INSERVICE FEMALE TEACHERS' ANXIETIES ABOUT MATHEMATICS: A REFLECTIVE STUDY ON MATHEMATICS CLASSROOM PRACTICE AT A COLLEGE OF EDUCATION

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Abstract

The incidence of mathematics anxiety manifesting itself in in-service college students is generally on the increase. Such anxiety does not only affect the mathematical performance of students but also their teaching of the subject. Thus a need exists to investigate measures to alleviate mathematics anxiety as displayed by practising teachers.

It is with these factors in mind that I have embarked upon a study to analyse the role of my teaching practice in the context of mathematics anxiety and learning theories.

The research indicates that a social constructivist approach to teaching mathematics contributes to allaying students' mathematics anxiety. It also raises pertinent questions for further investigation. For example, would the teaching practice of in-service mathematics teachers be influenced by their own positive experiences?

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CHAPTER 1

INTRODUCTION

Many practising teachers wishing to upgrade their teaching qualifications attend classes at a college of education on a part-time basis. These teachers, most of them women, obtained teaching certificates by following a two-year training course when standard eight was an entrance requirement for a pre-service student teacher. Later, when entrance requirements were revised and a Senior Certificate became a prerequisite to a teacher diploma course, most of the existing "standard eight teachers" were labelled as "underqualified". To upgrade their qualifications and status they had to follow a path of first obtaining a Senior Certificate in order to be accepted into the second year of a three-year teacher diploma course. As from 1 January 1993 the Department of Education and Culture, House of Representatives, also considered qualified teachers with a Junior Certificate (standard eight) and at least eight years satisfactory teaching experience for admission to the second year of the existing diploma course (Education Bulletin, 5/92).

Some of these teachers, for some or other reason, avoided doing mathematics in secondary school and/or avoided teaching mathematics to senior primary classes. Thus, when they are accepted at college, they are shocked and perturbed to find that mathematics is a compulsory subject up to the second year of study. Consequently I find that their attitude towards and their perception of mathematics make their learning and teaching of mathematics almost an impossible task. So, when the supervisor of the part-time classes at the college of education where I lecture informed me that many of the in-service teachers upon registering as part-time students request that I be assigned as their mathematics lecturer, it gave me food for thought. I thought that a possible reason for their request could be that my classroom practice had played some role in helping previous students in changing their negative attitudes towards mathematics.

Being shocked and perturbed - at having to do mathematics - is an emotional reaction which can broadly be described as "mathematics anxiety". So, if I were an agent in helping some students overcome their mathematics anxiety, without consciously being aware of this, would I not be better "equipped" if I reflect on - and analyse - my own class-room practices? Would a better understanding of some theories regarding women and mathematics, mathematics anxiety and learning not assist me in such an analysis of my classroom practices? Would a systematic study of this nature not help other mathematics teachers who are faced with similar problems/situations?

Thus the purpose of this study is to analyse the role of my classroom practice as an agent in the formation of certain attitudes and perceptions relating to the learning and teaching of mathematics of mature in-service women students studying at a college of education.

Before I make a systematic study of my classroom practice in Chapter 4, an analysis is made in Chapter 3 of the attitudes and perceptions with regard to mathematics of a current (1993) cohort of my students. But, in order to make this analysis, it is necessary to look at the emerging literature on "mathematics anxiety" and "gender and mathematics". Thus Chapter 2 addresses these issues. Learning and doing mathematics is the subject of Chapter 5, and in Chapter 6 I reflect on my classroom practice using constructivism as the framework for this reflection.

CHAPTER 2

MATHEMATICS ANXIETY, GENDER AND MATHEMATICS

A considerable research effort has been devoted to developing the construct of "mathematics anxiety". However, it remains difficult to define this construct in a clear and simple fashion without first giving an explanation or a description of the construct anxiety.

2.1 On Anxiety

In view of the complexity of anxiety and its manifestations, it is not surprising that there exist in the literature many differing views of anxiety. A few of the views that have been proposed about the nature of anxiety are as follows:

(a) Roy Grinker (1966) proposes that because anxiety has a tendency to feed upon itself, it has a special role in the adaptive operations of the human organism. Anxiety serves both as an indicator of reaction to stress as well as an indicator of further stress reactions. The more we become aware of our own ineffectiveness and ineptitude the more anxious we become.

(b) Carroll Izard and Silvan Tomkins (1966), in a paper to present their conceptual analysis of the relationship between affect and behaviour, conceptualized anxiety as an affect. The affect system, as one of the five subsystems that form the personality of an individual, they define as the primary motivational system and propose it to be the principal provider for cognition and action.

In order to make their conceptualization of anxiety as an affect meaningful, Izard and Tomkins (1966, p.99), put it into context as follows:

1. Anxiety is a negative affect, but anxiety and negative affect are not interchangeable terms. We posit five negative affects: (1) fear-terror, (2) distress-anguish, (3) shame-humiliation, (4) anger-rage and (5) contempt-disgust.
2. Anxiety is subsumed under the affect fear-terror; it may be generated by innate activators, drives, other affects, cognition, and external conditions.
3. Fear-terror (anxiety), like each of the eight primary affects, is subserved by its own innate mechanisms.
4. Fear-terror, as one of the eight primary affects, is part of the affect system. To determine the significance of anxiety, it must be considered one of the major affects.

Furthermore, Izard and Tomkins also propose that "anxiety is experienced as apprehension, uneasiness, uncertainty, insecurity" (p.107).

(c) George Mandler and David Watson (1966) argue that anxiety is evoked in an individual when an organized behavioural sequence is interrupted by a situation which does not offer an alternate course of action. An emotion of helplessness and disorganization, which we call anxiety, is the result of the interruption.

(d) Charles Spielberger (1966) distinguishes between anxiety as a "transitory state" and anxiety as a relatively stable "personality trait". According to Charles Spielberger

> anxiety states (A-states) are characterized by subjective, consciously perceived feelings of apprehension and tension, accompanied by or associated with activation or arousal of the autonomic nervous system. Anxiety as a personality trait (A-trait) would seem to imply a motive or acquired behavioral disposition that predisposes an individual to perceive a wide range of objectively nondangerous circumstances as threatening, and to respond to these

with A-state reactions
disproportionate in intensity to the
objective danger.

(Charles Spielberger, 1966, pp.16-17)

From Roy Grinker's assertion that anxiety serves as an indicator of stress, it is expedient to briefly explore the interrelationship of stress and anxiety.

Charles Spielberger et al (1978) propose that <u>stress</u> be defined in terms of the objective stimulus properties of situations (stressors). Irrespective of the presence of stress (real or objective danger), a person who perceives a situation as dangerous or threatening will experience an increase in <u>state anxiety</u>. Whereas stress can often be attributed to an individual's interpretation of a situation, anxiety is a reaction to perceived inability to handle a challenge in a satisfactory manner.

Also, with regard to the effects of anxiety, Charles Spielberger et al (1978, p.196) note the following to be among the characteristics of anxiety responses:

1. The situation is seen as difficult, challenging and threatening.
2. The individual sees himself or herself as ineffective in handling, or inadequate to, the task at hand.

3. The individual focuses on undesirable consequences of personal inadequacy.
4. Self-deprecatory preoccupations are strong and interfere or compete with task-relevant cognitive activity.
5. The individual expects and anticipates failure and loss of regard by others.

Furthermore, although some psychologists maintain that a "little" anxiety may facilitate performance they are all in agreement that intense anxiety is detrimental to sustaining constructive, effective functioning and creative learning. For example, in a volume of research utilizing the Taylor Manifest Anxiety Scale, Janet Spence and Kenneth Spence (1966) note that although emphasis on doing well may lead most individuals to increased effort and attention it "may also arouse anxiety (fear of failure) and negative evaluations of performance (failure reports) to intensify it" (p.313). They also found that as anxiety increases in intensity, so do the frequency and intensity of task-irrelevant responses. Raymond Cattell (1966, p.46) succinctly puts it when he says:

> Anxiety is more frequently a cognitive disorganizer than an aid to learning and will reduce the

capacity for immediate memory, and
may also result in poorer
calculation performance.

These explanations, views and definitions are, for the most part not contradictory, but rather, deal with somewhat different sets of variables and concepts. Taken together they are more complementary than contradictory in what they say about stress and anxiety. Anxiety, thus, seems to be an unpleasant emotional reaction to a perceived presence of danger - either from the external environment or from internal feelings or thoughts.

