{127}	4	-4	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{128}	-4	4	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0
χ_{129}	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	-4	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	4	-4	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{134}	-4	4	0	0	0	-2	2	0	0	0	0	0	0	0	0	0	0
χ_{135}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	2	-2	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	6D		6E					6F				6G	
	$6J$	$12F$	$6K$	$6L$	$6M$	$12G$	$12H$	$6N$	$12I$	$12J$	$12K$	$6O$	$12L$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
χ_3	-1	-1	0	0	0	0	0	0	0	0	0	-1	-1
χ_4	-1	-1	0	0	0	0	0	0	0	0	0	1	1
χ_5	2	2	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
χ_6	2	2	1	1	1	1	1	1	1	1	1	0	0
χ_7	0	0	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_8	0	0	-1	-1	-1	-1	-1	1	1	1	1	0	0
χ_9	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1
χ_{12}	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1
χ_{13}	-1	-1	1	1	1	1	1	-1	-1	-1	-1	1	1
χ_{14}	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_{15}	1	1	1	1	1	1	1	1	1	1	1	-1	-1
χ_{16}	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1
χ_{17}	0	0	1	1	1	1	1	1	1	1	1	0	0
χ_{18}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
χ_{19}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
χ_{20}	0	0	1	1	1	1	1	1	1	1	1	0	0
χ_{21}	-2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	-2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_{24}	0	0	-1	-1	-1	-1	-1	1	1	1	1	0	0
χ_{25}	0	0	-1	-1	-1	-1	-1	1	1	1	1	0	0
χ_{26}	0	0	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_{27}	0	0	1	1	1	1	1	1	1	1	1	0	0
χ_{28}	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0
χ_{29}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	2	2	-1	-1	-1	-1	-1	1	1	1	1	0	0
χ_{31}	2	2	1	1	1	1	1	-1	-1	-1	-1	0	0
χ_{32}	-1	-1	0	0	0	0	0	0	0	0	0	-1	-1
χ_{33}	-1	-1	0	0	0	0	0	0	0	0	0	1	1
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1
χ_{39}	1	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
χ_{40}	-1	1	0	0	0	0	0	0	0	0	0	-1	1
χ_{41}	-1	1	0	0	0	0	0	0	0	0	0	1	-1
χ_{42}	2	-2	-1	-1	1	1	-1	-1	1	1	-1	0	0
χ_{43}	2	-2	1	1	-1	-1	1	1	-1	-1	1	0	0
χ_{44}	0	0	1	1	-1	-1	1	-1	1	1	-1	0	0
χ_{45}	0	0	-1	-1	1	1	-1	1	-1	-1	1	0	0

Table 9.7 (continue)

	6D		6E					6F				6G	
	$6J$	$12F$	$6K$	$6L$	$6M$	$12G$	$12H$	$6N$	$12I$	$12J$	$12K$	$6O$	$12L$
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	-1	1	-1	-1	1	1	-1	1	-1	-1	1	1	-1
χ_{49}	-1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	1
χ_{50}	-1	1	1	1	-1	-1	1	-1	1	1	-1	1	-1
χ_{51}	-1	1	1	1	-1	-1	1	-1	1	1	-1	-1	1
χ_{52}	1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1
χ_{53}	1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	-1
χ_{54}	0	0	1	1	-1	-1	1	1	-1	-1	1	0	0
χ_{55}	0	0	-1	-1	1	1	-1	-1	1	1	-1	0	0
χ_{56}	0	0	-1	-1	1	1	-1	-1	1	1	-1	0	0
χ_{57}	0	0	1	1	-1	-1	1	1	-1	-1	1	0	0
χ_{58}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	1	1	-1	-1	1	-1	1	1	-1	0	0
χ_{61}	0	0	-1	-1	1	1	-1	1	-1	-1	1	0	0
χ_{62}	0	0	-1	-1	1	1	-1	1	-1	-1	1	0	0
χ_{63}	0	0	1	1	-1	-1	1	-1	1	1	-1	0	0
χ_{64}	0	0	1	1	-1	-1	1	1	-1	-1	1	0	0
χ_{65}	0	0	-1	-1	1	1	-1	-1	1	1	-1	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	2	-2	-1	-1	1	1	-1	1	-1	-1	1	0	0
χ_{68}	2	-2	1	1	-1	-1	1	-1	1	1	-1	0	0
χ_{69}	-1	1	0	0	0	0	0	0	0	0	0	-1	1
χ_{70}	-1	1	0	0	0	0	0	0	0	0	0	1	-1
χ_{71}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	2	2	0	0	-2	2	0	0	-2	0	0
χ_{76}	0	0	0	0	2	-2	0	0	2	-2	0	0	0
χ_{77}	0	0	-2	-2	0	0	2	-2	0	0	2	0	0
χ_{78}	0	0	0	0	-2	2	0	0	-2	2	0	0	0
χ_{79}	0	0	2	2	0	0	-2	2	0	0	-2	0	0
χ_{80}	0	0	0	0	2	-2	0	0	2	-2	0	0	0
χ_{81}	0	0	-2	-2	0	0	2	-2	0	0	2	0	0
χ_{82}	0	0	0	0	-2	2	0	0	-2	2	0	0	0
χ_{83}	0	0	-2	-2	0	0	2	-2	0	0	2	0	0
χ_{84}	0	0	0	0	2	-2	0	0	2	-2	0	0	0
χ_{85}	0	0	2	2	0	0	-2	2	0	0	-2	0	0
χ_{86}	0	0	0	0	-2	2	0	0	-2	2	0	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	6D		6E					6F				6G	
	$6J$	$12F$	$6K$	$6L$	$6M$	$12G$	$12H$	$6N$	$12I$	$12J$	$12K$	$6O$	$12L$
χ_{92}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	2	2	0	0	-2	-2	0	0	2	0	0
χ_{112}	0	0	2	2	0	0	-2	-2	0	0	2	0	0
χ_{113}	0	0	-2	-2	0	0	2	2	0	0	-2	0	0
χ_{114}	0	0	-2	-2	0	0	2	2	0	0	-2	0	0
χ_{115}	0	0	0	0	-2	2	0	0	2	-2	0	0	0
χ_{116}	0	0	0	0	-2	2	0	0	2	-2	0	0	0
χ_{117}	0	0	0	0	2	-2	0	0	-2	2	0	0	0
χ_{118}	0	0	0	0	2	-2	0	0	-2	2	0	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	-2	-2	0	0	2	2	0	0	-2	0	0
χ_{124}	0	0	2	2	0	0	-2	-2	0	0	2	0	0
χ_{125}	0	0	0	0	2	-2	0	0	-2	2	0	0	0
χ_{126}	0	0	0	0	-2	2	0	0	2	-2	0	0	0
χ_{127}	0	0	4	-4	0	0	0	0	0	0	0	0	0
χ_{128}	0	0	-4	4	0	0	0	0	0	0	0	0	0
χ_{129}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	0	0	-4	4	0	0	0	0	0	0	0	0	0
χ_{132}	0	0	4	-4	0	0	0	0	0	0	0	0	0
χ_{133}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	0	0	4	-4	0	0	0	0	0	0	0	0	0
χ_{136}	0	0	-4	4	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	6H			8A		8B		10A		12A		12B	
	$6P$	$6Q$	$6R$	$8M$	$8N$	$8O$	$8P$	$10C$	$20A$	$12M$	$24A$	$12N$	$24B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	-1
χ_3	-1	-1	-1	-1	-1	1	1	0	0	0	0	0	0
χ_4	1	1	1	-1	-1	-1	-1	0	0	0	0	0	0
χ_5	0	0	0	-1	-1	1	1	0	0	-1	-1	-1	-1
χ_6	0	0	0	-1	-1	-1	-1	0	0	1	1	1	1
χ_7	0	0	0	0	0	0	0	-1	-1	1	1	-1	-1
χ_8	0	0	0	0	0	0	0	-1	-1	-1	-1	1	1
χ_9	0	0	0	1	1	1	1	-1	-1	0	0	0	0
χ_{10}	0	0	0	1	1	-1	-1	-1	-1	0	0	0	0
χ_{11}	-1	-1	-1	0	0	0	0	0	0	1	1	-1	-1
χ_{12}	1	1	1	0	0	0	0	0	0	1	1	-1	-1
χ_{13}	-1	-1	-1	0	0	0	0	0	0	-1	-1	1	1
χ_{14}	1	1	1	0	0	0	0	0	0	-1	-1	1	1
χ_{15}	-1	-1	-1	0	0	0	0	0	0	1	1	1	1
χ_{16}	1	1	1	0	0	0	0	0	0	-1	-1	-1	-1
χ_{17}	0	0	0	1	1	1	1	0	0	-1	-1	-1	-1
χ_{18}	0	0	0	-1	-1	1	1	0	0	1	1	1	1
χ_{19}	0	0	0	1	1	-1	-1	0	0	1	1	1	1
χ_{20}	0	0	0	-1	-1	-1	-1	0	0	-1	-1	-1	-1
χ_{21}	0	0	0	0	0	0	0	1	1	0	0	0	0
χ_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	0	0	0	0	0	0	0	1	1	1	1	-1	-1
χ_{24}	0	0	0	0	0	0	0	1	1	-1	-1	1	1
χ_{25}	0	0	0	0	0	0	0	0	0	-1	-1	1	1
χ_{26}	0	0	0	0	0	0	0	0	0	1	1	-1	-1
χ_{27}	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1
χ_{28}	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_{29}	0	0	0	0	0	0	0	-1	-1	0	0	0	0
χ_{30}	0	0	0	0	0	0	0	0	0	1	1	-1	-1
χ_{31}	0	0	0	0	0	0	0	0	0	-1	-1	1	1
χ_{32}	1	1	1	0	0	0	0	0	0	0	0	0	0
χ_{33}	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	1	1	1	1	0	0	0	0	0	0
χ_{35}	0	0	0	1	1	-1	-1	0	0	0	0	0	0
χ_{36}	0	0	0	-1	-1	-1	-1	0	0	0	0	0	0
χ_{37}	0	0	0	-1	-1	1	1	0	0	0	0	0	0
χ_{38}	1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
χ_{39}	-1	-1	1	1	-1	-1	1	1	-1	-1	1	-1	1
χ_{40}	-1	-1	1	-1	1	1	-1	0	0	0	0	0	0
χ_{41}	1	1	-1	-1	1	-1	1	0	0	0	0	0	0
χ_{42}	0	0	0	-1	1	1	-1	0	0	-1	1	-1	1
χ_{43}	0	0	0	-1	1	-1	1	0	0	1	-1	1	-1
χ_{44}	0	0	0	0	0	0	0	-1	1	1	-1	-1	1
χ_{45}	0	0	0	0	0	0	0	-1	1	-1	1	1	-1

Table 9.7 (continue)

	6H			8A		8B		10A		12A		12B	
	$6P$	$6Q$	$6R$	$8M$	$8N$	$8O$	$8P$	$10C$	$20A$	$12M$	$24A$	$12N$	$24B$
χ_{46}	0	0	0	1	-1	1	-1	-1	1	0	0	0	0
χ_{47}	0	0	0	1	-1	-1	1	-1	1	0	0	0	0
χ_{48}	-1	-1	1	0	0	0	0	0	0	1	-1	-1	1
χ_{49}	1	1	-1	0	0	0	0	0	0	1	-1	-1	1
χ_{50}	-1	-1	1	0	0	0	0	0	0	-1	1	1	-1
χ_{51}	1	1	-1	0	0	0	0	0	0	-1	1	1	-1
χ_{52}	-1	-1	1	0	0	0	0	0	0	1	-1	1	-1
χ_{53}	1	1	-1	0	0	0	0	0	0	-1	1	-1	1
χ_{54}	0	0	0	1	-1	1	-1	0	0	-1	1	-1	1
χ_{55}	0	0	0	-1	1	1	-1	0	0	1	-1	1	-1
χ_{56}	0	0	0	1	-1	-1	1	0	0	1	-1	1	-1
χ_{57}	0	0	0	-1	1	-1	1	0	0	-1	1	-1	1
χ_{58}	0	0	0	0	0	0	0	1	-1	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	0	0	0	0	0	0	0	1	-1	1	-1	-1	1
χ_{61}	0	0	0	0	0	0	0	1	-1	-1	1	1	-1
χ_{62}	0	0	0	0	0	0	0	0	0	-1	1	1	-1
χ_{63}	0	0	0	0	0	0	0	0	0	1	-1	-1	1
χ_{64}	0	0	0	0	0	0	0	0	0	-1	1	-1	1
χ_{65}	0	0	0	0	0	0	0	0	0	1	-1	1	-1
χ_{66}	0	0	0	0	0	0	0	-1	1	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	1	-1	-1	1
χ_{68}	0	0	0	0	0	0	0	0	0	-1	1	1	-1
χ_{69}	1	1	-1	0	0	0	0	0	0	0	0	0	0
χ_{70}	-1	-1	1	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	0	1	-1	1	-1	0	0	0	0	0	0
χ_{72}	0	0	0	1	-1	-1	1	0	0	0	0	0	0
χ_{73}	0	0	0	-1	1	-1	1	0	0	0	0	0	0
χ_{74}	0	0	0	-1	1	1	-1	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{76}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{77}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{78}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{79}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{80}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{81}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{82}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{83}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{84}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{85}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{86}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{87}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{88}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{89}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{90}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{91}	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 9.7 (continue)

