An analysis of learners' ways of working in high stakes mathematics examinations: *Quadratic equations and inequalities*

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A thesis submitted in partial fulfilment for the degree Magister Educationis in the Faculty of Education, University of the Western Cape.

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Declaration

I declare that ***an analysis of learner's ways of working in high stakes mathematics examinations:*** *Quadratic equations and inequalities* is my own work, that it has not been submitted before for any degree or examination in any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

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Date: May 2012

Signed

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This study is dedicated to my parents, Mr. and Mrs. T. W. Godden.

Abstract

Every year there is a national outcry by educationists in South Africa relating to the poor performance of grade 12 mathematics learners. This is an unsatisfactory state of affairs in a country where mathematics is seen as playing a pivotal role in the preparation of students in disciplines for careers in science and technology. Interventions by the Department of Basic Education as well as by provincial education departments do not seem to be successful in stemming the tide in the decline of standards in the mathematical performance of learners. It is this which has motivated this study.

The aim of this study was to identify the types of errors committed by students in their responses to question one of Paper 1 in the final Grade 12 mathematics examinations of 2010. By reviewing the work done by different authors, an analytical framework was compiled that was used to identify and to label errors in the written responses of learners. This study has adopted a documentary analysis approach and has selected a representative sample of examination scripts of Western Cape students who wrote the first paper of the grade 12 mathematics examination in 2010.

The result shows that during the analysis of 1959 scripts (the sample taken from different educational departments and districts), 4163 errors were identified. These errors have been identified based on the above-mentioned analytical framework. Comparisons were labeled according to the different types of errors, across the different ex- department schools and per districts. If the percentage is calculated based on the number of scripts analyzed, the number of errors varies from 12% careless errors to 40% calculation errors. It was also noticed that the number of errors found in the urban districts was higher than all of the errors found in the different rural districts. In return, in the urban districts, the numbers of errors found were evenly distributed over the four districts. When the different ex-department schools were compared, the numbers of application and procedural errors were significantly higher in the DET schools (Department of Education and Training, which consists of mostly black disadvantaged learners) than all the other schools. The rest of the errors were evenly distributed over all the ex-department schools. The inability of learners solving inequality equations stood out as a major concern.

Key words

Alternative ways of working

Assessment

Errors

Misconceptions

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CHAPTER 1

1.1 Introduction

Currently in South Africa there exists, and rightly so, concern about the poor performance of grade 12 learners in the area of Mathematics. From 2008 to 2010, the number of learners that wrote mathematics during the grade 12 examination decreased by 40 000 candidates nationally, even though the total number of learners increased by more than 50 000. The 2010 mathematics results showed that at the 30% mark, only 47% of learners had passed. Also, only 24 % obtained a university entrance or 50% pass. The Mathematics examination paper consists of about 40% routine procedures and even though the pass requirement for Mathematics in grade 12 is only 30%, the failure rate in total is still over 50% (Department of Basic Education, 2011).

To address the poor results in Mathematics, the Western Cape Education Department (WCED) has implemented many programmes and activities to improve the pass rate in the final examination, but with little success. As a Mathematics educator at a previously disadvantaged school, I am also striving to improve the results of my learners but I have found that learners hand in examination and test scripts without checking their answers. The consequence of this is that learners losing marks due to unnecessary mistakes they make that could have been rectified if they had revisited there scripts.

In spite of these interventions, the concern is that the Mathematics results are still not improving. This study highlights the misconceptions which learners have in mathematics and errors which they commit in high-stakes examinations. The analysis of these errors will hopefully provide the insight which will assist learners minimize the future occurrence of these errors.

1.2 Motivation

According to the Assessment Guidelines for Mathematics in South Africa, one of the general purposes of assessment is to revisit or revise those sections of the mathematics curriculum where learners have difficulties (Curriculum and Assessment Policy, 2012).

This is extended by Niss (1993, p. 9), when he states the following, “to understand and master Mathematics there is a need to provide genuine assistance to each individual learner in monitoring and improving his or her acquisition of mathematical insight and power”. Provision should be made to assess, in a valid and reliable way, the knowledge, insights, abilities and skills related to the understanding of Mathematics in its essential aspects (Niss, 1993).

He further states that feedback to the educator may serve several purposes:

- to enable the educator to inform and advise the student;
- to assist the educator in assessing his or her teaching and its outcomes in relation to that particular learner, in order to adjust, develop, or fundamentally change it to what would better meet the needs of the learner;
- to take decisions and actions to influence the learner’s behaviour;
- to provide a basis for the educator’s reporting to the parents, the school, the authorities, further educational institutions, and employers of the performance of the individual student (Niss, 1993, p. 7).

The Western Cape Education Department (WCED) has implemented a number of strategies to improve results in general and Mathematics in particular. As a strategy, feedback is given to educators and learners that focus on the analysis of learner responses in examinations. However, this distribution of statistical information only focuses on the pass and failure rates of learners. A tabulated analysis pertaining to learner performance is completed and disseminated to each school after the examination. It serves to illustrate to educators which questions were answered well and which were not. The concern with this kind of response or feedback is that it does not necessarily deal with the reasons why learners do badly. It merely states that learners did not answer specific questions satisfactorily or well.

The table is primarily statistically-based and does not provide educators with information as to how learners go about answering questions. Such feedback does not indicate the ways of working of learners nor does it indicate how they interacted with the text. Also, it does not identify which errors and misconceptions were evident in the written responses of the learners.

The following is an example of such a statistical analysis of a specific school.

Table 1.1: Question analysis of final Mathematics (Nov 2009) Paper 1 & 2.

MATHEMATICS (Negative (-) difference is below provincial average)										Paper Av. Diff.
Questions	1	2	3	4	5	6	7	8	9	
Paper 1	6.9	-18.9	-6.5	-7.5	24.8	-0.1	-2.8			0.8
Number of learners	11	11	10	11	11	11	11			11
Paper 2	-8.5	-5.2	-7.6	-2.2	-9.1	-11.1	-14.1	-11.5	-7.5	-7.6
Number of learners	11	11	11	11	10	11	10	10	10	11

The tabulated analysis gives a clear indication of the learners' performance in each question in relation to the provincial average. The negative numbers show that the learner's performances in the different questions are below the provincial average. A positive number will indicate a performance above provincial average. In questions 5, 7, 8 and 9 in paper two the number of learners who attempted to answer the specific questions was less than 11. This indicates that in those questions not all the learners attempted the question.

According to the above table, as provided by the WCED, there is an emphasis on the mistakes made by learners in individual questions, rather than a diagnostic approach. This clearly is an area of concern since it does not effectively contribute to the learners' future performance in that it does not provide educators with the necessary tools to understand the root of the problem. The problem worsens when Mathematics teachers are only concerned with students acquiring facts and performing skills prescribed by the syllabus rather than being concerned about broader educational goals (Boris, 2003).

After every final examination, the WCED also releases a moderator's report which gives a more detailed analysis of how each question was answered by learners. It should be noted that this report is based on a sample of 100 randomly selected scripts. However, this report does not identify the errors made by learners.

Another method implemented by the Western Cape Education Department is visits by curriculum advisors which are intended to boost the Mathematics results. Unfortunately their visits only serve to basically check if the syllabus is being taught and implemented by the educators involved. Their input is purely in an advisory capacity and does not deal with the in-depth analysis of the written responses of learners in Mathematics.

A third method used by the WCED in their attempt to improve the pass rate of grade 12 Mathematics learners is curriculum contraction. Curriculum contraction involves the complete removal of topics or relegation to a non-compulsory status. For learners this means that the particular topics become part of an optional examination paper, which is, in this case, known as Paper 3. The designation of Euclidean Geometry as an optional paper is an example of such curriculum contraction in the National Curriculum Statement (NCS). Currently there are moves afoot to remove linear programming from the curriculum in the Curriculum and Assessment Policy Statement (CAPS) which was implemented at grade 10 levels in 2012.

This study subscribes to the notion of feedback as stated by Niss (1993) above, where it takes on a much more holistic approach. The feedback given by WCED does not fulfil this purpose because it does not supply assistance to the individual learner. This research which involves the analysis of errors and misconceptions and the reasons for these errors, fills this gap and provides a detailed analysis of the errors made in high-stakes examinations in the question on quadratic equations and inequalities.

During a study at the International Commission on Mathematical Instruction (ICMI), Niss (1993), states that by giving additional information to students, one provides them with assistance in monitoring and controlling their learning activity and behaviour. Normally, there are consistent patterns in errors. Hence, the most valuable clues to the nature of human information processing can be found by studying and analysing those errors.

The importance of identifying learner's errors is especially important for learners with learning disabilities and low performing learners (Fuchs, Fuchs, & Hamlett, 1990).

This study focuses on analysing the ways in which learners work in the final Mathematics examinations. It concentrates on summative assessment as it is put forward in the National Curriculum Statement (NCS) Assessment Guidelines of 2008.

1.3 Aim of this Study

The aim of this study is to identify how learners process and solve mathematical problems posed to them in their final examinations by identifying the errors exhibited in their responses to question one in the final Grade 12 Mathematics Paper 1 examinations. This is done to give comprehensive feedback to educators and learners in order to improve the teaching and learning of Mathematics.

The question one that is referred to will include the following sub-questions as laid down in the November Paper one of 2010.

Question 1

1.1 Solve for x, correct to TWO decimal places, where necessary:

1.1.1 $(3 - x)(5 - x) = 3$

1.1.2 $3x^2 = 2(x + 2)$

1.1.3 $4 + 5x > 6x^2$

1.2 Solve for x and y simultaneously:

$3y = 2x$

$x^2 + y^2 + 2x - y = 1$ (DBE/November 2010).

According to the Curriculum and Assessment Policy (2012) of the Department of Basic Education, question one falls under Algebra, and makes up the learning outcome of 11.4.4, this reads:

Solve:

- quadratic equations (by factorization, by completing the square, and by using the quadratic formula);
- quadratic inequalities in one variable and interpret the solution graphically;

- equations in two unknowns, one of which is linear the other quadratic, algebraically or graphically (Curriculum and Assessment Policy, p.39, 2012).

This learning outcome forms the basis to answer questions in 8 out of the 10 main topics in the Mathematics (FET) Curriculum, namely:

- Functions,
- Number patterns, sequences and series
- Finance, growth and decay
- Algebra
- Differential calculus
- Euclidean geometry and measurement
- Analytical geometry
- Trigonometry (Curriculum and Assessment Policy, p. 7, 2012).

The decision to choose this specific question for the research was strengthened by the fact that the cognitive level for this learning outcome only involves routine procedures, which total 40% of all tasks, tests and examinations in the grade 12 syllabus for Mathematics (Curriculum and Assessment Policy, 2012).

1.4 Research Questions

The following is the question which this study will seek answers to:

What errors are detectable in the written responses of learners for the solution of quadratic equations and inequalities in the National Senior Certificate Mathematics examination in 2010?

1.5 Key Terms of Study

1.5.1 Assessment

The research deals with assessment, which is thus one of the key terms. In Mathematics education, assessment refers to the judging of mathematical capability, performance and achievement of learners (Niss, 1993). This study aimed to enhance and support the guidelines as set out in the National Curriculum Statement Assessment Guidelines for General Education and Training in Mathematics in 2008.

The National Curriculum Statement puts forward some purposes of assessment needed to achieve the following:

- Enable teachers to reflect on their practice
- Identify learners' strengths and weaknesses
- Provide additional support
- Revisit or revise certain sections where learners seem to have difficulties

In support of the above, my focus was on diagnostic assessment as defined by the National Curriculum Statement and which reads as follows:

Diagnostic assessment is used to identify, scrutinize and classify learning difficulties so that appropriate remedial help and guidance can be provided. It should be administered by specialists and is followed by expert guidance, support and intervention strategies.