Whilst in this section I have endeavoured to provide a historical overview of research on emotional reactions to stress and also to review theory and research on anxiety, I shall look at the relationship between mathematics and anxiety in the next section.

2.2 Mathematics and Anxiety

In the previous section the construct of anxiety and the subsequent anxiety responses by individuals were discussed. We have seen that anxiety is an emotional reaction by individuals to certain internal or external stimuli. If the external sources involve mathematics and/or mathematical skills then the resulting anxiety will be referred to as mathematics anxiety. Thus, from this view of mathematics anxiety, it is understood to manifest itself as a cognitively demanding activity marked by self-occupation, self-depreciation, attentional blocks and neglect or misinterpretation of informational cues that may be easily available.

Most researchers have viewed mathematics anxiety and test anxiety as highly related constructs, while Lorelei Brush (1981) describes mathematics anxiety as no more than subject-specific test-anxiety.

Ray Hembree (1990), on the other hand, concludes, from a study to integrate the findings of the existing research on mathematics anxiety, that mathematics anxiety, like test anxiety, seems to be a learned condition more behavioural than cognitive in nature. Hembree also observes that the construct appears to comprise a general fear of contact with mathematics including classes, homework and tests.

Ray Hembree reports the following regarding the nature and effects of mathematics anxiety:

1. higher mathematics anxiety consistently relates to lower mathematics performance;
2. higher achievement consistently accompanies reduction in mathematics anxiety;
3. positive attitudes towards mathematics consistently relate to lower mathematics anxiety, with strong

inverse relations observed for an enjoyment of mathematics and self-confidence in the subject;

4. mathematics anxiety relates directly to debilitating test anxiety and inversely to the anxiety drive that facilitates performance during testing;
5. the highest levels of mathematics anxiety occur for students preparing to teach in elementary school;
6. across all grades, but especially in college, female students report higher levels of mathematics anxiety than males; and
7. at precollege levels, especially in high school, the negative effects of mathematics anxiety seem more pronounced in males than females.

The fact that male students at secondary school seem to exhibit stronger negative behaviours than their female counterparts as far their reported mathematics anxiety levels are concerned, makes it expedient to take a closer look at the affects and effects of mathematics and mathematics teaching on males and females. Thus I shall devote the last section of this chapter to literature on mathematics and gender.

2.3 Mathematics and Gender

Although mathematics as a school subject is compulsory, for boys and girls, up to standard seven (age 14 years and older), competence in mathematics is not a prerequisite for entering the teaching profession. Contrary to this, a mathematics qualification is a necessary prerequisite for entering many of the areas of study where gender inequality is particularly marked, such as technology, engineering and science. This issue becomes even more relevant to the topic under discussion if we consider the fact that the teaching of young children - especially junior primary classes - is generally accepted to be a woman's domain. Could the choice of a career, or the "impossibility" of some careers, be a relevant factor which deflects girls away from mathematics?

Although girls are supposedly "free" to choose their own careers and supposedly "free" to make their own choice of school subjects, they are often stereotyped, by society and the school, as potential wives and mothers, steered into the so-called caring professions - social work, nursing and teaching - and seldom encouraged to have career aspirations in directions where mathematics is clearly going to be needed. Also, when women choose a career without apparently being influenced from outside, they themselves often anticipate futures in which marriage and family responsibilities are seen in the present as major modifiers of anticipated careers (David Maines, 1985), careers in which mathematics is often accepted to be an interesting but socially irrelevant area of activity. Another dimension of the same view was expressed by Lorelei Brush (1980, p.15) when she commented:

> It is logical that students who see no application for mathematics in their own future careers or personal lives are less likely to take optional or advance courses in the subject.

Boys, on the other hand, who are projected as the future breadwinners of the family and whose careers are accepted to be life-long, are encouraged to consider their careers important and are guided towards those kinds of choices - physics, engineering, architecture, accountancy, chemistry or computer science - for which mathematics is a service subject. "So boys ... are also encouraged, explicitly and implicitly, to work at their mathematics" (Zelda Isaacson, 1986, p.234).

Small wonder then that more men than women - most of the men having embarked on teaching as a last option - have a relatively sound mathematical background, when entering colleges of education. No wonder, also, that Ray Hembree (1990) could report that mathematics anxiety has a more adverse effect on the performance of boys, relative to the performance of girls, in secondary school. After all, judging from the above, fear of failure in mathematics in secondary school should have a much more devastating effect on boys than girls.

Valerie Walkerdine and her team (1989) noted that, although, from primary school to fourth year secondary school, girls were more successful in mathematics, relative to boys, and performed better than boys overall, girls were often not entered for the more prestigious examinations. In South Africa, also, a similar occurrence can be observed: in national mathematics olympiads arranged for senior primary school pupils, the number of girls reaching the finals usually compares well with the number of boys reaching the finals; on the other hand, hardly any girls are entered for mathematics olympiads organized for senior secondary school pupils. For example: the team representing South Africa at the International Mathematics Olympiad in 1992 consisted of boys only while, in 1993, in the mathematics olympiad organized by the Cape Professional Teachers Association (CPTA), more girls than boys reached the finals. What happens to girls' "mathematical" ability during the intervening years between standards five and ten?

On analysing the interviews held with teachers, Valerie Walkerdine et al (1989) report that teachers, and even some of the students, attribute girls' success to hard work and rule-following, while boys are the ones seen to have natural ability - and will therefore, supposedly, stand a better chance of passing those examinations for which a sound understanding of mathematics as well as a keen insight is of vital importance! These double standards used in attributing mathematics achievement could be another factor contributing to girls' negative attitude towards mathematics. And, "... negative attitudes tend to accompany failure to participate in optional or advanced mathematics courses" (Lorelei Brush, 1980, p.15).

Also, in analysing conversations between teachers and pupils, they (Valerie Walkerdine et al, 1989) found that in a mathematics lesson, teachers' reactions - even the reactions of female teachers - to boys' challenges of the rules of mathematical discourse differed from their reactions to those of girls. Boys' contributions were frequently elaborated and extended while the contributions of girls were neither elaborated nor extended but rather thwarted or ignored. Thus it would seem that while girls are discouraged to challenge the rules of mathematical discourse, they are at the same time "accused" of rule-following with no mathematical "ideas" of their own.

Another factor deflecting women from mathematics could be the way mathematics is perceived and taught. Mathematical meanings are part of practices which are inherently social and political in nature, and yet it is often found "that 'mathematical' meanings and forms are accomplished only by prising them out of and suppressing the practices of which they once were a part" (Walkerdine et al, 1989, p.185). By suppressing and transferring these practices - many with which girls can easily identify - by prising the mathematical meanings out of the social world and presenting mathematics as an axiomatic system, alienate many students, but especially girls, from mathematics. To this end Dorothy Buerk (1985, p.59) writes:

> For many women, mathematics is a collection of right answers with correct methods and exact symbols. While this view may provide security to those who can use the symbols correctly, it is devastating for those who cannot.

In the preceding sections some psychologists' and researchers' views regarding anxiety and attitudes towards mathematics were described and discussed. From these discussions it can, perhaps logically, be deduced that social and emotional factors as well as classroom practices play a major role in the attitudes students adopt towards mathematics. The extent to which these, or other, constructs have influenced my current cohort of students' attitude towards mathematics will be investigated and discussed in the next chapter.

CHAPTER 3

STUDENTS' CONCEPTIONS ABOUT LEARNING MATHEMATICS

It has always been my practice to ascertain the fears and anxieties with which the students embark on my mathematics courses. I do this because I believe that having a better insight of my students' fears makes me more capable of countering negative experiences they might have had with mathematics by subjecting them to "new" ways of learning and doing mathematics.

My current (1993) second year part-time mathematics class is composed of 30 female students. Their average age is 38 years, and all of them are over 30 years old (see figure 1). The students are all primary school teachers - either junior primary or senior primary - averaging 18 years teaching experience (see figure 2).