	6H			8A		8B		10A		12A		12B	
	$6P$	$6Q$	$6R$	$8M$	$8N$	$8O$	$8P$	$10C$	$20A$	$12M$	$24A$	$12N$	$24B$
χ_{92}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{93}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{94}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{95}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{96}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{98}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{99}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{100}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{101}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{102}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{103}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{104}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{105}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{106}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{107}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{108}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{109}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{110}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{111}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{112}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{113}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{114}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{115}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{116}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{117}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{118}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{119}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{120}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{121}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{122}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{123}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{124}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{125}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{126}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{127}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{128}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{129}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{130}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{131}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{132}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{133}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{134}	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{135}	-2	2	0	0	0	0	0	0	0	0	0	0	0
χ_{136}	2	-2	0	0	0	0	0	0	0	0	0	0	0
χ_{137}	0	0	0	0	0	0	0	0	0	0	0	0	0

We restrict characters of $Irr(2^6{:}SP_6(2))$ to $\overline{G}$ and also compute the structure constants (using GAP) for the set $Irr(\overline{G})$ such that the consistency checks which were implemented by Programme C are satisfied. The information about the conjugacy classes found in Table 9.3 can be used to compute the power maps for the elements of $\overline{G}$ and the Programme C is used to confirm that our Table 9.7 produces the unique p-powers listed in Table 9.8.

Table 9.8: The power maps of the elements of $2^6{:}(2^5{:}S_6)$

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5
1A	1A				2A	2D	1A		
	2A	1A				2E	1A		
	2B	1A				4A	2A		
	2C	1A							
2B	2F	1A			2C	2H	1A		
	2G	1A				2I	1A		
	4B	2A				4D	2A		
	4C	2B				4E	2B		
2D	2J	1A			2E	2L	1A		
	2K	1A				2M	1A		
	4F	2A				2N	1A		
	4G	2B				2O	1A		
	4H	2B				4I	2B		
						4J	2B		
2F	2P	1A			2G	2S	1A		
	2Q	1A				4M	2A		
	2R	1A				4N	2B		
	4K	2B				4O	2B		
	4L	2B							
2H	2T	1A			2I	2U	1A		
	4P	2A				2V	1A		
	4Q	2B				4U	2B		
	4R	2B				2W	1A		
	4S	2B				4V	2B		
	4T	2B				4W	2B		
						4X	2B		
2J	2X	1A			3A	3A		1A	
	4Y	2B				6A	3A	2A	
	4Z	2B				6B	3A	2B	
	4AA	2A				6C	3A	2C	
	4AB	2B							
	4AC	2B							
3B	3B		1A		4A	4AD	2H		
	6D	3B	2A			4AE	2H		
	6E	3B	2C			8A	4D		
4B	4AF	2H			4C	4AH	2F		
	4AG	2H				4AI	2G		
	8B	4D				4AJ	2G		
4D	4AK	2F			4E	4AN	2F		
	4AL	2G				4AO	2G		
	4AM	2G				4AP	2G		
						4AQ	2G		

Table 9.8 (continue)

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	$[g]_G$	$[x]_{\overline{G}}$	2	3	5
$4F$	$4AR$	$2H$			$4G$	$4AT$	$2U$		
	$8C$	$4D$				$8E$	$4V$		
	$8D$	$4D$				$4AU$	$2V$		
	$4AS$	$2I$				$8F$	$4V$		
$4H$	$4AV$	$2U$			$4I$	$4AX$	$2U$		
	$4AW$	$2V$				$4AY$	$2U$		
	$8G$	$4V$				$4AZ$	$2U$		
	$8H$	$4V$				$8I$	$4V$		
						$8J$	$4V$		
$4J$	$4BA$	$2U$			$5A$	$5A$			$1A$
	$4BB$	$2U$				$10A$	$5A$		$2B$
	$4BC$	$2U$				$10B$	$5A$		$2C$
	$8K$	$4V$							
	$8L$	$4V$							
$6A$	$6F$	$3A$	$2D$		$6B$	$6H$	$3A$	$2F$	
	$6G$	$3A$	$2E$			$12B$	$6A$	$4B$	
	$12A$	$6A$	$4A$			$12C$	$6B$	$4C$	
$6C$	$6I$	$3A$	$2H$		$6D$	$6J$	$3B$	$2D$	
	$12D$	$6A$	$4D$			$12F$	$6D$	$4E$	
	$12E$	$6B$	$4E$						
$6E$	$6K$	$3A$	$2L$		$6F$	$6N$	$3A$	$2J$	
	$6L$	$3A$	$2M$			$12I$	$6B$	$4A$	
	$6M$	$3A$	$2O$			$12J$	$6A$	$4F$	
	$12G$	$6B$	4I			$12K$	$6B$	$4G$	
	$12H$	$6B$	4J						
$6G$	$6O$	$3B$	$2S$		$6H$	$6P$	$3B$	$2P$	
	$12L$	$6D$	$4M$			$6Q$	$3B$	$2Q$	
						$6R$	$3B$	$2R$	
$8A$	$8M$	$4AH$			$8B$	$8O$	$4AK$		
	$8N$	$4AI$				$8P$	$4AL$		
$10A$	$10C$	$5A$		$2D$	$12A$	$12M$	$6I$	$4AD$	
	$20A$	$10A$		$4A$		$24A$	$12D$	$8D$	
$12B$	$12N$	$6I$	$4AF$						
	$24B$	$12D$	$8B$						

9.7 The Fusion of $2^6{:}(2^5{:}S_6)$ into $2^6{:}SP_6(2)$

Let $\chi(2^6{:}SP_6(2)|\overline{G})$ be the permutation character of degree 63 of $2^6{:}SP_6(2)$ acting on $2^6{:}(2^5{:}S_6)$. We obtain that $\chi(2^6{:}SP_6(2)|2^6{:}(2^5{:}S_6)) = 1a + 27a + 35b$. We are able to obtain the partial fusion of $2^6{:}(2^5{:}S_6)$ into $2^6{:}SP_6(2)$ by using the information provided by the conjugacy classes of the elements of $2^6{:}(2^5{:}S_6)$ and $2^6{:}Sp_6(2)$, their power maps, together with the permutation character of $2^6{:}SP_6(2)$ of degree 63, the fusion map of $2^5{:}S_6$ into $SP_6(2)$ (see Table 9.9), Proposition 8.51 and Remark 8.52. We used the technique of set intersections for characters to restrict the ordinary irreducible characters $63a$, $315a$, $63b$, $315b$, $440a$ and $440b$ of $(2^6{:}SP_6(2))$ to $2^6{:}(2^5{:}S_6)$ to determine fully the fusion of the classes of $2^6{:}(2^5{:}S_6)$ into $2^6{:}SP_6(2)$.

Table 9.9: The fusion of $2^5{:}S_6$ into $SP(6,2)$

$[h]_{2^5:S_6}$	$\longrightarrow$ $[g]_{SP(6,2)}$	$[h]_{2^5:S_6}$	$\longrightarrow$ $[g]_{SP(6,2)}$	$[h]_{2^5:S_6}$	$\longrightarrow$ $[g]_{SP(6,2)}$	$[h]_{2^5:S_6}$	$\longrightarrow$ $[g]_{SP(6,2)}$
$1A$	$1A$	$2J$	$2D$	$4G$	$4E$	$6E$	$6B$
$2A$	$2D$	$3A$	$3C$	$4H$	$4C$	$6F$	$6G$
$2B$	$2D$	$3B$	$3A$	$4I$	$4E$	$6G$	$6D$
$2C$	$2A$	$4A$	$4C$	$4J$	$4B$	$6H$	$6F$
$2D$	$2B$	$4B$	$4B$	$5A$	$5A$	$8A$	$8A$
$2E$	$2D$	$4C$	$4D$	$6A$	$6E$	$8B$	$8B$
$2F$	$2C$	$4D$	$4D$	$6B$	$6B$	$10A$	$10A$
$2G$	$2A$	$4E$	$4A$	$6C$	$6D$	$12A$	$12A$
$2H$	$2D$	$4F$	$4E$	$6D$	$6A$	$12B$	$12B$
$2I$	$2C$						

Let ζ be the character afforded by the regular representation of $2^5{:}S_6$. We obtain that $\zeta = \Sigma_{i=1}^{37}\alpha_i\Phi_i$, where $\Phi_i \in Irr(2^5{:}S_6)$ and $\alpha_i = deg(\Phi_i)$.Then ζ can be regarded as a character of $2^6{:}(2^5{:}S_6)$ which contains 2^6 in its kernel such that

$$\zeta(x) = \begin{cases} |2^5{:}S_6| & \text{if } x \in 2^6 \\ 0 & \text{otherwise} \end{cases} .$$

If ϕ is a character of $2^6{:}SP_6(2)$ than we have that

$$\begin{aligned}
<\zeta,\phi>_{2^6:(2^5:S_6)} = & \frac{1}{|2^6{:}(2^5{:}S_6)|}\{\zeta(1A)\phi(1A)+\zeta(2A)\phi(2A)+30\zeta(2B)\phi(2B)+32\zeta(2C)\phi(2C)\} \\
= & \frac{1}{|2^6{:}(2^5{:}S_6)|}\{|2^5{:}S_6|(\phi(1A)+\phi(2A)+30\phi(2B)+32\phi(2C))\} \\
= & \frac{1}{64}\{\phi(1A)+\phi(2A)+30\phi(2B)+32\phi(2C)\} \\
= & <\phi_{2^6}, 1_{2^6}> .
\end{aligned}$$

Here 1_{2^6} is the identity character of 2^6 and ϕ_{2^6} is the restriction of ϕ to 2^6.We obtain that

$$\phi_{2^6} = a_1\theta_1 + a_2\theta_2 + a_3\theta_3 + a_4\theta_4,$$

where $a_i \in \mathbb{N}\cup\{0\}$ and θ_i are the sums of the irreducible characters of 2^6 which are in the same orbit under the action of $(2^5{:}S_6)$ on $Irr(2^6)$, for $i \in \{1,2,3,4\}$. Let $\varphi_j \in Irr(2^6)$, where $j \in \{1,2,3,...,137\}$. Then we obtain that

$$\begin{aligned}
\theta_1 = \varphi_1 \quad &, \quad deg(\theta_1) = 1 \\
\theta_2 = \varphi_2 \quad &, \quad deg(\theta_2) = 1 \\
\theta_3 = \sum_{j=3}^{32}\varphi_j \ &, \quad deg(\theta_3) = 30 \\
\theta_4 = \sum_{j=33}^{64}\varphi_j \ &, \quad deg(\theta_4) = 32 .
\end{aligned}$$

Hence

$$\phi_{2^6} = a_1\varphi_1 + a_2\varphi_2\varphi_j + a_3\sum_{j=3}^{32}\varphi_j + a_4\sum_{j=33}^{64}\varphi_j,$$

and therefore

$$\begin{aligned}
\langle\phi_{2^6},\phi_{2^6}\rangle = & a_1^2 + a_2^2 + 30a_3^2 + 32a_4^2 \\
= & \frac{1}{64}\{\phi(1A)\phi(1A)+\phi(2A)\phi(2A)+30\phi(2B)\phi(2B)+32\phi(2C)\phi(2C)\} ,
\end{aligned}$$

where $a_1 = < \zeta, \phi >_{2^6:(2^5:S_6)}$.