1.5.2 Errors

During the analyses of errors in mathematical tasks, which were described during the 39th meeting of the International Commission for the study and Improvement of Mathematics teaching: "an error takes place when a person chooses the false as the truth" (Booker, 1988).

Davis (1984) described errors in general as mistakes made by learners, preventing them from reaching their set outcomes.

The following is an example that Matz (1980) refers to as the Zero Product Principle:

After factorising, learners learn to solve $(x-2)(x-3)=0$ by either letting $x-2=0$, in which case $x=2$, or letting $x-3=0$, in which case $x=3$ but the error below is caused by the principle that the zero product rule is not used where the learner did not write the equation in the form $ax^2+bx+c=0$.

The following error is commonly found when my learners solve quadratic equations. Some of my learners will solve the following expression by not rewriting the equation into the standard form: $x^2-5x+6=2$, after factorising they get $(x-2)(x-3)=2$.
Therefore, either $x-2=2$, in which case $x=4$, or else $x-3=2$, in which case $x=5$.

1.5.3 Misconceptions

Misconceptions are the next key concept, described by Oliver (1992), as conceptual structures that will interact with new concepts, and influence new learning, mostly in a negative way. He further goes on by concluding that in most cases, misconceptions are the cause of errors. According to Davis (1984), misconceptions arise from background knowledge gained from experiences which are stored in the passive memory. Learners' misconceptions are the result of their efforts to construct their own knowledge which result in misconceptions which are intelligent constructions based on correct or incomplete (but not wrong) background knowledge. Misconceptions, therefore, cannot be avoided.

Elbrink (2008) also concludes that if misconceptions are not addressed, learners will try to build their knowledge base of Mathematics on misunderstood concepts, which leads to learners committing errors.

To get a clearer understanding of why learners have these misconceptions, it would be ideal to interview the specific learner, but the final examination is structured in such a way that it is impractical, within our education context, to obtain such an interview.

A common case of misconception that was referred to in more than one study by Rosnick (1981) reads as follows:

Use "S" for the number of learners and "P" for the number of professors. Write down an equation where there are six (6) times as many learners as professors. The correct answer is S = 6P.

Learners answered 6S = P.
It's evident that learners confused the variables as labels where "S" stands for learners and "P" stands for professors, rather than interpreting them correctly as variables that are containers of numerical values (Rosnick, 1981).

1.5.4 Alternative ways of working.

There are learners who use "non-standard" methods to solve problems, but still manage to get the correct answer. This brings up the next key concept, namely, alternative ways of working.

The same type of example which Matz (1980) refers to as the zero product principle is used below:
To solve for x in the equation $-x^2 + x + 6 = 4$, the next step is $-x^2 + x + 6 - 4 = 0$. After factorising, learners learn to solve $(2 - x)(x + 1) = 0$ by either $2 - x = 0$, in which case $x = 2$, or else $x + 1 = 0$, in which case $x = -1$.

A non-standard method to solve the following expression by not using the zero product principle and still obtain the correct answer:

$-x^2 + x + 6 = 4$, after factorising they get $(3 - x)(x + 2) = 4$.

They put $3 - x = 4$, in which case $x = -1$, or else $x + 2 = 4$, in which case $x = 2$.

In these examples learners use mathematically incorrect methods but end up with correct answers. Learners need to discuss these methods to understand why they render correct answers.

1.6 Organization of the Study

This study is organized into five chapters.

- Chapter 1 deals with the introduction and motivation of the study.
- Chapter 2 discusses the literature review which includes explanation on quadratic theory and literature on errors supported by examples. These errors are then compiled in a conceptual framework to be used as a guide for the identification of errors in learners' written responses.
- Chapter 3 focuses on the research methodology, sampling and data analysis.
- Chapter 4 deals with the presentation and analysis of the collected data against research question.
- Chapter 5 focuses on the conclusion of the study and possible recommendations that will emerge from this study.

Chapter 2

Literature Review

2.1 Introduction

According to Borasi (1996), the concept of error-making in education has not yet received adequate analysis and consequently the education community has so far failed to find innovative ways of using errors constructively in formal instruction. He goes further by stating that "errors can be used as a tool to identify learning difficulties and to plan curriculum and teaching material accordingly, or more generally, as a means to understand students' conceptions and learning processes" (Borasi, 1996, p. 3).

The purpose of this chapter is to present a review of the work done on errors in Mathematics in an attempt to better understand the different types of errors, and consequently create an analytic framework which will be utilized to identify errors and errors caused by misconceptions. The discussion/review will focus on quadratic theory as prescribed in the national curriculum statement of grade 12 Mathematics in South Africa.

2.2 Quadratic theory

The earliest evidence of solving quadratic equations can be traced back to the ancient Babylonians. The proof for solving quadratic equations was found between 1800 BC and 1600 BC on clay tablets. The Babylonians were also the first people to use the method of completing the square to solve quadratic equations with positive roots, but did not have a general formula. The first mathematician to find negative solutions with the general algebraic formula ($x = \frac{\sqrt{4ac+b^2}-b}{2a}$, for finding one solution of the equation $ax^2 + bx + c = 0$) was Brahmagupta (India, 7th century).

He was an Indian mathematician-astronomer, who solved quadratic equations with more than one unknown and is considered the originator of the equation.

Shidhara (India, 9th century) was one of the first mathematicians to give a general rule for solving quadratic equations (www.mytutoronline.com/history-of-quadratic-equation).

A quadratic function is a type of polynomial function which is a function that can be expressed in the form:

$y = p_n(x) = a_n x^n + a_{n-1}x^{n-1} + \dots a_1 x + a_0$ for all x in $\boldsymbol{R}$

For $n = 2$ we get a quadratic polynomial

$y = p_2(x) = a_2 x^2 + a_1 x^1 + a_0$

The general form in which a quadratic polynomial appears in school Mathematics is as

$y = f(x) = ax^2 + bx + c,\ where\ \ a, b\ and\ c\ \ are\ real\ numbers\ with\ a \neq 0$

A quadratic function in *x* is also called a second-degree polynomial function in x.

Currently, there are different methods for solving quadratic equations:

2.2.1 Factorization

The equation is first written in the standard form $y = ax^2 + bx + c$, for any real numbers a, b, c where $a \neq 0$. Factored quadratic equations can be solved using the zero product principle. If the product of two numbers (variables, algebraic expressions) $A * B = 0$, then $A = 0$ or $B = 0$ or A and B are both 0.

Factoring is an important process in algebra which is used to simplify expressions, simplify fractions, and solve equations. For the quadratic function, of a real variable x, the coordinates of the points where the graph intersects the x-axis are the solutions of the quadratic equation.

2.2.2 Square root method

If u is an algebraic expression and d is a positive real number, then the equation $u^2 = d$ has exactly two solutions: $u = +\sqrt{d}$ and $u = -\sqrt{d}$.

But, if the equation $u^2 = d$, where $d < 0$ then the equation has no real solutions.

This method is mostly used to solve quadratic equations.

2.2.3 Completing the square

This method is done by transforming the trinomial to a perfect square by adding $(\frac{1}{2}\text{ co}-\text{efficient of x})^2$ to both sides of the equation. Write the perfect square and simplify the constant. In addition to finding roots, completing the square is also used for transforming an equation in standard form ($ax^2 + bx + c = 0$) to vertex form$[a(x-d)^2 = k\ ]$. This method is used to solve equations and to graphing quadratic functions. Completing the square will find the roots of a quadratic equation even when those roots are irrational or complex.

2.2.4 Using the quadratic formula

A quadratic equation with real or complex coefficients has two solutions. These two solutions may or may not be distinct, and they may or may not be real.

Given a quadratic equation in standard form, the solutions can be found by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The learners also have the option to use the quadratic formula to solve a quadratic equation.

More information about the nature of the roots of a quadratic equation can be obtained from studying the value of the Discriminant ($\Delta = \sqrt{b^2 - 4ac}$) along with those of 'a', 'b' and 'c' in the quadratic equation (in the standard form, $ax^2 + bx + c = 0$, where $a \neq 0$) as follows:

If a, b, and c are real numbers, then:

- If $\Delta < \mathbf{0}$, the roots are a pair of complex conjugates. This means that the roots are complex numbers, and they are conjugates of each other.
- If $\Delta = 0$, the roots are real and equal.
- If $\Delta > 0$, the roots are real and unequal.

If a, b, and c is rational numbers, then:

- If $\Delta < 0$, the roots are a pair of complex conjugates.
- If $\Delta = 0$, the roots are rational and equal.
- If $\Delta > 0$ and Δ is a perfect square, the roots are rational and unequal.
- If $\Delta > 0$ and Δ is not a perfect square, the roots are irrational and unequal (Germeshuizen, 2007).

Considering that a few methods are taught in grade 12 to solve quadratic equations, the errors made in solving these equations should lead to fewer errors caused by misconceptions.
The lack of feedback to the learners might be one of the major contributors to the poor performance in solving quadratic equations.

2.3 Feedback

Giving and getting feedback is essential to the whole process of learning. It is a two-way process. Educators should give learners feedback on their performance and learning, and in return learners should reflect on their experience of learning and give feedback in return. There are immediate benefits from this. Learners who are given good feedback on their learning are more likely to:

- make decisions and solve problems for themselves
- learn from their mistakes
- see learning as positive – something they are involved in rather than something that is done for them
- think about their own needs and development
- consult others or work as part of a team when learning (Dweck, 2003).

Learners are likely to gain confidence if they get regular feedback on their progress. Taking the time to correct or congratulate a learner with a task, no matter how small, helps to foster a sense of achievement within the learner thus conditioning learners to want to master their skills (Dornyei, 2001).

It is also important to know what the learners think. It is not enough to just give learners feedback on their progress and performance. Educators should encourage learners to give feedback on their own experience of learning.

Educators who see their relationship with learners as a partnership place learners at the centre of their own learning. It is all part of encouraging learners to make their own, informed decisions with appropriate help and guidance. A good way of finding out what learners think is by asking them to reflect on what they have been doing. It does not have to be a drawn-out process, rather just an attempt to help them reflect on their own actions. Simple questions like the following can pave the way to more effective learning:

- Was there anything you found difficult, or would do differently next time?

- How could we help you in a better way?

In order to eliminate errors and errors caused by misconceptions, feedback is a valuable tool used to rectify and eliminate such mistakes. The next section deals with different types of errors identified in the literature.

2.4 Types of errors

2.4.1 Careless Errors

This type of error can be identified automatically upon reviewing one's work (Hodes, 1998, Mason, 2000). This includes missing answers, changing answers from the correct ones to the incorrect ones and miscopying.

Davis (1984) refers to the same type of error as a “slip” or “silly” mistake. Clemens (1982) deemed a careless error to be an error that was made on one occasion but not on another similar occasion. Nischal (2007) concurs but only if it satisfies the following two conditions:

- The error is not a result of a conceptual misunderstanding in the student and
- It is not repeated in every attempt of the solution.

This example of a careless error is taken from one of my learners. While busy with a question the error was made while solving for x:

$$x^2 - x - 6 = 0 \qquad (1)$$

$$(x - 3)(x + 2) = 0 \qquad (2)$$

$$x - 3 = 0 \quad and \quad x - 2 = 0 \qquad (3)$$

The careless error is committed when the equation $x + 2 = 0$ in step (2) was miscopied in the next step (3) and written as $x - 2 = 0$ while in the process of solving x.

2.4.2 Procedural errors

Elbrink (2008) refers to procedural errors when a learner computes or applies a procedure incorrectly. Hodes (1998) refers to these errors as concept errors, where the learner does not understand the properties or principles.

Orton (1983) concurs with the previous definitions by referring to this error as structural errors, which arise from a failure to appreciate the relationship involved in the problem or to grasp some principle essential to the solution. Mason (2000) also makes mention of errors where learners have difficulties with concepts and Davis (1984) simply named his error with defective procedure impeding mathematical logic, as procedural errors.

