AN ANALYSIS OF WORK TEAM LEARNING PROCESSES OF SECOND YEAR UNIVERSITY MATHEMATICS STUDENTS IN RWANDA WHEN DEALING WITH A MATHEMATICAL MODELLING PROBLEM

by

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A mini-thesis submitted in partial fulfilment of the requirements for the degree of Magister Philosophiae in the Department of Didactics, Faculty of Education at University of the Western Cape.

Promoter: Professor Cyril Julie

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MARCEL GAHAMANYI

August 2001

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To my wife Jeanne Marie Chantal

To my daughter Delphine

To my parents

To my brothers and sisters.

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Abstract

In our daily lives, we always have to tackle various problems in order to survive. The outcome of our effort is then our daily bread. Surely all of us are concerned. We solve some of these problems through research in different fields. Mathematics, in which the current study is embedded, is one of these fields. This study focuses particularly on mathematical modelling. Solving day-to-day real problems has become almost an obsession. Mathematical modelling is not only a way of solving some of these daily real-life problems, but it is also a way of demonstrating how mathematics can be applied in this way. The present research is concerned with the learning processes in mathematical modelling. It shows how students have been more creative in mathematics. At the same time it challenges the activity system in terms of learning mathematical modelling.

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Acknowledgments

We constantly need help, encouragement and motivation in our daily activities. "Although a thesis is written by one person, it is never the work of only one person" (Leonard, 1998: ii). The following people deserve recognition for their participation in this study.

I would like to warmly express my thanks to my supervisor, Professor Cyril Julie, for his help, support, guidance and advice during the time I was doing the coursework and research work at University of the Western Cape (UWC).

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A special word of thanks to my family, my child Delphine and a very understanding and supportive spouse, J.M. Chantal Mukayigamba, who encouraged me to pursue my studies at UWC.

Finally, I wish to express my sincere gratitude to our Creator, who wishes for us the best every day.

I thank all of you very much.

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Declaration

I declare that AN ANALYSIS OF WORK TEAM LEARNING PROCESSES OF SECOND YEAR UNIVERSITY MATHEMATICS STUDENTS IN RWANDA WHEN DEALING WITH A MATHEMATICAL MODELLING PROBLEM is my own work, that it has not been submitted for any degree or examination at any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

MARCEL GAHAMANYI Date: 08 · 11 · 2001

Signed: ____________

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Abbreviations

NUR	National University of Rwanda
S_1	Student number one
S_2	Student number two
S_3	Student number three
S_4	Student number four
S_5	Student number five
S_6	Student number six
S_7	Student number seven
S_8	Student number eight
S_9	Student number nine
S_{10}	Student number ten
S_{11}	Student number eleven
S_{12}	Student number twelve
S_{13}	Student number thirteen
USA	United States of America
UWC	University of the Western Cape

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CHAPTER 1

INTRODUCTION AND MOTIVATION OF THE STUDY

1.1 Introduction

From time to time, the needs of society encourage us to improve or innovate something in terms of new tools of work. In other words our moral and material daily need should be understood as a promoter of challenge of our daily work.

The main aim of researchers is to contribute or to make an improvement in their own field. Presently scientific and non-scientific developments are very advanced because much research has been done in the different domains. Worldwide, mathematics is one of these areas where huge discoveries have been made throughout the ages. Historically, "mathematics has developed or evolved through four ages, namely the Babylonian age (around 2000 BC), the Greek age (about 600 BC), the Newtonian age (1642-1727), and finally the Golden age (1800-date)" (Emenalo, 1994: 363). Each age corresponded with the societal needs at that time.

As we are aware, mathematics has been, and still is, continuing to be a useful tool in human society. "…the importance of mathematics in the cultural development of a given society has manifested itself in the application of mathematics to daily societal needs, especially in industries, sciences and technology" (Emenalo, 1994: 363). In society the utility of mathematics is recognized through several activities. These are socio-economic activities, political activities, business activities, planning activities and so on. Because of

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the concerns for dealing with fore-mentioned activities, the need exists for knowledge-ability on how mathematics is applied to societal affairs. Considering these societal needs, there is a continuous culture of change in mathematics curricula. "What we teach and how we teach must change as the needs of society change" (Pollak, 1984: xv). Ale (1989) realises that mathematics curriculum development is a dynamic process. This is because from time to time, society is confronted with new issues that require new mathematical formulation.

Over thousands of years, the only well-known approach in mathematics education has been pure mathematics. Mathematicians have begun to think about applied mathematics recently. In the 1960's, at the time of new mathematics, the mathematics curriculum in the United States, France and many other countries was devoid of applications. Those who called for an increased emphasis on applications were viewed as outsiders, who did not really understand what mathematics was about. Applied mathematics was viewed as inferior mathematics, and applied mathematicians as inferior mathematicians. Usiskin (1993) and his colleagues confirm that in their mathematics education there were few applications. An effort to think of applications as necessary components in the mathematics curriculum took place in the early 1970's.

Furthermore, over the past few decades experience has shown that there has been a considerable increase in the use of mathematical analysis, for solving everyday problems. In this regard, another branch of mathematics has been innovated, namely mathematical modelling.

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Mathematical modelling itself is an activity or process that involves the interaction between a set of real-world phenomenon and a set of mathematical models i.e. a given real-world problem is translated into a mathematical formulation from its initial context. The mathematical model is solved in order to "be translated back into the original context"(Burghes; Wood, 1980: 13).

1.2 Motivation of the study

1.2.1 Introduction

Most of us consider mathematics as consisting of a set of distinct branches that are both well compartmentalised and self-sufficient in the sense that they don't really need help from other areas of science. This view is reinforced by the majority of problems we solve as students or teachers, which tend to be well defined with unique solutions. Each problem is complete in itself; it contains all the necessary information and only requires a modification of a routine application of a mathematical technique to obtain only one correct answer.

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However, such a state of affairs rarely manifests itself in real-life applications of mathematics. This is particularly true when we use mathematics in order to better understand some processes. Among others, the processes may be social, physical, economic, chemical or involves a combination of some of these factors. In such cases we are, generally, trying to answer a question by finding a reasonable interpretation of a mass of mostly irrelevant data or information. In this context, the challenge is to make sense of the question and determine a solution. "When the process of problem involves transforming some idealised

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form of the real-world situation into mathematical terms, it goes under the generic name of mathematical modelling" (Cross; Moscardini, 1985: 15).

1.2.2 The meaning of mathematical modelling

The word model is often used in different contexts. In common parlance, it usually means a small object built to represent some existing or yet to be produced object. All the different usages, however, have one underlying feature in common, namely approximation to some real situation. A mathematical model is a representation or transformation of a real situation into mathematical terms, in order to understand more precisely, analyse and possibly predict where they are from. Thus "mathematical modelling is an art or exercise of building and working with mathematical models" (Arona; Rogerson, 1991: 112).

1.2.3 The mathematical modelling process

The process of mathematical modelling requires imagination and skills. A modeller always needs the so-called "two worlds": real world and mathematical world, which interact with each other during the process. The real world sometimes needs to be modelled mathematically. The obtained mathematical model must be solved, interpreted and validated in the context of the original real world situation. This is shown in Figure 1 below:

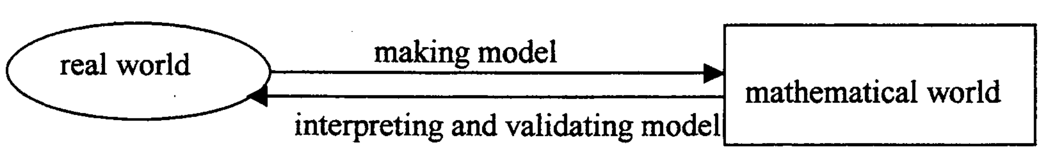

Figure 1: Interaction between the real world and the mathematical world.

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For a given real situation (phenomenon), we make some specific observations about the behaviour being studied in order to formulate the real-world problem. We then identify the factors that seem to be involved. In other words, we make simplifying assumptions that eliminate certain factors. We establish the relationships among the selected factors for creating the mathematical model. The model is solved, and its solution is interpreted and validated. The validated model is thus used to explain or to predict the original real problem. We may however find that there is a need to go back in order to refine and improve its predictive or descriptive capabilities. The above modelling process is summarised in Figure 2 below:

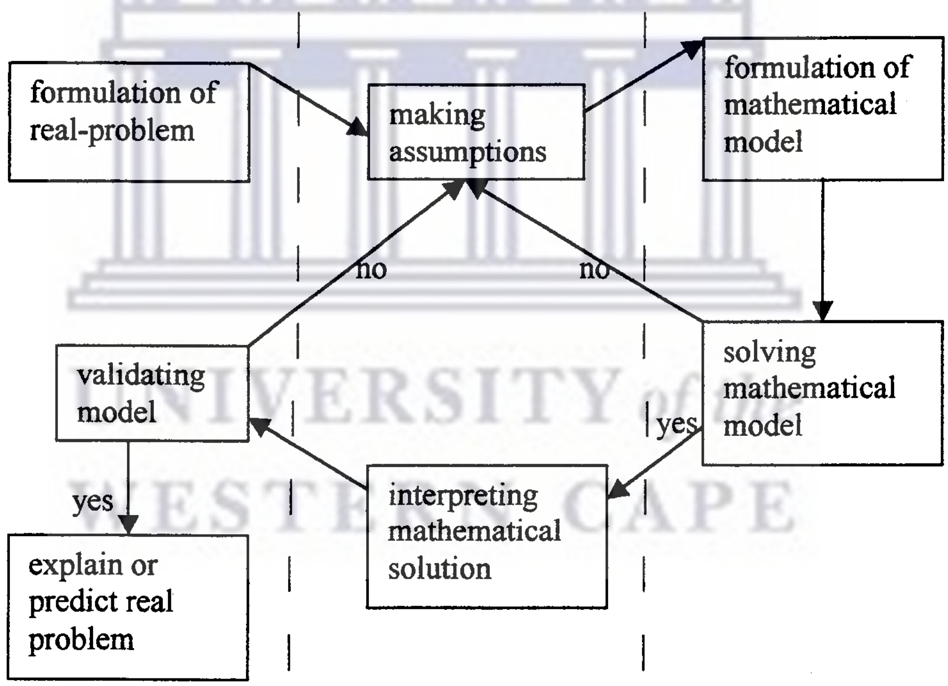

Figure 2: The modelling process.

The left hand column represents the real world, the right hand column the mathematical world, and the middle column the connection between these two worlds.

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1.2.4 The need for modelling in mathematics curriculum.

Pollak (1984) stipulated that we provide the time to teach mathematics not only in school but also at university every year, not because mathematics is beautiful, but because it is useful. One of teacher's aims is to prepare learners to function confidently and knowledgeably in the real world. Mathematical modelling is a powerful instrument of communication between the real world and the mathematical world. It is one of several ways of solving real problems. A modelling approach to problem solving "focuses a variety of mathematical skills on finding a solution and helps a learner see mathematics in a broad spectrum of applications" (Swetz; Hartzler 1991: 6). A modelling process provides useful information for dealing with the original problem. Formative, critical competence, utility, picture of mathematics, promoting mathematics learning are five arguments suggested by Blum and Niss (1991) for the inclusion of application, modelling and problem solving in mathematics curriculum. This also was supported by Julie (1998). Learning mathematical modelling enables one to understand and cope with real world situations. It allows learners to develop "a general competence such as to tackle problems or openness towards new situations" (Blum, 1993: 6). The modelling process helps to increase abilities in discussion and deepen the understanding of techniques in solving real world problems. Mathematical content is then consolidated by suitable modelling examples. These "may contribute towards deeper understanding and long retention of mathematical topics, or they may improve attitudes towards mathematics" (Blum, 1993: 6). In other words, the modelling process is a way of proving the utility of mathematics in our daily lives.

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The importance of mathematical modelling is increasing and has pushed some educators to include it in the mathematics curriculum at all levels of mathematics education. At the moment, it has been included in certain university mathematics curricula as a compulsory subject. Modelling in entry-level university mathematics is indispensable. However, "in most of the countries, mathematical modelling has a secondary, if not negligible, place not only in school curriculum but also in university mathematics curriculum" (da Ponte, 1993: 220).

This study aims to focus on the feasibility of teaching mathematical modelling at the third education level, at the National University of Rwanda (NUR). Rwanda, neighbouring country of Uganda, Democratic Republic of Congo (DRC), Burundi and Tanzania, is amongst the countries that do not yet include mathematical modelling in its mathematics curriculum at all levels. As a lecturer of mathematics at the NUR for four years, my curiosity now is to analyse the learning process of university mathematics students when studying mathematical modelling. I believe the results and recommendations of this study will allow mathematics lecturers at universities where mathematical modelling is not yet part of the curriculum, to develop the strategies to start a course on this subject.

1.2.5 Statement of the problem

After being introduced to and understanding the meaning of mathematical modelling at University of the Western Cape (UWC), my intention was to investigate the feasibility of teaching mathematical modelling in an environment

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where it is not yet a part of the mathematics curriculum. From this perspective, the current research focuses on the analysis of the learning processes during which the university students construct a mathematical model in a team, after being guided through the exploration of an existing model. Hopefully, the outcome of this research could provide insight into the nature of learning in the work team of university students studying mathematical modelling problems.

1.3 Conclusion

In this chapter, I introduced the role of mathematics in social development. The chapter describes chronologically how mathematics has been challenged over thousands of years. A second focus of chapter one was to provide motivation for including mathematical modelling in mathematics curricula.

The following chapter is an overview of the evolution of activity theory upon which the study is built. It then proposes how the learning of mathematical modelling should be framed within the theory of the human activity system. And it clarifies some of the methods of teaching mathematical modelling.

In the third chapter I present the methodology used in the current research. In the fourth chapter, the learning consists of two main stages namely the stage of discussion and the outcome of the discussion. From the video transcripts, I describe the learning processes i.e. I present what happened during the time of

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discussion. From the outcomes of the discussions the mathematical models constructed by students are presented. From this data, the results were obtained.