We apply the above results to some of the irreducible characters of 2^6:$SP_6(2)$, which in this case are $\phi_1 = 63a$, $\phi_2 = 315a$, $\phi_3 = 63b$ and $\phi_4 = 315b$. Their respective degrees are 63, 315, 63 and 315. For ϕ_1 we calculate that

$$< \zeta, \phi_1 >_{2^6:(2^5:S_6)} = \frac{1}{64}\{63 + (-1) + 30(-1) + 32(-1)\} = 0\ .$$

Now $a_1 + a_2 + 30a_3 + 32a_4 = 63$, since $deg\phi_1 = 63$. Since $a_1 = 0$, we must have that $a_2 = a_3 = a_4 = 1$. Note that 2^6:(2^5:S_6) does not have irreducible characters of degree 63. We obtain that $(\phi_1)_{2^6:(2^5:S_6)} = \chi_1 + \chi_{27} + \chi_{128}$ if the partial fusion of 2^6:(2^5:S_6) into 2^6:$SP_6(2)$ is taken into consideration. Similarly for ϕ_2, ϕ_3 and ϕ_4 we obtain that

$$(\phi_2)_{2^6:(2^5:S_6)} = \chi_{17} + \chi_{111} + \chi_{119}$$
$$(\phi_3)_{2^6:(2^5:S_6)} = \chi_2 + \chi_{28} + \chi_{127}$$
$$(\phi_4)_{2^6:(2^5:S_6)} = \chi_{18} + \chi_{112} + \chi_{120}.$$

By making use of the values of ϕ_1, ϕ_2, ϕ_3 and ϕ_4 on the classes of 2^6:$SP_6(2)$ and the values of $(\phi_1)_{2^6:(2^5:S_6)}$, $(\phi_2)_{2^6:(2^5:S_6))}$, $(\phi_3)_{2^6:(2^5:S_6)}$ and $(\phi_4)_{2^6:(2^5:S_6)}$ on the classes of 2^6:(2^5:S_6) together with the partial fusion, the complete fusion map of 2^6:(2^5:S_6) into 2^6:$SP_6(2)$ is given in the Table 9.10.

Table 9.10: The fusion of $2^6{:}(2^5{:}S_6)$ into $2^6{:}SP_6(2)$

$[g]_{(2^5:S_6)}$	$[x]_{2^6:(2^5:S_6)}$ $\longrightarrow$	$[y]_{2^6:SP_6(2)}$	$[g]_{(2^5:S_6)}$	$[x]_{2^6:(2^5:S_6)}$ $\longrightarrow$	$[y]_{2^6:SP_6(2)}$
$1A$	$1A$	$1A$	$2A$	$2D$	$2B$
	$2A$	$2A$		$2E$	$2B$
	$2B$	$2B$		$4A$	$4A$
	$2C$	$2C$			
$2B$	$2F$	$2D$	$2C$	$2H$	$2F$
	$2G$	$2D$		$2I$	$2F$
	$4B$	$4B$		$4D$	$4C$
	$4C$	$4B$		$4E$	$4C$
$2D$	$2J$	$2D$	$2E$	$2L$	$2H$
	$2K$	$2D$		$2M$	$2H$
	$4F$	$4B$		$2N$	$2H$
	$4G$	$4B$		$2O$	$2H$
	$4H$	$4B$		$4I$	$4E$
				$4J$	$4E$
$2F$	$2P$	$2F$	$2G$	$2S$	$2B$
	$2Q$	$2F$		$4M$	$4A$
	$2R$	$2F$		$4N$	$4A$
	$4K$	$4C$		$4O$	$4A$
	$4L$	$4C$			
$2H$	$2T$	$2H$	$2I$	$2U$	$2F$
	$4P$	$4F$		$2V$	$2F$
	$4Q$	$4F$		$4U$	$4C$
	$4R$	$4F$		$2W$	$2F$
	$4S$	$4F$		$4V$	$4C$
	$4T$	$4F$		$4W$	$4C$
				$4X$	$4E$
$2J$	$2X$	$2H$	$3A$	$3A$	$3C$
	$4Y$	$4E$		$6A$	$6B$
	$4Z$	$4E$		$6B$	$6B$
	$4AA$	$4E$		$6C$	$6B$
	$4AB$	$4E$			
	$4AC$	$4E$			
$3B$	$3B$	$3A$	$4A$	$4AD$	$4L$
	$6D$	$6A$		$4AE$	$4L$
	$6E$	$6A$		$8A$	$8B$
$4B$	$4AF$	$4J$	$4C$	$4AH$	$4N$
	$4AG$	$4J$		$4AI$	$4N$
	$8B$	$8A$		$4AJ$	$4N$
$4D$	$4AK$	$4N$	$4E$	$4AN$	$4H$
	$4AL$	$4N$		$4AO$	$4H$
	$4AM$	$4N$		$4AP$	$4H$
				$4AQ$	$4H$

Table 9.10 (continue)

$[g]_{(2^5:S_6)}$	$[x]_{2^6:(2^5:S_6)}$ $\longrightarrow$	$[y]_{2^6:SP_6(2)}$	$[g]_{(2^5:S_6)}$	$[x]_{2^6:(2^5:S_6)}$ $\longrightarrow$	$[y]_{2^6:SP_6(2)}$
$4F$	$4AR$	$4Q$	$4G$	$4AT$	$4Q$
	$8C$	$8C$		$8E$	$8C$
	$8D$	$8C$		$4AU$	$4Q$
	$4AS$	$4Q$		$8F$	$8C$
$4H$	$4AV$	$4C$	$4I$	$4AX$	$4Q$
	$4AW$	$4C$		$4AY$	$4Q$
	$8G$	$8B$		$4AZ$	$4Q$
	$8H$	$8B$		$8I$	$8C$
				$8J$	$8C$
$4J$	$4BA$	$4J$	$5A$	$5A$	$5A$
	$4BB$	$4J$		$10A$	$10A$
	$4BC$	$4J$		$10B$	$10A$
	$8K$	$8A$			
	$8L$	$8A$			
$6A$	$6F$	$6H$	$6B$	$6H$	$6D$
	$6G$	$6H$		$12B$	$12B$
	$12A$	$12E$		$12C$	$12B$
$6C$	$6I$	$6G$	$6D$	$6J$	$6C$
	$12D$	$12C$		$12F$	$12A$
	$12E$	$12C$			
$6E$	$6K$	$6D$	$6F$	$6N$	$6K$
	$6L$	$6D$		$12I$	$12F$
	$6M$	$6D$		$12J$	$12F$
	$12G$	$12B$		$12K$	$12F$
	$12H$	$12B$			
$6G$	$6O$	$6G$	$6H$	$6P$	$6I$
	$12L$	$12C$		$6Q$	$6I$
				$6R$	$6I$
$8A$	$8M$	$8E$	$8B$	$8O$	$8G$
	$8N$	$8E$		$8P$	$8G$
$10A$	$10C$	$10B$	$12A$	$12M$	$12G$
	$20A$	$20A$		$24A$	$24A$
$12B$	$12N$	$12H$			
	$24B$	$24B$			

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Appendix A

Progamme A $V := VectorSpace(FiniteField(q), n);$

$S < g1, g2 >:= MatrixGroup < n, Finite\ Field(GF(2))|generators >;$

$c := classes(S);$

$O_o := Orbit(S, elt < V|\alpha_1, ..., \alpha_n >);$

$O_1 := Orbit(S, elt < V|\beta_1, ..., \beta_n >);$

$\vdots$

$O_{k'} := Orbit(S, elt < V|\delta_1, ..., \delta_n >);$

$O := O_o\ join\ O_1\ join\ O_2\ join...join\ O_{k'};$

$for\ i\ to\ n(c)\ do;$

$print\ c[i, 1];$

$w := elt < V|0_1, 0_2, ..., 0_n >;$

$e := \{\}$

$while(O \text{ diff } e)\ ne\ \{\}\ do$

$d := \{\};$

$for\ x\ in\ O\ do;$

$y := \{x + w + (x * c[i, 3])\};$

$d :=\ d\ join\ y;$

$end\ for;$

```
print d;

e := d join e;

if(O diff e) ne {} then

w = Representative(O diff e);

end if;

end while;

r := {}

; u := elt < V|0, 0, ..., 0 >;

while(O diff r) ne {} do;

m := {};

for g in Centralizer(S, c[i, 3]) do

l := u * g;

m = m join l;

end for;

print"A block for the vectors under the action of centralizer : ";

print m;

r := m join r;

if (O diff r) ne {} then

u := Representative(O diff r);

end if;
```

end while;

print" ********************************";

end for ;

Progamme B

$V := VectorSpace(FiniteField(q), n);$

$S < g1, g2 >:= MatrixGroup < n, Finite\ Field(GF(2))|generators >;$

$c := classes(S);$

$g := c[i, 3];$

$d\ =\ elt < V|\alpha_1, ..., \alpha_n >$

$w\ = `d + d * g + d * (g^2) + d * (g^3) + d * (g^3) + ... + d * (g^{m-1})$

print w

Progamme C

```
gap>ct:=fuction()local ct;ct:=rec();

>ct.SizesCentralizers:=[m Centralizer Orders];

>ct.OrdersClassRepresentatives:=[m Class Representatives Orders];

>ct.Irr:=[[m × m irreducibles]]; >ct.UnderlyingCharacteristic:=0;ct.Id:=G̅;

>ConvertToLibraryCharacterTable NC(ct);return ct;end;ct:=ct();
```

gap>SetInfoLevel(InfoCharacterTable,2);

gap>IsInternallyConsistent(ct);

gap>PossiblePowerMaps(ct,p); (p-prime divisor of $|G|$).

Appendix B

Character Tables of inertia factor groups of $2^9{:}(L_3(4){:}2)$

Table 9.11: The Character table of H_1

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Size	1	120	315	2240	1260	2520	2520	8064	6720	2880	2880	5040	2880	2880
Order	$1A$	$2A$	$2B$	$3A$	$4A$	$4B$	$4C$	$5A$	$6A$	$7A$	$7B$	$8A$	$14A$	$14B$
$P=2$	1	1	1	4	3	3	3	8	4	10	11	5	11	10
$P=3$	1	2	3	1	5	6	7	8	2	11	10	12	14	13
$P=5$	1	2	3	4	5	6	7	1	9	11	10	12	14	13
$P=7$	1	2	3	4	5	6	7	8	9	1	1	12	2	2
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	1	1	-1	1	-1	1	1	-1	-1	-1
χ_3	20	6	4	2	0	0	2	0	0	-1	-1	0	-1	-1
χ_4	20	6	4	2	0	0	-2	0	0	-1	-1	0	1	1
χ_5	35	7	3	-1	3	-1	-1	0	1	0	0	-1	0	0
χ_6	35	-7	3	-1	3	-1	1	0	-1	0	0	1	0	0
χ_7	45	3	-3	0	1	1	-1	0	0	A	$\overline{A}$	1	$-A$	$-\overline{A}$
χ_8	45	3	-3	0	1	1	1	0	0	$\overline{A}$	A	1	$-\overline{A}$	$-A$
χ_9	45	-3	-3	0	1	1	1	0	0	A	$\overline{A}$	1	A	$\overline{A}$
χ_{10}	45	-3	-3	0	1	1	-1	0	0	$\overline{A}$	A	1	$\overline{A}$	A
χ_{11}	64	8	0	1	0	0	0	-1	-1	1	1	0	1	1
χ_{12}	64	-8	0	1	0	0	0	-1	1	1	1	0	-1	-1
χ_{13}	70	0	6	-2	-2	2	0	0	0	0	0	0	0	0
χ_{14}	126	0	-2	0	-2	-2	0	1	0	0	0	0	0	0

Where $A = -1 - b7$.