There are two errors mentioned by Matz (1980), namely: re-directed errors and linear extrapolation errors, which are categorized by the author as “an error that goes hand - in - hand with control errors and procedural errors” (p.45).

An algebraic manipulation error is referred to by Mason (2000), as an error that was created when learners perform an unheeded and incorrect act. Two errors mentioned by Brodie and Berger (2010), namely halting signal and keyword trigger, can also be incorporated as procedural errors. A halting signal is classified if the learner selects an answer which is the answer to an intermediate (not final) step of a procedure.

The second error referred to be a “keyword trigger”, which distracts the learner to follow the wrong procedure in answering the question.

All the above errors occur when a learner computes or applies a procedure incorrectly. These errors will be collectively referred to as procedural errors. The following is an example of a procedural error:

A learner solves, $x^2 + 2x - 3 = 0$ as follows,

$(x - 1)(x + 3) = 0$ (step a),

$x = 1$ or $x = 3$ (step b),

Step (a) was correctly performed but in step (b) the rule for the procedure for solving the second linear equation was not applied. When solving $(x + 3) = 0$ the next step is $x + 3 = 0$ which lead to $x = -3$.

2.4.3 Calculation errors

According to Elbrink’s (2008) calculation errors can be generalized as mistakes in addition, subtraction, multiplication and division of numbers. Calculation errors will only include arithmetic calculations involving the above mentioned four basic operations. The above referenced author did not put forward any examples to explain the errors; hence my own example from learners at my school is used to clarify the specific error.

Solve: $x^2 - x = -4 - 3$ (1)

$x^2 - x = -1$ (2)

$x^2 - x + 1 = 0$ (3)

A calculation error is found in (2) $(-4 - 3 \rightarrow -1)$ of the equation, yet the third (3) part of simplifying, transposing the terms to the left hand side of the equal sign, is done correctly. The correct procedure is followed when answering the question, but the calculation was wrong. Calculation errors will only focus on the basic operation.

2.4.4 Application errors

Application errors are defined by Hodes (1998) as mistakes made by learners when they know the concept but cannot apply it to a specific situation or question. Orton (1983) refers to errors which involve failure to execute the procedures correctly, though the principles involved may have been understood. Orton (1983) names these errors as executive errors.

A learner may see that a particular information-processing act is incorrect within a given context, and yet continues to make the same error repeatedly. Davis (1984) has named this control error. These errors will be referred to as application errors.

Davis (1984) supplied one of the most common examples of an application error in Mathematics. The learner is asked to re-write $x(y + z)$, many learners will write

$$x(y + z) \rightarrow xy + z$$

This is, of course, wrong, the correct re-writing being

$$x(y + z) \rightarrow xy + xz$$

2.4.5 Symbolic errors

Symbolic errors occur when learners falsely relate mathematical problems that use similar symbols. Learners try to create meaning in the patterns of mathematical symbols and signs that they see in front of them rather than trying to understand what they are actually doing or mean. Elbrink (2008) refers to "Linear extrapolation errors that are probably grounded in an overgeneralization of the distributive property, which learners often encounter in arithmetic and in introductory algebra" (Matz, 1980. p. 13).

Mason (2000) mentions notation in his errors in Mathematics as learner difficulties, describing them as learning barriers to the learner due to lack of associations, images and meaning. Other learning barriers include symbols which refer to the use of different names for the same concepts which have the same value and ratio. These errors I will refer to as symbolic errors.

The following is an example of a symbolic error from one of my learners:

Find $f(x) + 3$ if $f(x) = x^2 - 4x - 2$

Due to a misinterpretation of the function's symbolic representation, the response is given as $(3)^2 - 4(3) - 2$ then the meaning assigned to $f(x) + 3$ was that that of $f(3)$ and not $f(x) + 3$.

2.5 Developing a conceptual framework

Due to the similarities in the definition of errors identified by the different authors, the analytic framework is constructed by considering the explanation of those errors and formulating a definition to incorporate all of them. I managed to identify the five different errors below with appropriate definitions.

The framework that I am going to use for identification of errors is as follows:

- ***Careless Errors*** **–** unnecessary mistakes made by learners.

- ***Procedural Errors*** - occur when a learner computes or applies a procedure incorrectly.

- ***Calculation Errors*** **-** mistakes in addition, subtraction, multiplication and division of numbers.

- ***Application Errors*** **-** mistakes made by learners when they know the concept but cannot apply it to a specific situation or question.

- ***Symbolic Errors* –** when learners falsely relate mathematical problems that use similar symbols

The next chapter deals with the research design used to identify the different errors in the written responses of learners.

Chapter 3

Research Design

3.1 Introduction

This chapter gives an overview of the research design adopted for this research. Research methods are guided by the nature of the research question posed and the existing knowledge about it (York, 1998). The choice of method is governed mainly by a general explanation pertaining to learners' understanding and their responses by considering various errors committed. The emphasis is on the systematic analysis of data in the national senior certificate Mathematics examination of 2010.

3.2 The research approach

This study has adopted a qualitative research approach. In qualitative research, the main objective is to describe events or experiences of individuals in their natural setting such as a home, school or an organisation. The participants' and researcher's interpretation are extremely important to the research process and the prediction of outcomes is not a meaningful goal of the research study.

The reason for choosing a qualitative paradigm is because this approach is best suited to understand the phenomenon in this field of study. In a qualitative study, the variables are not usually controlled and it is not intended to apply the generalization of findings to other populations. Qualitative studies require sufficient freedom and scope to unlock the natural development of action and representation that the researcher wishes to capture (Henning, 2004).

The focus of the study was to determine how learners performed in a controlled environment. In return, the researcher analysed this natural behaviour by focusing specifically on errors that have been committed by learners.

"The researcher thus identifies a particular phenomenon, isolates it, and after analysing it, gives an interpretation of the phenomenon" (Mbekwa, 2003, p.63). The phenomenon of interest that referred to in this study were the errors that learners commit on answering their final Mathematics question paper in grade 12.

Qualitative research allows in this case, among other benefits, the analysis of data generated in a natural setting of a high-stakes examination. It accounts for phenomena in context-specific settings, such as "real world settings where the researcher does not attempt to manipulate the phenomenon of interest" (Patton, 2002, p.39). The grade 12 final examination is labelled as a high-stakes examination where the learners write under strict supervision.

Qualitative research is defined by Strauss and Corbin (1990) as any kind of research that produces findings not deduced by means of statistical procedures or other means of quantification, but instead produces findings deduced from real-world settings where "phenomenon of interest unfolds naturally" (p.17). The examination papers were analysed after having been written under controlled conditions with no interference from outside.

With the aid of this research approach, error identification will be done within the initial framework previously described in chapter 2. Finally, a qualitative research method also assists in providing appropriate recommendations to educators.

3.3 Document analysis in qualitative research.

Document analysis is a systematic procedure for reviewing or evaluating documents. Also, document analysis was mostly used to complement other research methods, but for this specific study, it will be the preferred method of use.

Document analysis has previously been used as a stand-alone method (Bowen, 2009).

Like other methods in qualitative research, document analysis requires that data be examined and interpreted, in order to elicit meaning, gain understanding and develop knowledge (Strauss & Corbin, 1990).

Document analysis is also systematic, controlled, directed, organized, explicit and objective (Quade, 1970). During this study, the documents which were analysed are the final examination scripts of grade 12 Mathematics learners of 2010.

In relation to other qualitative research methods document analysis has both advantages and limitations.

Some of the advantages are:

- Efficient method - document analysis is less time consuming and therefore more efficient than other research methods. It requires data selection, instead of data collection.
- Constancy - documents can be reviewed repeatedly.
- Availability - many documents in this study are already available.
- Cost effectiveness - document analysis is less costly than other research methods.
- Lack of obtrusiveness and reactivity - documents are unaffected by the research process, therefore document analysis counters the concerns related to reflexivity (or the lack thereof) inherent in other qualitative research methods.
- Stability - the researcher's presence does not alter or manipulate the result of what is being studied.
- Coverage - documents provide the broad and wide range of coverage. (Yin, 2003, p. 80)

During document analysis there are also limitations:

- Insufficient detail - some documents do not provide sufficient detail to answer a research question as required in the research framework.
- Restricted accessibility - some access to documents may be deliberately denied.
- Biased selectivity - incomplete selection of documents (Yin, 2003, p. 80).

3.4 Sampling

The type of sampling used in conducting this project is stratified sampling. Stratified sampling refers to the process of dividing members of the population into homogeneous sub-groups before sampling. The strata should be mutually exclusive: every element in the population must be assigned to only one stratum. The strata should also be collectively exhaustive: no population element can be excluded. Then random or systematic sampling is applied within each stratum. This often improves the representativeness of the sample by reducing sampling errors. Marshall (1996) stressed the fact that choosing a study sample is an important step in any research project since it is rarely practical, efficient or ethical to study whole populations. He also added that the aim of all qualitative sampling approaches is to draw a representative sample from the population.

If population density varies greatly within a region, stratified sampling will ensure that estimates can be made with equal accuracy. During this study, randomized stratification was also used, which improved the population representativeness.

There are a few potential benefits to choosing this type of sampling, namely:

- Dividing the population into distinct, independent strata can enable researchers to draw inferences about specific sub-groups that may be lost in a more generalized random sample.
- Utilizing a stratified sampling method can lead to more efficient statistical estimates, provided that each stratum is proportional to the group's size in the population.
- It is sometimes the case that the data is more readily available for individual, pre-existing strata within a population than for the overall population.
- Increased likelihood of being selected as sample group thus leading to more conclusive results (Marshall, 1996).

During the apartheid era, education in South Africa was segregated according to race, with different government departments administering schools for each race.
What is now the Western Cape was at that time part of the Cape Province, and schools for white students were managed by the Education Department of the Cape Provincial Administration (CED).

Schools for coloured students were managed by the House of Representatives Education Department (HOR) and schools for black students were managed by the Department of Education and Training (DET); and schools for Indian students were managed by the House of Delegates (HOD).

During this study, the above- mentioned schools will be referred to as Ex-departmental schools, inclusive of the following types of schools: Education of learners with special needs (ELSEN), Private Schools (IND - non-public schools) and Special Schools (WCED – special focus on Mathematics and science).

The Western Cape in return is divided into eight education districts; of which four are rural districts, namely: the Cape Winelands, Eden and Central Karoo, West coast and Overberg. The other four are urban or metro districts within the City of Cape Town. These districts are Metro North, Metro South, Metro East and Metro Central.

The districts are responsible for the management of education, with policy and planning being managed by the head office in Cape Town. The examination papers under investigation are the Mathematics Paper 1 of 2010 of candidates who wrote in the Western Cape region.

There were 17 544 candidates who wrote the 2010 grade 12 final Mathematics examination in the Western Cape region. To acquire a 2% tolerate error with a 99% confidence level, 12% of the total candidates were chosen as a sample size. The sample totals 2105 scripts.

In a proportionate stratified sample design the number of observations in the total sample is allocated among the strata of the population in proportion to the relative number of elements in each stratum of the population.

That is, a stratum containing a given percentage of the elements in the population would be represented by the same percentage of the total number of sample elements (Ross, 2001).

To compile the sample size, the number of students for each district were taken and expressed as a percentage of the total number of the students.

Therefore, the sample of students from each district forms part of the 12% (2105). The percentage spread for each of the ex-departments (and for each of the districts), was found. The numbers of learners that must be selected for each district were calculated.

This number of learners was requested, through a random selection of schools from each ex-department. The schools that were randomly selected were limited to learner totals, where only schools were selected where learner totals were between 20 and 60. This was done to create fairness when it comes to class sizes when teaching is happening and eliminate the opportunity for individual tutoring that can happen with smaller classes.