In the fifth chapter, I deal with the interpretation and discussion of the findings in terms of the human activity system. The final point of my research are recommendations for teaching mathematical modelling in universities where the subject is not yet a part of the mathematics curriculum.

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CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The present research deals with educational development. It deals with innovative learning processes in mathematics education. It especially focuses on work team learning of mathematical modelling. This research is framed within activity system theory. Although activity system theory is a huge field, the above frame is focusing particularly on the expansive learning theory as given by Engeström (1996_a). But as usual, learning and teaching are two dependent activities.

2. 2 Activity system

Goodchild (1997) stipulates that the fundamental unit of study within activity theory is activity, which is identified as the link between the acting person and the problem upon which he or she acts to achieve some desired outcome.

The cultural-historical activity theory as described by Engeström (1996_b) has been initiated by the Russian psychologists Vygotsky, Leont'ev and Luria in the 1920's and 1930's. Up to now, the evolution of human activity theory has been marked by three generations. In the human activity system, Vygotsky pointed out a "triangular model of a complex mediated act" (Engeström, 1996_b: 132). For Vygotsky, subject, object and mediating artefacts or instruments are the major interrelated components of human activity.

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Although Vygotsky was concerned with individual action, Leont'ev's secondary explanation was about "the crucial difference between an individual action and a collective activity" (Engeström, 1996$_b$: 132). However, as Engeström (1996$_b$) explains, Leont'ev never explicitly expanded Vygotsky's original model into a collective activity system. Engeström depicts in the model of the basic structure of a human activity system in Figure 3. By this time, the concept of an activity system took an enormous step forward in that, it turned the focus on complex interrelations between the individual subject and his or her community.

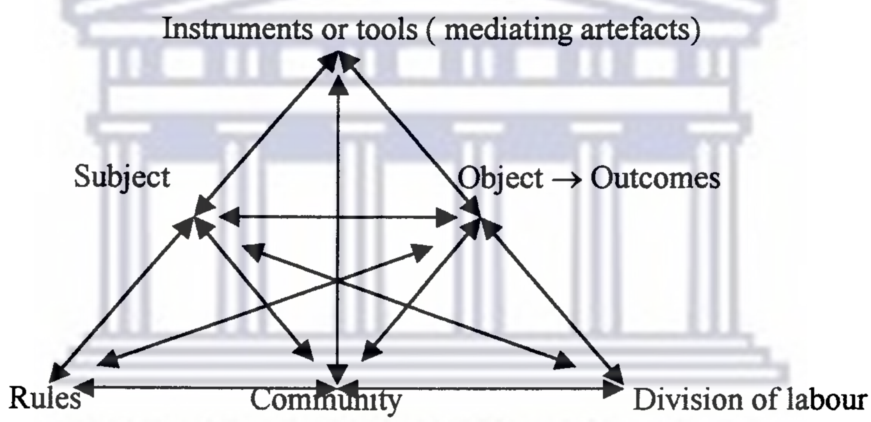

Figure 3. Engeström's model of the structure of a human activity system (Engeström, 1996$_b$:132)

In this model, Engeström defines its different components as follows:

> The **subject** refers to the individual or subgroup whose agency is chosen as the point of view in the analysis. The **object** refers to the raw material or problem space at which the activity is directed and which is molded or transformed into **outcomes** with the help of physical

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> and symbolic, external and internal **tools** (mediating instruments and signs). The **community** comprises multiple individuals and or subgroups who share the same general object. The **division of labour** refers to both the horizontal division of tasks between the members of the community and to the vertical division of power and status. Finally the **rules** refer to the explicit and implicit regulations norms and conventions that constrain actions and interactions within the activity system (Engeström, 1993: 67).

Many educators and psychologists contributed and still continue to challenge Leont'ev's generation. Through these challenges, the third generation of activity theory is ongoing. In this way Engeström (1996$_b$) specifies that the third generation of activity theory needs to develop conceptual tools in order to understand dialogue, multiple perspectives and network of interacting activity systems.

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Engström is one of the contributors of the third generation. Over fifteen years, with his colleagues, referring to the first and second generations, he witnessed the radical changes in the world of work. Among the famous results produced by him, the theory of expansive learning is qualified as an outcome of developmental educational research.

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2.3 Expansive learning

The concept of expansive learning that has been used and developed by Yrjö Engeström is one of the theoretical cornerstones of developmental work research. Julie (2000) defines expansive learning as a theoretical orientation dealing with innovative learning. Engeström describes expansive learning as follows:

> Activity systems periodically face situations in which their internal contradictions are aggravated and demand a qualitative reorganization, or re-mediation, of the entire activity. When an activity system –a workplace, for example –goes through such a reorganization and constructs a historically new mode of practice for itself, it learns something that was not there at the outset, something that no authority was able to transmit and teach. This is collective learning in which internalisation and externalisation, appropriation and creation, routinization and innovation, take place as parallel and intertwined processes. It is a type of learning that is systematically neglected in standard learning theories (1996_b: 134-135).

Expansive learning is a kind of learning in which the task is accomplished most often collectively, through the use of mediating instruments and signs. This process manifests itself in the form of discourse, in which the participants are not necessarily supposed to be aware of some specific background of the task. The participants themselves may discover the desired outcome with help of their

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environmental disposition. The achievement of this goal is often a result of a long discussion. The activity is dominated by the members' discourse convictions related to the given task.

"The theory of expansive learning is based on the dialectics of ascending from the abstract to the concrete" (Engeström, 1996_a: 382). Inspired by Engeström's theory Julie (2000: 1) describes the method of ascending from the abstract to the concrete as follows:

> In ascending from abstract to concrete, a basic initial idea is first formed of the observed phenomenon to be learnt. This initial idea is called a "germ cell", which expresses the genetically original inner contradictions of the system under scrutiny (Engeström, 1987: 245). The germ cell becomes multi-faced, enhanced and more secured through the subjects' engagement with the object of learning. During this engagement the initial abstract idea is transformed "into a concrete system of multiple, constantly developing manifestations" Engeström (1996_a: 382).

Ascending from the abstract to the concrete is not a usual method of learning. In an expansive leaning process, the initial simple idea is transformed into a complex object, which is a new form of learning. Engeström (1996_a) asserts that in dialectical-theoretical thinking, based on ascending from the abstract to the concrete, an abstraction captures the smallest and simplest genetically primary unit of the whole functionally interconnected system. The expansive learning

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process begins with individual subjects questioning, which is the accepted traditional learning, and it gradually expands into a collective movement.

The expansive learning process, in terms of dialectics of ascending from the abstract to the concrete, is accomplished through seven cyclical learning actions suggested by Engeström (1996_a).

Firstly, the process starts with the action called questioning. The concerned participants, after being aware of the task or problem under scrutiny, have an automatic reaction of questioning, criticizing or rejecting some aspects of the accepted practice and existing wisdom.

The second stage of the process Engeström calls the action of analysing the situation. To analyse a situation or a phenomenon requires the involvement or intervention of the mental or discursive transformation of the situation in order to find the causes or explanatory mechanisms. Two types of analysis suggested by Engeström are historical-genetic, which explains the situation by tracing its origination and evolution, and actual-empirical, which explains the situation by constructing a picture of its inner systemic relations.

The third action in expansive learning is the modelling of the newly found explanatory relationship in some publicly observable and transmittable medium. This means constructing an explicit simplified model of the new idea that explains and offers a solution to the problematic of the situation (phenomenon).

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Examining the model is the fourth stage of expansive learning. At this stage the participants need to run, to operate and to experiment with the constructed model in order to fully understand its dynamics, potentials and limitations. This action is followed by the action of implementing the model. This supposes an action of concretising the model by means of practical applications, enrichments, and conceptual extensions.

The sixth and seventh actions are those of reflecting and evaluating the process and consolidating its outcomes into a new, stable form of practice.

2.4 Human activity system as a framework for studying the mathematical modelling process

According to Engeström's model of the structure of a human activity system, mathematical modelling learning should be classified in the set of human activity systems. In the mathematical modelling process we find all elements found in the human activity system i.e. subject, tools, object, division of labour, community and rules. In the same way, the learning of mathematical modelling should then be analysed according to the human activity system because most of the time mathematical modelling learning is done collectively. The analysis of this kind of learning means to describe and interpret human behaviour and discourse within and between the members of the work team learning. In this regard Engeström (1993) affirms that from an activity-theoretical viewpoint, a collective activity system can be taken as the unit of analysis. Julie (2000) adds that this implies that the data about observable behaviour is analysed using the phases of expansive learning. Because we are dealing with the learning, and as

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we are aware that there is no sense in dealing with learning without teaching, the following section is centred on the teaching of mathematical modelling.

2.5 Some ways of teaching mathematical modelling

2.5.1 Introduction

According to the Worldbook Encyclopedia (1994: 65) teaching is defined as a "process by which a person helps other people learn". Teaching helps people to gain knowledge. Everybody teaches. For instance, "parents teach their children, employers teach their employees, coaches teach their players, wives teach their husbands (and vice versa), and of course professional teachers teach their students" (Gordon; Burch, 1974: 1). Here I am going to focus particularly on the teaching of professional teachers.

Teaching and learning are two dependent activities. As teachers we always wish that teaching could become remarkably more effective than it usually is. In other words, we wish that teaching could bring more knowledge and maturity to learners. On the one hand, the nature of learning could depend on the method of teaching a given subject. On the other hand, the way of teaching could depend on several factors. We do not teach mathematics as we teach sport. Didactically the two subjects are taught differently. This supposes that the teacher or lecturer has to be aware of the nature of the given subject before beginning to teach. Another factor to be considered is the learner's background in the subject. Because, as teachers or lecturers, if our duty is attempting to maximize the deeper understanding of the learners, the checking of the learner's background

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could provide a certain orientation to deal with the beginning of teaching. The teacher's behaviour in small and big classrooms is quite different. In other words to follow, for example, the learning process of sixty learners is more difficult than twenty. Thus the class-size is one of various considerations, which needs to be taken into account before starting to teach the subject. Furthermore, we should enumerate more factors such as the environment, equipment and so on. The point to be emphasized here is that all of these factors have a great influence on the nature of the learning process through the chosen way of teaching. For this purpose the teacher may choose an appropriated method of teaching according to the observed factors. Mathematical modelling, as other areas in mathematics, could be taught through several methods. The next section provides some of these possible methods.

2.5.2 Integrationist teaching

Integrationist teaching can be defined as a type of teaching where the teacher gives or recommends to the students the necessary resources (programs, books, and so on). In integrationist teaching all the components of the mathematical modelling process are integrated. The students are thus requested to read and understand the content of the subject for assessment. It is in this way, for instance, that Giordano and Weir (1985) present a course as follows:

For them it is not necessary to introduce the meaning of mathematical modelling. They simply start by constructing the different examples of graphical models in the real world situations and end by giving exercises. Then, they indicate clearly the several stages of the modelling process and end by giving the exercises again as well as some examples. Basically, for the authors, the

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learners are supposed to be aware not only of some concepts in mathematical modelling, but they are also supposed to have sufficient knowledge about higher mathematics.

Because tackling the real problem requires much time, learners may do research to find information about the given problem. In mathematical modelling this kind of teaching favours the development of self-responsibility. It gives learners the opportunity to be more creative and to understand everything without the continuous guidance of the teacher. However, this method would be feasible in the case where students are sufficiently mature and already have a background in mathematics.

2.5.3 Separatist teaching

This method of teaching consists of teaching the subject, where the teacher explains everything and gives examples in the classroom. In this sort of teaching, according to the level of the students and their background in the subject, the teacher may introduce the course by giving the background, definition and motivation for studying the subject. From the beginning until the end of course, the teacher is always supposed to stay with the students in the classroom in order to guide them in their learning process. For example, Edwards and Hamson (1996) presented this method of teaching: They start the course of mathematical modelling by introducing the subject: the meaning of mathematical modelling, what the real problems are, the process of mathematical modelling and the motivation of studying mathematical modelling. After the introduction, they divide the mathematical modelling

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process into four chapters, namely collecting and interpreting data, setting up models, developing models and checking models. The chapters that follow deal with the types of possible models which should be met. These are discrete models, continuous models, periodical models etc. For each chapter the authors give the background, worked examples, exercises and their answers.

In separatist teaching, the different components are taught separately and then integrated into the whole. Contrary to integrationist teaching, separatist teaching is applied from the first level until the third level of education.

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CHAPTER 3

RESEARCH METHODOLOGY

3.1 Sample and fieldwork

This research is based essentially on the teaching of mathematical modelling. During the fieldwork, I have chosen a class of second year university mathematics students. The class size is thirteen students. One of them is female and twelve are males. All of them are not only both French and English speakers, but also they have Kinyarwanda as a mother tongue. At the beginning I handed out to the class a paper entitled "Introduction to mathematical modelling course ". In order to facilitate a deeper understanding, a selected reference list and some articles on mathematical modelling were handed to the students. All of these papers are written in English.

I then introduced them (for about four academic hours) to the mathematical modelling subject, which included the background, the meaning and the process. Finally, I gave them a solved mathematical modelling problem. In the session, I randomly formed two groups of six and seven students. Each group received their own mathematical modelling problem as an exercise. The students were allowed to use any documentation. They could even request assistance from anybody on what they don't understand. They were allowed to speak in any language. Then each group was requested to construct a mathematical model and submit to me a report after one week.

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3.2 Data collection

During the in-class problem solving time, I made observation notes and collected the rough work. I videotaped each group for about one academic hour. For the analytical purposes, the transcript of this videotape is available (see appendices 1 and 2). Finally, after one week, each group submitted a copy of their mathematical modelling report (see appendices 3 and 4).