Table 9.12: The Character table of H_2

Class	1	2	3	4	5	6	7	8	9	10	11	12
Size	1	15	40	60	320	60	120	120	240	384	320	240
Order	$1A$	$2A$	$2B$	$2C$	$3A$	$4A$	$4B$	$4C$	$4D$	$5A$	$6A$	$8A$
$P=2$	1	1	1	1	5	2	2	2	4	10	5	6
$P=3$	1	2	3	4	5	6	7	8	9	10	3	12
$P=5$	1	2	3	4	1	6	7	8	9	1	11	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	1	1	-1	-1	1	-1	-1
χ_3	4	4	2	0	1	0	0	2	0	-1	-1	0
χ_4	4	4	-2	0	1	0	0	-2	0	-1	1	0
χ_5	5	5	1	1	-1	1	1	1	-1	0	1	-1
χ_6	5	5	-1	1	-1	1	1	-1	1	0	-1	1
χ_7	6	6	0	-2	0	-2	-2	0	0	1	0	0
χ_8	15	-1	3	3	0	-1	-1	-1	1	0	0	-1
χ_9	15	-1	3	-1	0	3	-1	-1	-1	0	0	1
χ_{10}	15	-1	-3	3	0	-1	-1	1	-1	0	0	1
χ_{11}	15	-1	-3	-1	0	3	-1	1	1	0	0	-1
χ_{12}	30	-2	0	-2	0	-2	2	0	0	0	0	0

Table 9.13: The Character table of H_3

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Size	1	3	4	6	6	12	12	32	12	12	12	24	24	32
Order	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$3A$	$4A$	$4B$	$4C$	$4D$	$4E$	$6A$
$P=2$	1	1	1	1	1	1	1	8	2	2	2	5	4	8
$P=3$	1	2	3	4	5	6	7	1	9	10	11	12	3	3
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	1	-1	1	-1	-1	1	-1	1	-1
χ_3	1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	1
χ_4	1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	-1
χ_5	2	2	2	2	2	0	0	-1	2	0	0	0	0	-1
χ_6	2	2	-2	2	2	0	0	-1	-2	0	0	0	0	1
χ_7	3	3	3	-1	-1	-1	-1	0	-1	-1	-1	1	1	0
χ_8	3	3	3	-1	-1	1	1	0	-1	1	1	-1	-1	0
χ_9	3	3	-3	-1	-1	1	-1	0	1	-1	1	1	-1	0
χ_{10}	3	3	-3	-1	-1	1	-1	0	1	-1	1	1	-1	0
χ_{11}	6	-2	0	-2	2	0	-2	0	0	2	0	0	0	0
χ_{12}	6	-2	0	-2	2	0	2	0	0	-2	0	0	0	0
χ_{13}	6	-2	0	2	-2	-2	0	0	0	0	2	0	0	0
χ_{14}	6	-2	0	2	-2	2	0	0	0	0	-2	0	0	0

Table 9.14: The Character table of H_4

Class	1	2	3	4	5	6	7	8	9
Size	1	9	12	8	18	36	24	18	18
Order	$1A$	$2A$	$2B$	$3A$	$4A$	$4B$	$6A$	$8A$	$8B$
$P=2$	1	1	1	4	2	2	4	5	5
$P=3$	1	2	3	1	5	6	3	8	3
χ_1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	-1	1	-1	-1
χ_3	1	1	-1	1	1	-1	-1	1	1
χ_4	1	1	-1	1	1	1	-1	-1	-1
χ_5	2	2	0	2	-2	0	0	0	0
χ_6	2	-2	0	2	0	0	0	A	$-A$
χ_7	2	-2	0	2	0	0	0	$-A$	A
χ_8	8	0	2	-1	0	0	-1	0	0
χ_9	8	0	-2	-1	0	0	1	0	0

Where $A = -2i$.

Character Tables of inertia factor groups of $2^9{:}(L_3(4){:}3)$

Table 9.15: The Character table of H_1

Class	1	2	3	4	5	6	7	8	9	10	11
Size	1	315	336	336	960	960	2240	3780	4032	4032	5040
Order	$1A$	$2A$	$3A$	$3B$	$3C$	$3D$	$3E$	$4A$	$5A$	$5B$	$6A$
$P=2$	1	1	4	3	6	5	7	2	10	9	3
$P=3$	1	2	1	1	1	1	1	8	10	9	2
$P=5$	1	2	4	3	6	5	7	8	1	1	12
$P=7$	1	2	3	4	5	6	7	8	10	9	11
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	A	$-\overline{A}$	A	$-\overline{A}$	1	1	1	1	A
χ_3	1	1	A	A	A	A	1	1	1	1	$\overline{A}$
χ_4	20	4	5	5	-1	-1	2	0	0	0	1
χ_5	20	4	$-A$	$-\overline{A}$	F	$\overline{F}$	2	0	0	0	A
χ_6	20	4	$-\overline{A}$	$-A$	$\overline{F}$	F	2	0	0	0	$\overline{A}$
χ_7	45	-3	0	0	3	3	0	1	0	0	0
χ_8	45	-3	0	0	3	3	0	1	0	0	0
χ_9	45	-3	0	0	B	$\overline{B}$	0	1	0	0	0
χ_{10}	45	-3	0	0	B	$\overline{B}$	0	1	0	0	0
χ_{11}	45	-3	0	0	$\overline{B}$	B	0	1	0	0	0
χ_{12}	45	-3	0	0	$\overline{B}$	B	0	1	0	0	0
χ_{13}	63	-1	3	3	0	0	0	-1	H	$^*H$	-1
χ_{14}	63	-1	3	3	0	0	0	-1	$^*H$	H	-1
χ_{15}	63	-1	B	$\overline{B}$	0	0	0	-1	H	$^*H$	$-A$
χ_{16}	63	-1	B	$\overline{B}$	0	0	0	-1	$^*H$	H	$-A$
χ_{17}	63	-1	$\overline{B}$	B	0	0	0	-1	H	$^*H$	$-\overline{A}$
χ_{18}	63	-1	$\overline{B}$	B	0	0	0	-1	$^*H$	H	$-\overline{A}$
χ_{19}	64	0	4	4	1	1	1	0	-1	-1	0
χ_{20}	64	0	G	$\overline{G}$	A	$\overline{A}$	1	0	-1	-1	0
χ_{21}	64	0	$\overline{G}$	G	$\overline{A}$	A	1	0	-1	-1	0
χ_{22}	105	9	0	0	0	0	-3	1	0	0	0

Where $A=-1-b3$, $B=-3-b3$, $C=-1-b7$, $D=E(21)^5+E(21)^{21}+E(21)^{20}$, $E=E(21)^2+E(21)^8+E(21)^{11}$, $F=-5-5b3$, $G=-2-2i3$, $H=-b5$, $I=E(15)^{11}+E(15)^{14}$ and $J=E(15)^2+E(15)^8$.

Table 9.15 (Continue)

Class	12	13	14	15	16	17	18	19	20	21	22
Size	5040	2880	2880	4032	4032	4032	4032	2880	2880	2880	2880
Order	$6B$	$7A$	$7B$	$15A$	$15B$	$15C$	$15D$	$21A$	$21B$	$21C$	$21D$
$P=2$	4	13	14	17	18	15	16	20	19	22	21
$P=3$	2	14	13	10	9	9	10	13	13	14	14
$P=5$	11	14	13	4	4	3	3	21	22	19	20
$P=7$	12	1	1	16	15	18	17	6	5	5	6
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	$-\overline{A}$	1	1	$\overline{A}$	A	$\overline{A}$	A	A	$\overline{A}$	A	$\overline{A}$
χ_3	A	1	1	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	A	$\overline{A}$	A
χ_4	1	-1	-1	0	0	0	0	-1	-1	-1	-1
χ_5	$\overline{A}$	-1	-1	0	0	0	0	$-A$	-$\overline{A}$	$-A$	$-\overline{A}$
χ_6	A	-1	-1	0	0	0	0	-$\overline{A}$	$-A$	-$\overline{A}$	$-A$
χ_7	0	C	$\overline{C}$	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C
χ_8	0	$\overline{C}$	C	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$
χ_9	0	C	$\overline{C}$	0	0	0	0	D	$\overline{E}$	E	$\overline{D}$
χ_{10}	0	$\overline{C}$	C	0	0	0	0	E	$\overline{D}$	D	$\overline{E}$
χ_{11}	0	C	$\overline{C}$	0	0	0	0	$\overline{E}$	D	$\overline{D}$	E
χ_{12}	0	$\overline{C}$	C	0	0	0	0	$\overline{D}$	E	$\overline{E}$	D
χ_{13}	-1	0	0	$^*H$	$^*H$	H	H	0	0	0	0
χ_{14}	-1	0	0	H	H	$^*H$	$^*H$	0	0	0	0
χ_{15}	$-\overline{A}$	0	0	I	$\overline{I}$	J	$\overline{J}$	0	0	0	0
χ_{16}	$-\overline{A}$	0	0	J	$\overline{J}$	I	$\overline{I}$	0	0	0	0
χ_{17}	$-A$	0	0	$\overline{I}$	I	$\overline{J}$	J	0	0	0	0
χ_{18}	$-A$	0	0	$\overline{J}$	J	$\overline{I}$	I	0	0	0	0
χ_{19}	0	1	1	-1	-1	-1	-1	1	1	1	1
χ_{20}	0	1	1	$-\overline{A}$	$-A$	$-\overline{A}$	$-A$	A	$\overline{A}$	A	$\overline{A}$
χ_{21}	0	1	1	$-A$	$-\overline{A}$	$-A$	$-\overline{A}$	$\overline{A}$	A	$\overline{A}$	A
χ_{22}	0	0	0	0	0	0	0	0	0	0	0

Where $A=-1-b3$, $B=-3-b3$, $C=-1-b7$, $D=E(21)^5+E(21)^{21}+E(21)^{20}$, $E=E(21)^2+E(21)^8+E(21)^{11}$, $F=-5-5b3$, $G=-2-2i3$, $H=-b5$, $I=E(15)^{11}+E(15)^{14}$ and $J=E(15)^2+E(15)^8$.

Table 9.16: The Character table of H_2

Class	1	2	3	4	5	6	7	8	9	10
Size	1	15	60	16	16	80	80	180	320	192
Order	$1A$	$2A$	$2B$	$3A$	$3B$	$3C$	$3D$	$3E$	$4A$	$5A$
$P=2$	1	1	1	5	4	7	6	8	2	11
$P=3$	1	2	3	1	1	1	1	1	9	11
$P=5$	1	2	3	5	4	7	6	8	9	1
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	A	$\overline{A}$	A	$\overline{A}$	1	1	1
χ_3	1	1	1	$\overline{A}$	A	$\overline{A}$	A	1	1	1
χ_4	3	3	-1	3	3	0	0	0	-1	E
χ_5	3	3	-1	3	3	0	0	0	-1	$*E$
χ_6	3	3	-1	B	$\overline{B}$	0	0	0	-1	E
χ_7	3	3	-1	3	3	0	0	0	-1	$*E$
χ_8	3	3	-1	$\overline{B}$	B	0	0	0	-1	$*E$
χ_9	3	3	-1	$\overline{B}$	B	0	0	0	-1	E
χ_{10}	4	4	0	4	4	1	1	1	0	-1
χ_{11}	4	4	0	C	$\overline{C}$	1	1	1	0	-1
χ_{12}	4	4	0	$\overline{C}$	C	A	$\overline{A}$	1	0	-1
χ_{13}	5	5	1	5	5	-1	-1	-1	1	0
χ_{14}	5	5	1	D	$\overline{D}$	$-\overline{A}$	$-A$	-1	1	0
χ_{15}	5	5	1	$\overline{D}$	D	$-A$	$-\overline{A}$	-1	1	0
χ_{16}	15	-1	3	0	0	3	3	0	-1	0
χ_{17}	15	-1	3	0	0	$\overline{B}$	B	0	-1	0
χ_{18}	15	-1	3	0	0	B	$\overline{B}$	0	-1	0
χ_{19}	45	-3	-3	0	0	0	0	0	1	0

Where $A = -1 - b3$, $B = -3 - 3b3$, $C = -2 - 2i3$, $D = -5 - 5b3$, $E = -b5$, $F = -E(15)^2 - E(5)^8$ and $G = -E(15)^{11} - E(5)^{14}$.

Table 9.16 (Continue)

Class	11	12	13	14	15	16	17	18	19
Size	192	240	240	240	240	192	192	192	192
Order	$5B$	$6A$	$6B$	$6C$	$6D$	$15A$	$15B$	$15C$	$15D$
$P=2$	10	5	4	6	7	19	18	17	16
$P=3$	10	3	3	2	2	10	11	10	11
$P=5$	1	13	12	15	14	4	4	5	5
χ_1	1	1	1	1	1	1	1	1	1
χ_2	1	A	$\overline{A}$	A	$\overline{A}$	$\overline{A}$	A	A	$\overline{A}$
χ_3	1	$\overline{A}$	A	$\overline{A}$	A	A	$\overline{A}$	$\overline{A}$	A
χ_4	$*E$	-1	-1	0	0	E	E	$*E$	$*E$
χ_5	E	-1	-1	0	0	$*E$	$*E$	E	E
χ_6	$*E$	$-A$	$-\overline{A}$	0	0	F	$\overline{F}$	$\overline{G}$	G
χ_7	E	$-A$	$-\overline{A}$	0	0	$\overline{G}$	G	F	$\overline{F}$
χ_8	E	$-\overline{A}$	$-A$	0	0	$\overline{G}$	G	F	$\overline{F}$
χ_9	$*E$	$-\overline{A}$	$-A$	0	0	$\overline{F}$	F	G	$\overline{G}$
χ_{10}	-1	0	0	1	1	-1	-1	-1	-1
χ_{11}	-1	0	0	A	$\overline{A}$	$-\overline{A}$	$-A$	A	$-\overline{A}$
χ_{12}	-1	0	0	$\overline{A}$	A	$-A$	$-\overline{A}$	$-\overline{A}$	$-A$
χ_{13}	0	1	1	-1	-1	0	0	0	0
χ_{14}	0	A	$\overline{A}$	$-A$	$-\overline{A}$	0	0	0	0
χ_{15}	0	$\overline{A}$	A	$-\overline{A}$	A	0	0	0	0
χ_{16}	0	0	0	-1	-1	0	0	0	0
χ_{17}	0	0	0	0	$-A$	$-\overline{A}$	0	0	0
χ_{18}	0	0	0	$-\overline{A}$	$-A$	0	0	0	0
χ_{19}	0	0	0	0	0	0	0	0	0

Where $A = -1 - b3$, $B = -3 - 3b3$, $C = -2 - 2i3$, $D = -5 - 5b3$, $E = -b5$, $F = -E(15)^2 - E(5)^8$ and $G = -E(15)^{11} - E(5)^{14}$.