Table 3.1 Actual sample of learners that wrote Mathematics in each District per Ex- Department

District	**CED**	**DET**	**HOD**	**HOR**	**IND**	**WCED**	**TOTAL**
Cape Winelands	126	13	0	69	0	0	208
Metropole Central	149	72	0	236	143	0	600
Metropole East	27	189	0	7	12	82	317
Metropole North	151	9	0	81	14	56	311
Eden & Central Karoo	57	11	0	31	0	0	99
Metropole South	55	38	20	96	11	46	266
Westcoast	35	0	0	41	0	0	76
Overberg	19	0	0	19	0	44	82
	619	332	20	580	180	228	**1959**

After the Western Cape finalized the sample size, the total number of scripts that they send, came to 1959.

3.5 Data analysis

The template analysis method was used to analyse the grade 12 examination scripts using the analytical framework constructed in chapter 2. The analysis that was solely conducted by me, first entailed colour-coding the identified errors in each script from ex-department and also per district. These errors were identified according to the five initial errors as described in the analytical framework in chapter 2. During analysis of the scripts, it became evident that certain errors could be further sub divided into more descriptive common occurrence type of errors. For example, careless errors were further divided into instances where learners copied incorrectly from the question paper, or incorrect transfer from a previously-obtained result or the insertion of an arbitrary value.

Furthermore, calculation errors in return were also divided into basic calculation errors and a calculator error. Application errors were split into errors where learners factorised but applied the process wrongly or substituted incorrectly into the quadratic formula. The greatest variety of errors was identified during the labelling of the procedural errors. These were further divided into categories were the quadratic formula was not used, procedure errors were divided into quadratic formula errors, simultaneous errors, in a quality sign error and general procedural errors. Errors were also identified outside the analytical framework classified as non-completion errors and an inequality solution non-completion error. All these errors were tabulated according to ex-department and districts in the Western Cape.

The table below summarise these errors and the extensions.

Table 3.2 Initial Framework with Errors and the extensions

Errors	Extensions	Definition
Careless	Incorrect Transcribe	Incorrect transcribe a symbol or number from the question paper
	Incorrect Transfer	Incorrect transfer from a previously-obtained result
	Arbitrary	The insertion of an arbitrary number
Calculation	Basic Calculation	When simple multiplication, division, subtraction and addition errors are made
	Calculator	Mistakes when using a calculator
Application	Factorization	Factorize, but applied the process wrongly
	Substitution	Where substitution was done incorrectly
Procedural	Quadratic Formula	When the quadratic formula is the hinted procedure, but the learner use an inappropriate procedure
	Simultaneous	Use wrong procedure to solve simultaneous equations
	Wrong method	Wrong procedure when solving quadratic equations
	Inequality sign	Where the inequality sign was not reversed after both sides were multiplied by -1
Non-Completion	Inequality Solution	While solving the inequality, but stop after factorizing and do not follow the procedure to complete the question
	Non-Completion	Where the question was started correctly, but not completing the question

In qualitative template analysis, the initial template is applied in order to analyse the text through the process of coding, but is itself revised in the light of the on-going analysis (King, 2004).

All errors in the answer scripts, focussing on questions dealing with quadratic theory, were identified, labelled and categorised according to the definitions provided in the analytic framework.

These errors were first labelled in the five different categories namely: careless errors, procedural errors, application errors, calculation errors and symbolic errors.

There errors were then divided into ex-departments and districts. The final step was to count each error and complete a table for each error per ex-department per district.

Also, during template analysis, the researcher can assume that there are always multiple interpretations to be made of any phenomenon (King, 2004). Template analysis is a more flexible technique with fewer specified procedures, thus permitting researchers to tailor their research to match their own requirements (King, 2004).

3.6 Reliability and Validity

Validity and reliability are pivotal to ensuring that any research is deemed authentic, precise, impartial and valid. During qualitative research, the credibility depends on the ability and effort of the researcher (Patton, 2001). Also, reliability and validity are conceptualized as trustworthiness, rigor and quality in qualitative paradigm (Golafshani, 2003). The question of reliability and validity will be answered if the findings of the study are acceptable and credible both to researcher and other readers of the research.

McMillan & Schumacher (1993) refer to reliability as the extent to which other researchers, given the same set of circumstances could come to the same kind of conclusions as a particular researcher does.

To enhance reliability, the researcher was a relative outsider who was unknown to the participants and the data is visible and verifiable. In other words, the data will be readily available for verification and to corroborate the findings.

According to Guba & Lincoln (1985), the researcher must be explicit as to the process by which evidence is interpreted and provide access to the data for the verification of findings. This is also emphasised by MacDonald & Tipton (1996) who concluded that, “the most important part of the research is that there is a capacity for it to be replicated, to validate the information that you collect” (Creswell & Miller, 2000, p. 126). This is pivotal to ensure that charges of non-authenticity, imprecision and partiality are non-existent.

Hammersley (1990) describes validity thus “By validity, I mean truth: interpreted as the extent to which an account accurately represents the social phenomena to which it refers” (p.57). To prove the validity, as mentioned above, the data is being readily available to for scrutiny.

Also, after identification and analysis of data, all errors identified were discussed with a peer/expert and a consensus reached. If no consensus can be reached, a third person/expert will be conferred with in the presence of both prior persons involved. Consensus will be reached and this will be accepted as valid. “Credibility of findings will further be confirmed valid by way of inter-rater agreement as to provide a confluence of evidence” (Eisner, 1991, p. 110). Inter-rater agreement is a step taken by researchers to involve peer researchers’ interpretation of the data at a different time or location (Johnson, 1997).

During the analysis, when consensus on an error was not established, a colleague was consulted to confirm the validity of the error. These errors were only labelled as valid once consensus was reached. Scripts used in the analysis are readily available for confirmation and validation of all errors identified.

3.7 Ethical considerations

The WCED was approached by letter to obtain permission for the use of the answer scripts for Mathematics Paper One of 2010. They were provided with confirmation of confidentiality, emphasising that no district numbers, candidate numbers or schools names would be divulged.

3.8 Conclusion

This study has adopted a qualitative research approach. Qualitative research allows for the analysis of data to be generated in a natural setting of a high stakes examination where learners write in a controlled environment under strict supervision. The documents that were analysed were the final Grade 12 mathematics examination scripts of 2010.

The selected sample of scripts was obtained over all districts and ex-department schools in the Western Cape by a stratified sample model. The data was analysed based on a Template based model. This model was created by considering definitions of relevant errors to construct a framework that was used to identify and analyse the errors committed by learners. The issue of viability and validity as well as ethical considerations were fully addressed. The following chapter looks at the findings of the research.

Chapter 4

Research Findings

4.1 Introduction

The previous chapter discussed the research design, sampling and data collection procedures. It also looked at document analysis in qualitative research and offered a motivation on why a stratified sample was used. The issues of reliability and validity were discussed and an indication of how these were addressed was also given.

The questions that were focused on in the grade 12 Mathematics paper of 2010 were the following:

1.1 Solve x, correct to TWO decimal places, where necessary:

1.1.1 $(3-x)(5-x)=3$

1.1.2 $3x^2=2(x+2)$

1.1.3 $4+5x>6x^2$

1.2 Solve x and y simultaneously:

$3y=2x$

$x^2+y^2+2x-y=1$ (DBE/November 2010).

The grade 12 Mathematics scripts were analysed, focusing on learner's errors.

4.2 Analysis of errors

4.2.1 Careless Errors

In the conceptual framework in chapter 2, careless errors were defined as incorrect transcriptions made by learners. Considering the definition of careless errors in the framework and together with analysing the scripts, a more specific description for careless errors were derived.

Careless errors are those instances where learners incorrectly transcribe a symbol or number from the question paper to his/her script or a prior-obtained number or symbol is incorrectly copied. These errors were labelled as incorrect transcription from the question paper to the learner's script (figure 4.1), incorrect transfer from a previously-obtained result (figure 4.2) and the insertion of an arbitrary number (figure 4.3). An arbitrary number is a number that is chosen for no apparent reason.

Figure 4.1, 4.2 and 4.3 are examples of the three different kind of careless errors that were identified in the work of the examinees.

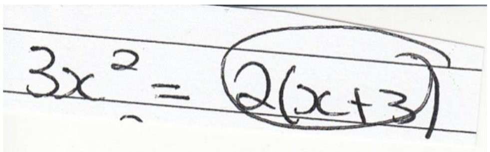


Figure 4.1: Incorrect transfer from question paper.

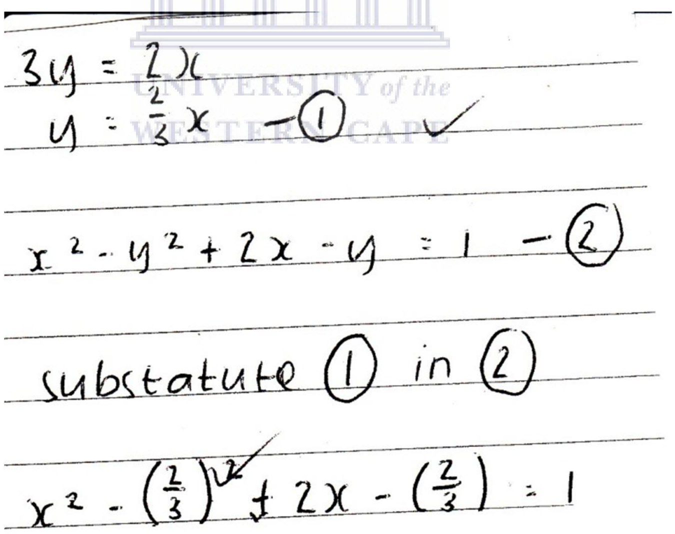


Figure 4.2: Incorrect transfer from a previously-obtained result.

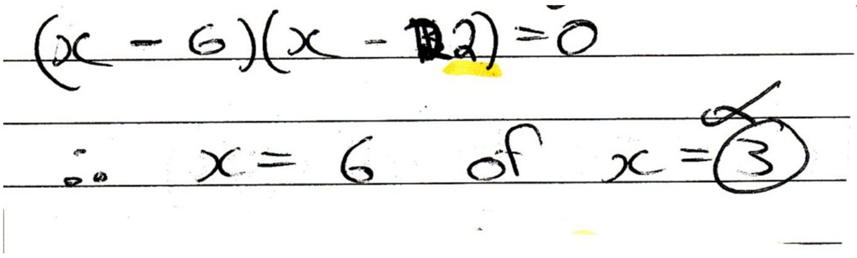


Figure 4.3: Arbitrary error.

It seems that the learner managed to correct the factorization, but forgot to rectify the solution. The number “3” that was scratched out is labelled as arbitrary, because it was chosen for no apparent reason.

Table 4.1: Careless Errors per Districts.

Districts	Transcription from examination paper	Transfer from previously-obtained results	Arbitrary	Total
Cape Winelands	0	4	5	9
Metropole Central	6	40	40	86
Metropole East	0	14	27	41
Metropole North	2	12	8	22
Eden & Centrale Karoo	1	4	13	18
Metropole South	3	11	17	31
West Coast	0	5	9	14
Overberg	0	7	4	11
	12	**97**	**123**	**232**

Table 4.2: Careless Errors per Ex-Departments.

Ex- Departments	Transcription from examination paper	Transfer from previously-obtained results	Arbitrary	Total
CED	5	24	20	49
DET	1	20	22	43
HOD	0	0	4	4
HOR	5	32	49	86
IND	0	7	12	19
WCED	1	14	16	31
	12	**97**	**123**	**232**

4.2.2 Calculation Errors

Calculation errors are often the result of a lack of attention carelessness or short attention span (Elbrink, 2008). For this study the focus was only on mistakes in addition, subtraction, multiplication and division of numbers during the answer of the questions mentioned above. Two different calculation errors were separated, namely a basic calculation error and a calculator error. A basic calculation error is when a simple multiplication, division, subtraction and addition error is made (Figure 4.4). The calculator error is where the learner made a mistake when using a calculator to complete the calculation. This error only occurs for solution of quadratic equations when using the quadratic formula.