FIGURE 1
AGE DISTRIBUTION OF STUDENTS

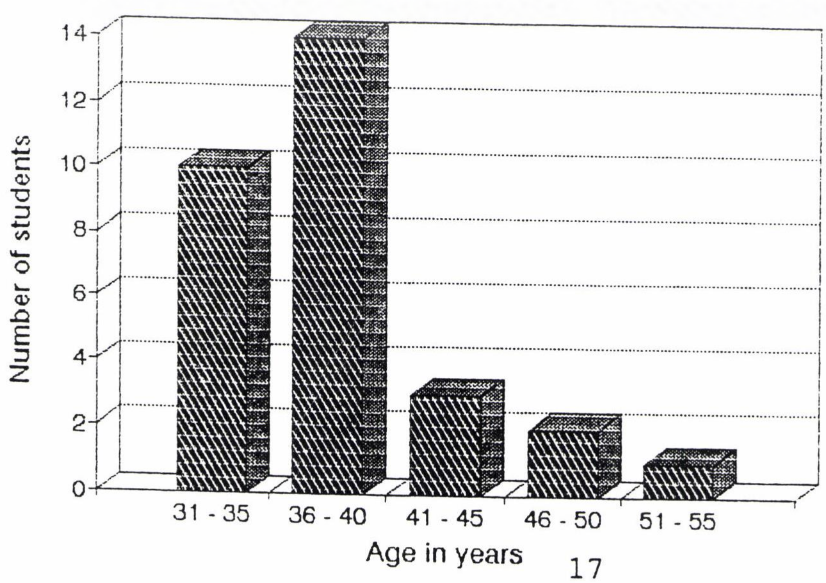
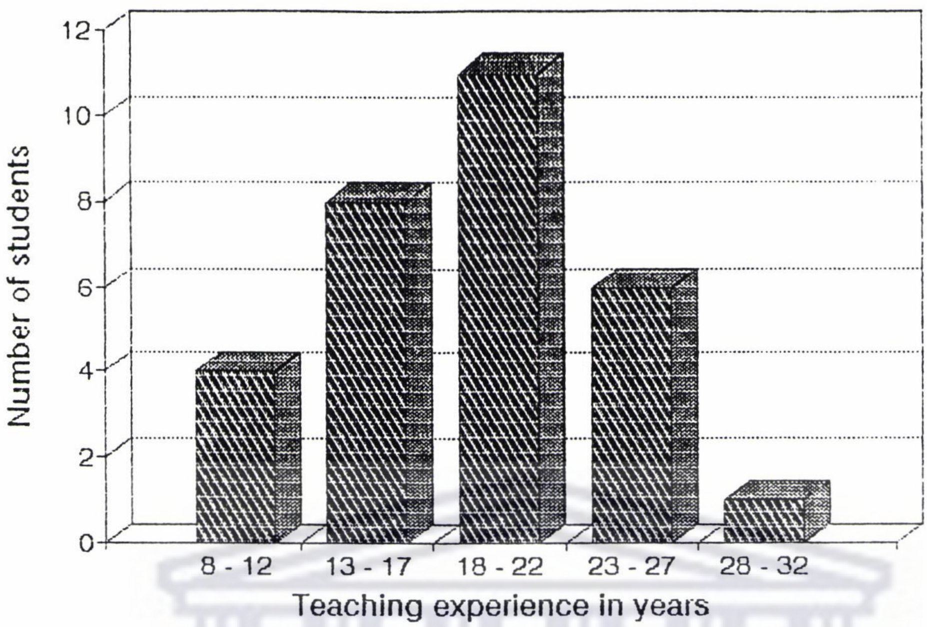


FIGURE 2
TEACHING EXPERIENCE OF STUDENTS



At the beginning of the year, when the students reported for their first mathematics lesson, I realized that most of them were very apprehensive about mathematics and the mathematics class. During our discussion of the syllabus and the course requirements their fears and their "negative" attitudes toward mathematics became more pronounced and they even voiced a subsequent disillusionment about the whole diploma course.

The content of the syllabus for Mathematics for the second year of the senior primary diploma course issued by the Department of Education and Culture in the House of Representatives (Appendix 1) includes

1. <u>sets</u> - properties; representations by means of Venn diagrams; solving problems using Venn

diagrams; sets of ordered pairs including graphical representations;

2. <u>algebra</u> - different interpretations as well as historical background of Algebra; important concepts; the four basic operations performed on algebraic terms and expressions; products by inspection of binomials; factors and lowest common multiple of polynomials; simplification of algebraic fractions; substitution in formulas; change of the subject of a formula; solving linear equations and inequalities in one unknown;
3. <u>arithmetic</u> - compound interest; ratio and proportion; roots; number systems with other bases;
4. <u>geometry</u> - simple scale drawings; geometric transformations; axioms and theorems (without formal proofs) regarding triangles (including congruency and the theorem of Pythagoras) and a parallelogram with simple applications; introduction to circles;
5. <u>mensuration</u> - circumference and area of circles; calculating the exterior areas and volumes of right solid cylinders; and also
6. <u>subject didactics</u> - teaching techniques based on the subject content of the school syllabus for Mathematics standards 2 to 4 and relevant learning theories.

In addition in-service teachers - since they are exempted from the first year of study - also have to study those topics, included in the first year's syllabus, which serve as essential pre-knowledge for the second year's work.

For these part-time students the situation is further compounded by the fact that the course requirements stipulate that for them to proceed to their third year of study for the diploma, they have to obtain at least 33,3% in mathematics - full-time students are exempted from this stipulation. Furthermore, all students (full-time and part-time) who wish to continue with mathematics at third year level have to pass the subject with at least 50% at second year level.

Now I am what one can call a "generally" anxious person - prone to fears and worries. In learning to deal with my fears and worries I realized that the first thing to do is to put my fear (or worry) into words in order to know exactly what I fear or what I am worried about. This practice, and my twenty eight years' teaching experience, have convinced me that most mathematics anxiety can be overcome, or at least reduced, by first of all knowing precisely what one fears.

Thus, at our second class meeting, I requested those students who wished to do so to answer a questionnaire I had constructed. I had constructed the questionnaire in order to firstly, give them an opportunity to "voice" their fears and anxieties and secondly, for me to obtain information about:

(a) the nature of their feelings and fear(s) regarding mathematics, the learning of mathematics and the mathematics class;

(b) possible causes of the fear(s);

(c) possible effects of the fear(s) on them - as students and as teachers.

Furthermore, because the responses to the questionnaire (Appendix 2) were not only collected as part of my practice but also for this mini-thesis, I requested the students to hand in their completed questionnaires anonymously.

In analysing the responses I received, I thought that it might be expedient to first "group" them as I perceived them to be indicative of specific factors which might cause cognitive blocks to mathematics learning. Secondly I shall try to relate, where possible, students' conceptions about the learning of mathematics to these six perceived affective factors and "anxieties". Most of the conceptions mentioned below are also highlighted by Marilyn Frankenstein (1989, pp.18-21) as <u>misconceptions</u> about the learning of mathematics. (I refer to the respondents as Student 1, Student 2, and so forth).

A. **Lack of self-confidence**

- Student 1

  ▬ "Onsekerheid omtrent my vermoë m.b.t. my kennis. Beskou myself as 'n persoon wat nooit akademies in wiskunde sal presteer nie." (Uncertain about my ability regarding my knowledge. Consider myself as a person who will never be able to have academic success in mathematics.)

- Student 2

  ▬ "Onsekerheid. Bang as gevolg van die feit dat 'n termyn van 22 jaar laas met wiskunde te doen gehad het, was ek nie seker dat ek die wiskunde sal kan baasraak nie." (Uncertainty. Afraid. Because 22 years went by since I last had something to do with mathematics, I was not sure whether I would master the mathematics.)

- Student 4

  ▬ "Gevrees dat ek nie gou genoeg sou snap om by te hou by die ander studente nie." (Anxious that I would not grasp quickly enough to keep up with the other students.)

- Student 6

  ▬ "Was 24 jr. gelede nog in kontak met Wiskunde op die vlak." (My last contact with Mathematics at this level was 24 years ago.)

- Student 7

  ▬ "Onsekerheid. As gevolg van die feit dat 'n termyn van 13 jaar verbygegaan het, sedert ek laas met Wiskunde op sekondêre vlak kontak gehad het, was ek nie seker of ek moontlike vernuwingselemente sou kon baasraak nie." (Uncertainty. Because a period of 13 years had elapsed since I last had contact with mathematics at secondary level, I had my doubts as to whether I would be able to master possible new aspects.)