Table 9.17: The Character table of H_3

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Size	1	6	9	12	8	8	16	16	32	36	24	24	48	48
Order	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$3C$	$3D$	$3E$	$4A$	$6A$	$6B$	$6C$	$6D$
$P = 2$	1	1	1	1	6	5	8	7	9	3	5	6	8	7
$P = 3$	1	2	3	4	1	1	1	1	1	10	2	2	4	4
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	1	1	1	1	1	-1	1	1	-1	-1
χ_3	1	1	1	1	A	$\overline{A}$	$\overline{A}$	A	1	-1	A	$\overline{A}$	$-A$	$-\overline{A}$
χ_4	1	1	1	-1	$\overline{A}$	A	A	$\overline{A}$	1	-1	$\overline{A}$	A	$-\overline{A}$	$-A$
χ_5	1	1	1	1	A	$\overline{A}$	$\overline{A}$	A	1	1	A	$\overline{A}$	A	$\overline{A}$
χ_6	1	1	1	-1	$\overline{A}$	A	A	$\overline{A}$	1	1	$\overline{A}$	A	$\overline{A}$	A
χ_7	2	2	2	0	-1	-1	2	2	-1	0	-1	-1	0	0
χ_8	2	2	2	0	$-A$	$-\overline{A}$	C	$\overline{C}$	-1	0	$-A$	$-\overline{A}$	0	0
χ_9	2	2	2	0	$-\overline{A}$	$-A$	$\overline{C}$	C	-1	0	$-\overline{A}$	$-A$	0	0
χ_{10}	6	2	-2	0	3	3	0	0	0	0	-1	-1	0	0
χ_{11}	6	2	-2	0	B	$\overline{B}$	0	0	0	0	$-A$	$-\overline{A}$	0	0
χ_{12}	6	2	-2	0	$\overline{B}$	B	0	0	0	0	$-\overline{A}$	$-A$	0	0
χ_{13}	9	-3	1	-3	0	-2	0	0	0	-1	0	0	0	0
χ_{14}	9	-3	1	3	0	2	0	0	0	1	0	0	0	0

Where $A = -1 - b3$, $B = -3 - 3b3$ and $C = 2b3$.

Table 9.18: The Character table of H_4

Class	1	2	3	4	5	6	7	8	9	10
Size	1	9	8	12	12	24	24	54	36	36
Order	$1A$	$2A$	$3A$	$3B$	$3C$	$3D$	$3E$	$4A$	$6A$	$6B$
$P = 2$	1	1	3	5	4	7	6	2	5	4
$P = 3$	1	2	1	1	1	1	1	8	2	2
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	$\overline{A}$	A	A	A	1	$\overline{A}$	$-\overline{A}$
χ_3	1	1	1	A	$\overline{A}$	$\overline{A}$	A	1	A	A
χ_4	2	-2	2	-1	-1	-1	-1	0	1	1
χ_5	2	-2	2	$-\overline{A}$	$-A$	$-A$	$-\overline{A}$	0	$\overline{A}$	A
χ_6	2	-2	2	$-A$	$-\overline{A}$	$\overline{A}$	$-A$	0	A	$\overline{A}$
χ_7	3	3	3	0	0	0	0	-1	0	0
χ_8	8	0	2	2	2	-1	-1	0	0	0
χ_9	8	0	-1	B	$\overline{B}$	$\overline{A}$	$-A$	0	0	0
χ_{10}	8	0	-1	$\overline{B}$	B	A	$-\overline{A}$	0	0	0

Where $A = b3$ and $B = 2b3$.

Character Tables of inertia factor groups of $2^8{:}(U_4(2){:}2)$

Table 9.19: The Character table of H_1

Class	1	2	3	4	5	6	7	8	9	10	11	12	13
Size	1	36	45	270	540	80	240	480	540	540	1620	3240	5184
Order	$1A$	$2A$	$2B$	$2C$	$2D$	$3A$	$3B$	$3C$	$4A$	$4B$	$4C$	$4D$	$5A$
$P=2$	1	1	1	1	1	6	7	8	4	3	4	4	13
$P=3$	1	2	3	4	5	1	1	1	9	10	11	12	13
$P=5$	1	2	3	4	5	6	7	8	9	10	11	12	13
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	1	1	-1	1	1	1	-1	1	-1	1	1
χ_3	6	4	-2	2	0	-3	3	0	-2	2	2	0	1
χ_4	6	-4	-2	2	0	-3	3	0	2	2	-2	0	1
χ_5	10	0	-6	2	0	1	-2	4	0	2	0	-2	0
χ_6	15	-5	7	3	-1	-3	0	3	-3	-1	1	1	0
χ_7	15	-5	-1	-1	3	6	3	0	-1	3	-1	-1	0
χ_8	15	5	7	3	1	-3	0	3	3	-1	-1	1	0
χ_9	15	5	-1	-1	-3	6	3	0	1	3	1	-1	0
χ_{10}	20	10	4	4	2	2	5	-1	2	0	2	0	0
χ_{11}	20	-10	4	4	-2	2	5	-1	-2	0	-2	0	0
χ_{12}	20	0	4	-4	0	-7	2	2	0	4	0	0	0
χ_{13}	24	4	8	0	4	6	0	3	0	0	0	0	-1
χ_{14}	24	-4	8	0	-4	6	0	3	0	0	0	0	-1
χ_{15}	30	-10	-10	2	2	3	3	3	4	-2	0	0	0
χ_{16}	30	10	-10	2	-2	3	3	3	-4	-2	0	0	0
χ_{17}	60	10	-4	4	2	6	-3	-3	-2	0	-2	0	0
χ_{18}	60	-10	-4	4	-2	6	-3	-3	2	0	2	0	0
χ_{19}	60	0	12	4	0	-3	-6	0	0	4	0	0	0
χ_{20}	64	16	0	0	0	-8	4	-2	0	0	0	0	-1
χ_{21}	64	-16	0	0	0	-8	4	-2	0	0	0	0	-1
χ_{22}	80	0	-16	0	0	-10	-4	2	0	0	0	0	0
χ_{23}	81	9	9	-3	-3	0	0	0	3	-3	-1	-1	1
χ_{24}	81	-9	9	-3	3	0	0	0	-3	-3	1	-1	1
χ_{25}	90	0	-6	-6	0	9	0	0	0	2	0	2	0

Table 9.19 (Continue)

Class	14	15	16	17	18	19	20	21	22	23	24	25
Size	720	1440	1440	1440	1440	2160	4320	6480	5760	5184	4320	4320
Order	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$8A$	$9A$	$10A$	$12A$	$12B$
$P=2$	6	8	7	7	8	7	8	10	22	13	19	14
$P=3$	3	3	3	2	2	4	5	21	6	23	9	10
$P=5$	14	15	16	17	18	19	20	21	22	2	24	25
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	1	-1	-1	1	-1	-1	1
χ_3	1	-2	1	1	-2	-1	0	0	0	-1	1	-1
χ_4	1	-2	1	-1	2	-1	0	0	0	1	-1	-1
χ_5	-3	0	0	0	0	2	0	0	1	0	0	-1
χ_6	1	1	-2	-2	1	0	-1	1	0	0	0	-1
χ_7	2	2	-1	1	-2	-1	0	1	0	0	-1	0
χ_8	1	1	-2	2	-1	0	1	-1	0	0	0	-1
χ_9	2	2	-1	-1	2	-1	0	-1	0	0	1	0
χ_{10}	-2	1	1	1	1	1	-1	0	-1	0	-1	0
χ_{11}	-2	1	1	-1	-1	1	1	0	-1	0	1	0
χ_{12}	1	-2	-2	0	0	2	0	0	-1	0	0	1
χ_{13}	2	-1	2	-2	1	0	1	0	0	-1	0	0
χ_{14}	2	-1	2	2	-1	0	-1	0	0	1	0	0
χ_{15}	-1	-1	-1	-1	-1	-1	-1	0	0	0	1	1
χ_{16}	-1	-1	-1	1	1	-1	1	0	0	0	-1	1
χ_{17}	2	-1	-1	1	1	1	-1	0	0	0	1	0
χ_{18}	2	-1	-1	-1	-1	1	1	0	0	0	-1	0
χ_{19}	-3	0	0	0	0	-2	0	0	0	0	0	1
χ_{20}	0	0	0	-2	-2	0	0	0	1	1	0	0
χ_{21}	0	0	0	2	2	0	0	0	-1	-1	0	0
χ_{22}	2	2	2	0	0	0	0	0	-1	0	0	0
χ_{23}	0	0	0	0	0	0	0	1	0	-1	0	0
χ_{24}	0	0	0	0	0	0	0	-1	0	1	0	0
χ_{25}	-3	0	0	0	0	0	0	0	0	0	0	-1

Table 9.20: The Character table of H_2

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Size	1	9	36	2	24	18	36	18	72	54	54	36	36	36
Order	$1A$	$2A$	$2B$	$3A$	$3B$	$4A$	$4B$	$6A$	$6B$	$8A$	$8B$	$12A$	$12B$	$12C$
$P=2$	1	1	1	4	5	2	2	4	5	6	6	8	8	8
$P=3$	1	2	3	1	1	6	7	2	3	10	11	6	7	7
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	1	1	1	-1	1	-1	1	1	1	-1	-1
χ_3	1	1	1	1	1	1	-1	1	1	-1	-1	1	-1	-1
χ_4	1	1	-1	1	1	1	1	1	-1	-1	-1	1	1	1
χ_5	2	2	0	2	2	-2	0	2	0	0	0	-2	0	0
χ_6	2	-2	0	2	2	0	0	-2	0	B	$-B$	0	0	0
χ_7	2	-2	0	2	2	0	0	-2	0	$-B$	B	0	0	0
χ_8	6	-2	0	-3	0	2	-2	1	0	0	0	-1	1	1
χ_9	6	-2	0	-3	0	2	2	1	0	0	0	-1	-1	-1
χ_{10}	6	-2	0	-3	0	-2	0	1	0	0	0	1	A	$-A$
χ_{11}	6	-2	0	-3	0	-2	0	1	0	0	0	1	$-A$	A
χ_{12}	8	0	2	8	-1	0	0	0	-1	0	0	0	0	0
χ_{13}	8	0	-2	8	-1	0	0	0	1	0	0	0	0	0
χ_{14}	12	4	0	-6	0	0	0	-2	0	0	0	0	0	0

Where $A = -3i$ and $B = 2i$.