3.3 The use of transcripts

The transcripts are presented in the form of tables. Although, the rows are horizontally presented according to the intervals of time, the tables contain five columns. In the first column we find several intervals of time. For example, the interval 0:23:00-0:50:00 means the time between twenty-three minutes and fifty minutes. In the second column we find the different participants of the work team learning i.e. S_1 is student number one, S_2 student number two, S_3 student number three and so on. In the third column we find the various actions made by different participants. Some of these actions are, for example, moving the pen, looking at the paper, talking and so on. In the fourth column we find the words said by the participants, and finally in the last column we find the codes. According to the actions and words of different participants, I came to several conclusions. For instance at 0:00:10-0:01:30 interval of time, S_8 (from the second group) was looking at the paper on the table, taking a pen in the right hand; he was not talking and moving the pen from left to right on the exercise paper. I concluded that he was reading the exercise paper, keeping silent. This implies individual reading.

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CHAPTER 4

THE NATURE OF THE LEARNING PROCESS IN WORK TEAM

4.1 Introduction

An ongoing analysis was made of the transcript of the videotape and students' reports of the mathematical model. At the NUR, a class of second year mathematics students was randomly divided into two groups. Each group received problem 1 and problem 2 respectively. Both problems dealt with the construction of a mathematical model. For the first problem, the students were requested to find a mathematical model which could help and provide some information on the estimation of the number of fish in any pond (see appendix 3). In the second problem, students were requested to find a mathematical model that could be used as a tool to estimate the growth of a deer population over five years (see appendix 4). Each group was videotaped separately. During this time the participants of each team had an opportunity to make sense of the task through reading and discussion. However, both teams submitted their final task (mathematical modelling report) after one week as I requested them. My analysis deals with two main issues namely, making sense of the task through reading and discussion and the consequences or outcomes of reading and discussing together.

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4.2 Making sense of the task through reading and discussion

4.2.1 Reading time

As is usual when learners are requested to produce an outcome for a given exercise, they first need to cope with its meaning in order to clarify and possibly remember which things they may utilise. When the learning is in progress, the understanding or meaning of the problem is a crucial stage. This stage in our case goes through both reading and discussion together. The learning process in both teams was dominated by several types of behaviour: individual and collective reading, discussion and writing, the use of notes and books, some questions addressed to me, etc. Whatever they were doing appeared to be an attempt at making sense of the exercise in order to solve it.

When the students received the exercise paper, they looked at it, taking the pen in the right hand, moved it for some time on the paper from left to right, keeping silent. This behaviour seemed to indicate individualised reading. Because the exercise was not too short and written in English, which is not their usual class language, the students quickly decided to check another way of making sense of the problem. Within the first group, for example, after three minutes and thirty seconds the following conversation occurred:

> S_4: … is it possible to read loudly?…
>
> S_2: …what?…
>
> S_4: ...I think we can make an open reading…
>
> S_2: …ok, no problem…

In the same way, from the second group, after one minute and thirty seconds:

> S_8: …someone can read loudly…

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S_{13}: ...ok, I am going to read...

Both teams agreed to read loudly. From the first group, S_4 read loudly without interruption. During his reading S_1, S_2, S_3, S_5, S_6 and S_7 were looking at the exercise paper, still moving their pens and did not talk. They were listening and following him. However, the situation was not exactly the same for the second group. S_{13} was the first to make this kind of reading, but just after about one minute he stopped himself and looked at his colleagues. Then S_{11} started again from the beginning of the exercise and every time he read the sentence, he translated it into French mixed with his mother tongue. During the reading S_8, S_9, S_{10}, S_{12} and S_{13} followed as in the first group. After about six minutes, S_{11} was reading loudly and suddenly, when he read the word "herd", the following conversation took place:

S_{11}: ...herd...

S_{13}: ...herd ni amashyo (herd means amashyo in mother tongue)...

S_{10}: ...herd ni troupeau (herd means troupeau in French), eeh...

S_{11}: ...aah herd means troupeau...

After this conversation S_{11} continued to read, but after eight minutes he read the word "fawns". He stopped and S_{10} took the English-French dictionary. Together with S_{11} and S_9, they checked the meaning of the word. Then the next conversation took place:

S_{11}: ...fans, faawwns...

S_9: ...ni akana ka biche (this means baby or young deer in mother tongue)...

S_{10}: ...yee, ni akana kayo (yes, it means young deer in mother tongue)...

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Within each team, this seems to indicate a collective reading. Even in the first group there was no intervention by S_1, S_2, S_3, S_5, S_6 and S_7. They did not disagree with S_4 to start reading. This supposes that the collective reading could probably have helped somehow because of some non-understandable English words. Basically the English language was an obstacle to quickly understanding the meaning of the exercise.

4.2.2 Discussion time

4.2.2.1 Discussion within the first group

Within the first group, a short time (three minutes) after the collective reading, the discussion was centred on "to know the number of fish in the pond". After eight minutes and thirty seconds, the following dialogue took place:

> S_2: …a company of fish breeding would like to know the number of fish in the pond…
>
> S_4: …eeh, c'est ça le problem (eeh, the problem is that) to know the number of fish in the pond…
>
> S_2: …this information would be valuable for stocking the pond and for studying the availability of the fish in the pond…
>
> S_4: …connaître le nombre (to know the number)…
>
> S_1: …tugomba kureba umubare company ishobora gustockinga (we have to check the number that the company could stock)…

At this moment the students got a sense of what the problem is. They were convinced that their task is to find the number of fish in the pond. According to

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Engeström (1993), this seems to be an "object" of the task. Because from the exercise text, it is written that the estimation of the number of fish in any pond would be valuable information for stocking and studying the availability of the fish in the pond; they therefore needed to assess the feasibility for achieving this. Gradually, their discussion was changing not far from the object, but around the manner which should be used in order to find this number. To achieve this, they concentrated on certain expressions such as: "catch some fish" and "tag and release them in the pond". The evolution of this process can be observed in the following conversation:

S_2 : …you are allowed to catch the…

S_5: …suivez l'information suivante

(…check the following information):

this information would be valuable for stoking the pond and

studying the availability of fish in the pond.

C'est ce que nous allons voir (this is what we need to check)…

S_2: …eeh, you are allowed to catch some fish place plastic tags on

their tails in a way that will not hurt them and then

release them…

S_3: …catch its…

S_4: ...ukazishyiraho plastic ukongera ukazisubizamo…

(…place plastic tags and then release them back…)…

S_6: …ukongera ukaroba izindi

(…you should catch again other fish)…

S_2: …the company have made the fish security that any one can't

catch fish and take it home…

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S_3: …ni ukuvuga ko ntakuzijyana imuhira? (it means that it is not allowed to take its home?)…

S_4: …yes…

S_2: … so we have to catch the fish when we are allowed…

S_4: …yes…

S_3: …For a certain pond the employees catch, tag, and release ten fish…

S_4: …ten…

S_2: …catch and tag ten fish…

S_4: …then they apply the capture/recapture method and obtain the following data…

This discussion lasted about eight minutes. Despite the effort, it was still tough for them to know the number of fish in any pond. The suggested method of "capture and recapture" in the exercise was a main cause or obstacle to moving forward. After sixteen minutes, I intervened in the discussion to explain the capture/recapture method, which was given to them as a way to achieve the goal. My intervention caused the discussion to move in another direction. Between eighteen and twenty three minutes, the members of the team tried to interpret or clarify the available data for the given case. On one hand they wanted to know how could they use the capture/recapture method. On the other hand some of them (S_1, S_2, S_4, S_5) were worried that this method would not finally help them to reach their goal. But others, such as S_6 and S_7, believed that by using the proposed method many times, they could achieve their goal. In the meantime, when S_4 was attempting to convince S_1 how he can proceed in order to know the number of fish, S_5 reminded all of the colleagues that they do not

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have to forget something which is very important: "you are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond". This means S_5 is warning them that whatever they do their task is to build the mathematical model. This was noted from the following dialogue:

S_3: ... they wanted to know the number of fish in the pond...

S_2:... it could be possible to catch the tagged and no-tagged fish at the same time...

S_6: ...ntakibazo kuberako ari ubwambere (... no problem because it is the first time)...

S_7: ...ntakibazo kuberako ari ubwambere (... no problem because it is the first time)...

S_4: ... ok, you can make a catch and if you get tagged and no-tagged fish at the same time, you may then tag the no-tagged ones ok!... but you can catch the fish, place the tag, first time, second time third time and so on, you see!...

S_1: ... just hano (just here) let us suppose they caught fifteen fish and three of them were tagged, the remained twelve ones have been tagged also... Then next time the caught fifteen again; but no one was tagged... they tagged and released them also... then how can we know the number of fish in the pond?...

S_5: ... but you forget something, which is very important: you are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond... then if your mathematical model is working....

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S_4: ...apply it...

S_5: ...apply it for the following particular case...

this is what we are going to see!...

S_2: ...let reread here...you are allowed to catch some fish,

place plastic tags on their tails in...a way that will not hurt

them... and then release them back into the pond...

S_4: ...That is a problem...we have to know the number of fish

in the pond...

S_2: ...in aaaany pond...

S_5: ...how? ...

S_4: ...how? ...

After this period of time until the end of the session the learning process focused specifically on the idea of S_5, which emphasizes their purpose. They were convinced that to construct a mathematical model is their main task. But how could they find this model? After looking at the existing notes on mathematical modelling, S_3 and S_2 began to think and write about the assumptions. The size of the pond was the first topic. S_4 suggested that they do not need to consider the size because their task is to estimate the number of fish in any pond. S_4 asked S_2 how they can estimate the number of fish if they arrive, for example, at Rwasave (the name of a place where NUR is doing the fish-breeding). S_3 responded by saying that it is simple - they may use the capture/recapture method. When S_5 proposed that they should remember that the fish should produce the newborns, all of them agreed that it could be possible to make several catches in a short time, so that they do not need to consider the appearance of newborns.

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Gradually during the discussion, when they had the common idea, S_3 wrote it down.

4.2.2.2 Discussion within the second group

Within the second group the discussion began after eleven minutes. After reading aloud, the members of the group did not immediately have an open discussion. On the contrary, all of them looked at the exercise paper, kept silent and wrote something on the empty papers. However, after fifteen minutes the learning process changed. They started to make sense of the task through the given numerical quantities. During this time of discussion they were busy with numerical quantities. They wanted to find the number of the whole deer population. For this reason they were obliged to make some calculations. Every time they had a common idea they wrote something.

> S_{11}: … each year for every hundred does, 150 fawns are produced…
>
> S_9: … ikibazo rero…(then the problem is…)…
>
> S_{11}: …we have 3714 females… each year every 100 of them produce 150 fawns…
>
> S_{12}: …it means 1920 females fawns…. 100 produce 150…
>
> S_{13}: …they will be produced and added to the existing number…
>
> S_{11}: …after 3 years the newborns are also able to produce the newborns…
>
> S_{13}: …two-thirds of the males and females are newborns…when we will find the number of whole population…

After about thirty-two minutes, they remembered the problem of how and when the adult females produce the newborns. In order to harmonise their ideas they

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assumed that all females produce the newborns at the same time. This is described in the following dialogue:

> S_9: …the problem is that, each year the newborns are produced.
> So …you see? And we don't know how and when
> the adult females produce the newborns…
>
> S_8: …in this way we assume that all adult females produce
> the newborns at the same time…
>
> S_{10}: …ok, we assume that all adult females produce
> the newborns at the same time…
>
> S_9: …ok, we assume that all adult females produce
> the newborns at the same time…

Almost at the end of the session, S_8 suggested that it should be possible that some of the adult females do not produce newborns. Finally, towards the end of session, S_9 asked me to explain to them why the given numerical quantities are not harmonised with the five given considerations. I told him that they might consider the problem in terms of a mathematical modelling problem, not in terms of a purely mathematical problem. This could show that to work with the numerical quantities had a big influence on the learning process. Actually the learners tried to solve the problem as they usually do for a purely mathematical problem. But a look at their reports shows that they seemed to change their way of solving the problem.

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4. 3 Outcomes of discussion (report)

After the discussion, the students had a deadline of one week to submit their reports. Looking at their report structure, both teams seemed to follow the mathematical modelling process. In fact both teams called the first phase "**Identification of the problem**" (see appendices 3 and 4). They briefly describe not only the problematic of the problem, but also how and why they have to construct one of the possible mathematical models, which can help to solve the given mathematical modelling problem. From the first group, for example, the students wrote that to estimate the number of fish in any pond, they should firstly catch a certain number of fish, tag and release them in the pond. Secondly, they should repeat this operation a certain number of times. And finally they should use the simple proportions between the tagged and non-tagged fish. Then in order to develop a model that could help to estimate the number of fish in the pond and make the problem practical, it is necessary to make some assumptions.

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For the second group, the students wrote that the growth of the deer population there meets a number of obstacles such as hunting, starvation and other kinds of death. Fawns are most affected: only a few of them can survive to two years of age. The study of growth is then delicate and considers the rate of death, survival and birth rates. But to develop a mathematical model to study the County's deer population, they had to base it on available statistical indications and make assumptions to make the problem practical.

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In both teams this stage took a long time during the discussion. Because "in real-life situations no one simply hands us a mathematical problem to solve. Usually we have to sort through large amounts of data and identify some particular aspect of situation we wish to study" (Giordano & Weir, 1985: 35). It is then understandable why the students have spent a large amount of time at this stage.

In their reports, the second phase is called **assumptions**. In the mathematical modelling process, the assumptions play a major role. They constitute the link between the first stage and the third stage (**formulation of a mathematical model**). For instance, from the first group we find the following assumptions:

- Because we need to find the number of fish in any pond, that why we are not going to consider size of the pond.
- We are allowed to catch and tag a certain number of fish and release them immediately in the pond.
- All fish of the first catch will be tagged.
- The capture/recapture method will be applied after the first catch.
- The catch could be made from anywhere in the pond.
- The fish are not too small so that it is feasible to catch any fish.
- In the pond the fish are secured under good conditions (not take its home, not diseases etc).
- The number of catches and number of fish per catch are not limited.
- Once a fish is tagged, the tag cannot be lost.
- We are going to use the simple proportion between the tagged and non-tagged fish.