To my mind, judging from the above responses, it is evident that the following conceptions are some of the factors causing the students to doubt their own abilities:

1. the conception that current gaps in mathematics knowledge reflect lack of intelligence;
2. the conception that slow-thinkers are stupid people;
3. the conception that if you had difficulty in learning mathematics in the past, it does not matter how hard you study or how much effort you apply, you will never be able to do mathematics; and
4. the older you get the more difficult it is to do mathematics.

B. The fear of appearing foolish amongst fellow-students

- Student 1

  ▬ <u>"Skaam vir vernedering as ek iets verkeerd beantwoord. Bevrees dat mede-studente 'n gek van my sal maak."</u> (Ashamed of humiliation if I should answer something wrong. Scared that fellow students will make a fool of me.)

- Student 6

  ▬ <u>"...Die gevolg was dat ek voel my handeling is uiters stadig - 'n verleentheid volg."</u> (The result was that I feel my action is extremely slow - an embarrassment follows.)

Many people are under the impression that all mathematics is done "fast" and that all problems relating to mathematics can and should be solved quickly, and, if they cannot do this then they are stupid. They mistakenly feel embarrassed about thinking too long about a mathematics problem. Another conception is that these feelings of inadequacy are unique; that other people do not make mistakes and will therefore have a low regard for the one making mistakes - to the extend of subjecting the learner to ridicule!

On the contrary, solving mathematics problems is a slow process. It involves careful thinking and planning:

information has to be assimilated; existing mathematical knowledge that has bearing on the problem has to be considered; a strategy for solving the problem has to be devised and acted upon, and, even after a solution has been obtained, it still has to be reflected on. Terenzio Scapolla (1988), in his paper addressing the basic ideas of the works of Imre Lakatos (1922-1974), gives Lakatos's view of the solution of a mathematics problem as:

> a process of trial and error where the problem is a problematic situation, that is a moment of open aggregation rather than a closed set of operations.

Many people, also, mistakenly believe that there is always only one "correct" answer to a mathematics problem. This belief was highlighted by Munir Fasheh at the second international Political Dimensions of Mathematics Education (PDME) conference held in April 1993. Munir Fasheh, in summarizing his paper, urged mathematics educators to move away from the usual practice that there is only one answer to a problem. He suggests that we look at the **context** to decide what the best answer under certain conditions should be.

Marilyn Frankenstein (1989) also refers to this "misconception" of "only one correct answer to each maths problem." She, in turn, argues that the answers to real-life mathematics application problems will be affected by the assumptions made as well as the accuracy with which data are collected.

Two further conceptions about the nature of mathematics that might underlie my students' responses are:

1. the belief that if an answer to a problem is "wrong" it is **all** wrong, with no aspect of the reasoning that led to a "wrong" answer to be correct;
2. the conception that there is only one correct way (usually the teacher's formal way) of solving a mathematics problem.

To my mind, if an answer to a problem is wrong it need not necessarily be due to a lack of mathematical ability. Besides having some aspect of the reasoning correct, a wrong answer might have resulted from a careless computational error. Or, it could be the correct answer to a different question (see also Robert Davis, 1984).

Furthermore, I believe, that students' "intuitive" solutions to problems should be encouraged, respected and accepted by mathematics teachers. In this regard, David Henderson (1990) refers to "Direct Solutions" and "Formal Mathematical Solutions". He first illustrates, by means of an example, that the "Direct Solution" can be more powerful, more creative, more generalizable and more mathematical, and then he poses the question:

> What happens to human beings when they are convinced that the Formal Mathematical Solution is THE solution?
>
> (David Henderson, 1990, p.117)

C. Novelty of curriculum progress

- Student 1

  ▬ "Verskillende benaderings tot werk dan voorheen toe ek op Kollege was. Baie reëls het verander." (Different approaches to work then previously, when I was at College. Many rules have changed.)

Many teachers are under the impression that there is only one correct way of teaching a specific mathematical concept or topic. A different teaching approach/technique is thus mistakenly seen as the "new" "correct" approach/technique, with all other teaching approaches/techniques to be wrong. I believe that, in order to "challenge" this conception, teachers in training should be exposed to as many teaching approaches as possible regarding the concepts to be taught in primary school. Unfortunately the current curriculum content of the diploma course does not make allowance for the time needed to accomplish this.

Furthermore, I am of the opinion that teachers and teacher educators should participate in the curriculum-making process. I argue that if we participate in the creation of something, it is more likely that we will contribute to the success of its implementation. Also, it is my belief that teachers will have much more confidence in the appropriateness and effectiveness of a mathematics curriculum of whose design they themselves were part of. Cyril Julie (1990, p.145) shares this belief by, after a description of the coming-into-being of the South African school mathematics curriculum, asserting that:

> this separation of conception and interpretation from execution (of the mathematics curriculum) reduces teachers to mere technicians to implement someone else's ideas.

Another conception that might underlie the student's response is that learners cannot create their own rules and techniques to make the doing and learning of mathematics easier. Many students, especially those who were successful at school mathematics because of their ability to memorize and follow rules, believe that doing mathematics involves the following of explicit rules which were generated by some person in authority. These rules are often applied "automatically" without understanding why the rule works or why a specific rule is applied to a particular situation.

D. **Anticipating failure and fear of failure**

- Student 1

■ "Bevrees dat ek die werk nie sal kan baasraak nie." (Afraid that I will not be able to cope with the work.)

- Student 5

■ "Onsekerheid. Bevrees dat as ek antwoord die antwoord verkeerd sal wees." (Uncertainty. Afraid that if I answer the answer will be wrong.)

- Student 6

■ "Bang - ek moes die klas bywoon en het daarna uitgeput gevoel. Ek het gevoel om alles te staak maar het besef dat wiskunde 'n noodsaaklikheid is." (Afraid - I had to attend the class and felt exhausted afterwards. I felt like abandoning everything but realized that mathematics is a necessity.)

According to Marilyn Frankenstein (1989, p.19), many people mistakenly believe

> that 'smart' people never make mistakes and do not have to ask questions: 'smart' people figure out everything correctly, on their own.

As a matter of fact I believe that it is the "smart" person who asks questions; who is willing to admit when she does not understand something; who realises that one learns through one's mistakes; who realises the importance of discussions and perseverance in trying to solve problems.

E. Debilitating anger at one's inadequacy

- Student 6

  ■ "Ek was baie onseker en voel kwaad as ek nie my doel bereik nie." (I was very uncertain and feel cross when unable to achieve my objective.)

Some people are by nature perfectionists. Thus, according to my interpretation of this response of my student, people's inability to handle a task at hand effectively (so as to maintain their own high standards) may generate anger at their own imperfection. This preoccupying anger may strongly interfere with task-relevant cognitive activity. It is my belief that ample experiences of personal achievement or success will counteract the debilitating effect anger has on performance. The writings of Sheila Tobias (1978) supports this view. To this end she writes: "Experience of success makes people more tolerant of failure" (p.155).

F. Authoritarian approach of past teachers

- Student 2

  ■ "Alle vorige Wiskunde onderwysers het vrees vir die vak by my ingeboesem. Ek kon nooit werklik sê wanneer ek iets nie verstaan nie." (All previous Mathematics teachers instilled fear of the subject in me. I could never really say when I did not understand something.)

- Student 3

  ■ "Jy as 'n persoon was bang omdat die leerkragte streng was. Bang om 'n antwoord te gee - as dit verkeerd was, was jy gestraf." (You as a person was afraid because the teachers were strict. Afraid to give an answer - if it was wrong you were punished.)

- Student 4

  ■ "Op hoërskool het die wiskunde onderwyser net een keer 'n som verduidelik en dan het hy verwag dat jy die volgende dag die som moet verstaan." (At high school the mathematics teacher explained a sum once only and then you were expected to understand it the next day.)

- Student 5

  ■ "Wiskunde onderwyser was baie streng. Kon nooit sê as jy nie verstaan met die gevolg het ek 'n afkeur in Wiskunde gehad in st. 8." (Mathematics teacher was very strict. You could never say if you did not understand, with the result that I developed an aversion toward Mathematics in Std 8.)

Many people are subjected to authoritarianism in mathematics education. This authoritarianism could be the result of some mathematics teachers' dualistic views of mathematics. Dualism regards mathematics as concerned with facts, rules, correct procedures and simple truths determined by absolute authority. Mathematics is viewed as having a unique structure - being fixed and exact - and doing mathematics is following rules (Paul Ernest, 1991). Thus students are told what to learn, are told when to learn, are told how to learn, are told what to do and how to do it. Authority is threatening, especially in its power to make judgements, and it is a misconception that the teachers are the only ones who know the "correct" way of solving mathematical problems and performing mathematical operations.