Table 9.21: The Character table of H_3

Class	1	2	3	4	5	6	7	8	9	10
Size	1	1	4	4	6	12	12	12	24	32
Order	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$2G$	$2H$	$3A$
$P=2$	1	1	1	1	1	1	1	1	1	10
$P=3$	1	2	3	4	5	6	7	8	9	1
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	-1	1	-1	-1	1
χ_3	1	1	-1	-1	1	-1	1	-1	1	1
χ_4	1	1	-1	-1	1	1	1	1	-1	1
χ_5	2	2	-2	-2	2	0	2	0	0	-1
χ_6	2	2	2	2	2	0	2	0	0	-1
χ_7	3	3	-3	-3	3	-1	-1	-1	1	0
χ_8	3	3	-3	-3	3	1	-1	1	-1	0
χ_9	3	3	3	3	3	1	-1	1	1	0
χ_{10}	3	3	3	3	3	-1	-1	-1	-1	0
χ_{11}	4	-4	-2	2	0	2	0	-2	0	1
χ_{12}	4	-4	-2	2	0	-2	0	2	0	1
χ_{13}	4	-4	2	-2	0	2	0	-2	0	1
χ_{14}	4	-4	-2	2	0	-2	0	2	0	1
χ_{15}	6	6	0	0	-2	2	2	2	0	0
χ_{16}	6	6	0	0	-2	-2	2	-2	0	0
χ_{17}	6	6	0	0	-2	0	-2	0	-2	0
χ_{18}	6	6	0	0	-2	0	-2	0	2	0
χ_{19}	8	-8	-4	4	0	0	0	0	0	-1
χ_{20}	8	-8	4	-4	0	0	0	0	0	-1

Table 9.21 (Continue)

Class	11	12	13	14	15	16	17	18	19	20
Size	12	12	12	24	24	48	32	32	32	48
Order	$4A$	$4B$	$4C$	$4D$	$4E$	$4F$	$6A$	$6A$	$6A$	$8A$
$P=2$	2	5	5	5	5	7	10	10	10	11
$P=3$	11	12	13	14	15	16	2	3	4	20
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	-1	1	-1	1	1	1	-1
χ_3	1	1	1	-1	-1	-1	1	-1	-1	1
χ_4	1	-1	-1	1	-1	1	1	-1	-1	-1
χ_5	2	0	0	0	-2	0	-1	1	1	0
χ_6	2	0	0	0	2	0	-1	-1	-1	0
χ_7	-1	1	1	-1	1	1	0	0	0	-1
χ_8	-1	-1	-1	1	1	-1	0	0	0	1
χ_9	-1	1	1	1	-1	-1	0	0	0	-1
χ_{10}	-1	-1	-1	-1	-1	1	0	0	0	1
χ_{11}	0	-2	2	0	0	0	-1	1	-1	0
χ_{12}	0	2	-2	0	0	0	-1	1	-1	0
χ_{13}	0	-2	2	0	0	0	-1	1	-1	0
χ_{14}	0	2	-2	0	0	0	-1	-1	1	0
χ_{15}	-2	0	0	-2	0	0	0	0	0	0
χ_{16}	-2	0	0	2	0	0	0	0	0	0
χ_{17}	2	2	2	0	0	0	0	0	0	0
χ_{18}	2	-2	-2	0	0	0	0	0	0	0
χ_{19}	0	0	0	0	0	0	1	-1	1	0
χ_{20}	0	0	0	0	0	0	1	1	-1	0

Character Tables of inertia factor groups of $2^6 : (2^5{:}S_6)$

Table 9.22: The Character table of $H_1 = H_2$

Class	1	2	3	4	5	6	7	8	9	10	11	12
Size	1	1	15	15	30	30	60	60	180	180	180	160
Order	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$2G$	$2H$	$2I$	$2J$	$3A$
$P = 2$	1	1	1	1	1	1	1	1	1	1	1	12
$P = 3$	1	2	3	4	5	6	7	8	9	10	11	1
$P = 5$	1	2	3	4	5	6	7	8	9	10	11	12
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	1	1	1
χ_3	5	5	5	5	1	1	-3	-3	1	1	1	-1
χ_4	5	5	5	5	3	3	-1	-1	3	1	1	2
χ_5	5	5	5	5	-3	-3	1	1	-3	1	1	2
χ_6	5	5	5	5	-1	-1	3	3	-1	1	1	-1
χ_7	6	-6	2	-2	-4	4	0	0	0	-2	2	3
χ_8	6	-6	2	-2	4	-4	0	0	0	-2	2	3
χ_9	9	9	9	9	3	3	3	3	3	1	1	0
χ_{10}	9	9	9	9	-3	-3	-3	-3	-3	1	1	0
χ_{11}	10	10	10	10	2	2	-2	-2	2	-2	-2	1
χ_{12}	10	10	10	10	-2	-2	2	2	-2	-2	-2	1
χ_{13}	10	-10	-2	2	-4	4	-4	4	0	-2	2	1
χ_{14}	10	-10	-2	2	-4	4	4	-4	0	-2	2	1
χ_{15}	10	-10	-2	2	4	-4	-4	4	0	-2	2	1
χ_{16}	10	-10	-2	2	4	-4	4	-4	0	-2	2	1
χ_{17}	15	15	-1	-1	5	5	-3	-3	-3	-1	-1	3
χ_{18}	15	15	-1	-1	7	7	3	3	-1	3	3	3
χ_{19}	15	15	-1	-1	-7	-7	-3	-3	1	3	3	3
χ_{20}	15	15	-1	-1	-5	-5	3	3	3	-1	-1	3
χ_{21}	16	16	16	16	0	0	0	0	0	0	0	-2
χ_{22}	20	-20	-4	4	0	0	0	0	0	4	-4	2
χ_{23}	24	-24	8	-8	-8	8	0	0	0	0	0	3
χ_{24}	24	-24	8	-8	8	-8	0	0	0	0	0	3
χ_{25}	30	30	-2	-2	2	2	6	6	2	2	2	-3
χ_{26}	30	30	-2	-2	-2	-2	-6	-6	-2	2	2	-3
χ_{27}	30	-30	10	-10	-4	4	0	0	0	-2	2	-3
χ_{28}	30	-30	10	-10	4	-4	0	0	0	-2	2	-3
χ_{29}	36	-36	12	-12	0	0	0	0	0	4	-4	0
χ_{30}	40	-40	-8	8	-8	8	0	0	0	0	0	1
χ_{31}	40	-40	-8	8	8	-8	0	0	0	0	0	1
χ_{32}	40	-40	-8	8	0	0	-8	8	0	0	0	-2
χ_{33}	40	-40	-8	8	0	0	8	-8	0	0	0	-2
χ_{34}	45	45	-3	-3	3	3	3	3	-5	-3	-3	0
χ_{35}	45	45	-3	-3	9	9	-3	-3	1	1	1	0
χ_{36}	45	45	-3	-3	-9	-9	3	3	-1	1	1	0
χ_{37}	45	45	-3	-3	-3	-3	-3	-3	5	-3	-3	0

Table 9.22 (continue)

Class	13	14	15	16	17	18	19	20	21	22	23	24
Size	640	120	120	180	180	360	720	720	720	720	720	2304
Order	$3A$	$4A$	$4B$	$4C$	$4D$	$4E$	$4F$	$4G$	$4H$	$4I$	$4J$	$5A$
$P = 2$	13	3	3	4	4	4	10	3	10	10	10	24
$P = 3$	1	14	15	16	17	18	19	20	21	22	23	24
$P = 5$	13	14	15	16	17	18	19	20	21	22	23	24
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	-1	-1	1	1	1	-1	1
χ_3	2	1	1	1	1	-3	-1	1	-1	-1	-1	0
χ_4	-1	3	3	1	1	-1	1	1	-1	-1	1	0
χ_5	-1	-3	-3	1	1	1	-1	1	-1	-1	-1	0
χ_6	2	-1	-1	1	1	3	1	1	-1	-1	1	0
χ_7	0	2	-2	2	-2	0	-2	0	0	0	2	1
χ_8	0	-2	2	2	-2	0	2	0	0	0	-2	1
χ_9	0	3	3	1	1	3	-1	1	1	1	-1	-1
χ_{10}	0	-3	-3	1	1	-3	1	1	1	1	1	-1
χ_{11}	1	2	2	-2	-2	-2	0	-2	0	0	0	0
χ_{12}	1	-2	-2	-2	-2	2	0	-2	0	0	0	0
χ_{13}	1	-2	2	-2	2	0	0	0	2	-2	0	0
χ_{14}	1	-2	2	-2	2	0	0	0	-2	2	0	0
χ_{15}	1	2	-2	-2	2	0	0	0	-2	2	0	0
χ_{16}	1	2	-2	-2	2	0	0	0	2	-2	0	0
χ_{17}	0	1	1	3	3	1	1	-1	-1	-1	1	0
χ_{18}	0	-1	-1	-1	-1	-1	1	-1	1	1	1	0
χ_{19}	0	1	1	-1	-1	1	-1	-1	1	1	-1	0
χ_{20}	0	-1	-1	3	3	-1	-1	-1	-1	-1	-1	0
χ_{21}	-2	0	0	0	0	0	0	0	0	0	0	1
χ_{22}	2	0	0	4	-4	0	0	0	0	0	0	0
χ_{23}	0	4	-4	0	0	0	0	0	0	0	0	-1
χ_{24}	0	-4	4	0	0	0	0	0	0	0	0	-1
χ_{25}	0	-2	-2	2	2	-2	0	-2	0	0	0	0
χ_{26}	0	2	2	2	2	2	0	-2	0	0	0	0
χ_{27}	0	2	-2	2	-2	0	2	0	0	0	-2	0
χ_{28}	0	-2	2	2	-2	0	-2	0	0	0	2	0
χ_{29}	0	0	0	-4	4	0	0	0	0	0	0	1
χ_{30}	-2	-4	4	0	0	0	0	0	0	0	0	0
χ_{31}	-2	4	-4	0	0	0	0	0	0	0	0	0
χ_{32}	1	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	1	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	3	3	1	1	-1	-1	1	1	1	-1	0
χ_{35}	0	-3	-3	-3	-3	1	-1	1	-1	-1	-1	0
χ_{36}	0	3	3	-3	-3	-1	1	1	-1	-1	1	0
χ_{37}	0	-3	-3	1	1	1	1	1	1	1	1	0

Table 9.22 (continue)

Class	25	26	27	28	29	30	31	32	33	34	35	36	37
Size	160	480	480	640	960	960	1920	1920	1440	1440	2304	960	960
Order	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$	$6H$	$8A$	$8B$	$10A$	$12A$	$12B$
$P = 2$	12	12	12	13	12	12	13	13	16	17	24	26	26
$P = 3$	2	3	4	2	6	5	8	7	33	34	35	14	15
$P = 5$	25	26	27	28	29	30	31	32	33	34	2	36	37
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	1	1	-1	-1
χ_3	-1	-1	-1	2	1	1	0	0	-1	-1	0	1	1
χ_4	2	2	2	-1	0	0	-1	-1	1	-1	0	0	0
χ_5	2	2	2	-1	0	0	1	1	-1	-1	0	0	0
χ_6	-1	-1	-1	2	-1	-1	0	0	1	-1	0	-1	-1
χ_7	-3	-1	1	0	-1	1	0	0	0	0	-1	1	-1
χ_8	-3	-1	1	0	1	-1	0	0	0	0	-1	-1	1
χ_9	0	0	0	0	0	0	0	0	-1	1	-1	0	0
χ_{10}	0	0	0	0	0	0	0	0	1	1	-1	0	0
χ_{11}	1	1	1	1	-1	-1	1	1	0	0	0	-1	-1
χ_{12}	1	1	1	1	1	1	-1	-1	0	0	0	1	1
χ_{13}	-1	1	-1	-1	-1	1	1	-1	0	0	0	-1	1
χ_{14}	-1	1	-1	-1	-1	1	-1	1	0	0	0	-1	1
χ_{15}	-1	1	-1	-1	1	-1	1	-1	0	0	0	1	-1
χ_{16}	-1	1	-1	-1	1	-1	-1	1	0	0	0	1	-1
χ_{17}	3	-1	-1	0	-1	-1	0	0	-1	1	0	1	1
χ_{18}	3	-1	-1	0	1	1	0	0	-1	-1	0	-1	-1
χ_{19}	3	-1	-1	0	-1	-1	0	0	1	-1	0	1	1
χ_{20}	3	-1	-1	0	1	1	0	0	1	1	0	-1	-1
χ_{21}	-2	-2	-2	-2	0	0	0	0	0	0	1	0	0
χ_{22}	-2	2	-2	-2	0	0	0	0	0	0	0	0	0
χ_{23}	-3	-1	1	0	1	-1	0	0	0	0	1	-1	1
χ_{24}	-3	-1	1	0	-1	1	0	0	0	0	1	1	-1
χ_{25}	-3	1	1	0	-1	-1	0	0	0	0	0	1	1
χ_{26}	-3	1	1	0	1	1	0	0	0	0	0	-1	-1
χ_{27}	3	1	-1	0	-1	1	0	0	0	0	0	1	-1
χ_{28}	3	1	-1	0	1	-1	0	0	0	0	0	-1	1
χ_{29}	0	0	0	0	0	0	0	0	0	0	-1	0	0
χ_{30}	-1	1	-1	2	1	-1	0	0	0	0	0	1	-1
χ_{31}	-1	1	-1	2	-1	1	0	0	0	0	0	-1	1
χ_{32}	2	-2	2	-1	0	0	-1	1	0	0	0	0	0
χ_{33}	2	-2	2	-1	0	0	1	-1	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	1	-1	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	1	1	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	-1	1	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	-1	-1	0	0	0