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From the second group the following assumptions were suggested:

- Fawns are considered adult at two years old.
- Fawns include less than one-year old fawns (newborn) and one-year old fawns.
- Every time there is a constant proportionality:

  - k between adult females and the whole population.

  - l between all adults and the whole population.
- $\frac{2}{3}$ of adults are fawns and each year $\frac{15}{10}$ of adult females are newborns.
- Each year $\frac{45}{100}$ of newborns die.
- Each year $\frac{40}{100}$ of one-year old fawns die.
- Each year $\frac{10}{100}$ of adult population die and adult females die after giving birth.

After making assumptions about the problem, the students established the relationships between the selected factors. In order to develop a model, they used the letters in terms of variables as they usually do in the normal course of mathematics. For example, the first group wrote that to estimate the number of fish in any pond, they needed to set up the following variables:

Z = total number of tagged fish at the first catch before the capture/recapture method.

n = the desired number of catches.

m_i = number of tagged fish for i^{th} catch.

k_i = total number of fish for i^{th} catch.

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Y = total number of non-tagged fish in the pond.

X = total number of fish in the pond.

In the same way, from the report of the second group, the students set up the following variables:

H_0 = the whole population.

F_0 = adult females.

A_0 = adult population.

f_0 = fawns that are less than one-year old.

f_1 = fawns that are one year old.

They then used mathematical logic and symbols in order to find the desired mathematical model. For this purpose they identified the relationships between those variables and at the same time, referring to their respective assumptions, they solved the established mathematical expressions. From the first report, for example, the students wrote that for each catch the number (m_i) of tagged fish over the total number (Z) of tagged fish at the first catch before the capture/recapture method is directly proportional to the number ($k_i - m_i$) of non-tagged fish over the total number (Y) of non-tagged fish in the pond i.e. $\frac{m_i}{Z} = \frac{k_i - m_i}{Y}$. Then, according to the students, after n catches they got $\sum_{i=1}^{n} \frac{m_i}{Z} = \sum_{i}^{n} \frac{k_i - m_i}{Y}$. They then used the arithmetical average and got the following mathematical equation:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{m_i}{Z} = \frac{1}{n}\sum_{i=1}^{n}\frac{k_i - m_i}{Y} \qquad (1)$$

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Basically their logic is that the total number (X) of fish in the pond equals the sum of the total number (Z) of tagged fish at the first catch before the capture/recapture method and the total number (Y) of non-tagged fish in the pond i.e. $X = Z + Y$.

According to this logic, they had to find the value of Y from the above equation (1) because Z is already given. They did it and got $Y = Z\frac{\sum_{i=1}^{n} k_i - m_i}{\sum_{i}^{n} m_i}$. Finally, using their mathematics background, they got the following formula (2) as a mathematical model:

$$X = Z + Y = Z\left(1 + \frac{\sum_{i=1}^{n} k_i - m_i}{\sum_{i=1}^{n} m_i}\right) \qquad (2)$$

From the second report, the students wrote that at the initial situation:

$$F_0 = \mathrm{k} H_0$$

$$A_0 = l H_0$$

$$f_0 = \frac{15}{10} F_0 = \frac{15}{10} \mathrm{k} H_0$$

$$f_1 = \frac{2}{3} A_0 - \frac{15}{10} \mathrm{k} H_0 = \left(\frac{2}{3} l - \frac{15}{10} k\right) H_0$$

As with the first group, using their background in mathematics, the members of the second group made the calculations and got:

$$H_n = (1 + \alpha)^n H_0 \qquad (3)$$

as a generalised model to find the number of the deer population over n years (appendix 4). Then, in the case of five years, which is their concern they got

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$$H_5 = (1+\alpha)^5 H_0 \tag{4}$$

At this third stage the students in both teams were using their background in mathematics. They got the general models that could be applied in any case, as they wrote in their reports. For instance, in the first report they wrote that, according to their assumptions, the mathematical model should be applied to any pond. In the second report they wrote that the model could be applied to estimate the deer population at any time (over a certain number of years). However, in order to make sure that their mathematical models were working, they applied it to the particular cases respectively. So in the first group they applied model (2) as follows:

$Z = 10$

$n = 10$

$k_1 = k_2 = k_3 = k_4 = \ldots = k_{15} = 15$

$\sum_{i=1}^{10} m_i = 27$

$\sum_{i=1}^{10} k_i - m_i = 123$

$$X = Z + Y = Z\left(1 + \frac{\sum_{i=1}^{10} k_i - m_i}{\sum_{i=1}^{10} m_i}\right) = 55.5 \approx 56$$

They concluded that the number of fish in the pond could be estimated at 56.

In the second group, model (3) was applied as follows:

H_0 = 9399 deer

F_0 = 3714 adult females

A_0 = 5421 adults (deer)

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Fawns = 3978

n = 5

$$H_5 = (1+\alpha)^5 H_0 = 42{,}773 \text{ deer}$$

They called this stage **validating and implementing the model.** They agreed that their computed models could be used to estimate the number of fish and the growth of the deer population respectively. But at the end of the work both teams wrote that most of the time the models have some **weaknesses**. The first team specified that with the constructed model, it is impossible to determine the exact number of catches that could allow finding the exact number of fish in the pond. The second group wrote that their model is based on the statistical data, which are not really facts but estimations because it is difficult to count animals in the forest.

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4.4 The research findings

From the learning process in the work team, the data indicate that:

During the reading and discussion, in both teams the students were automatically obliged to use three languages, namely English, French and their mother tongue (Kinyarwanda). For their reports, the first group wrote in French (appendix 3) and the second group in English (appendix 4). This indicates that the language could be an obstacle for carrying out the task.

In their reading and discussion (appendices 1 and 2), instead of individual work, in both teams the collective work (reading and discussion) was the best way of making sense of the task.

In both teams the discussion focused on the main goal and the assumptions of the problem as given (appendices 1 and 2).

In both teams the reports are structured as a mathematical modelling process as given (appendices 3 and 4).

In both teams the exercise paper, the notes on mathematical modelling, and myself were the main sources of information.

In both teams the students got their mathematical model through their background in mathematics.

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CHAPTER 5

INTERPRETATION, DISCUSSION AND RECOMMENDATIONS

5.1. Interpretation and discussion

5.1.1 Interpretation

Once again coming back to the data of the current learning process, we should check for some similarities or changes compared to the traditional (normal) activity system for the purely mathematical learning process. With the traditional mathematics learning process we can consider the following example: In order to check the students' performance in mathematics, the teacher would request the class to compute integrals in the classroom for the whole period. In this case the teacher may follow everybody.

According to Engeström (1993), a human activity system has subject(s), instrument(s), object(s) that lead(s) to outcome(s), division of labour, community and rules as its essential elements (see Figure 3). For the suggested traditional mathematics learning process, these elements should be defined as follows: Subject refers to the whole class of mathematics students learning mathematics. Tools (instruments) should be the papers, pen or pencil, books, classroom notes, their memory and language. Object refers to the initiation to the computation of integrals. Outcome refers to the performance of students in mathematics (particularly to the computation of the given integrals). Division of labour should refer firstly to the individual work, secondary to giving advice to

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colleagues upon request and comparing the produced answers. The community refers to their colleagues, their teacher of mathematics and their parents. Rules refer to one single correct answer for each integral within a restricted time. This is depicted in Figure 4.

In the emerging activity system, for the current learning process, the above elements are identified as follows: The subjects are students themselves (two groups). The tools used by the students are firstly their ability or background in mathematics. Secondly, there are the books or classroom notes, pens and language. The object refers to the learning of mathematical modelling through a real world problem. Consequently their outcome becomes a report on their mathematical model without references. The division of labour refers to the collaborative work and checking of advice or information from anybody outside of the classroom. The community refers to the colleagues, their teacher of mathematical modelling, parents and other interested people in the given real world problem. Finally, the rules set for them were such that each group of students found only one mathematical model (one answer) in the extended time. This is depicted in Figure 5.

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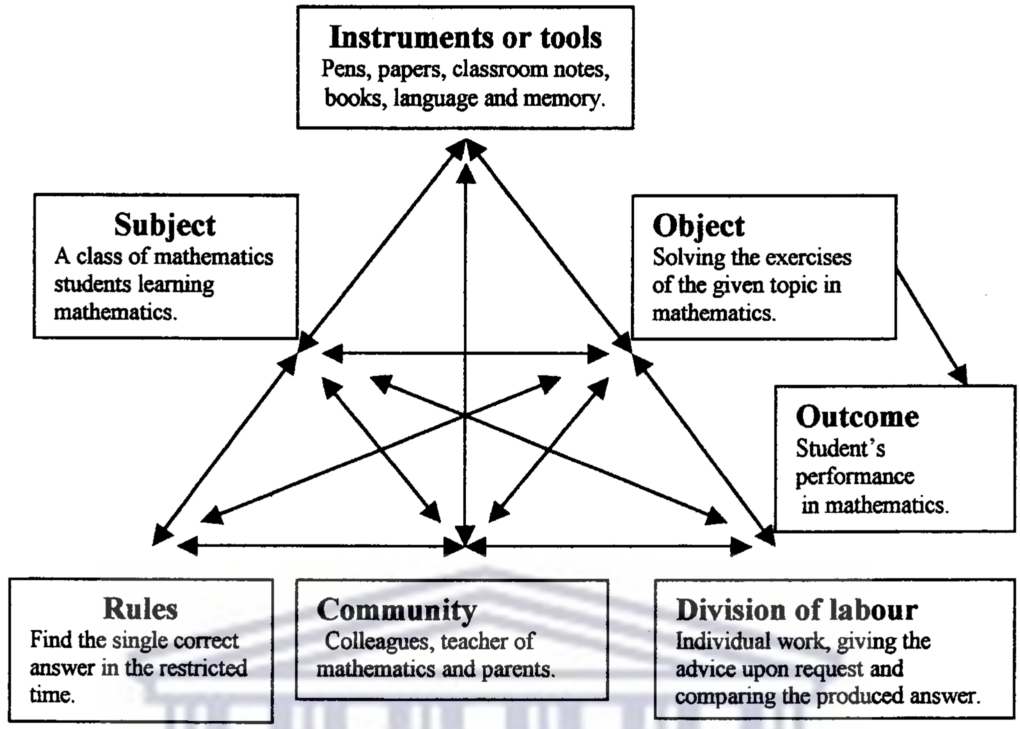

Figure 4: Traditional activity system of learning mathematics.

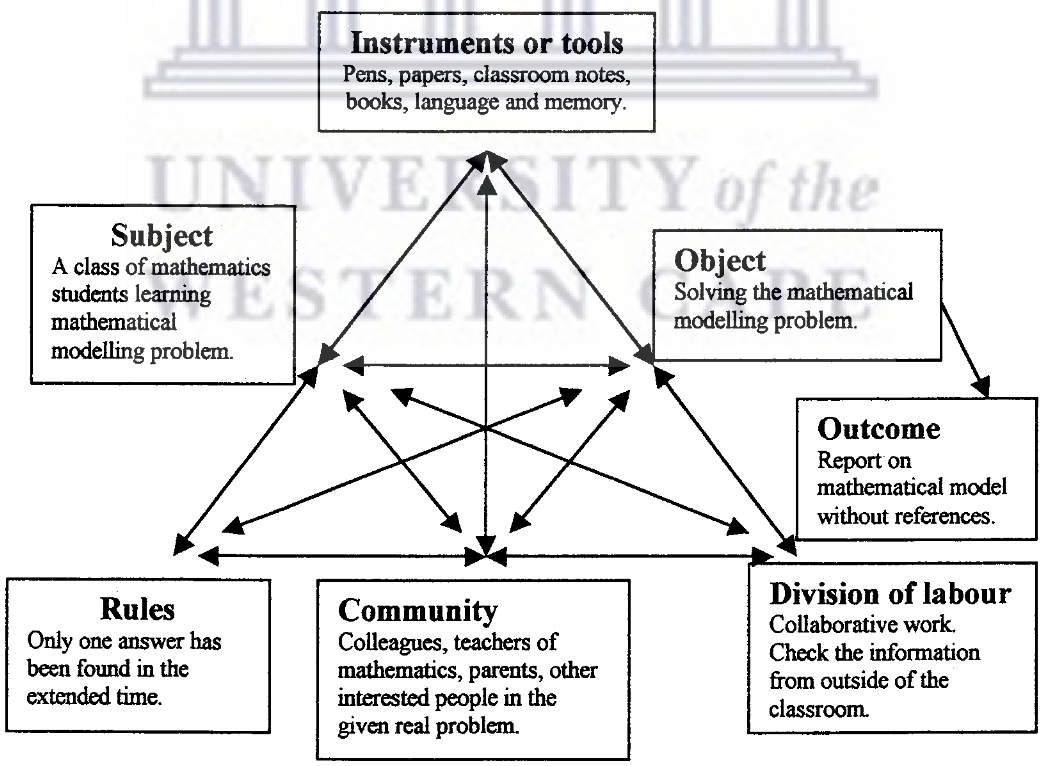

Figure 5: Emerging activity system of learning mathematical modelling.