Another misconception, to my mind, is the belief that punishment and/or instilling fear will cause students to produce correct answers or enhance students' understanding of mathematics. It is not only overt punishment that is of concern. Students can also feel "punished" and threatened by the attitude and behaviour of the teacher - shouting, sarcastic, condescending, ignoring the student, and so forth.

These recorded feelings, attitudes and beliefs which motivate my students' behaviour in mathematics and toward mathematics strengthened my conviction that the learning/teaching environment plays a great role (if not the major role) in engendering mathematics anxiety.

This conviction was further strengthened by the following remarks made by four of my students when they completed the aforementioned questionnaire - a questionnaire, as can be recalled, that was completed after our <u>second</u> session as a mathematics class:

- Student 2:

  "<u>U aanbieding en verduideliking het my egter baie gou op my gemak gestel. My verhouding tot Wiskunde het heeltemal verander en ek vind dit nou aangenaam."</u> (Your presentation and explanation, however, put me at ease very quickly. My attitude toward Mathematics has changed completely and I now find it enjoyable.)

- Student 3:

  "<u>Ek het rêrig nog nooit wiskunde klasse so geniet soos nou nie. Ek het gedink dat die dosent die lesse net gaan aanbied of jy verstaan of nie. Gelukkig is dit nie so nie. Sy verduidelik die werk op so 'n manier dat jy dit moet verstaan. 'n Grap word soms gemaak as jy nie verstaan nie. Op skool was die gesigsuitdrukking van die aanbieder stuurs en nors. Die manier van aanbieding van die dosent speel 'n groot rol."</u> (Really, I have never enjoyed mathematics classes as much as now. I thought that the lecturer would just present the lessons whether you understand or not. Fortunately this is not so. She explains the work in such a manner that you must understand it. A joke is sometimes made if you do not understand. At school the facial expression of the presenter was surly and grim. The manner of presentation of the lecturer plays a great role.)

- Student 5:

  "U rustige stemtoon en rustige manier van les aanbieding het my laat ontspan. Geduld wat u aan die dag lê deur elkeen individueel te verduidelik wat nie mooi verstaan nie, het my laat besef Wiskunde is glad nie so aaklig nie maar heel aangenaam." (Your tranquil tone of voice and calm way of presenting a lesson, caused me to relax. The patience that you display by explaining individually to each one who does not understand, made me realize that Mathematics is not so bad at all but quite enjoyable.)

- Student 6:

  "Daar's nog vrees maar ek voel dat dit oorbrug kan word. Ek begin stadig van die vak te hou veral na 'n suksessie in die klas wat deur myself gedoen is." (There is still anxiety but I feel that it can be bridged. I am slowly beginning to like the subject especially after a small success achieved by myself in class.)

The above remarks and the six constructs discussed earlier in this chapter, as well as the assertion made by the supervisor of the part-time classes and referred to in Chapter 1, necessitate that I closely reflect on my own classroom practice. This reflection is the concentration of the next chapter.

CHAPTER 4

A REFLECTION ON MY CLASSROOM PRACTICE AND TEACHING STRATEGIES

4.1 My Goals as a Teacher

Reflecting on my teaching strategies and classroom approaches, I perceive the following to be my primary goals when planning a learning and teaching situation:

1. facilitating the doing, understanding and the knowing of mathematics;
2. ensuring mathematics achievement; and
3. preventing or reducing mathematics anxiety in my students.

Of course, if mathematics anxiety has already manifested itself in a student, the ideal aim would, of necessity, be to help the student **overcome** her mathematics anxiety. But, reflecting on the lessons learned from my many years of teaching, I am inclined to agree with the following assertion made by the educational psychologist Steve Dossel:

> Strategies for the prevention of maths anxiety are more effective than attempts at curing the problem, since anxiety, once established, tends to be self-reinforcing.

(Steve Dossel, 1993, p.6)

The question that now comes to mind is: "In trying to achieve these goals what role do I play in the learning situation?"

4.2 My Role as a Teacher in the Learning Situation

The role that I, as a mathematics teacher, play (and should play) in the learning/teaching situation was put into perspective at a course I attended in 1991 on the theory and practice of Instrumental Enrichment - ways of developing thinking skills. Instrumental Enrichment is based on what the Israeli professor of psychology, Reuven Feuerstein, calls the Mediated Learning Experience (MLE), which describes a special kind of interaction in a learning situation which aims at promoting effective learning. Feuerstein extended Piaget's formula of **S**(timulus) - **O**(rganism) - **R**(esponse) further to include a human mediator (**H**) between the world of stimuli (**S**), the organism or individual learner (**O**) and the response (**R**). His new formula for mediated learning is **S - H - O - H - R** (Mervyn Skuy, 1991).

Thus, my role in the learning/teaching situation, I believe, is that of a **mediator,** with mediation as a kind of interaction that develops the basic attitudes and competence for self-directed learning. Also, in mediation, learning is an interaction between learner and mediator and when the learner experiences difficulty with learning the fingers have to be pointed in both directions and not with what Reuven Feuerstein calls a "stiff finger". This "stiff finger" attitude is where the index finger points stiffly in the direction of the student indicating that the problem and failure are fixed firmly with the student.

Having clarified my goals and role as a teacher, and bearing the responses I had received from my present students in mind, I conducted an unstructured, informal interview with Ursela, one of the students in my second year mathematics class of 1992. Ursela was forty three years old and, at the start of the interview, described herself as having been a "very average student in secondary school with very little confidence in my mathematics ability."

I: If this is a fact, how did you feel about the excellent mark you obtained at the end of last year?

Urs: Surprised and excited. Although I enjoyed the classes tremendously I did not expect to do so well.

I: Can you perhaps explain why you enjoyed the classes?

Urs: We were a nice group and the atmosphere was relaxed. Though we did not know each other to start off with, we became firm friends. Even now, when it comes to a Saturday morning, like this morning, I miss the mathematics class.

I: I also enjoyed our classes very much. Was there anything in particular, that you can think of, which made the learning of mathematics, for instance, easier and/or interesting.

Urs: Yes. The practical work we did. When we did the addition and subtraction of integers for instance. I struggled until I could deduce rules for myself - you did not get impatient and was always so encouraging. The exchanging of ideas also made the work very interesting. Also the examples we used. They were so everyday - we could relate to all the issues. Some of the examples were things we had already experienced, we knew the mathematics involved although we did not realize it. I don't know whether I'm expressing myself clearly enough, but you know what I mean, so help me out!

I: I'm not suppose to put words into your mouth - it will defeat the whole purpose of this conversation. But, are you perhaps saying that because of the everyday examples used, the mathematics you learned became more meaningful?

Urs: Yes, that's it! It also gave me confidence in my own ability of doing mathematics because I had already been applying some of the mathematics we were learning. Like the morning you were late for class because of the unexpected fog and you got us to work on the relationship between speed and time. I remember it was still early in the year, because that is the day we found out exactly where you lived. Also, when we had to determine the amount of material needed to cover that paint bucket. The story you told us about that bucket was really funny. That is another thing about our classes; there was never a dull moment. I really looked forward to Saturday mornings. We often still talk about some of those incidents.

I: Your saying that you still talk about some of those incidents brings another question to mind. How did you feel about working in groups and learning mathematics within the group situation?

Urs: Being able to exchange viewpoints and being able to reason with each other made the work much easier to understand. You could risk suggestions which were discussed by the group, if the suggestion was not acceptable you could correct yourself!

I: What made you feel free to risk suggestions?

Urs: Because we respected each others' viewpoints. But, I must add, I don't think working in groups would have been successful if we did not understand each other so well. To think that at the beginning of last year we did not know each other at all! I think that the coffee-breaks we shared contributed a lot to the congenial atmosphere that existed inside the classroom. Socializing and sharing everyday personal experiences made us more caring for each other. This made us feel more free to participate in activities inside the classroom. Thus the discussions, and suggestions of ways to solve problems became much more spontaneous. Even during the class discussions one could ask questions and volunteer answers freely. I appreciated this because things then became clearer since you would explain something again and the class would discuss it further. You really wanted all of us to understand what we were doing.