Table 9.23: The Character table of H_3

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Size	1	1	1	1	1	1	1	1	6	6	6	6	6	6	6	6	6
Order	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$2F$	$2G$	$2H$	$2I$	$2J$	$2K$	$2L$	$2M$	$2N$	$2O$	$2P$
$P=2$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$P=3$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1
χ_4	1	1	-1	1	1	-1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1
χ_5	1	-1	-1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	-1
χ_6	1	-1	-1	1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	-1
χ_7	1	-1	1	1	-1	-1	-1	1	1	1	-1	1	1	-1	-1	1	1
χ_8	1	-1	1	1	-1	-1	-1	1	1	1	-1	1	1	-1	-1	1	1
χ_9	2	2	-2	2	2	-2	-2	-2	-2	2	-2	2	2	2	-2	-2	-2
χ_{10}	2	-2	-2	2	-2	2	2	-2	-2	2	2	2	2	-2	2	-2	-2
χ_{11}	2	-2	2	2	-2	-2	-2	2	2	2	-2	2	2	-2	-2	2	2
χ_{12}	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_{13}	3	3	3	3	3	3	3	3	-1	-1	-1	-1	3	-1	-1	3	-1
χ_{14}	3	3	3	3	3	3	3	3	3	-1	-1	3	-1	3	3	-1	-1
χ_{15}	3	3	3	3	3	3	3	3	-1	3	3	-1	-1	-1	-1	-1	3
χ_{16}	3	3	3	3	3	3	3	3	-1	-1	-1	-1	3	-1	-1	3	-1
χ_{17}	3	3	3	3	3	3	3	3	3	-1	-1	3	-1	3	3	-1	-1
χ_{18}	3	3	3	3	3	3	3	3	-1	3	3	-1	-1	-1	-1	-1	3
χ_{19}	3	-3	3	3	-3	-3	-3	3	3	-1	1	3	-1	-3	-3	-1	-1
χ_{20}	3	-3	3	3	-3	-3	-3	3	-1	3	-3	-1	-1	1	1	-1	3
χ_{21}	3	-3	3	3	-3	-3	-3	3	-1	-1	1	-1	3	1	1	3	-1
χ_{22}	3	-3	3	3	-3	-3	-3	3	3	-1	1	3	-1	-3	-3	-1	-1
χ_{23}	3	-3	3	3	-3	-3	-3	3	-1	3	-3	-1	-1	1	1	-1	3
χ_{24}	3	-3	3	3	-3	-3	-3	3	-1	-1	1	-1	3	1	1	3	-1
χ_{25}	3	-3	-3	3	-3	3	3	-3	1	3	3	-1	-1	1	-1	1	-3
χ_{26}	3	-3	-3	3	-3	3	3	-3	-3	-1	-1	3	-1	-3	3	1	1
χ_{27}	3	-3	-3	3	-3	3	3	-3	1	-1	-1	-1	3	1	-1	-3	1
χ_{28}	3	-3	-3	3	-3	3	3	-3	1	3	3	-1	-1	1	-1	1	-3
χ_{29}	3	-3	-3	3	-3	3	3	-3	-3	-1	-1	3	-1	-3	3	1	1
χ_{30}	3	-3	-3	3	-3	3	3	-3	1	-1	-1	-1	3	1	-1	-3	1
χ_{31}	3	3	-3	3	3	-3	-3	-3	1	-1	1	-1	3	-1	1	-3	1
χ_{32}	3	3	-3	3	3	-3	-3	-3	1	3	-3	-1	-1	-1	1	1	-3
χ_{33}	3	3	-3	3	3	-3	-3	-3	-3	-1	1	3	-1	3	-3	1	1
χ_{34}	3	3	-3	3	3	-3	-3	-3	1	-1	1	-1	3	-1	1	-3	1
χ_{35}	3	3	-3	3	3	-3	-3	-3	1	3	-3	-1	-1	-1	1	1	-3
χ_{36}	3	3	-3	3	3	-3	-3	-3	-3	-1	1	3	-1	3	-3	1	1
χ_{37}	4	4	4	-4	-4	4	-4	-4	0	0	0	0	0	0	0	0	0
χ_{38}	4	4	4	-4	-4	4	-4	-4	0	0	0	0	0	0	0	0	0
χ_{39}	4	-4	4	-4	4	-4	4	-4	0	0	0	0	0	0	0	0	0
χ_{40}	4	-4	4	-4	4	-4	4	-4	0	0	0	0	0	0	0	0	0
χ_{41}	4	-4	-4	-4	4	4	-4	4	0	0	0	0	0	0	0	0	0
χ_{42}	4	-4	-4	-4	4	4	-4	4	0	0	0	0	0	0	0	0	0
χ_{43}	4	4	-4	-4	-4	-4	4	4	0	0	0	0	0	0	0	0	0
χ_{44}	4	4	-4	-4	-4	-4	4	4	0	0	0	0	0	0	0	0	0
χ_{45}	6	6	-6	6	6	-6	-6	-6	2	-2	2	-2	-2	-2	2	2	2
χ_{46}	6	-6	-6	6	-6	6	6	-6	2	-2	-2	-2	-2	2	-2	2	2
χ_{47}	6	-6	6	6	-6	-6	-6	6	-2	-2	2	-2	-2	2	2	-2	-2
χ_{48}	6	6	6	6	6	6	6	6	-2	-2	-2	-2	-2	-2	-2	-2	-2
χ_{49}	8	8	8	-8	-8	8	-8	-8	0	0	0	0	0	0	0	0	0
χ_{50}	8	-8	8	-8	8	-8	8	-8	0	0	0	0	0	0	0	0	0
χ_{51}	8	-8	-8	-8	8	8	-8	8	0	0	0	0	0	0	0	0	0
χ_{52}	8	8	-8	-8	-8	-8	8	8	0	0	0	0	0	0	0	0	0

Table 9.23 (continue)

Class	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
Size	6	6	6	12	12	12	12	12	12	12	12	32	12	12	12	12	24
Order	$2Q$	$2R$	$2S$	$2T$	$2U$	$2V$	$2W$	$2X$	$2Y$	$2Z$	$2AA$	$3A$	$4A$	$4B$	$4C$	$4D$	$4E$
$P=2$	1	1	1	1	1	1	1	1	1	1	1	29	8	8	8	8	12
$P=3$	18	19	20	21	22	23	24	25	26	27	28	1	30	31	32	33	34
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	-1
χ_3	1	-1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	1
χ_4	1	-1	1	-1	-1	-1	1	1	1	-1	1	1	1	1	-1	-1	-1
χ_5	-1	1	-1	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	1	1
χ_6	-1	1	-1	-1	1	1	1	1	-1	-1	-1	1	1	-1	-1	1	-1
χ_7	-1	-1	-1	1	-1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1
χ_8	-1	-1	-1	-1	1	1	-1	-1	1	-1	1	1	1	-1	1	-1	-1
χ_9	2	-2	2	0	0	0	0	0	0	0	0	-1	2	2	-2	-2	0
χ_{10}	-2	2	-2	0	0	0	0	0	0	0	0	-1	2	-2	-2	2	0
χ_{11}	-2	-2	-2	0	0	0	0	0	0	0	0	-1	2	-2	2	-2	0
χ_{12}	2	2	2	0	0	0	0	0	0	0	0	-1	2	2	2	2	0
χ_{13}	-1	3	3	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1
χ_{14}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	1
χ_{15}	3	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	1
χ_{16}	-1	3	3	1	1	1	1	1	1	1	1	0	-1	-1	-1	-1	1
χ_{17}	-1	-1	-1	1	1	1	1	1	1	1	1	0	-1	-1	-1	-1	-1
χ_{18}	3	-1	-1	1	1	1	1	1	1	1	1	0	-1	-1	-1	-1	-1
χ_{19}	1	1	1	1	-1	-1	1	1	-1	1	-1	0	-1	1	-1	1	-1
χ_{20}	-3	1	1	1	-1	-1	1	1	-1	1	-1	0	-1	1	-1	1	-1
χ_{21}	1	-3	-3	1	-1	-1	1	1	-1	1	-1	0	-1	1	-1	1	1
χ_{22}	1	1	1	-1	1	1	-1	-1	1	-1	1	0	-1	1	-1	1	1
χ_{23}	-3	1	1	-1	1	1	-1	-1	1	-1	1	0	-1	1	-1	1	1
χ_{24}	1	-3	-3	-1	1	1	-1	-1	1	-1	1	0	-1	1	-1	1	-1
χ_{25}	-3	-1	1	-1	1	1	1	1	-1	-1	-1	0	-1	1	1	-1	1
χ_{26}	1	-1	1	-1	1	1	1	1	-1	-1	-1	0	-1	1	1	-1	1
χ_{27}	1	3	-3	-1	1	1	1	1	-1	-1	-1	0	-1	1	1	-1	-1
χ_{28}	-3	-1	1	1	-1	-1	-1	-1	1	1	1	0	-1	1	1	-1	-1
χ_{29}	1	-1	1	1	-1	-1	-1	-1	1	1	1	0	-1	1	1	-1	-1
χ_{30}	1	3	-3	1	-1	-1	-1	-1	1	1	1	0	-1	1	1	-1	1
χ_{31}	-1	-3	3	1	1	1	-1	-1	-1	1	-1	0	-1	-1	1	1	1
χ_{32}	3	1	-1	1	1	1	-1	-1	-1	1	-1	0	-1	-1	1	1	-1
χ_{33}	-1	1	-1	1	1	1	-1	-1	-1	1	-1	0	-1	-1	1	1	-1
χ_{34}	-1	-3	3	-1	-1	-1	1	1	1	-1	1	0	-1	-1	1	1	-1
χ_{35}	3	1	-1	-1	-1	-1	1	1	1	-1	1	0	-1	-1	1	1	1
χ_{36}	-1	1	-1	-1	-1	-1	1	1	1	-1	1	0	-1	-1	1	1	1
χ_{37}	0	0	0	-2	-2	2	2	-2	-2	2	2	1	0	0	0	0	0
χ_{38}	0	0	0	2	2	-2	-2	2	2	-2	-2	1	0	0	0	0	0
χ_{39}	0	0	0	2	-2	2	-2	2	-2	-2	2	1	0	0	0	0	0
χ_{40}	0	0	0	-2	2	-2	2	-2	2	2	-2	1	0	0	0	0	0
χ_{41}	0	0	0	-2	2	-2	-2	2	-2	2	2	1	0	0	0	0	0
χ_{42}	0	0	0	2	-2	2	2	-2	2	-2	-2	1	0	0	0	0	0
χ_{43}	0	0	0	2	2	-2	2	-2	-2	-2	2	1	0	0	0	0	0
χ_{44}	0	0	0	-2	-2	2	-2	2	2	2	-2	1	0	0	0	0	0
χ_{45}	-2	2	-2	0	0	0	0	0	0	0	0	0	2	2	-2	-2	0
χ_{46}	2	-2	2	0	0	0	0	0	0	0	0	0	2	-2	-2	2	0
χ_{47}	2	2	2	0	0	0	0	0	0	0	0	0	2	-2	2	-2	0
χ_{48}	-2	-2	-2	0	0	0	0	0	0	0	0	0	2	2	2	2	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0

Table 9.23 (continue)