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However, beside the traditional and emerging activity systems of learning processes, we should also look at the hypothesised activity system of learning mathematical modelling. In my opinion, the hypothesised activity system could be carried out as depicted in Figure 6 below:

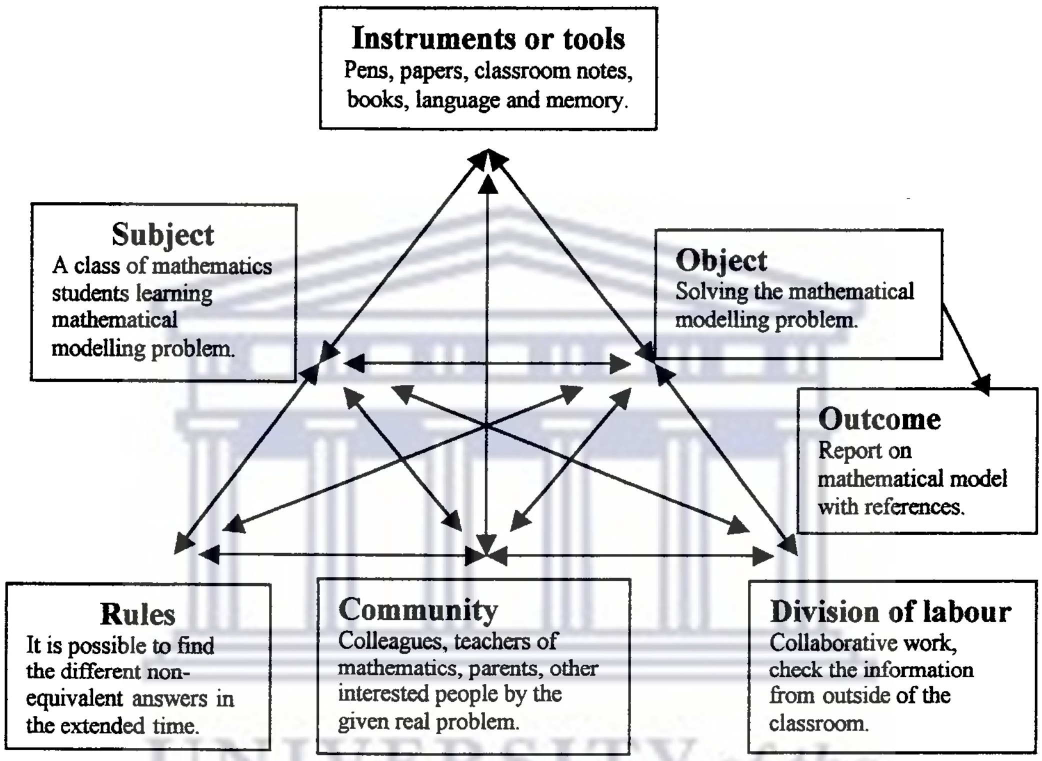

Figure 6: Hypothesised activity system of learning mathematical modelling.

As we can see, the emerging and hypothesised systems are not too different. In fact, except for outcomes and rules, which constitute the difference between the two systems, both systems seem to be similar. For this reason, I am going to emphasize the traditional and emerging systems.

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In both traditional and emerging activity systems, the subject refers to the learners. However, we should note that in the traditional one the students seem to work individually, whereas in the mathematical modelling activity system the learners need to collaborate because the result of their task is supposed to be shared with people from outside of the classroom.

In both systems the tools are quite similar. Meanwhile, some of them were used differently. For instance, the language in the traditional activity system is less used than in the mathematical modelling activity system. In fact it is used not only within the group but also outside of the group in order to get more information for the given real problem. Unfortunately, in our case, the students did not do it. However, after twenty-eight minutes, after discussion in the first group, S_4 referred to the place (Rwasave) where the NUR does the fish breeding. This means that if they could go there, they should necessarily talk to the officials responsible for the fish breeding.

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Furthermore, the use of the learners' memory in both systems was different such that, for example, in normal mathematics learning, the students should refer only to the theory of the current subject. On the contrary, in mathematical modelling, the given problem does not only involve many mathematical areas but also things from other fields. That is why in the modelling process it can be argued that the learners expected to be more creative.

The object consists of solving the given exercises in the traditional system and constructing a mathematical model in the other system. The task in the

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traditional activity system is less than in the activity system of mathematical modelling. In fact, in mathematical modelling the subjects firstly constructed the mathematical model. In other words, the subjects had to find the desired theory in mathematics for the given mathematical modelling problem in order to carry out the desired model. Secondly, they solved the found model and finally applied it to the given real situation. In the traditional system, the subject has to solve the given mathematics exercise in terms of the application of the known existing theory of mathematics.

Within the traditional activity system, the outcome refers to the learners' performance in mathematics (specifically in the given mathematics topic). Within the emerging activity system, the outcome is a report [without references] on a mathematical model. As mentioned in the previous paragraph, once the tasks are different, the outcomes are also different. The important point is that in both activity systems the subjects demonstrate their performance. On the one hand, in the traditional activity system, the subjects are expected to demonstrate their performance in mathematics through the given exercise. On the other hand, through their report the subjects demonstrate their performance not only in mathematics when they have chosen the appropriate mathematics topics for the given mathematical modelling problem, but also in mathematical modelling when they carry out the model for the given mathematical modelling problem.

The division of labour refers to the individual task in the traditional activity system, whereas in the modelling system the collaborative task and the checking

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of information from outside are recommended. However, from the data there is no evidence of checking the information from outside.

Except those interested in the given problem in the modelling system, the community in both systems seems to be the same. Finally, according to the rules of the normal activity system, the learners are expected to find a single answer for each exercise in the limited time. In the modelling activity system it should be possible to produce the various non-equivalent answers for each real problem in the extended time. Although both teams were aware that it is possible to carry out various mathematical models for one problem, the data shows that the students did not do it.

5.1.2 Discussion

In the usual mathematics learning, the time spent making sense of the problem is normally shorter than in mathematical modelling. This is because, firstly, most of the time in pure mathematics the learners refer to their notes or teacher's explanations. Therefore they don't need to check references outside. Secondly, the exercises or problems are often similar to the solved examples in the classroom. However, in mathematical modelling the situation is not so. The important issue is that the problems are always related to a real situation. The problems are often written under some form of description of the phenomenon or situation. This means that the problem could be a long text. At the same time the learners need to understand the meaning of the language and the environment where the problem is situated. Then what happened in the current study?

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In our case, at first when the students received the exercise paper, they automatically read individually as they usually do in pure mathematics. But this method of making sense did not work for a long time, perhaps because of the problem of a foreign language. The collective reading was chosen as a normal way of making sense of the task. During reading, the mixture of three languages (English, French and mother tongue) took place. The English was used as a new class-language, French as a usual class-language and mother tongue as a moderator-language between English and French. The translation in the second group for instance facilitated deeper the understanding of the problem. This way of working allowed students to move forward fluently. Unfortunately this way of making sense of the task took a long time. Probably if the problem was written in their mother tongue, they would not have taken a long time to read.

The beginning of the open discussion within both teams ended the loud reading. The discussion focused essentially on the main task. For the first group, the task was "to construct a mathematical model that could help to know the number of fish in any pond". For the second group the task was "to develop a mathematical model that could help to study the growth of deer population in Dauphin County (USA)". The repetition of the main task dominated their dialogue. The procedure of making the dialogue with repetitions was the best way to achieve the goal. They made assumptions, which in their report formed the basis for formulating the model.

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The report's structure seems to be similar to the mathematical modelling process. This should not be surprising, firstly because the students had in their mind that they were requested to construct a mathematical model. Secondly, through my teaching they were aware of the mathematical modelling process. They followed the mathematical modelling process as was explained in class. I think the major cause of this outcome is that the students did what they usually do in pure mathematics. On the one hand they were still using the routine method of mathematics learning. On the other hand, as the instructor, I sought to investigate their performance in mathematical modelling. Although it was the first time this class learnt mathematical modelling, their reports showed that they successfully solved the problem.

Due to several relevant tools the students achieved success in solving the problem. In fact, besides my participation, the students used other relevant tools to reach this achievement. Firstly, the exercise paper was the primary source of information. Secondly, the existing notes on mathematical modelling contributed meaningfully to the outcome. Thirdly and finally, their wisdom or background in mathematics played a significant role in developing the model. However, all of these tools should be used even in the normal mathematics learning. In my opinion, in their discussion or reports, they could talk or write about references such as the library or other source of information. Although they asked me some clarifications, the data showed no evidence that they checked other references. Once again this suggests that the routine method of mathematics learning was used sufficiently.

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In normal mathematics learning, the learners don't solve the problem within the framework of expansive learning. Their task is just to solve the concrete problem by applying existing theory. However, in mathematical modelling the learners may find the appropriate mathematics theory for the given modelling problem. That is why within the current study the students went through the dialectics of ascending from the abstract to the concrete. At the beginning of the learning process, the participants of both teams were asking the questions, looking right and left, deciding to make a collective reading and repeating some sentences or words; it was what Engeström (1996_a) calls in expansive learning questioning and analysing the situation. From their reports, making assumptions, relationships between the variables, and constructing the model, is an action of modelling in the expansive cycle. This was followed by the actions of validating and implementing the model. And finally, the students made a kind of application and identified weaknesses of the constructed model. This is done to deal with reflecting and consolidating a postulated model in the expansive cycle.

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Furthermore, the students have shown that their learning process is a concrete example of ascending from the abstract to the concrete when both teams have developed a model in general and shown how it can work for the given particular case. Looking again at the discussion within the first group, for example, the students' object was to find the number of fish in any pond. Initially they were worried about how the capture/recapture method [which is the accepted practice] could help them in order to find the number of fish in any pond. For example:

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S_1:...just hano (just here) let us suppose they caught fifteen fish and three of them were tagged, the remained twelve ones have been tagged also… Then next time the caught fifteen again; but no one was tagged…they tagged and released them also… then how can we know the number of fish in the pond?…

Time spent recording was short, therefore it was impossible to follow their discussion until the end of the task. But their report provides crucial evidence of how they proceeded to construct the model. According to their report, when they remembered that they may use the proportionality between the number (m_i) of tagged fish over the total number (Z) of tagged fish at the first catch before the capture/recapture method and the number ($k_i - m_i$) of non-tagged fish over the total number (Y) of non-tagged fish in the pond i.e. $\frac{m_i}{Z} = \frac{k_i - m_i}{Y}$; referring to their background in mathematics, at the end of the time set, they got a mathematical model $X = Z + Y = Z\left(1 + \frac{\sum_{i=1}^{n} k_i - m_i}{\sum_{i=1}^{n} m_i}\right)$, which is a general model for n catches. In order to validate this model they applied it for ten (n=10) catches as a concrete or particular case.

In the second group the same situation occurred when they got $H_n = (1+\alpha)^n H_0$ as a general model for n years. They also applied it for five years as a particular case.

As we can see, we dealt with the learning as a given human activity. Our interpretation and discussion centred around three types of activity systems of

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learning: traditional learning of mathematics, emerging learning of mathematical modelling and hypothesised learning of mathematical modelling. The following table could give us the picture of differences and common points between the three activity systems of learning mathematics and mathematical modelling.

	Traditional activity system	**Emerging activity system**	**Hypothesised activity system**
Subject	A class of mathematics students learning mathematics.	A class of mathematics students learning mathematical modelling problem.	A class of mathematics students learning mathematical modelling problem.
Tools	Pens, papers, books classroom notes, language, books and memory.	Pens, papers, books, language, classroom notes, books and memory.	Pens, papers, books, language, classroom notes, books and memory.
Object	Solving the exercises of the given topic in mathematics.	Solving the mathematical modelling problem.	Solving the mathematical modelling problem.
Outcome	Student's performance in mathematics.	Report on mathematical model without the references.	Report on mathematical model with the references.
Division of labour	Individual work, comparing the produced answer.	Collaborative work, check the information from outside of the classroom.	Collaborative work, check the information from outside of the classroom.
Community	Colleagues, teacher of mathematics and parents.	Colleagues, teacher of mathematics, parents and other interested people by the given real problem.	Colleagues, teacher of mathematics, parents and other interested people by the given real problem.
Rules	Find the single correct answer in the restricted time.	Only one answer has been found in the extended time.	It is possible to find the different non-equivalent answers in extended time.

Table 1: The comparison between the activity systems of learning mathematics and mathematical modelling.

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5.2. Recommendations

Mathematics and its applications are really important for human societal needs. Human beings have never been satisfied in the sense that our needs are unrestricted. In this way, then, we could reduce some of our daily problems. Mathematical modelling is proposed as one of the different instruments that we should utilise. But like other subjects such as economics, history, information systems, chemistry and so on, mathematical modelling needs to be taught at all levels of education. The present research focuses on the feasibility of teaching this topic where it is not yet introduced. The results are satisfactory such that we as teachers or lecturers can catch a first glimpse of what happens when we include modelling in our mathematics curriculum.

In fact, according to the given mathematical modelling problem, the students learn not only to use several references in mathematics [as they do in traditional mathematics learning], but they also make use of fieldwork. They could work with not only the abstract data but also with visible, concrete data. Although in the current research, the proposed problems didn't give students the opportunity to collect these kinds of data, they told me that the subject is more interesting than what they usually do in normal mathematics learning.

Furthermore, in mathematical modelling the students learn to be more creative towards mathematics. This can be seen, for example, when the students themselves develop the assumptions that will help them to formulate the model. Then they come up with a generalised formula, which was unknown before. This formula could be applied to several cases.

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However, mathematical modelling learning requires that the teacher has to draw attention to certain factors. The manner of teaching is of prime importance. In this research I used the "separatist way" of teaching, not only because it was the first time that the students engaged with the subject, but I also introduced to them a course of mathematics which I had taught traditionally.

The free choice of language should be the basis for the quality of outcomes. If the students are multi-lingual, it is better to give them the free choice of language. At the NUR, French and English are used as the academic language. But outside the university, the mother tongue is the most spoken language. Therefore, in my research, the voice of the students was of crucial importance. I told them that in their discussion there is no limit of language. But I told them the report must be written in French or in English. Effectively their discussion was dominated by the mother tongue and the reports were written in both French and English.

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As teachers or lecturers our main duty is to initiate the learners to tackle real-life problems. In other words, through the learning process, the students may themselves discover that mathematical modelling would be a helpful tool in our daily lives. For this reason, we [teachers or lecturers] have to teach them in a way that should give them the opportunity to work with visible and concrete data. To achieve this goal, we may choose the problem which is related to the environmental area and encourage the students that in order to tackle those problems they need to check some data or information from there. We also have

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to consider that besides mathematical modelling, the learners are always busy with other courses such that they need to be oriented or guided. Therefore, the two ways of teaching proposed in chapter two should be combined to obtain a new method of teaching: Integro-separatist teaching.