I: I am glad to hear this because, sometimes, when a student asks a question or make a mistake and I use this to stimulate further discussion, I am quite apprehensive of boring the rest of the class. Is there anything else you appreciated?

Urs: The regular exercises you gave us to do gave me an opportunity to work on my own to see whether I had understood what we were doing. Even then we used to compare and discuss solutions amongst each other.

The above interview with Ursela puts into perspective what could be considered the three most important aspects of my classroom practice. These aspects will be discussed in some detail in the next section.

4.3 My Classroom Practice

4.3.1 The learning environment

My first and major task in any learning situation is at all times to work toward creating an environment where it would be "safe" for students to open up to themselves, to each other and to me.

Thus, at the first class meeting, instead of introducing myself to the class and asking them to introduce themselves, we try to learn not only each others' names but more about each other as persons. This is accomplished by dividing the class, myself included, into pairs - I find it is important to stress that the two persons forming the partnership should not previously have known each other intimately. Each couple is then given five minutes to learn as much as possible about each other regarding name, background, likes, dislikes, aspirations, and so forth. After the five minutes of "getting to know something about one other person", each person gets the opportunity of "introducing" her partner to the rest of the class. It is amazing how relaxed and congenial the atmosphere in the class becomes after this activity.

Besides learning the names of the other persons in the class and at the same time establishing a more relaxed atmosphere, this method of learning to know each other holds further advantages for me. First, with my very bad memory - I have even bought myself books on how one can improve ones' memory - though it still takes me a long time to know every student by name, I find that this exercise actually aids my memory. This could be because the students become more then just a name on the register. They become persons, each one with her own identity. Second, and definitely not less important, I find that because each student has to "open up" to only one other person fears and dislikes are voiced quite freely. Thus, when requested afterwards to write, or talk, about their fears regarding mathematics and the mathematics class, the students comply readily with much less fear of victimization or the notion of being patronized.

Experience has taught me that people have different learning "preferences". Some people prefer working in a group while others prefer working on their own. I therefore give my students the freedom of choice: either form groups consisting of two or three students with subsequent learning to take place within such a group or work individually. What I have discovered though, is that the initial choice of the "immediate" learning environment seldom remains static. A student who at first might have indicated a preference of learning on her own will sometimes join in on the discussions and arguments of an existing group; and a student who generally prefers learning in a group will at times move apart to work on her own. To me this is always very heartening. First, it shows that the students, instead of being totally dependent on the teacher to take charge of their learning, are becoming more willing to take on the responsibility of satisfying their own learning needs. Second, if a student's initial decision to work on her own was mainly driven by the fear of appearing foolish in public, joining a group at a later stage might be an indication that the student is beginning to overcome this fear - she is prepared to have her arguments refuted in public, albeit in a small group.

4.3.2 Clarity of mathematical concepts

To facilitate an understanding of mathematics and mathematical concepts I try, as far as possible, to confront my students with real contexts in which the relevant concepts are embedded. Because the students - at least the majority of them - can identify with these contexts I believe that, not only will they be able to make better sense of the mathematics, they will also find the mathematics they do more interesting, meaningful and applicable. But, having said all this, I wish to relate a certain learning-teaching experience. An experience that taught me a very valuable lesson.

Two years ago, in an effort to introduce a discussion on negative numbers, I gave each of my students a copy of one of my bank statements - a statement that indicated a negative balance.

STATEMENT

R70,11- PUR

R44,63- CASH

R28,00- PUR

R43,00- CASH

R1,50- PUR

R577,77 CRED

R5 312,93- BALANCE

A discussion of the meaning of the symbols used by the bank ensued my students' scrutiny of the statement. Because the students were familiar with bank statements, they knew that the amount indicated by the "CRED" symbol signified an amount I had paid into the account. After some discussion they decided that the amounts shown as "PUR" and "CASH", and indicated by a "-" sign, had to be amounts that were paid out by the bank. The meaning they attached to the amount stated as my "balance" was of crucial importance to the lesson, because, I believed, it would establish a "making sense" of a negative number.

Unfortunately the students could not agree on the "meaning" of this particular balance. There were two conflicting arguments. The one hypothesis was that the balance signified an amount that I owed the bank because: "some financial institutions indicate amounts you are 'in the red' with '-' signs." On the other hand, it was argued, that, since the other negative amounts indicated on the statement signified amounts paid **out by the bank**, the "-" sign accompanying the amount shown as my balance meant that the bank owed me that money! I was flustered. Although I knew that the first argument precisely described the reality of my financial status regarding that account - and was what I wanted them to deduce in the first place - the second argument, to my mind, definitely showed more logic in the reasoning.

Fortunately for me I had another transaction record in my possession, a transaction record from another account and one which I had earlier decided **not** to use because this statement indicated a "positive" balance.

I congratulated the students on both their arguments, circulated the second statement amongst them and then copied it on the chalkboard.

R7,85- ACB	R500,00- ATMW
R300,00- SO	R161,96- 1105
R43,00- ACB	R569,77- ACB
	R2.547,06+ 07/02

I then informed them that the two statements were from different accounts: the first one from my credit account and the second one from my current account.

Judging from the questions then raised, it was evident that, although they were all familiar with "current accounts", the same could not be said about "credit accounts". After I had explained how credit accounts - and, of course, the so-called "plastic money" - functioned, they were all in agreement that the balance indicated on my credit account was in actual fact an amount I owed the bank. Nobody refuted the suggestion that the "+" sign accompanying the amount shown as the balance of my current account indicated money "you could spend."

To make sure that the students' subsequent understanding of the meaning of a negative number was the desired one, I confronted them with a hypothetical transaction: all the money from my current account to be transferred to my credit account. The following responses to questions I asked caused me to sigh with relief:

"R0,00 in your current account";
"your credit card statement will indicate a balance of R2 765,87-"; and finally
"financially you're **worth less than nothing.**"

I learned the following valuable lessons that memorable day:

1. in my efforts of knowledge-shaping, NEVER to take anything for granted but to make sure, beforehand, that the students understand the

"material" they have to work with - what may be "obvious" to me may not be "obvious" to the students;

2. to, whenever possible, confront my students with different contexts concerned with the same concept; and
3. to ensure that my students, when engaged in constructing their own meaning of a mathematical concept, have at their disposal the means to make some or other comparison during the meaning-making process.

4.3.3 Recognizing patterns and relationships

Cultivating and developing the skill of recognizing patterns and relationships is an important mathematical skill which enhances the learning and doing of mathematics. Also, I believe, that

1. if a student recognizes a pattern or a mathematical relationship herself, she will at the same time experience a sense of achievement. These experiences of achievement are important if I wish to counteract the feeling of "helplessness regarding mathematics" which has manifested itself in some of the students;
2. since the recognition of patterns and relationships takes place within the small-group situation, collaborative learning is encouraged;
3. by first describing the specific relationships (or

patterns), either verbally or in writing, will make it so much easier - having reflected on their observations - to formulate and internalize a general, symbolized presentation of the concept; and

4. any mathematics, rule or formula to be deduced would be generated by the students themselves. This will be their OWN mathematics, rule or formula. This, I believe, counteracts the "blindly" following of explicit rules - rules generated by someone in authority - without knowing <u>why</u> a rule works or <u>when</u> to apply a particular rule.

Furthermore, because I have to live with my own very bad memory, I know from experience that knowing <u>how</u> a rule or formula was generated makes one so much more self-confident and less dependent on memory. And, as will be recalled, in Chapter 3 "lack of self-confidence" was one of the six constructs displayed by my students and identified as a factor which might cause cognitive blocks to mathematics learning. Thus, I firmly believe, any teaching strategy that will develop, amongst other things, students' confidence in their own mathematical abilities should be pursued. Some of my teaching strategies, aimed at assisting the students in making the mathematics they learn their "own", are illustrated by the following case study.

4.4 Presentation of Case Study of Current (1993) Cohort of Students

My thirty part-time students are divided into two groups of fifteen. Each group meets for one three-hour session of mathematics per week - the one group on a Tuesday afternoon and the other group on a Saturday morning. This case study is of the Tuesday afternoon class and is based on three lesson episodes of teaching and learning mathematics that occurred during the course of this year.