Class	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Size	24	24	24	24	24	24	24	24	24	24	24	32	32	32	32	32	32	32
Order	$4F$	$4G$	$4H$	$4I$	$4J$	$4K$	$4L$	$4M$	$4N$	$4O$	$4P$	$6A$	$6B$	$6C$	$6D$	$6E$	$6F$	$6G$
$P=2$	11	13	12	13	13	12	11	13	11	11	12	29	29	29	29	29	29	29
$P=3$	35	36	37	38	39	40	41	42	43	44	45	8	7	6	3	4	2	5
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1
χ_3	-1	1	-1	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	-1
χ_4	1	-1	1	-1	1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1
χ_5	1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1
χ_6	-1	1	1	-1	-1	-1	1	-1	1	1	1	1	-1	-1	-1	1	1	-1
χ_7	-1	-1	1	1	-1	1	1	-1	1	-1	-1	-1	1	-1	-1	1	-1	1
χ_8	1	1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
χ_9	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	-1	-1	1	1
χ_{10}	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	-1	-1	1
χ_{11}	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	1	-1	1	-1
χ_{12}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1	-1
χ_{13}	-1	1	-1	1	1	1	1	1	1	1	-1	0	0	0	0	0	0	0
χ_{14}	1	-1	1	1	1	-1	-1	-1	1	1	1	0	0	0	0	0	0	0
χ_{15}	1	1	1	-1	-1	1	1	1	-1	-1	1	0	0	0	0	0	0	0
χ_{16}	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1	0	0	0	0	0	0	0
χ_{17}	-1	1	-1	-1	-1	1	1	1	-1	-1	-1	0	0	0	0	0	0	0
χ_{18}	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	0	0	0	0	0	0	0
χ_{19}	1	-1	-1	-1	1	1	1	-1	-1	1	1	0	0	0	0	0	0	0
χ_{20}	1	1	-1	1	-1	-1	-1	1	1	-1	1	0	0	0	0	0	0	0
χ_{21}	-1	1	1	-1	1	-1	-1	1	-1	1	-1	0	0	0	0	0	0	0
χ_{22}	-1	1	1	1	-1	-1	-1	1	1	-1	-1	0	0	0	0	0	0	0
χ_{23}	-1	-1	1	-1	1	1	1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{24}	1	-1	-1	1	-1	1	1	-1	1	-1	1	0	0	0	0	0	0	0
χ_{25}	1	-1	-1	-1	-1	1	-1	1	1	1	-1	0	0	0	0	0	0	0
χ_{26}	1	1	-1	1	1	-1	1	-1	-1	-1	-1	0	0	0	0	0	0	0
χ_{27}	-1	-1	1	1	1	1	-1	1	-1	-1	1	0	0	0	0	0	0	0
χ_{28}	-1	1	1	1	1	-1	1	-1	-1	-1	1	0	0	0	0	0	0	0
χ_{29}	-1	-1	1	-1	-1	1	-1	1	1	1	1	0	0	0	0	0	0	0
χ_{30}	1	1	-1	-1	-1	-1	1	-1	1	1	-1	0	0	0	0	0	0	0
χ_{31}	-1	-1	-1	-1	1	-1	1	1	1	-1	1	0	0	0	0	0	0	0
χ_{32}	1	-1	1	1	-1	-1	1	1	-1	1	-1	0	0	0	0	0	0	0
χ_{33}	1	1	1	-1	1	1	-1	-1	1	-1	-1	0	0	0	0	0	0	0
χ_{34}	1	1	1	1	-1	1	-1	-1	-1	1	-1	0	0	0	0	0	0	0
χ_{35}	-1	1	-1	-1	1	1	-1	-1	1	-1	1	0	0	0	0	0	0	0
χ_{36}	-1	-1	-1	1	-1	-1	1	1	-1	1	1	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	-1	1	1
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	1	-1	1	1
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	-1	1
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	-1	-1	-1	1
χ_{41}	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	-1	1	-1
χ_{42}	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	-1	-1	1	-1
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	-1	-1	-1
χ_{44}	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1	-1	-1	-1
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-1	1	-1	-1
χ_{50}	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	1	1	-1
χ_{51}	0	0	0	0	0	0	0	0	0	0	0	1	-1	-1	1	1	-1	1
χ_{52}	0	0	0	0	0	0	0	0	0	0	0	-1	-1	1	-1	1	1	1

Table 9.24: The Character table of H_4

Class	1	2	3	4	5	6	7	8	9	10	11
Size	1	15	15	45	40	40	90	90	144	120	120
Order	$1A$	$2A$	$2B$	$2C$	$3A$	$3B$	$4A$	$4B$	$5A$	$6A$	$6B$
$P = 2$	1	1	1	1	5	6	4	4	9	5	6
$P = 3$	1	2	3	4	1	1	7	8	9	2	3
$P = 5$	1	2	3	4	5	6	7	8	1	10	11
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	-1	-1	1	1	1	-1	1	1	-1	-1
χ_3	5	-3	1	1	2	-1	-1	-1	0	0	1
χ_4	5	-1	3	1	-1	2	1	-1	0	-1	0
χ_5	5	1	-3	1	-1	2	-1	-1	0	1	0
χ_6	5	3	-1	1	2	-1	1	-1	0	0	-1
χ_7	9	3	3	1	0	0	-1	1	-1	0	0
χ_8	9	-3	-3	1	0	0	1	1	-1	0	0
χ_9	10	-2	2	-2	1	1	0	0	0	1	-1
χ_{10}	10	2	-2	-2	1	1	0	0	0	-1	1
χ_{11}	16	0	0	0	-2	-2	0	0	1	0	0

The conjugacy classes of $SP(6, 2)$

Table 9.25: The conjugacy classes of $SP(6, 2)$

$[g]_{SP(6,2)}$	Class representative	$\lvert[g]_{SP(6,2)}\rvert$	$[g]_{SP(6,2)}$	Class representative	$\lvert[g]_{SP(6,2)}\rvert$
$1A$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \end{pmatrix}$	1	$2A$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&1&1&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 1&0&0&1&0&1 \end{pmatrix}$	63
$2B$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 1&1&0&0&0&0 \\ 1&0&1&0&0&0 \\ 1&0&0&1&0&0 \\ 1&0&0&0&1&0 \\ 0&1&1&1&1&1 \end{pmatrix}$	315	$2C$	$\begin{pmatrix} 1&1&1&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&1&1&0&1 \\ 0&1&0&0&1&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	945
$2D$	$\begin{pmatrix} 0&1&0&1&1&1 \\ 1&1&0&0&0&1 \\ 1&0&1&0&0&1 \\ 0&1&1&1&0&0 \\ 1&1&1&0&1&1 \\ 1&1&0&1&1&0 \end{pmatrix}$	3780	$3A$	$\begin{pmatrix} 0&1&1&0&1&1 \\ 0&1&1&0&1&0 \\ 0&0&1&0&0&0 \\ 0&0&1&1&1&0 \\ 1&1&1&0&0&1 \\ 1&1&1&0&1&0 \end{pmatrix}$	672
$3B$	$\begin{pmatrix} 0&0&1&0&1&1 \\ 0&1&1&1&0&1 \\ 1&1&0&0&1&0 \\ 0&1&0&1&1&1 \\ 1&0&1&1&0&0 \\ 1&1&0&1&0&1 \end{pmatrix}$	2240	$3C$	$\begin{pmatrix} 0&1&1&1&1&1 \\ 0&0&1&1&0&1 \\ 1&1&1&0&1&0 \\ 1&1&0&1&1&0 \\ 1&0&0&0&1&1 \\ 0&1&0&0&0&1 \end{pmatrix}$	13440
$4A$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 1&1&1&0&1&0 \\ 0&1&1&1&0&0 \\ 0&0&1&1&1&0 \\ 1&1&0&1&1&0 \\ 1&0&1&1&0&1 \end{pmatrix}$	3780	$4B$	$\begin{pmatrix} 1&0&1&0&0&1 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&1&1&0&1 \\ 0&0&0&0&0&1 \end{pmatrix}$	7560
$4C$	$\begin{pmatrix} 0&1&0&0&1&1 \\ 1&0&0&0&1&1 \\ 0&1&1&1&1&1 \\ 0&1&1&0&1&1 \\ 1&1&0&0&0&1 \\ 1&0&0&1&0&1 \end{pmatrix}$	7560	$4D$	$\begin{pmatrix} 1&0&0&0&0&0 \\ 1&0&0&1&0&0 \\ 1&1&0&0&1&0 \\ 0&1&0&0&0&0 \\ 1&0&1&1&0&0 \\ 0&1&1&0&0&1 \end{pmatrix}$	11340

Table 9.25 (continue)

$[g]_{SP(6,2)}$	Class representative	$\lvert[g]_{SP(6,2)}\rvert$	$[g]_{SP(6,2)}$	Class representative	$\lvert[g]_{SP(6,2)}\rvert$
$4E$	$\begin{pmatrix} 0&1&1&0&1&1\\ 1&0&1&0&1&1\\ 0&1&0&1&0&0\\ 1&0&0&0&1&1\\ 1&1&1&1&1&0\\ 0&1&0&0&0&1 \end{pmatrix}$	45360	$5A$	$\begin{pmatrix} 0&1&0&0&1&0\\ 0&1&1&1&1&0\\ 1&1&0&0&1&1\\ 1&1&1&1&1&0\\ 1&1&0&1&1&1\\ 1&1&0&1&0&0 \end{pmatrix}$	48384
$6A$	$\begin{pmatrix} 0&1&0&0&0&1\\ 1&1&1&1&1&1\\ 0&1&0&1&0&0\\ 1&0&0&0&1&0\\ 0&1&0&1&0&1\\ 0&0&1&0&1&0 \end{pmatrix}$	10080	$6B$	$\begin{pmatrix} 1&1&1&0&0&0\\ 1&1&0&0&1&1\\ 0&0&1&0&0&0\\ 1&0&1&1&1&0\\ 0&1&0&0&1&1\\ 1&0&0&0&1&0 \end{pmatrix}$	10080
$6C$	$\begin{pmatrix} 0&0&1&1&1&0\\ 0&1&1&0&1&0\\ 1&0&1&1&1&0\\ 1&1&1&0&1&1\\ 1&1&1&0&0&0\\ 1&1&1&1&0&1 \end{pmatrix}$	20160	$6D$	$\begin{pmatrix} 0&0&0&1&1&1\\ 1&0&1&1&1&0\\ 1&1&0&1&1&0\\ 1&1&0&0&1&1\\ 1&0&1&1&0&1\\ 1&1&1&1&1&1 \end{pmatrix}$	30240
$6E$	$\begin{pmatrix} 1&1&0&0&1&0\\ 1&1&1&1&0&1\\ 1&0&0&1&1&1\\ 1&1&1&1&1&0\\ 1&0&1&1&0&0\\ 1&1&1&0&1&1 \end{pmatrix}$	40320	$6F$	$\begin{pmatrix} 1&1&0&0&0&0\\ 1&0&1&0&1&1\\ 1&0&0&0&1&1\\ 0&1&0&1&0&0\\ 1&0&1&1&1&1\\ 1&0&0&0&0&1 \end{pmatrix}$	40320
$6G$	$\begin{pmatrix} 1&0&1&0&1&1\\ 1&1&0&0&1&0\\ 1&0&0&1&0&0\\ 0&1&0&0&0&1\\ 0&1&0&1&0&0\\ 0&0&0&1&0&0 \end{pmatrix}$	120960	$7A$	$\begin{pmatrix} 0&1&0&0&1&0\\ 1&0&0&1&0&1\\ 0&1&1&0&1&1\\ 1&0&1&0&0&0\\ 1&0&1&1&0&1\\ 1&1&1&0&0&0 \end{pmatrix}$	207360
$8A$	$\begin{pmatrix} 1&0&0&0&0&0\\ 1&1&0&1&1&0\\ 1&0&0&1&0&0\\ 1&1&1&1&0&0\\ 0&1&0&0&0&0\\ 1&0&1&0&0&1 \end{pmatrix}$	90720	$8B$	$\begin{pmatrix} 0&0&1&1&1&0\\ 0&0&1&1&0&1\\ 1&1&0&1&1&1\\ 1&1&0&1&0&1\\ 1&0&0&0&1&0\\ 1&0&0&1&1&0 \end{pmatrix}$	90720

Table 9.25 (continue)

$[g]_{SP(6,2)}$	Class representative	$\vert[g]_{SP(6,2)}\vert$	$[g]_{SP(6,2)}$	Class representative	$\vert[g]_{SP(6,2)}\vert$
$9A$	$\begin{pmatrix} 0&1&1&1&0&0\\ 1&1&0&1&1&1\\ 0&1&1&0&1&1\\ 0&0&1&0&1&0\\ 0&1&1&1&0&1\\ 1&1&1&1&1&0 \end{pmatrix}$	161280	$10A$	$\begin{pmatrix} 1&0&0&1&1&0\\ 0&0&1&0&1&1\\ 1&1&0&0&1&1\\ 0&0&0&0&1&0\\ 0&0&1&1&1&1\\ 1&0&0&1&0&1 \end{pmatrix}$	145152
$12A$	$\begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&1&1&1\\ 1&0&1&1&1&1\\ 1&0&1&1&0&0\\ 1&1&1&0&1&1\\ 1&1&1&1&1&0 \end{pmatrix}$	60480	$12B$	$\begin{pmatrix} 1&1&0&1&0&0\\ 0&1&1&1&0&1\\ 1&0&0&1&0&0\\ 0&0&0&1&1&1\\ 0&0&0&1&0&0\\ 1&1&0&0&1&0 \end{pmatrix}$	60480
$12C$	$\begin{pmatrix} 0&1&0&1&0&0\\ 1&1&0&1&0&0\\ 1&0&0&1&0&0\\ 1&1&1&0&1&0\\ 1&1&1&1&1&1\\ 1&1&1&0&0&1 \end{pmatrix}$	120960	$15A$	$\begin{pmatrix} 1&0&0&1&0&0\\ 1&0&0&0&0&0\\ 1&0&0&0&1&0\\ 0&1&0&1&1&0\\ 0&1&1&0&0&1\\ 0&0&1&1&1&0 \end{pmatrix}$	96768