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APPENDICES

APPENDIX 1: Video transcript for the first group

This group has seven students (participants) i.e. S_1, S_2, S_3, S_4, S_5, S_6 and S_7. They were seating from right to left i.e. from rightS_1 until S_7.

Time	Partici-pant	Actions	Words	Codes
0:00:10 - 0:00:30	Myself	Moving from left to right and handing out the exercise paper each student Do not say anything.		Silent within the members of the team.
	S_1, S_2, S_3 S_4, S_5, S_6 S_7	Seating near the table and keeping some papers in front of them. Receiving the exercise Paper and do not say anything.		
0:00:30 – 0:01:55	S_1 S_2 S_3 S_4 S_5 S_6 S_7	Taking the received paper in the hand and pointing their eyes on this paper. Do not look at left or right. Taking a pen in the right hand and moving it on the received paper from left to right. Do not say anything.		Individual reading
0:01:55 – 0:03:30	S_1	Still moving the pen from left to right on the exercise paper. Does not say anything and keeping the look at S_2.		Beginning of the collective reading.
	S_2	Looking in front of him while he is talking to S_3 and S_4 with the low voice. Making the up and down head's movements at the same time. Looking and moving the pen on the paper.	…what?… …ok, no problem… (0:03:30)	

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:01:55 – 0:03:30	S_3	Still moving a pen from left to right on the received paper. Turning many times a head from right to left and talking to S_4, S_2 and S_5 in the low voice. Making the up and down head's movements.		Beginning of the collective reading.
	S_4	Still moving a pen from left to right on the received paper. Turning many times the head from left to right and talking to S_3 and S_2 with the low voice Making the up and down head's movements	…is it possible to read loudly?… …I think we can make an open reading (0:03:25)	
	S_5	Still moving the pen from left to right on the received paper. Showing S_3 some words on the paper and talking to him with in low voice.		
	S_6	Still moving a pen from left to right on the received paper. Talking to S_7 in low voice and showing him some words on the paper.		
	S_7	Still moving a pen from left to right on the received paper. Listening S_6 and talking to him in low voice.		

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:03:30-0:05:30	S_4	Opening and closing a mouth regularly and making noise until he finished to speak all written words on the exercise paper.	A company of fish breeding would like to know the number of fish in a pond. This information would be valuable for stoking the pond and for studying the availability of fish in the pond. You are allowed to catch some fish, place plastic tags on their tails in a way that will not hurt them, and release them back into the pond. The company have made the fish security that any one can't catch fish and take it home. So no-allowed fishing is not acceptable. There is not dead fish because the company have a habit to put regularly the drugs against fish's diseases in the pond. You are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond. If your mathematical model is working apply it for the following particular case: For a certain pond the Employees catch, tag, and release ten fish. Then they apply the capture /recapture method and obtain the following data (0:05:30)	Loud collective reading.
	S_1, $S_{2, 3}$ S_4, S_5, S_6 S_7	Pointing the eyes on the paper and didn't talk anything while S_4 was reading loudly.		

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:05:30-0:16:00	S_1	Looking at the paper of S_2 and talking to him in the mother tongue. Following the dialogue between S_2 and S_4.	…turareba umubare company ishobora gustockinga… (we are going to find the number of fish that the company could stock…) (0:09:33)	Making sense of the problem through collaborative explanations.
	S_2	Still looking at the paper on the table and showing S_1 with the pen some words. Talking loudly very slowly and making the up and down movements of arms. Most of the time he was talking in the mother tongue.	-…a company of fish irashaka kw'estima thenumber of fish in the pond… (0:07:45) -… kuzikuramo bakongera bakazisubizamo…(0:07:50) -.. a company of fish-breeding would like to know the number of fish in the pond (0:08:30). This information would be valuable for stocking the pond and for studying the availability of fish in the pond. - ..stoking the pond and studying the number of fish - …you are allowed to catch some fish place plastic tags on their tails in a way that will not hurt them , and then release them (0:10:00) -…you catch its… (0:10:27) - ntamuntu numwe wemerewe kuroba ifi ngo ayijyene imuhira .Nicyo bivuga ngo the company have made the fish security that any one can't catch fish and take it home(0:11:18) -…we have to catch the fish when we are allowed… - catch and tag ten fish(0:14:23)	

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APPENDIX 1: Video transcript for the first group(to continue)

Time	Partici-pant	Actions	Words	Codes
0:05:30-0:16:00	S_3	Taking the papers from the table and putting down its on the table. Looking at left and right. Looking at the papers on the table. Moving a pen on the papers. Listening the dialogue between S_1, S_2 and S_4. Up and down head's movement.	- …aaa..(**0:08:00**) -…catch its.. - …ni ukuvuga ko izo bakuyemo zose bagomba kuzisubizamo (It means that all of caught fish must be released back in the pond) (**0:11:45**) -…ni ukuvugako udashobora kuzijyana imuhira (It means that you can not take its home)…(**0:11:46**) -…for a certain pond the employees catch, tag, and release ten fish…(**0:13:50**)	Making sense of the problem through collaborative explanations.
	S_4	Taking a pen in the right hand and moving it from left to right. Looking at left and right, listening the voice of S_2.	- …eeh, c'est ça le problem(in English this means: eeh the problem is that..) to know the number of fish in the pond… (**0:08: 42**) - ok, ok - ok, the problem is following: to know the number of fish in the pond - ok -…connaître le nombre..(...to know the number…)(**0:08:59**) - ntushobora kuzijyana imuhira(you can not take its home)	

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APPENDIX 1: Video transcript for the first group(to continue)

Time	Partici-pant	Actions	Words	Codes
0:05:30-0:16:00	S_5	Taking a pen in the right hand and moving it on the papers. Looking and listening S_2.	- ..suivez l'information suivante(check the following information):this information would be valuable for stocking the pond and studying the availability of fish in the pond. C'est ce que nous ollons voir (this is what we have find or we need to check this)(**0:09:52**)	Making sense of the problem through collaborative explanations.
	S_6	Taking the papers and pen in the hands. Looking at S_2 and the papers. Moving a pen on the papers and listening the voice of S_2	…ukongera ukaroba izindi (you catch again others fish…)(**0:10:38**)	
	S_7	Taking a pen in the right hand and moving it on the paper. Listening and looking at S_2 .		

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:16:00-0:18:10	Myself	Going in the way of student's table and talking to them, moving the arms and touching on the papers of S_4. Going back to my place.	On the beginning they caught ten fish, tagged them and released all of them in the pond. After they made another catch of fifteen fish; three of them were tagged. They released again all of them. They have made ten times this operation. This is a catch that have been made to estimate the number of fish in the pond. You are not sure to catch and tag fish until the last fish. Unless the pond is too small size. - Do you understand? - Ok	Teacher as resource of information.
	S_1, S_2, S_3, S_4, S_5, S_6, S_7	Taking a pen in their hands. Looking and listening me.	- yes - ok	
0:18:10 –0:23:00	S_1	Taking a pen in right hand and moving it. Looking at the exercise paper of S_2 and following him when he was reading. Talking and looking other participants.	- just here suppose they caught these fifteen fish. Three of them were tagged. The remained twelve ones could be tagged also, ok! (0:20:15) - next time they caught fifteen again but no one was tagged, then they tagged them also and released them in the pond(0:20:20) - then how can we know the number of fish in the pond?(0:20:33)	Making sense of the problem through collaborative explanations.

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APPENDIX 1: Video transcript for the first group(to continue)

Time	Partici-pant	Actions	Words	Codes
0:18:10 –0:23:00	S_2	Taking a pen in right hand and moving it. Talking and looking at others participants. Up and down head's movement and looking in front of him (0:22:00)	-… it could be possible to catch the tagged and no-tagged fish at the same time… (0:19:10) - …you allowed to catch some fish, place plastic tags on their tails in a way that will not hurt them… and then release them back into the pond - let reread here!… you are allowed to catch some fish place plastic tags on their tails in a way that will not hurt them , and then release them(0:21: 55) **- in aaaany pond**(0:22:24)	Making sense of the problem through collaborative explanations. The main goal of the problem.
	S_3	Taking a pen in this right hand and moving it. Talking and looking at other participants.	… they wanted to know the number of fish in the pond… (0:18:55)	

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APPENDIX 1: Video transcript for the first group(to continue)

Time	Partici-pant	Actions	Words	Codes
0:18:10 –0:23:00	S_4	Taking the pen in right hand and moving it. Talking and looking at other participants.	- rightly! probably the tagged fish could be caught many times (0:19:00) - no while making catch and you get tagged and no-tagged fish at the same time, you may tag the no-tagged ones, ok!(0:19:21) - …no to know this: you can catch the fish, place the tag, first time, second time, third and so on, you see!(0:20:41) - …apply it…this is the problem… (0:22:15) **- we have to know the number of fish the pond(0:22:19)** **- how?(0:22:28)** - fishing them and putting on the fish the named (numbered) card (0:22:30)	Making sense of the problem through collaborative explanations. The main goal of the problem.
	S_5	Taking a pen in right hand and moving it. Talking and looking at other participants.	-… which is true according to the second catch…but you forget something, which is very important: you are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond. (0:21:23) **-how?** (0:22:24) -Ok! Then if your mathematical model is working apply it for the following particular case: - this is what we are going to see, ok! (0:21:50)	

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:18:10 –0:23:00	S_6	Taking a pen in right hand and moving it. Talking and looking at other participants.	- …ntakibazo kubera ko ari ubwambere(…no problem, it is the first time) (0:19:20)	Making sense of the problem through collaborative explanations.
	S_7	Taking a pen in right hand and moving it. Talking and looking at other participants.	- … ntakibazo kubera ko ari ubwambere (no problem, it is the first time)…(0:19:20) - which is true according to the second catch	
0:23:00-0:50:00	S_1	Taking the papers from the table and a pen in the hand. Talking in low voice Writing something Taking and reading the notes on mathematical modelling.	- suppose that you are catching until you see that all fish are tagged - every time you catch the fish , you tag them then one time you can catch all of them are tagged you can stop to catch(0:34:55).	Classroom notes as resource of information. Making assumptions of the problem.
	S_2	Taking the papers from the table and a pen in the hand. Talking in low voice Writing something Taking and reading the notes on mathematical modelling.	- this information would be valuable for stoking the pond studying mathematical model in the pond…(0:23:15) -let look at the notes on mathematical modelling (0:25:00) - my question is following how to make the fishing in the pond if we do not know the size of the pond, you see! - yes	

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APPENDIX 1: Video transcript for the first group(to continue)

<table>
<tr><th>Time</th><th>Partici-pant</th><th>Actions</th><th>Words</th><th>Codes</th></tr>
<tr><td rowspan="2">0:23:00-0:50:00</td><td>S_3</td><td>Taking the papers from the table and a pen in the hand.
Talking in low voice
Writing something
Taking and reading the notes on mathematical modelling.</td><td>- …assumptions…(0:24:20)
- no don't worry about this we can make several catches in short time that we do not consider the newborns(0:30:20).
- very simple! you can use the method of capture and recapture</td><td rowspan="2">Classroom notes as resource of information.

Making assumptions of the problem.</td></tr>
<tr><td>S_4</td><td>Taking the papers from the table and a pen in the hand.
Talking in low voice
Writing something
Taking and reading the notes on mathematical modelling.</td><td>- ariko ariya mafi ashobora kubyara (but, in the pond the fish could produce the newborns)(0:23:20)
- …the number of fish in the pond…assumptions …(0:24:20)
-no, we need to find out the number of fish in any pond (0:27:25)
- suppose that we arrive at Rwasave what can we do to estimate the number of fish the pond
- writ, writ it(0:28:50)
- the problem is that we must find out the mathematical model, which is similar to the formula , which must be usable in any pond not what you are saying!(0:39:30)</td></tr>
</table>

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APPENDIX 1: Video transcript for the first group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:23:00-0:50:00	S_5	Taking the papers from the table and a pen in the hand. Talking in low voice Writing something Taking and reading the notes on mathematical modelling.	-don't forget that the fish could produce the newborns(**0:30:20**)	Classroom notes as resource of information. Making assumptions of the problem.
	S_6	Taking the papers from the table and a pen in the hand. Talking in low voice Writing something Taking and reading the notes on mathematical modelling.	- we can consider a part of the pond and make the catch in this area (**0:27:20**).	
	S_7	Taking the papers from the table and a pen in the hand. Talking in low voice Writing something. Taking and reading the notes on mathematical modelling.	...but you are not sure if you can catch every fish. It can not happen, so in this case what you can do? (**0:30:55**)...but it is impossible to find the formula because...	

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APPENDIX 2: Video transcripts for the second group

This group has six students (participants) i.e. S_8, S_9, S_{10}, S_{11}, S_{11}, S_{12} and S_{13}. They were seated from right to left i.e. from right S_8 up to S_{13}.

Time	Partici-pant	Actions	Words	Codes
0:00:00 - 0:00:10	S_8, S_9 S_{10}, S_{11} S_{12}, S_{13}	Seating near the table. Keeping some papers in front of them on the table. Receiving the exercise paper and do not say anything.		Silent within the members of the team.
0:00:10 - 0:01:30	S_8, S_{10} S_{11}, S_{13}	Looking at the papers on the table with the pen in their hand and moving the pen on the paper from left to right. Don't talk anything.		Individual reading.
	S_9 S_{12}	Looking at the papers on the table with the pen in their hand. Are not talking and are not moving anything.		
0:01:30-0:11:00	S_8	Looking at S_9 and talking to him something in low voice (0:01:30). Looking at other members of team. Keeping quit while S_{13} is reading loudly and while S_{11} is translating. Looking at the exercise paper and moving the pen on the exercise paper.	…someone can read loudly… …fawns…	Process of making sense of problem through a loud collective reading and translation.