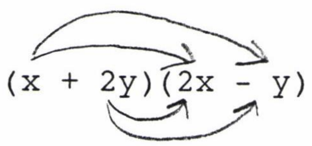
These examples of chronological lesson episodes are illustrative of the student-student as well as the teacher-student interaction that became common to almost every lesson during the course of the year. They also show that the students were progressively gaining more and more confidence in their own mathematical knowledge and their ability of doing mathematics. Furthermore it is apparent that the students were spontaneously voicing their opinions without any fear of being ridiculed.

During each of these episodes I made short notes of the events that transpired in the lesson and recorded these in more detail immediately afterwards. I have also taken the liberty of translating what transpired during the lessons into English.

Example 1:

The following description is of an "incident" that occurred during the earlier part of the year while we were busy with a lesson on "the product of two binomials". In lessons prior to this one the "meaning" of such products had already been established. Also, by then, certain "behaviours" were becoming classroom practice. One of the behaviours that had become customary was: explaining and motivating any writing activity that is done on the board. Furthermore, that I expected the <u>students</u> to rectify (or verify) each others' statements or arguments - this to be done with the necessary motivation - was also slowly being accepted as standard practice.

During the course of this lesson, which I hoped would terminate in the students being able to determine the products of binomials by inspection, a student offered to find the product: (x + 2y)(2x - y), on the board. First she drew in arrows while saying: "These are the terms I have to multiply."



She then wrote: $2x^2 - xy + 4xy - 2y^2$, saying, as she wrote each product: "x and 2x is 2x squared; a plus and a minus is a minus and x times y is xy; 2y and 2x is 4xy; a plus and a minus is a minus and 2y times y is y squared." Next she wrote $2x^2$ underneath the previous expression and, indicating the terms -xy and +4xy, said: "These terms are like terms and a minus and a plus is a minus," and wrote: -3xy. Another student interjected immediately: "I think that should be plus 3xy." The following dialogue then ensued:

First student:	Why?
Second student:	Because if you have 4 rand and you owe 1 rand you'll be worth 3 rand. Thus the sign should be a plus.
First student:	I know that! But a minus and a plus is a minus.
Second student:	I think you mean a minus times a plus, but we are adding.
First student:	I'll change it (changes the "-" sign to a "+" sign), but these minus signs confuse me.
Teacher:	I suggest that in future, for better clarity, we avoid using the word "and" when we perform a mathematical operation. When we are determining a product we should either use the words "multiply by" or "times", and, when we are adding terms I suggest we use the words "added to."
First student:	Should I say: "minus xy added to plus 4xy."
Teacher:	Yes. That says exactly what

operation you are performing. But, I think you and I should discuss these computations again later. Is there anybody else who wishes to join us? (This invitation to the rest of the class.)

Seven students raised their hands, and the first student completed her simplification of the expression by writing "$-2y^2$".

At this point, while the other students were working on an exercise, I spent some time with the eight students, trying to help them in their "sense-making" of the addition and the multiplication of integers. To verify their computations regarding addition, some of them used the numberline while the others made use of additive inverses (as in $-5 + 8 = -5 + 5 + 3 = 3$). One of the students conceded afterwards that "saying 'added to' and 'times' instead of 'and' definitely made things less confusing."

Example 2:

This lesson took place during the second term. To enable the students to deduce generalizations of the conditions for congruency of triangles, I gave each student worksheets (Appendix 3) on which I had drawn a number of different triangles. For every given triangle, and using three "measures" of that triangle (as specified), they were required to construct another triangle(s). They were further instructed to cut out their triangle(s); to superimpose their triangle(s) on the given triangle and to record their findings. For those cases where exact "fits" were obtained the given "measures" had to be generalized. Also, possible reasons for the non-congruency of the other triangles had to be supplied.

First of all, just constructing the triangles was a major challenge to the students - because they were exempted from the first year of study they had missed the work on accurate constructions of triangles. They, however, improvised the construction of one angle equal to another by using tracing paper - one group actually refined their technique to just making three dots on the tracing paper and then, by using a straight pin, mark the exact position of the second side of the required angle. Unfortunately, even after some "trial-and-error" methods, none of the students could figure out a way of constructing a triangle accurately by only using the three sides as "givens". Only after I had mediated by focusing their attention on the "locus" of a given line segment with a fixed endpoint, were they able to continue on their own again. Nonetheless, what should not be discounted, is the fact that the students, while debating and applying possible methods, realized that by simply tracing the sides of the given triangle would have assumed the measures of the angles also as "givens".

After completion of the worksheets I requested each group to present an observation - and the relevant generalization - to the whole class to be discussed and revised (if necessary). The following was the outcome of this particular learning experience.

1. Everybody was in agreement that: "all triangles with the lengths of their three sides 'fixed' are congruent."
2. The statement: "two angles and a side of one triangle constructed equal to two angles and a side of another triangle, will make the two triangles congruent", was refuted by the counterexample: "What about number 6? There we also used two angles and a side, but the triangles did not fit." After further analysis and some more discussions, the class agreed that for congruency "the <u>positions</u> of the sides have to be the same." Because, as a class, we had already encountered the term "corresponding", the suggestion (by one of the students) that we revise the statement to read: "all triangles with two angles and corresponding sides equal are congruent", was accepted.
3. The next group admitted that they could not come to a compromise regarding "two sides and a corresponding angle" as "givens", because "although

we think the triangles should be congruent, because in numbers 2 and 4 the triangles fitted, in number 3 we were able to construct two triangles with the one much bigger than yours." Because no one from the other three groups contradicted this observation, I commented: "Okay, so what you are saying is that these constructions were 'similar' because the positions of the angles did correspond. Did you notice any differences in the positions of the angles?" "Yes," one of the students responded, "I notice now that in number 2 the angle lies between the two sides" and "In number 4 the angle is a right angle", another student commented. After verifying these observations, the group concerned reached consensus and offered the following condition for congruency: "twee sye en hulle tussen-in hoek gelyk" (two sides and their "in-between" angle equal). This requisite for congruency, with the proviso: "If the angle is a right angle it need not be 'in-between'", was accepted by the whole class.

4. Nobody refuted the assertion: "If two (or more) triangles have only their respective angles equal they are not congruent." In reply to my question: "Why?" the student said: "All of us constructed different triangles because we decided to just guess the length of a side. None of these triangles fitted on the given triangle."

Example 3:

The lesson I describe here is the first lesson on solving linear equations in one unknown and occurred during the latter part of the year. This, incidentally, was also the students' first "formal" encounter with the concept "equation". The discussion started by my requesting the class (myself included) to create "number problems". Each person had to think of a number, do whatever mathematical operations she wanted to on this number and obtain an "answer". After this had been accomplished, some students using pen and paper while others used their calculators, I asked for a volunteer to state her number problem so that I could write it on the chalkboard. Most of the students volunteered. A short discussion on who was going to be first resulted in the decision that I should choose the presenters. I explained to them that each problem was going to challenge the rest of the class to try and determine the "original" number.

The first number problem (Annie's) I wrote on the chalkboard in words, the way it was stated, next to her name. (I also wrote, next to each subsequent problem, the student's name.)

> My number I first multiplied by 3 then I added 2 and got 17.

I then asked: "Should we leave the problem in this form?"

The replies were emphatic:

Naomi: No. It looks difficult and confusing like that.

Clare: No. Rather write the signs.

Then I asked: "What about this part?" indicating the "and got 17".

Annie: That's the same as "equal to" 17.

On the board: Number X 3 + 2 = 17

Clare: I think I know what Annie's number is, but I'll first wait. It could be wrong.

Norma: I also think I know what the answer is, but, can't we first write x instead of "number"?

Teacher: Will you please come and write it on the board.

On the board: 3x + 2 = 17

Teacher: Norma, you said that you know what the answer is. What do you mean by "answer"?

Ciréne: I think she means the number. Now it will be the number for which x is a place-holder.

At this point I brought the relevant conventional mathematical language (equation, variable/unknown and solution) into the discussion and negotiated their meaning with the students. Then I asked: "What is the solution to Annie's equation?"

Clare: It is 5. I know I'm right. Right, Annie?

Norma: I also knew it is 5.

Teacher: How do you know it is 5?

Clare: Because 15 added to 2 gives 17 and ...

Welma: And 3 times 5 equals 15.

Teacher: How did you get the 15?

Katrina: 17 minus 2.

Teacher: Can I write it like this?

Underneath the equation on the board I wrote:

$$3x = 17 - 2$$
$$3x = 15$$
$$x = 5$$

Class: Yes.