The following example in figure 4.4 was identified as a basic calculation error.

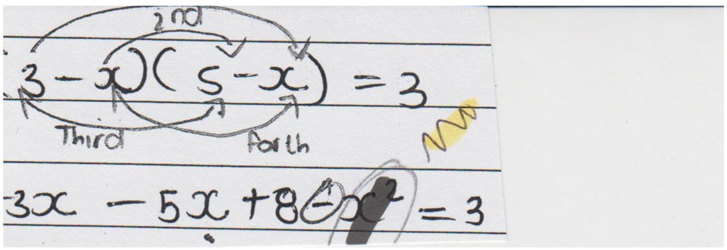


Figure 4.4: Basic Calculation Error.

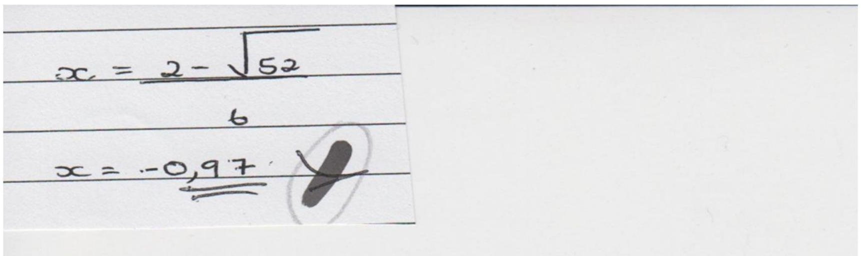


Figure 4.5: Calculator error.

The learner made an error using the calculator, the correct answer is -0.87. The equation was entered as $2 - \frac{\sqrt{52}}{6}$ into the calculator.

The table 4.3: Summary of basic calculation and calculator errors.

Errors	CED	DET	HOD	HOR	IND	WCED	TOTAL
Basic Calculation	128	91	4	121	70	78	492
Calculator	55	42	11	83	24	49	264
							756

4.2.3 Application errors

Application errors that were concentrated on, refer to errors characterized by a failure to execute a manipulation (Orton, 1983). A more apt description may be attributed to Hodes (1998), who referred to application errors as mistakes made by learners when they know the concept but cannot apply it to a specific situation or question. According to Kantor (1978), when there is a problem with a concept, the learners have not learned a rule or principle needed to solve a problem.

In this study, the only application errors that were identified were errors made when learners knew when to:

- factorize, but applied the process wrongly.

- use the quadratic formula, but substituted wrongly.
- solve the linear equation correctly, during simultaneously solving $for\ x\ and\ y$, but substituted incorrectly.

To get a clearer understanding of the different application errors that were identified, examples are provided. The error below in figure 4.6 is an example of an application error where the learner knows when to factorize, but applied the process wrongly. This error will be named application (factorization) error.

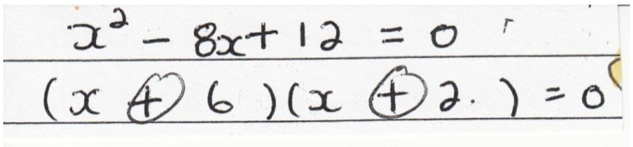


Figure 4.6: Application (Factorization) Error.

The next application error was identified in a situation in which the student knew that a quadratic formula had to be used but substituted the values of $a, b\ and\ c$ incorrectly. The following example illustrates the application error. Based on the standard formula, $ax^2 + bx + c = 0$, the values of $a, b\ and\ c$ were substituted into the quadratic formula in corrected.

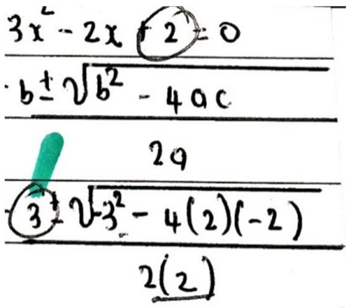


Figure 4.7: Application (Substitution) Error.

The third application error mentioned above was only found once. During the analysis of the scripts, looking specifically at simultaneously solving x and y, where the correct linear equation was calculated, the substitution of the equation ($y = \frac{2}{3}x\ or\ x = \frac{3}{2}y$) were all done correctly.

It was also evident that when there was a mistake made with the linear equation, the wrong equation was still substituted into the quadratic equation correctly.

Table 4.4: Application Errors per Ex-Department and Educational Districts.

	CED	DET	HOD	HOR	IND	WCED	**TOTAL**
Cape Winelands	3	2	0	32	0	0	**37**
Metropole Central	28	31	0	80	29	0	**168**
Metropole East	10	72	0	1	0	33	**116**
Metropole North	16	2	0	23	0	21	**62**
Eden & Central Karoo	11	2	0	14	0	0	**27**
Metropole South	7	15	3	29	0	4	**58**
West Coast	7	0	0	4	0	0	**11**
Overberg	1	0	0	4	0	14	**19**
Total	**83**	**124**	**3**	**187**	**29**	**72**	**498**

4.2.4 Procedural errors

Elbrink (2008) stated that procedural errors occur when a student computes or applies a procedure incorrectly. These types of errors suggest that students do not understand the concepts related to solving these questions. During the analyses of the Mathematics scripts, one noticed during the answering of the question where the quadratic equation was required to be used, the learners tried to factorize the equation.

During previous question papers, the’ key word’ next to the equation was “correct to two decimals”. That was the catalyst to use the quadratic formula for that specific equation. During the November 2010 question paper, the question read at the top, before the equations were stipulated “solve for x, correct to two decimal places, where necessary”. The learner had to make the choice when to use the quadratic formula from the following three equations: $(3-x)(5-x) = 3$, $3x^2 = 2(x+2)$ and $4 + 5x = 6x^2$.

Below in figure 4.8, is an example of what one learner has done to solve the equation. The learner had to use a particular procedure and the hinted procedure is to use the quadratic equation but the learner uses an inappropriate procedure i.e. factorization where the quadratic is not factorable.

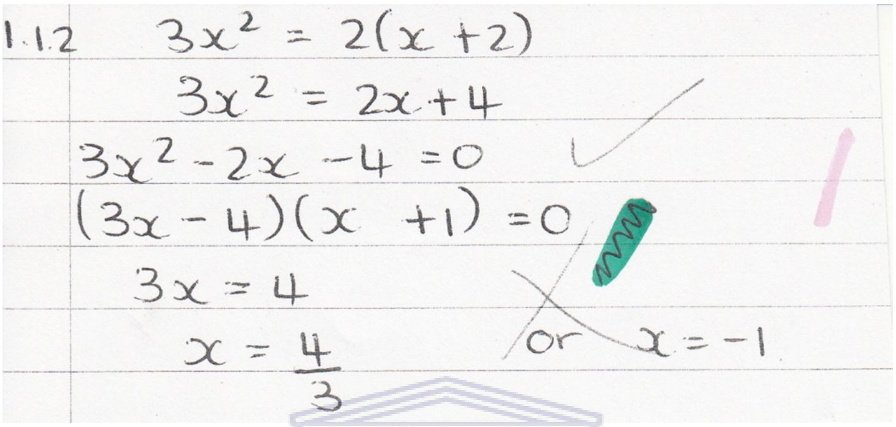


Figure 4.8: Procedural (Quadratic Formula) Error.

The total errors where the quadratic formula was not used on the one question { $3x^2 = 2(x + 2)$} are compiled in the table below.

Table 4.5: Procedural (Quadratic Formula) Error per Ex-Department.

Errors	CED	DET	HOD	HOR	IND	WCED	TOTAL
Procedural (Quadratic Formula) Error.	47	113	0	87	13	31	291

The following error also falls under procedural errors, but because of the similarities why learners commit this error, it was treated separately. There are two possible reasons for this error. One of the possible reasons can be that most learners shy away from using fractions or it might be that learners were influenced by previous examination questions.

The latter reason will be discussed after the example. This error was constantly made when the linear equation had to be solved, before the equation had to be substituted into the quadratic equation. The equation in question was solving x and y simultaneously.

The following is an example of this error:

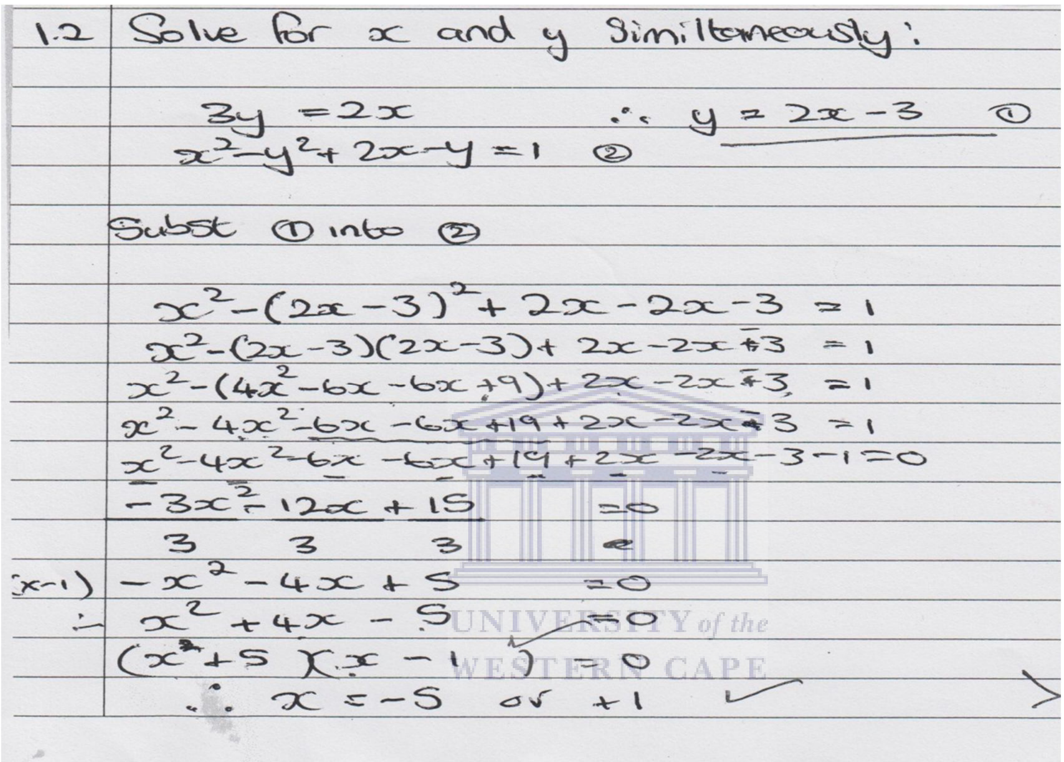


Figure 4.9: Procedural (Simultaneous) Error.

The error was made when y was made the subject of the equation. The correct solution is $y = \frac{2}{3}x$, but in all previous question papers that were used for preparation towards the final grade 12 Mathematics examination, focussing on solving simultaneous equations, the solved linear equation was only in the solved form of $y = ax + b \;\; or \;\; x = ay + b \;\;\; where \;\; a, b \in N$.

For example in the November 2008, paper one, the linear equation was $2x + y = 3$, expresses as $y = 3 - 2x$. The error was only committed when solving the linear part of the question, hence the focus on the linear equation. A quadratic equation is also part of each question.

November 2009 (1) the equation was $x - y = 3$, solved as: $x = y + 3 \ or \ y = x - 3$. November 2009 the equation was $y - x + 3 = 0$, solved as: $y = x - 3 \ or \ x = y + 3$, March 2009 the equation was $x - 3y = 1$ solved as: $x = 3y + 1$.