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APPENDIX 2: Video transcripts for the second group

This group has six students (participants) i.e. S_8, S_9, S_{10}, S_{11}, S_{11}, S_{12} and S_{13}. They were seated from left to right i.e. from left S_8 up to S_{13}.

Time	Partici-pant	Actions	Words	Codes
0:00:00 - 0:00:10	S_8, S_9 S_{10}, S_{11} S_{12}, S_{13}	Seating near the table. Keeping some papers in front of them on the table. Receiving the exercise paper and do not say anything.		Silent within the members of the team.
0:00:10 - 0:01:30	S_8, S_{10} S_{11}, S_{13}	Looking at the papers on the table with the pen in their hand and moving the pen on the paper from left to right. Don't talk anything.		Individual reading.
	S_9 S_{12}	Looking at the papers on the table with the pen in their hand. Are not talking and are not moving anything.		
0:01:30-0:11:00	S_8	Looking at S_9 and talking to him something in low voice (0:01:30). Looking at other members of team. Keeping quit while S_{13} is reading loudly and while S_{11} is translating. Looking at the exercise paper and moving the pen on the exercise paper.	…someone can read loudly… …fawns…	Process of making sense of problem through a loud collective reading and translation.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:01:30-0:11:00	S_9	Looking at S_8 and talking to him something with low voice (**0:01:45**). Keeping quit while S_{13} is reading loudly and while S_{11} is translating. Looking at S_{11}. Writing and talking something. Looking at the exercise paper and consulting the English- French dictionary with S_{10} and S_{11}.	… fawns…	Process of making sense of problem through a loud collective reading and translation.
	S_{10}	Looking at the papers on the table with the pen in his hand and talking. Keeping quit while S_{13} is reading loudly and while S_{11} is translating. Putting the right hand on his forehead. Fetching the English-French dictionary and consulting it with S_9 and S_{11}.	…yes, someone can read loudly… (**0:02:00**) …herd ni troupeau, eeh (**0:05:50**) … fawns… …eeh… akana kayo… (the deer's baby…)	

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:01:30-0:11:00	S_{11}	Looking at the papers on the table with the pen in his hand. Keeping quit while S_{13} is reading loudly. Putting the right hand on his forehead. Pointing the pen on the word deer. Looking on the exercise paper and talking in French the meaning of the first sentence. Making loud voice in English and French, some times in the mother tongue. Consulting the English-French dictionary with S_{10} and S_{11}.	…deer… …sur la côte est des Etats Unis d'Amérique les white-tailed deer se promenaient liblement et sont devenus une source d'aliments pour les Indiens… …with residential … …donc avec l… … …deer hunting provides recreation for hunters… …la chasse aux deer est une sorte de recreation pour les chasseurs… …the size of deer herds …herd means troupeau in French…(**0:06:00**) - fawns, faawwns… (**0:08:00**) … …how many fawns?…	Process of making sense of problem through a loud collective reading and translation.
	S_{12}	Listening S_{13} and talking to him something. Looking at the papers on the table. Keeping quit while S_{13} is reading loudly and while S_{11} is translating. Putting the right hand on his forehead. Fetching the English-French dictionary and consulting it alone. Talking and writing something.	…ok… …fawns… …donc ni akana kayo… (it means the deer's baby…)	Process of making sense of problem through a loud collective reading and translation.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:01:30-0:11:00	S_{13}	Looking at S_{12} and talking to him. Making a loud voice. Stopping to read.(0:02:54) Speaking in French Looking at S_{11}, S_9 and S_8. Putting the pen in the mouth. Keeping quit while S_{11} is translating and looking at the exercise paper.	…ok, I am going to read…(0:02:10) In the early years of settlement on the east coast of the United States, the white-tailed deer roamed freely and was a source of food for the Indians and settlers. At this time, the deer population probably numbered in the hundreds of thousands. Gradually, as the region developed, the size of the deer herds diminished. With residential, agricultural, and industrial development, the deer's living range has been greatly reduced to the available forestlands. American… (0:02:54) …installation… …la côte est desEtats Unis d' Amérique… … white-tailed deer… …herd ni amashyo… (0:05:55) …fawns…	Process of making sense of problem through a loud collective reading and translation.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_8	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Moving a pen on empty paper and talking to S_9 in low voice in the mother tongue. Talking many times to S_8, S_{10} and S_{11}.	…approximately each… …it mean $\frac{2}{3}$ of the males and females are newborns …(0:20:10) …3978 fawns, ok(0:21:04) …yes this is right …does this imply that 45% of all fawns born do not reach one year of age?…(0:26:11) …you see, 40%… …ok!(0:26:42) …in this way we assume that all adult females produce the newborns at the same time…(0:32:40) …we are considering that the available data are for the first year, ok! (0:37:40) …it is possible that some of the adult females do not produce the newborns each year, how can we kwon about it?…(0:41:00)	Loud collective discussion about the main goal and assumptions of the problem.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_9	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Moving the pen on empty paper and writing something.(0:30:00). Talking to S_8, S_{11} in the mother tongue. Looking at the papers, writing and talking in low voice.(0:42:30) Showing S_8 what he has written at the same time talking about it each other (0:42:45)	-…. Ikibazo rero…(this means in english: then the problem is…)(0:15:35) - these 100 does, next year they will produce 150 fawns() -ok - the problem is to distinguish the number of males and females. - $\frac{2}{3}$ of the males and females are newborns(0:19:45) - yes , it's ok(0:20:20) - yes fine 3978 fawns(0:21:04) - the problem is …(0:22:17) - … if so 40% of these die, ok!(0:25:22) - yes (0:26:42) -the problem is that each year there are produced newborns, so …, you see? (0:31:50) - and we do not know how and when the adult females produce the newborns(0:32:00) **- ok, we assume that adult females produce the newborns at the same time(0:32:40)** -yes these are for first year then for the next year, we are going to calculate ….but we do not forget that some of for this year will die (0:37:15) - I am going to ask the question about it(0:44:10)	Loud collective discussion about the main goal and assumptions of the problem.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_{10}	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Talking to S_{11} in the mother tongue. Opening the existing papers on the table and looking at these papers. (0:29:00) Closing the papers and does not say any thing (0:30:50)	- yes - what means - $\frac{2}{3}$ of the males and females are newborns(0:19:45) - no, …(0:19:53) - yes fine 3978 fawns (0:21:04) - …. are newborns (0:22:24) -yes(0:22:24) - where do you get these sixty?(0:26:25) **- ok, we assume that adult females produce the newborns at the same time(0:32:40)** - yes (0:37:20)	Loud collective discussion about the main goal and assumptions of the problem.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_{11}	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Talking to S_9. Looking at the paper, and writing something. (0:26:56)	- each year for every 100 does, 150 fawns are produced(0:15:11) - yes - each year we have 3714females, approximately each year 100 of them produce 150 fawns(0:16:01) - after 3 years the newborns are also able to produce the newborns - $\frac{2}{3}$ of the males and females are newborns(0:19:45) - how many fawns?(0:20:45) three thousands.. - yes fine 3978 fawns(0:21:04) - see, $\frac{2}{3}$ of the males and females are newborns(0:22:20) - the natural survival rate for adult is 90%(0:28:58) **- ok, we assume that adult females produce the newborns at the same time(0:32:40)**	Loud collective discussion about the main goal and assumptions of the problem.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_{12}	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Talking to S_{11} in the mother tongue. Looking at the papers and moving a pen like he is writing (**0:26:56**)	- yes - two-thirds of the males and females are newborns. - we must find out their number - it means 1920 females fawns (**0:18:06**) - 100 produce 150(**0:18:20**) - $\frac{2}{3}$ of the males and females are newborns(**0:19:45**) - yes fine 3978 fawns(**0:21:04**) - within 150 fawns produced each year there are females and males . Then we need to check the number of both each year(**0:24:30**) -each year 55%of all fawns born reach one year of age and of these, 60%survive to two years of age(**0:25:00**)	Loud collective discussion about the main goal and assumptions of the problem.

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APPENDIX 2: Video transcripts for the second group (to continue)

Time	Partici-pant	Actions	Words	Codes
0:11:00-0:44:00	S_{13}	Putting down the papers on the table. Taking a pen in the right hand and looking at the exercise paper. Up and down finger's movements Looking at the papers and moving a pen like he is writing (0:26:56) Talking to S_8 and S_9 about the results of calculations(0:43:40)	- they will be produced and added to the existing number -thus they say… two-thirds of the males and females are newborns(0:17:50) - $\frac{2}{3}$ of the males and females are newborns(0:19:45) - when we will find out the hole population …(0:20:19) - yes fine 3978 fawns - yes - probably yes(0:41:00) - this is for 1989 then for 1990 you have see number of died deer population(0:43:50)	Loud collective discussion about the main goal and assumptions of the problem.
0:44: 10-0:44-20	S_9	Looking at me and talking.	I would like to make more, eeh to distinguish between two because two are not ….	Teacher as a resource of information.
	Myself	Talking to him.	Because it is about an estimation of deer population in the forest.	
	S_8, S_{10}, S_{11},S_{12}, S_{13}	Looking at my face and do not talk any thing.		

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APPENDIX 3:

First group report on mathematical modelling

Problem 1

A company of fish breading would like to know the number of fish in a pond. This information would be valuable for stoking the pond and for studying the availability of the fish in the pond. You are allowed to catch some fish, place plastic tags on their tails in the way that will not hurt them, and release them the back into the pond. The company have made the fish security that any one can't catch fish and take it home. So non-allowed fishing is not acceptable. There is not dead fish because the company have a habit to put regularly the drugs against fish's diseases in the pond. You are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond. If your mathematical model is working apply it for the following particular case: For a certain pond the employees catch, tag, and release ten fish. Then they apply the capture/recapture method and obtain the following data:

Sample catch	Number of tagged fish	Size of sample catch
1	3	15
2	0	15
3	3	15
4	1	15
5	2	15
6	4	15
7	6	15
8	2	15
9	4	15
10	2	15

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UNIVERSITE NATIONALE DU RWANDA
FACULTE D'EDUCATION
DEPARTEMENT DES SCIENCES EXACTES
Bacc II MATH-PHYSIQUE
ANNEE ACADEMIQUE 2000/2001

83

A WORK ON MATHEMATICAL MODELLING

CONSTRUCT THE MATHEMATICAL MODEL TO SOLVE A PROBLEM

REALISE PAR GROUPE 1 :
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DATE : 23-01-2001

TITULAIRE DU COURS : Maître GAHAMANYI Marcel

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PROBLEM

A company of fish-breeding would like to know the number of fish in a pond. This information would be valuable for stocking the pond and for studying the availability of fish in the pond. You are allowed to catch some fish, place plastic tags on their ~~lag~~ tails in a way that will not hurt them, and then release them back into the pond. The company have made the fish security that any one can't catch fish and take it home. So non-allowed fishing is not acceptable. There is not dead fish because the company have a habit to put regularly the drugs against fish's diseases in the ponds You are requested to construct a mathematical model that can help the company to estimate the number of fish in any pond. If your mathematical model is working apply it for the following particular case: For a certain pond the employees catch, tag, and release ten fish. Then they apply the capture/recapture method and obtain the following data:

Sample catch	Number of tagged fish	Size of sample catch
1	3	15
2	0	15
3	3	15
4	1	15
5	2	15
6	4	15
7	6	15
8	2	15
9	4	15
10	2	15

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1. IDENTIFICATION DU PROBLEME (Identification of problem)

Estimer le nombre de poissons se trouvant dans n'importe quel étang (pond) revient à tirer un certain nombre de poissons, leur donner des signes et les remettre dans l'étang ; puis utiliser la moyenne de proportionnalité entre l'apparition des poissons ayant des signes et celle de ceux qui n'en ont pas. La grandeur de l'étang ne nous intéresse pas et elle n'aura pas d'influence sur notre modèle. Alors pour développer un modèle afin d'estimer le nombre de poissons dans un étang quelconque, il est nécessaire de faire des suppositions pour rendre le problème praticable.

2. SUPPOSITIONS (Assumptions)

- Tous les poissons dans l'étang sont dans les meilleures conditions de vie
- Tous les poissons ne sont pas trop petits pour être attrapés par le fillet
- Tirer un certain nombre de poissons et mettre les signes sur leurs queues et les remettre dans l'étang
- Ces poissons s'éparpillent dans l'étang
- Tous les signes ne peuvent pas se détacher
- Tous les poissons tirés lors des tirages sont remis dans l'étang sans rien transformer avant le tirage suivant
- Varier les lieux de tirage dans l'étang
- Le nombre de poissons tirés peut varier d'un tirage à l'autre
- Etablir une proportionnalité entre l'apparition des poissons ayant été donnés des signes et ceux qui n'en portent pas.
- Le coefficient de proportionnalité dans tous les tirages est égal à 1

3. FORMULER ET RESOUDRE LE MODELE (Formulate and solve the model)

Pour estimer le nombre de poissons dans un étang (pond) quelconque, nous partons des inconnues suivantes afin d'établir la formule du modèle.