The examiners did not examine learners on substituting fractions. Learners created the familiar equation themselves and from that the error was created. In the following table 6 are all the errors where the learners wrote the equation in the form $y = ax + b \ or \ x = ay + b \ \ where \ a, b \in N$.

Table 4.6: Procedural (Simultaneous) Error per Ex-Department and Educational Districts:

	CED	DET	HOD	HOR	IND	WCED	**TOTAL**
Cape Winelands	1	3	0	18	0	0	**22**
Metropole Central	21	33	0	80	39	0	**173**
Metropole East	3	52	0	3	0	20	**78**
Metropole North	11	4	0	26	0	14	**55**
Eden & Central Karoo	4	2	0	7	0	0	**13**
Metropole South	3	19	2	30	0	0	**54**
Westcoast	5	0	0	1	0	0	**6**
Overberg	0	0	0	3	0	22	**25**
Total	**48**	**113**	**2**	**168**	**39**	**56**	**426**

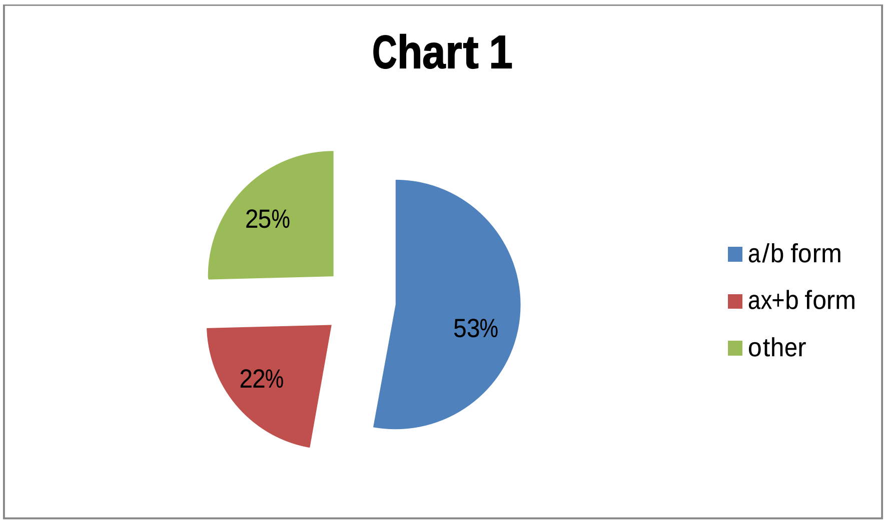
The total number of 426 errors is an indication of the above mentioned trend that was mentioned. All schools use previous question papers to do revision for preparation towards the final grade 12 examination. The next table 4.7 contain all the correct linear equations before the learner substitute it into the quadratic equation.

Table 4.7: The correct answer per Ex-Department and Educational Districts

	CED	DET	HOD	HOR	IND	WCED	**TOTAL**
Cape Winelands	59	7	0	37	0	0	**103**
Metropole Central	96	21	0	98	87	0	**302**
Metropole East	20	81	0	4	9	42	**156**
Metropole North	56	0	0	32	11	28	**127**
Eden & Central Karoo	49	9	0	21	0	0	**79**
Metropole South	49	6	17	43	11	41	**167**
Westcoast	36	0	0	33	0	0	**69**
Overberg	18	0	0	4	0	12	**34**
Total	**381**	**124**	**17**	**272**	**118**	**123**	**1035**

To get a clearer picture, the following pie-chart gave the totals of the two scenarios compared with the total of the number of errors analysed.

Table 4.8, Chart 1: The distribution in the form $y = ax + b$ or $x = ay + b$ versus the correct form of $y = \frac{a}{b}x$ where $a, b \in N$.



The next section looks at the total number of procedural errors that were not part of the above procedural errors but also falls under the definition of procedural errors.

During this study, the first equation in the question paper $\{(3-x)(5-x) = 3\}$, the learner had to manipulate the equation into the standard form$(ax^2 + bx + c = 0)$, and then solve the equation by simply factorizing it. That was the correct procedure to follow. The learners also have the option to use the quadratic formula and that also was seen as the correct procedure. The following errors were committed when the learner used wrong procedures when solving the quadratic equations.

In the figure 4.10 and 4.11 below are examples of procedural errors that were identified during the analyses of the scripts.

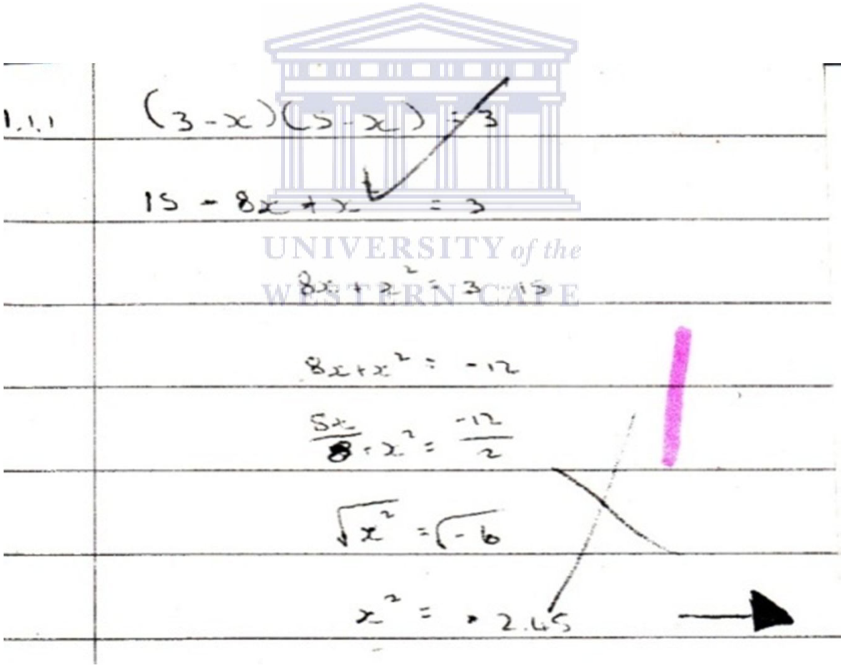


Figure 4.10: Procedural Error

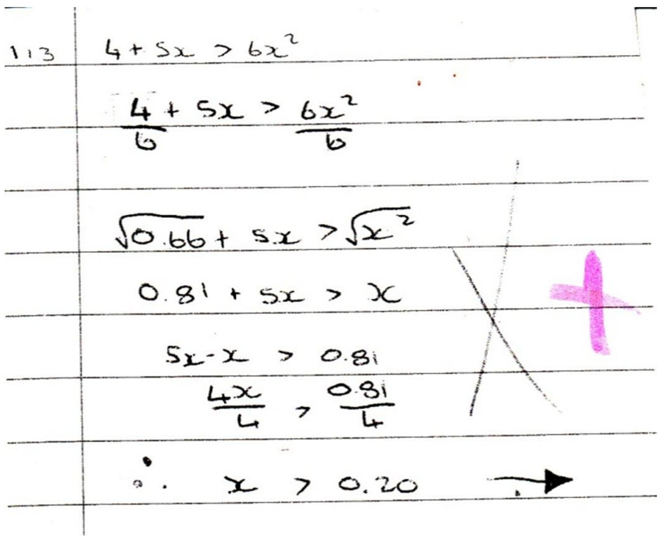


Figure 4.11: Procedural Error

During the analysis, it was noticed that when simultaneously solving for *x* and *y*, factorizing or the use of the quadratic formula were the only procedures that were used to solve for *x* or *y*.

Table 4.9 Procedural (Wrong method) Error

Errors	CED	DET	HOD	HOR	IND	WCED	TOTAL
Procedural	58	112	0	89	12	31	302

The next error can be classified as a procedural error, but during this study this error was treated separately, because there was another factor, explained below, that causes this error.
The procedural error learners commit is when they multiply an inequality by a negative number.

Booth (1989) on the other hand claims that if one wants to understand the difficulties in mathematical structures, then "our ability to manipulate algebraic symbols successfully requires that we first understand the structural properties of mathematical operations and relations" (p.57). The procedural error was committed when the learner did not understand the structural properties of inequalities. Hence the learner multiplied with a negative sign, but inequality sign was not changed.

The following example below show the specific error:

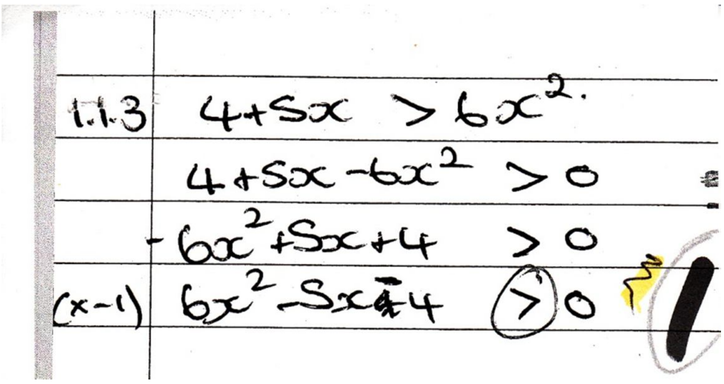


Figure 4.12: Procedural (Inequality sign) Error.

The table below provides the totals where the inequality sign was not reversed after both sides were multiplied by -1.

Table 4.10: Inequality Sign Errors per Ex-Department and Educational Districts.

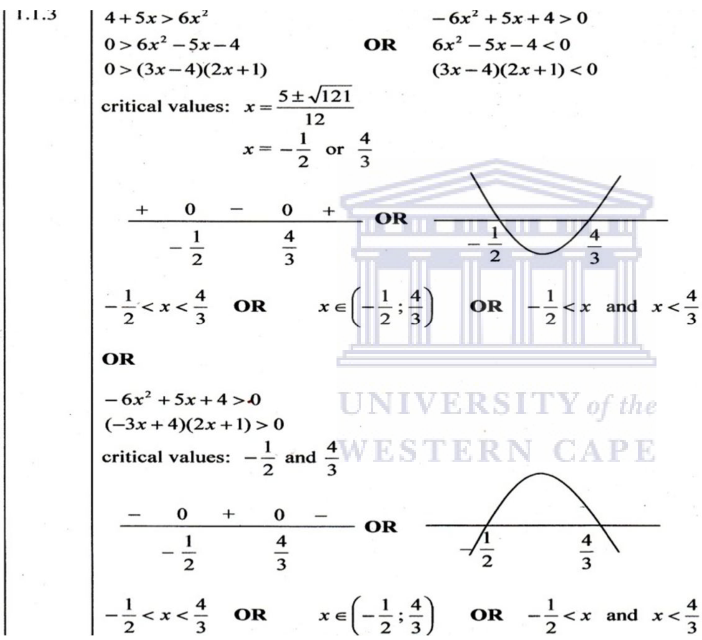
	CED	DET	HOD	HOR	IND	WCED	**TOTAL**
Cape Winelands	1	3	0	27	0	0	**31**
Metropole Central	19	22	0	53	44	0	**138**
Metropole East	7	31	0	6	1	22	**67**
Metropole North	14	0	0	12	3	22	**51**
Eden & Central Karoo	15	1	0	7	0	0	**23**
Metropole South	5	3	1	14	1	8	**32**
Westcoast	7	0	0	15	0	0	**22**
Overberg	1	0	0	1	0	7	**9**
Total	**69**	**60**	**1**	**135**	**49**	**59**	**373**

4.2.5 Non- completion errors

The next error was identified separately, and was not part of the analytic framework.

This error can be seen as a non-completion error and is named as “inequality solution non-completion error” error. This error is committed when the learners attempt to solve the inequality, but stop after the factorizing and do not follow the procedure to complete the question.

In the figure 4.13 below, learners stop at the section where the critical values are identified.



1.1.3

$4 + 5x > 6x^2$ OR $-6x^2 + 5x + 4 > 0$

$0 > 6x^2 - 5x - 4$ OR $6x^2 - 5x - 4 < 0$

$0 > (3x-4)(2x+1)$ OR $(3x-4)(2x+1) < 0$

critical values: $x = \frac{5 \pm \sqrt{121}}{12}$

$x = -\frac{1}{2}$ or $\frac{4}{3}$

+ 0 − 0 + ($-\frac{1}{2}$, $\frac{4}{3}$) OR ($-\frac{1}{2}$, $\frac{4}{3}$)

$-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$ OR $-\frac{1}{2} < x$ and $x < \frac{4}{3}$

OR

$-6x^2 + 5x + 4 > 0$

$(-3x+4)(2x+1) > 0$

critical values: $-\frac{1}{2}$ and $\frac{4}{3}$

− 0 + 0 − ($-\frac{1}{2}$, $\frac{4}{3}$) OR ($-\frac{1}{2}$, $\frac{4}{3}$)

$-\frac{1}{2} < x < \frac{4}{3}$ OR $x \in \left(-\frac{1}{2}; \frac{4}{3}\right)$ OR $-\frac{1}{2} < x$ and $x < \frac{4}{3}$

Figure 4.13: The answer from the memo on the inequality question.

Table 4.11: An “inequality solution non-completion error” per Ex-Department.

Errors	CED	DET	HOD	HOR	IND	WCED	TOTAL
inequality solution non-completion error	318	238	5	339	77	159	1136

The total number of this type of error found, namely 1136, shows that more than half of the learners did not follow the procedure and complete the question. The same scenario was found over all the ex-departments, on average 50% did not complete that specific question.

The last procedural error was found while solving the inequality equation. The literature was very limited on this specific error. This study recommends a further study on why learners commit this specific error.

The following error includes all calculations that were not completed. It excludes the "inequality solution non-completion error".

Table 4.12 Summary of non-completion errors.

Errors	CED	DET	HOD	HOR	IND	WCED	TOTAL
Non- completion	56	20	1	33	8	31	149

4.2.6 Errors per Ex-Department

Currently, in South Africa schools are also divided into quintiles where the CED and IND schools are mainly quintile 5 schools, HOR and HOD range from quintile 2 till 4 depending on the area where it is situated. The other two Ex-Department schools, namely CED and WCED consist mainly of quintile 1. Quintile 1 schools are mostly situated in the Townships (Informal Settlements) and Quintile 5 schools are mostly situated in the more affluent areas. The balance of the schools, namely HOR and HOD schools are situated either in previous disadvantaged areas and more working class areas. The following table will share light on what the situations is post-apartheid.

The values in the table below are percentages that were calculated based on the number of scripts analysed and the errors found in each Ex-Department. In total the number of scripts analysed were: CED - 619, DET - 332, HOD - 20, HOR - 580, IND - 180 and WCED - 228.

Table 4.13: Percentages of total scripts analysed per Errors and Ex-Department

Errors	CED	DET	HOD	HOR	IND	WCED
Careless	8	13	20	14	10	13
Basic Calculation	20	27	20	20	38	34
Calculator	8	12	55	14	13	21
Application (Substitution/Factorization)	13	37	15	32	16	31
Procedural (Quadratic Formula)	7	34	0	15	7	14
Procedural (Simultaneous)	7	34	5	28	21	24
Procedural (Wrong Method)	9	33	0	15	6	13
Procedural (Inequality)	11	18	5	23	27	25
Inequality Solution Non-Completion	51	71	25	58	43	70

4.2.7 Errors per District

It was very clear when errors were analysed the greater percentage of errors were identified in the urban areas than the rural areas. For example, only 4% of careless errors were found in the Cape Winelands District as compared to 15% in Metro Central. This phenomenon was also noted during the analysis of application errors e.g. Metro East 36% application errors were found where the West Coast only had 14% of these errors. The same phenomenon was also noted during the analysis of procedural errors where in the urban districts 25% on average where procedural errors as compared to 10% in the rural districts.

This phenomenon can possibly be attributed to the scenario that urban schools by nature have larger and more populated classes as compared to those in the rural areas.
Unfortunately, it is difficult to establish the true reasons for this phenomenon without physically visiting and interviewing the participants'.

4.3 Conclusion

During the analysis of only 1959 scripts, in total 4163 errors were found. These errors only included careless, application, calculation and procedural errors.
During this study, no symbolic errors were identified. If the percentage is calculated based on the number of scripts analysed, the number of errors vary from 12% careless errors to 40% for calculation errors. During the analyses of the scripts, a very prominent error was noticed which was not part of the initial framework. This error was labelled as a non-completion error, more specifically an "Inequality solution non-completion error".

This specific error was only identified from one particular question in the examination; hence only one error could have been reported per script. This error was committed when solving the inequality question. Based on the number of scripts analysed, 1136 instances of this error were found. In total, only 42% of learners could complete that specific question.

There were times when learners also attempted the other questions and only 149 non-completion errors were detected. Through the study it became clear that most of the errors that were made show a significant similarity. This makes the task to eliminate them easier because the focus for learners and teachers can now be more specific.

The next chapter deals with the interpretations and discussions of the findings. Also the final conclusion and recommendations for further study and interventions for the education department will be presented.

Chapter 5

Discussion, recommendations and conclusion.

5.1 Introduction

As stated in the first chapter, this study analyses and identifies errors made by learners during their grade 12 Mathematics examination. The following is the question which this study sought answers to:

- What errors are detectable in the written responses of learners for the solution of quadratic equations and inequalities in the National Senior Certificate Mathematics examination of 2010?

The documents analysed were the learner's examination scripts, focusing on quadratic theory and specifically Question One. The questions that were focused on in the grade 12 Mathematics paper of 2010 are set out in Chapter 4. The subtopics that were dealt with in this question were: the solution of quadratic equations, inequalities and simultaneous equations - one linear and one quadratic by factorization or the quadratic formula.

These errors were identified and named, based on the analytic framework that was compiled in chapter two. The study also revealed errors outside the framework that were not part of the initially constructed framework but contribute to a vast number of errors that were identified, namely non-completion error and inequality solution non-completion error.

This chapter discusses the results, makes recommendations for teaching practice, points to further research in the area and concludes the thesis.

5.2 Discussion of results

5.2.1 Careless Errors

The results for careless errors were divided into three categories: namely transcription from the examination paper, transfer from previously obtained results and arbitrary mistakes.

Figures 4.1, 4.2 and 4.3 in chapter 4, are examples of the three different kind of careless errors that were identified in the work of the examinees.

Of the 1959 scripts analysed, it was established that 12% of errors can be attributed towards careless errors. From the total number of errors found, namely 4163, careless errors constituted only 5, 6 %. These findings are in line with the findings of Yang and Wan (1991) and Elbrink (2008), who concluded that the numbers of careless errors in general are small.

Even though the numbers of careless errors are small, learners forfeit a substantial number of marks as result of careless errors. If a leaner transcribes the question incorrectly from the question paper, the learner will lose marks as a result of these types of errors These errors are normally the result of carelessness or short attention span (Elbrink, 2008). However, anxiety can also be a possible contributing factor.

I think that learners are in a hurry when the copy questions from the question papers or concentrating more on the next step of calculation when they transfer calculations from previously obtained results. There is a possibility that some learners may be mentally distracted and have difficulty focusing on multistep problems and procedures.

They can get side tracked and commit these careless errors without being aware of it. All learners want to do well and do not make careless errors on purpose. Davis (1984), for instance, mentioned that careless errors can simply be just “silly’ mistakes.

5.2.2 Calculation errors

For the purpose of this study, calculation errors were divided into: (1) basic calculation errors and (2) calculator errors The total number of basic calculation errors was 492, which comprises 65% of calculation errors. This is a substantial number of basic calculation errors, even though it is the easiest type of errors to address (Elbrink, 2008). The error can be rectified if the learner goes over his answer sheet when finished writing.

Based on the number of scripts analysed, 25% were calculation errors and that should be a big concern for teachers. This specific error can be attributed to the fact that the percentage of learners reaching at least the partially achieved level in Mathematics in grade 7 is 12% according to the annual national assessment of 2011 by the education department (http://www.education.gov.za). The majority of learners already start high school with a lack of basic calculation knowledge. It can be presumed that teachers are too busy completing the syllabus and do not focus on the basic calculation knowledge.

Another contributory factor may be the overcrowded classes which are found especially in formerly disadvantaged schools. Teachers find it difficult to give individual attention to learners lacking in computational proficiency. The general lack of parental support compounds the problem since learners do not have the opportunity to practice at home with parents giving the necessary guidance in basic calculations and hence strengthen their basic calculation skills. Innumeracy and illiteracy of parents may also to blame for this state of affairs.

Another factor can also be that learners do not revise their scripts before submission. One can attribute these errors to the omission of drilling and consolidating calculation techniques in lower grades. According to Barnes (2008), drilling relies and promotes rote memorization. It is of the opinion of teachers that these types of teaching gives rise to errors committed by learners (Department of Basic Education, 2011).

The calculator error which is referred to above was identified in questions involving the use of the quadratic formula. (Refer to figure 4.5 in chapter 4). The total number of calculator errors was 273. This constituted 14% of total scripts analysed. This error is made purely by the fact that many learners do not know how to use a calculator. Most of the learners in previous disadvantaged schools cannot afford scientific calculators and only have access to them at school, which may lead to the lack of competence in how to use them. Grade 12 Mathematics requires learners to own their own scientific calculator but for most disadvantaged learners these calculators are too expensive to buy because the cost is more than R100 each. During examinations, learners borrow each other's calculators but unfortunately lack the expertise to use these calculators effectively.

Teachers on the other hand, assume that learners know how to use calculators and do not teach the learners how to use calculators effectively and sensibly. In total, basic calculation errors plus calculator errors numbered, 762 calculation errors constituting 18% of the total errors committed.

More errors discussed are those that may be due to a lack of conceptual understanding as well as the candidate's lack of procedural (fluency or proficiency).

5.2.3 Procedural Errors

During the analyses, it was noted that there were five different procedural errors identified, which are explained below.

i. Question 1.1.2 specifically examines where the hinted procedure uses the quadratic formula. The total number of errors discovered was 291.

ii. The next type of procedural error analysed addresses the case when the learner changed the answer to match a known answer.

iii. This error was made while answering question 1.2 when 'y' was made the subject of the equation. In total, 426 errors were made while the learners made the error of transforming the linear equation from $3y = 2x$ to $y = 2x - 3$ .This reaction of learners can also be based on the fact that all previous questions that were asked on simultaneous questions was of the form $y = ax + b$. Learners were not exposed to deal with fractions during the solving of simultaneous equations. . Many learners changed $3y = 2x$ into $2x \pm 3$ instead of $y = \frac{2}{3}x$. The reason for this error is that learners are taught when solving equations to take numbers over. This 'rule without reason' approach prevents learners from developing a deeper conceptual understanding of the processes involved in Mathematics.

iv. While solving the inequality question, the error was committed when the inequality was multiplied by -1 without the inequality sign changing. Figure 4.14 explains the error. When multiplying, the inequality sign stayed the same. The total numbers of errors of this nature discovered equals 373.

v. The error type in which the procedure was applied incorrectly, which does not include any of the above type of errors, totals 302.

A clear and more concise description of the type of error referred to in (iii) and which focuses on procedural errors can be found in chapter 4 (figure 4.10 & 4.11).
This error may be committed because learners are of the opinion that the inequality sign behaves like an equal sign. This error is created due to the similarities when solving inequalities and equations.

The fact that one can multiply with the same number on both sides of an equation and produces equivalent equations can lead to pitfalls when solving inequalities. Garuti (2001) refer to inequalities as a difficult topic for learners to understand and to add to that, inequalities are taught as a subordinate subject in relation to equations. Another factor is that the learner has no or little understanding of the concepts behind the procedures, when solving inequalities (Hiebert, 1986).