Z = nombre de poissons à donner des signes avant l'opération des tirages

n = nombre de tirages effectués

m_i = nombre de poissons ayant des signes pour chaque ième tirage

K_i = nombre total de poissons tirés dans ième tirage (tagged and non tagged for a sample catch)

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- Y : nombre total de poissons n'ayant pas de signes dans tout l'étang (pond)

X = nombre total de poissons dans l'étang

c : coefficient de proportionnalité

Après n tirages on peut avoir un tableau suivant

N° de tirage (Sample catch)	Poissons ayant des signes (tagged fish)	Nombre total de poissons lors du tirage (Size of sample catch)
1	m_1	k_1
2	m_2	k_2
3	m_3	k_3
4	m_4	k_4
5	m_5	k_5
⋮	⋮	⋮
n	m_n	k_n

Nous pouvons établir des proportionnalités entre les apparitions des poissons ayant des signes et ceux qui n'en ont pas. Pour chaque tirage les poissons ayant de signes (m_i) par le nombre total des poissons ayant été donnés des signes avant l'opération des tirages (Z) sont directement proportionnels au nombre de poissons n'ayant pas de signes ($k_i - m_i$) par le nombre total des poissons n'ayant pas de signes dans l'étang (Y). Dans le langage mathématique $\frac{m_i}{Z} \sim \frac{(k_i - m_i)}{Y}$ $(i = 1, 2 \ldots n)$

En prenant $\frac{1}{n}\sum_{i=1}^{n}\frac{m_i}{Z}$ comme étant la moyenne d'apparitions des poissons ayant des signes et $\frac{1}{n}\sum_{i=1}^{n}\frac{(k_i - m_i)}{Y}$ la moyenne d'apparitions des poissons n'ayant pas de signes, nous établissons une proportionnalité entre ces deux moyennes :

$$\frac{1}{n}\sum_{i=1}^{n}\frac{m_i}{Z} \sim \frac{1}{n}\sum_{i=1}^{n}\frac{(k_i - m_i)}{Y}$$

En admettant que le coefficient de proportionnalité dans tous les tirages vaut 1 nous écrivons que $\frac{1}{n}\sum_{i=1}^{n}\frac{m_i}{Z} = c\,\frac{1}{n}\sum_{i=1}^{n}\frac{(k_i - m_i)}{Y}$ devient

$$\frac{1}{n}\sum_{i=1}^{n}\frac{m_i}{Z} = \frac{1}{n}\sum_{i=1}^{n}\frac{(k_i - m_i)}{Y}$$

$$\Rightarrow \frac{1}{nZ}\sum_{i=1}^{n} m_i = \frac{1}{nY}\sum_{i=1}^{n}(k_i - m_i)$$

$$\Rightarrow \frac{1}{Z}\sum_{i=1}^{n} m_i = \frac{1}{Y}\sum_{i=1}^{n}(k_i - m_i) \Rightarrow$$

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A partir de cette proportionnalité (égalité) nous pouvons facilement tirer la valeur de Y (nombre total de poissons n'ayant pas de signes dans tout l'étang).

$$\frac{1}{Z}\sum_{i=1}^{n} m_i = \frac{1}{Y}\sum_{i=1}^{n}(k_i - m_i)$$

$$Y\sum_{i=1}^{n} m_i = Z\sum_{i=1}^{n}(k_i - m_i)$$

$$Y = Z\,\frac{\sum_{i=1}^{n}(k_i - m_i)}{\sum_{i=1}^{n} m_i}$$

Ainsi le nombre total des poissons dans l'étang (X) est donné par la somme de Y et Z

$$X = Z + Z\,\frac{\sum_{i=1}^{n}(k_i - m_i)}{\sum_{i=1}^{n} m_i}$$

$$\boxed{X = Z\left(1 + \frac{\sum_{i=1}^{n}(k_i - m_i)}{\sum_{i=1}^{n} m_i}\right)}$$

4. VALIDER LE MODELE (Validate the model)

Suivant les critères que nous nous sommes fixés, le modèle trouvé (la formule mathématique) est applicable à n'importe quel étang (pond) ce qui répond à notre problème identifié au commencement. Nous avons calculé en supposant que le coefficient de proportionnalité est égal à 1. Sûrement il peut ne pas être égal à 1 pour tout tirage ce qui aurait une petite influence voir même négligeable sur l'estimation.

5. WEAKNESSES (FAIBLESSES)

Par ce modèle (la formule établie) on ne peut pas préciser le nombre de tirages qui permettraient de donner de bon résultats.

Application du modèle sur le cas particulier où $Z = 10$ et K_i est constant et vaut 15.

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Sample catch (i)	Number of tagged fish (m_i)	Size of sample catch (k_i)	($k_i - m_i$)
1	3	15	12
2	0	15	15
3	3	15	12
4	1	15	14
5	2	15	13
6	4	15	11
7	6	15	9
8	2	15	13
9	4	15	11
10	2	15	13

$$\sum_{i=1}^{10} m_i = 27$$

$$\sum_{i=1}^{10} (k_i - m_i) = 123$$

$$Z = 10$$

$$X = Z \left(1 + \frac{\sum_{i=1}^{10} (k_i - m_i)}{\sum_{i=1}^{10} m_i} \right)$$

$$X = 10 \left(1 + \frac{123}{27} \right) \Rightarrow X = 55{,}5 \simeq 56$$

Par ces dix tirages, en utilisant notre modèle, nous estimons qu'il y a ≃ 56 poissons dans l'étang (pond)

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APPENDIX 4:

Second group report on mathematical modelling

Problem 2

In the early years of settlement on the east coast of United States, the white-tailed deer roamed freely and was a source of food for Indians and settlers. At this time, the deer population probably numbered in the hundred of thousands. Gradually as the region developed, the size of deer hers diminished. With residential, agricultural, and industrial development, the deer's living range has been greatly reduced to the available forestlands. American white-tailed deer remain a natural resource whose beauty can be enjoyed in the wild. Deer hunting provides recreation for hunters every fall (autumn). The size of deer herds depends on a delicate balance. If the herds become too large, that is the number of deer exceeds the amount of available food, deer starve and die. The starving deer destroy agricultural crops and cause traffic hazards through their wide migration patterns. Therefore, the size and quality of deer hers must be carefully monitored and controlled by the wildlife officials. This control is accomplishing special hunting seasons; for example, in doe seasons the female deer may also be hunted.

Dauphin County, in 1989 had an estimated population of 9399 animals, distributed as follows:

Adult males (bucks) 1707. Male fawns 2058.
Adult females (does) 3714. Female fawns 1920.

Available game-commission statistic indicate the following:

1. Fawns are considered adults at two years of age.
2. Two-thirds (2/3) of the males and females are newborns.
3. Approximately each year for every 100 does, 150 fawns are produced.
4. Each year 55 percent of fawns born reach one year of age, and of these, 60 percent survive to two years of age.
5. The natural survival rate for adult is 90 percent.

Develop a mathematical model to study the growth of Dauphin County's deer population over five years.

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Homework on Mathematical Modelling.

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The growth of deers' population there, meets a number of obstacls. Hunting, starving and other kinds of death. Fawns are the most affected: only a few of them can survive to two years of age. The study of the growth is then delicate and it becomes to considering the rate of death, surviving and birth rate. But to develop a mathematical model to study county's deer population let base us on available statistical indications and make assumptions to make the problem practicable.

ASSUMPTIONS

1) Fawns are considered adults at two years old.
2) Fawns include less than-one-year-fawns (newborns) and one-year-fawns.
3). Everytime there is a constant proportionality:
   - k between adult females and the whole population.
   - l between all adults and the whole population.
4) 2/3 of adults are fawns and each year $\frac{15}{10}$ of adult females are new borns. Each year $\frac{45}{100}$ of new-borns die
5) Each year $\frac{40}{100}$ of one-year-fawns die.
6) Each year $\frac{10}{100}$ of Adult population die. Adult females die after giving birth.

INITIAL SITUATION

Let H_0 be the whole population
F_0 Adult females
A_0 Adult population
f_0 fawns that are less than one year old
f_1 fawns that are one year old

To express the above proportionalities:

$F_0 = k H_0$

$A_0 = l H_0$

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$$f_0 = \frac{15}{10}F_0 = \frac{15}{10}kH_0$$

$$f_1 = \frac{2}{3}A_0 - \frac{15}{10}kH_0 = \left(\frac{2}{3}l - \frac{15}{10}k\right)H_0$$

<u>First year</u> :

Newborns : $\frac{15}{10}F_0 = \frac{15}{10}kH_0$

Had died :
- $* \frac{1}{10}A_0 = \frac{1}{10}lH_0$
- $* \frac{45}{100} \cdot \frac{15}{10}kH_0$
- $* \frac{40}{100}\left(\frac{2}{3}l - \frac{15}{10}k\right)H_0 = \frac{2}{5}\left(\frac{2}{3}l - \frac{15}{10}k\right)H_0$

Then the population has increased of :

$$\frac{15}{10}kH_0 - \left[\frac{1}{10}lH_0 + \frac{45}{100}\cdot\frac{15}{10}kH_0 + \frac{2}{5}\left(\frac{2}{3}l - \frac{15}{10}k\right)H_0\right]$$

$$\left(\frac{15}{10}k - \frac{1}{10}l - \frac{45}{100}\cdot\frac{15}{10}k - \frac{4}{15}l + \frac{3}{5}k\right)H_0$$

$$\left(\frac{15}{10}k - \frac{45\cdot15}{1000}k + \frac{3}{5}k - \frac{1}{10}l - \frac{4}{15}l\right)H_0$$

$$(1{,}5k - 0{,}675k + 0{,}6k - 0{,}1l - 0{,}26l)H_0$$

$$(1{,}425k - 0{,}36l)H_0$$

Let α be equal to $(1{,}425k - 0{,}36l)$.

Then the population has increased of αH_0.

The population has then become $H_0 + \alpha H_0 = (1+\alpha)H_0$.

Let $H_1 = (1+\alpha)H_0$.

Let take H_1 as initial population for <u>the second</u> year as k and l are constant and consequently independent of time, we have $F_1 = kH_1$

$A_1 = lH_1$

And in the same way, the population increase of αH_1 during the second year and becomes $H_1 + \alpha H_1 = (1+\alpha)H_1$

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$$f_0 = \frac{15}{10} F_0 = \frac{15}{10} k H_0$$

$$f_1 = \frac{2}{3} A_0 - \frac{15}{10} k H_0 = \left(\frac{2}{3} l - \frac{15}{10} k\right) H_0$$

<u>First year</u> :

Newborns : $\frac{15}{10} F_0 = \frac{15}{10} k H_0$

Had died :
- $* \ \frac{1}{10} A_0 = \frac{1}{10} l H_0$
- $* \ \frac{45}{100} \cdot \frac{15}{10} k H_0$
- $* \ \frac{40}{100} \left(\frac{2}{3} l - \frac{15}{10} k\right) H_0 = \frac{2}{5} \left(\frac{2}{3} l - \frac{15}{10} k\right) H_0$

Then the population has increased of :

$$\frac{15}{10} k H_0 - \left[\frac{1}{10} l H_0 + \frac{45}{100} \cdot \frac{15}{10} k H_0 + \frac{2}{5}\left(\frac{2}{3} l - \frac{15}{10} k\right) H_0\right]$$

$$\left(\frac{15}{10} k - \frac{1}{10} l - \frac{45}{100} \cdot \frac{15}{10} k - \frac{4}{15} l + \frac{3}{5} k\right) H_0$$

$$\left(\frac{15}{10} k - \frac{45 \cdot 15}{1000} k + \frac{3}{5} k - \frac{1}{10} l - \frac{4}{15} l\right) H_0$$

$$\left(1{,}5 k - 0{,}675 k + 0{,}6 k - 0{,}1 l - 0{,}26 l\right) H_0$$

$$\left(1{,}425 k - 0{,}36 l\right) H_0$$

Let α be equal to $(1{,}425 k - 0{,}36 l)$.

Then the population has increased of αH_0.

The population has then become $H_0 + \alpha H_0 = (1+\alpha) H_0$.

Let $H_1 = (1+\alpha) H_0$.

Let take H_1 as initial population for <u>the second</u> year as k and l are constant and consequently independent of time, we have $F_1 = k H_1$

$A_1 = l H_1$

And in the same way, the population increase of αH_1 during the second year and becomes $H_1 + \alpha H_1 = (1+\alpha) H_1$

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Replacing H_1 by its value $(1+\alpha)H_0$

we obtain $H_2 = (1+\alpha)H_1$

$$= (1+\alpha)(1+\alpha)H_0$$

$$= (1+\alpha)^2 H_0.$$

$H_2 = (1+\alpha)^2 H_0$ is the population over two years.

<u>In the same way:</u>

$H_3 = (1+\alpha)^3 H_0$ is the population over 3 years

$H_4 = (1+\alpha)^4 H_0$ is the " " 4 years, and

$H_5 = (1+\alpha)^5 H_0$ is the population over five years.

⋮

$H_n = (1+\alpha)^n H_0$ is the population over n years.

Validate and impliment the model.

<u>Considering our case:</u>

$H_0 = 9399$ deers

$A_0 = 5421$ Adults (deers)

$F_0 = 3714$ Adult females

fawns : 3978.

⊛ $F_0 = kH_0 \Rightarrow k = \frac{F_0}{H_0} = \frac{3714}{9399} = 0{,}395 \simeq 0{,}4$

⊛ $A_0 = lH_0 \Rightarrow l = \frac{A_0}{H_0} = \frac{5421}{9399} = 0{,}576 \simeq 0{,}6$

$$\alpha = 1{,}425k - 0{,}36l = 1{,}425 \cdot 0{,}4 - 0{,}36 \cdot 0{,}6$$
$$= 0{,}354$$

$H_1 = (1+\alpha)H_0$

$H_1 = (1+0{,}354) \cdot 9399 = 12726$. deers

Then $H_5 = (1+0{,}354)^5\, 9399 = 42773$ deers

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(~~Weak~~ness)

The results of this model are realistic. The model can be applied to estimate the dears' population at any time (over a certain number of years).

Whenever there are initial statistical data of any given population, the model is also practical to compute the growth and the amount of population.

The population computed with this model corresponds to reasonable estimations.

Weakness

For practical reasons and complications that would be due to determining over more than two years the variation of different sections of the population, we considered the proportionalities between Adults, Adult female and the whole population to be constant. Whereas these constants of proportionalities k and l are not really constants because of natural conditions.

Our model is based on statistical data which are not really facts ~~[illegible]~~ but estimations because it is difficult to count animals in the forest.

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