Often learners are not aware that the inequality equation changes when multiplying with a negative sign. The reason for that is that positive and negative numbers are symmetric about zero. Multiplying by a positive number merely stretches each of the distances relative to zero, and the direction of the inequality is preserved. On the other hand, multiplying by a negative number invert the sense of the distance to zero, reflecting the configuration of the points over the y-axis at x=0. As a result, the direction of the inequality must also be inverted in order to preserve the correct sense of the magnitude relationship between the distances. The fact that division is considered the reciprocal of multiplication, the same argument can be used for division also.

The procedural error that is referred to in (iv) can also be committed because of learners performing pointless operations on meaningless symbols and that everyone, including the teacher, promotes the learning of Mathematics by memorizing (Oaks, 1990).

Another possible reason for learners to commit procedural errors is when a learner falsely applies an algorithm that worked successfully for one problem to a problem in which the algorithm is not appropriate (Vinner, 1981). The learner picks up similarities among a procedure and follows the same format to solve the equation.

An example to illustrate this is the situation when learners were required to solve the equation $(3-x)(5-x) = 3$. It was observed that learners would begin by writing $3-x = 3$ and $5-x = 3$, and then solving these. It is presumed that the learner likens these equations to the situation where $(3-x)(5-x) = 0$ (Figure 4.10).

The following reason for errors occurring constantly is when learners make procedural errors but the teacher does not correct the learner's mistakes. That learner will make the same procedural errors on another occasion.

5.2.4 Application Errors

Application errors refer to errors made by learners when they know the concept, but apply it incorrectly (Hodes, 1998). In this study, the application errors which were identified were limited to the following applications: (1) factorization and (2) substitution. During factorizing, the learners have some idea how to factorize, but made an error while doing it. This error can be seen in chapter 4, figure 4.8.

The substitution mistake on the other hand, was where the learners wrongly substituted the values of a, b, and c into the quadratic formula and also while solving the linear equation of the simultaneous equation question, substituted the solved equation wrongly into the quadratic equation. During the analysis, a total of 498 application errors were identified which contributes to 12% of all errors identified.

Teaching for procedural knowledge means teaching definitions, symbols, and isolated skills in an expository manner without first focusing on building deep, connected meaning to support those concepts (Skemp, 1987). Teaching for conceptual understanding on the other hand requires learners to reason flexibly. They are expected to make connections to what they already know and apply that knowledge to new situations. This is where misconceptions are more likely to appear then working with procedures.

At the moment, teachers seems to be more focused on procedures, but the idea is that one should first teach conceptual knowledge before procedural knowledge and not the other way around (Brown & Liedholm, 2002). The main objective is that learners need to understand when and how to apply certain procedures. Learners that understand Mathematics are less likely to make application errors than learners that are drilled with procedures.

5.2.5 'Non-completion' error.

There was one error type outside the analytic framework that was also noted, namely the 'non-completion' error. This refers to situations where some questions were attempted by the learner, the correct procedure followed but the process was stopped for no apparent reason. There are some possible reasons for that.

It could be that the learner did not have sufficient time to complete the question, or the learner possibly got stuck with the question, moved on and forgot to come back or did not have time to come back. There can be quite a few logical reasons, but if the learner is not interviewed, it's difficult to suggest the correct reason.

5.2.6 Errors per Ex-Department

The analysed question is generally the first question asked in the examination paper and that learners are exposed to this content for some time, because they get exposed to this question from grade 10 and this section of quadratic theory is finalised in the first quarter of grade 11.

According to the Curriculum and Assessment Policy (2012) of the Department of Basic Education this part of the syllabus is considered as a routine topic. The question educators need to ask themselves - what is an acceptable percentage of errors committed during the answering of those questions.

Although the number of scripts from HOD schools in the sample was only 20, it is noted with concern that the percentage of calculator errors is 55%; this translates to 11 errors committed from 20 scripts. This is much higher than the overall trend of calculator errors.

On the other hand, with the inequality solution non-completion error, the scripts from this ex-department indicated that 25% of learners made that error. This is again well below the average trend for that specific error.

Based on the number of scripts analysed, it was evident that there were no significant differences between the percentages of careless errors across the different Ex-Departments. The results confirm the findings of Wiens (2007) that states "all learners commit careless errors" (p.8). The biggest factor for learners to commit careless errors is their general attitude to mathematics (Clarkson, 1983).

Teachers need to take the time to show learners their careless errors and change their attitude towards not reviewing their work. All learners want to do well in tests and examinations and making them aware of these errors will eventually eliminate careless errors or significantly reduce the incidence of these errors.

The same phenomenon can be seen with the percentages of calculation errors per Ex-Department. However, the average percentage of basic calculation errors above 20% is a definite concern. According to Elbrink (2008), calculation errors are due to a result of carelessness and a short attention span, but I feel that there is a more definite concern with the basic operations of addition, subtraction, multiplication and division of numbers. To eliminate calculation errors, Elbrink (2008) suggest a checklist of the most common mistakes, such as: added wrong, dropped the negative, did not distribute the negative, copied wrong, and so on. I would add to this list the suggestion that learners should just learn the multiplication tables.

The errors, substitution, use of the quadratic formula, solving simultaneously and using the wrong method, are application and procedural errors. It is evident from the data that in the DET schools more of these errors were committed. Application depends highly on the skills of learners and procedure depends on the ability of the learner to follow steps. If learners do not understand the concepts, they will simply plug-and chug without applying meaning and understanding to mathematical procedures (Elbrink, 2008). She suggests teaching the mathematical concept before the mathematical procedure.

The fact that ex-DET's schools are mostly situated in townships, are generally characterised by overcrowded classes, non-equipped classrooms, under-qualified or unqualified teachers who lack the lack of basic content knowledge can possibly be a contributing factor to the number of application and procedural errors (Mji and Makgato, 2006). One can get carried away naming reasons, but there are schools with similar characteristics that have indeed produced good results, which implies fewer errors, that this results cannot be generalised to all historically disadvantaged schools (Mji and Makgato, 2006). Interviewing the teachers and learners at these schools using a scientific approach with instruments designed for that purpose will assist in understanding the learners' application and procedural errors.

The data was very specific, learners had a major problem solving inequality equations. Garuti (2001) was very clear that inequalities are a difficult subject for learners. The most crucial factor is that learners are under the impression that the inequality sign behaves like an equal sign. The confusion can start with the fact that the question reads, "solve for x".
That question is part of questions where the equal sign dominates the equations in paper one. That can give rise to the confusion with the equal sign.

The fact that in all the Ex-Department there were instances of learners not completing that specific question and were not aware that if multiplied with a negative value, the sign changes around, creates the possibly that the teaching of inequalities were not done sufficiently. Garuti (2001) refers to the reality that most secondary schools treat inequalities as a subordinate topic. The WCED will have to investigate the situation around inequalities. Teachers need to cover that section properly or develop their skills to effectively teach the topic.

5.3 Recommendations

To understand and recognize errors and misconceptions in their learners, teachers need to move away from procedural teaching approaches and focus more on conceptual understanding by employing their content knowledge and mathematical knowledge for teaching (Sheinuk, 2010).

A possible suggestion is to incorporate the understanding and identifying of errors and misconceptions as part of teacher training at tertiary institutions. An important aspect is an attempt to probe erroneous thinking and find solutions to address mathematical flaws, both of which require pedagogical content knowledge (Sheinuk, 2010).

The teacher should have a good understanding about the development of mathematical concepts and procedures, the connectedness of mathematical ideas within mathematical domains, the prior knowledge that learners bring into the classroom (from their understandings in previous years at school), and the constructions learners make in their thinking (Sheinuk, 2010).

Knowing that misconceptions cannot be prevented, we need to have skilled teachers who can give explanations so that those misconceptions are not encouraged or reinforced (Swan, 2002). Swan (2002) went on further to say that misconceptions should be welcomed in the classroom and made explicit to learners so that discussions about them can take place in order to produce more meaningful and longer lasting learning. Teachers also need to be aware that direct teaching to correct procedures does not eliminate the underlying causes for erroneous behaviour (Olivier, 1992). When learners are faced with an obstacle, they distort known schemas to overcome the hurdle. Oliver (1992) maintains that mistakes are made due to valid pre-existing knowledge and therefore if new ideas are to be built there need to be construction of new schema.

Teachers need to be in a position to first understand learner's errors and misconceptions and then employ different strategies in an attempt to transform learner's mathematical knowledge structures that lie behind errors (Sheinuk, 2010). This means that a teacher needs to have clarity on the goal of the Mathematics lesson, use approaches that transmit ideas and concepts, assess learner's responses and arguments, interpret their explanations, structure appropriate tasks, ask questions that promote thinking, and interpret curriculum resources (Sheinuk, 2010).

The concern is that learners learn procedures through practice and drill without any knowledge of the underlying structures behind the procedures (Kilpatrick et. al., 2001). To prevent this, a possible solution is to teach the mathematical concept before the mathematical procedure is introduced (Elbrink, 2008). The learner must be made aware of similarities and differences in the different procedures. This will rectify some misconceptions that the learner might have.

The reality is that learners want to do their best and do not intentionally rush through their tests to get them done (Wiens, 2007). Wiens (2007) suggests the first action is to teach learners what to look for when they go over their tests. Supporting that idea, Elbrink (2008) introduces a more constructed solution by incorporating a check list of all possible errors and misconceptions into the classroom routine. The idea is to make them aware of their errors and misconceptions, by highlighting all of these errors during tests and examinations. This exercise will help them reduce such errors and also prevent the learner from making the same mistake.

From the learners' side, it is important that they must be involved and responsible for their own learning and teachers must help them to be able to do this (Lee, 2006). " Teachers can accomplish this in several ways: by changing the ways in which learners interact with the work and each other; by giving them more challenging problems to solve ; and by asking them to express their mathematical ideas in writing" (Lee, 2006, p69). A possible method for this to work is to create an organized small group for them to work in.

Feedback is probably the most vital component when it comes to the assessment for learning (Lee, 2006). To prevent making the same errors, learners should be given an opportunity to rectify these mistakes and consequently improve their learning. If the learner does not act on the feedback of the educator it becomes a useless exercise (Lee, 2006). Currently there is a rush by teachers just to complete the Mathematics syllabus and that limits the time for constructive feedback (Department of Education, 1997). Feedback should be given immediately and not at the end of the quarter to be effective (Lee, 2006).

Learners need to spend time thinking through their mistakes and discover their misconceptions themselves to eliminate their own errors. Feedback is a time consuming practice.

Without time to read and respond to written feedback learners will not value it (Lee, 2006). There is also the option to use oral feedback. This will be more effective when learners talk about their difficulties and in return learn from one another what mistakes they made and how to eliminate them. Once again time constraints impact on its success. However, feedback does not have to be lengthy to be effective (Lee, 2006).

Feedback is the key for learners to discover their own errors and misconceptions, with the help of their peers and the teacher. The only concern is the time available to apply this aspect efficiently.

5.4 Conclusion

The objective of this study was to identify common errors that appear in the responses of learners in a high-stakes Mathematics examination.
In most cases, learners are not aware of their errors and keep on committing the same procedural errors. In addition, they often do not possess the conceptual knowledge needed to check their solutions (Vinner, 1981). Therefore, it is critical that, with the support of teachers, they can correct and prevent mathematical errors (Elbrink, 2008).

Learners need to recognize and successfully deal with their errors; more specifically they must first understand how to recognize their errors. In return, this study provides a substantial guide on identifying the different errors that learners make during solving quadratic equations and inequalities.

It was a real eye opener to realize the huge number of errors that occur during the answering of only four routine questions, but the question is why learners made these errors.

Further studies and a deeper analysis need to be completed, where learners are interviewed and observed to give more light on the reasons for these errors and misconceptions.

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