The discussion of the solution of the second equation basically followed the same pattern but this time, as the discussion proceeded, I asked the students to write the solution on the board. The problem: "I subtracted 5 from twice my number and got 35", produced the following on the board:

$$2x - 5 = 35$$
$$35 + 5 = 40$$
$$2x = 40$$
$$40 \div 2 = 20$$
$$2 \times 20 = 40$$
$$x = 20$$

Norma's number problem: "3 plus my number times 9 equals 30", was the next problem to be considered.

Noticing the ambiguity in the wording of the problem, I requested the students to first solve the equation by themselves before we discussed the solution together. They complied - some working on their own, while others debated the solution within their respective "small" groups.

When they were ready I asked Clare's group to put their solution on the board. As was customary by now the presentation on the board, without my having to ask for it, would be accompanied by a verbal explanation of the reasoning that went into obtaining the solution.

Clare told the class: "We decided to use y for the number", and wrote: $3 + y \times 9 = 30$

Marion interjected immediately by saying: "I used x as my number but put 3 + x in a bracket." My reaction to this was to ask Marion to put her equation, next to Clare's equation, on the board.

She wrote: $(3 + x) \times 9 = 30$

In answer to my subsequent enquiries, the class admitted that nobody else had inserted brackets, and Norma indicated Clare's equation to be the "right" one. To instigate an analysis of the wording of the problem I initiated a further discussion:

Teacher: Marion, why did you decide to insert brackets?

Marion: Because I thought the 3 had to be added to the number first.

Welma: Perhaps it does not matter.

Gertrude: It will matter because we will have to multiply 3 by 9 also.

Annie: I see now Norma could have meant a bracket.

Teacher: Norma?

Norma: I should rather have said "I added 9 times my number to 3."

Teacher: I think it's just fair that we get Marion's solution as well.

Clare's solution on the board:

$$3 + y \times 9 = 30$$
$$3 + 27 = 30$$
$$9y = 27$$
$$y = 3$$

Marion's solution on the board:

$$(3 + x) \times 9 = 30$$
$$3 + x = 30 \div 9$$
$$3 + x = 3\tfrac{1}{3}$$
$$x = 3\tfrac{1}{3} - 3$$
$$= \tfrac{1}{3}$$

Ciréne then said: "I worked out Marion's equation to see if it would also yield a 3, but I first removed the bracket. Could I do that?"

Teacher: Well, show us what you did.

Ciréne's solution on the board:

$$(3 + x) \times 9 = 30$$
$$3 \times 9 + 9x = 30$$
$$27 + 9x = 30$$
$$9x = 30 - 27$$
$$9x = 3$$
$$x = 3 \div 9$$
$$= \tfrac{1}{3}$$

After the class had established the validity of Ciréne's reasoning, they decided that both ways of solving Marion's equation were acceptable.

Next I asked Anita to present her number problem. She reckoned that her problem was basically the same as the first problem and would be too easy to solve. She then suggested that I present them with my number problem.

My problem: I added 5 to 6 times my number and found that the answer I obtained I could express as twice my number subtracted from 21.

The students were a bit nonplussed at first but leading questions like "Which part of the problem will give rise to the left hand side of the equation", "How can we express this symbolically" and "How can we write the right hand side of the equation", led to the following equation on the board:

$$5 + 6x = 21 - 2x$$

I then instructed the students to discuss the solution of the equation within their groups. I also reminded them of the importance of reflection - they had to reflect on their reasoning and the strategies they used to determine whether there was any "pattern" they could observe.

The following are some of the arguments I noted while moving among the groups:

First group: "The right hand side should just have been a number."

"'Teacher' said it was a number."

"Well, let's say it's 25, we'll then have to minus 5 to get 6x."

Second group: "6x is the difference and it means 2x must be added."

Third group: "2 works. Look 5 plus 12 is 17 and 21 minus 4 is also 17."

"How did you get the 2?"

Fourth group: "You cannot add 16 and minus 2x, they're not like terms."

"How will we get 6x alone?"

"Look here! 8x minus 2x equals 6x..."

"That means 8x must be the same as 16."

After a few more minutes all the groups had found the solution and I requested "group 4" to put their solution on the board.

(The solution was written on the board accompanied by the necessary verbal explanations):

$$21 - 5 = 16$$
$$6x = 16 - 2x$$
$$6x = 8x - 2x$$
$$8x = 16$$
$$x = 2$$

Then Norma (group 2) said: "That's a different way to ours. We said that because 2x is subtracted from 16 to give 6x, it means that 2x has to be added to 6x to give 16 - like how we defined subtraction." I then asked her to write their version of the solution on the board.

She wrote:

$$5 + 6x = 21 - 2x$$
$$6x = 21 - 2x - 5$$
$$6x = 16 - 2x$$
$$16 - 2x = 6x$$
$$6x + 2x = 16$$
$$8x = 16$$
$$x = 2$$

In answer to a question by me, the other two groups indicated that they had nothing to add to the discussion. Then I asked, to elicit further general "strategies" about solving equations: "If I give you a few more equations to solve on your own, will you be able to solve them quickly? Is there a noticeable 'pattern' in our arguments that you could perhaps apply?" Because there was no reaction from the students I started underlining all those terms that had "changed signs" in the process of solving the equations. After a few moments of reflection the students started responding. They talked about "if you had subtracted you must now add", "adding all the constants", "adding all the x's" and "if you had a plus on the left hand side you will make it a minus when you write it with the term on the other side."

Although these generalizations about the properties of equations were not expressed in conventional mathematical terms I was satisfied that some learning did take place. As a result of the verbalization of their reasoning, discussions, dialogue and reflection the students were able to "shape" their existing mathematical knowledge to "suit" the task at hand. I felt assured that the students, if not all, then at least the majority of them, would be able to solve simple linear equations in one unknown, in a somewhat "formal" manner, on their own.
(In a follow-up lesson, by applying the "balance-algorithm" to identities, the class corroborated the above-mentioned assumptions about the properties of equations.)

In the preceding sections of this chapter I have expounded my goals, role and practice as a mathematics teacher. But looking at one's practice compels one to consider the literature dealing with "practice". Thus, since in this mini-thesis the practice is mathematics education, literature dealing with the notions of how mathematics is learned and how mathematics is done is the consideration of the next chapter.

CHAPTER 5

LITERATURE ON LEARNING AND DOING MATHEMATICS

How is mathematics learned? There are various theories of how mathematics is learned. Of these, the behaviourist theories of learning, which guided much teaching, use the learning principle of conditioning in order to predict behaviour. Eleanor Nash et al (1990, p.6) describes the behaviourist view of learning as follows:

> This view of learning stresses that there are no innate determinants of behaviour and all changes that are observed are the outcome of the individual's experience in what is called 'tabula rasa' or clean slate approach.

This stimulus-response approach is also described by Paulo Freire (1972, p.58) as

> the "banking" concept of education, in which the scope of action allowed to the students extends only as far as receiving, filing and storing the deposits.

Thus, in this view, conscious thought has no bearing on learning, and, knowledge of the reality of the world outside "the mind" is passively received by the learner.

Recently, though, much attention has been given to the way humans construct knowledge from experiencing situations. One theory of knowledge with regard to knowledge construction is constructivism. Jeremy Kilpatrick (1987) describes constructivism as involving two principles:

1. Knowledge is actively constructed by the cognising subject, not passively received from the environment.
2. Coming to know is an adaptive process that organizes one's experiential world; it does not discover an independent, pre-existing world outside the mind of the knower.

Based on these two principles, two forms of constructivism are described by Jeremy Kilpatrick:

1. Trivial constructivism, embracing only the first principle, and "one to which most cognitive scientists outside the behaviourist tradition would readily give assent" (p.7).
2. Radical constructivism, embracing both the above principles, and which "rejects the metaphysical realism on which most empiricism rests" (p.7).

A proponent of Radical Constructivism is Ernst von Glasersfeld. He describes radical constructivism as follows:

> Radical constructivism must be interpreted as a possible model of knowing and the

> acquisition of knowledge in cognitive organisms that are capable of constructing for themselves, on the basis of their own experience, a more or less reliable world.
>
> (Ernst von Glasersfeld, 1984, p.39)

Furthermore, Ernst von Glasersfeld describes this kind of knowledge, derived by reason from observation and experience, as a knowledge that "fits" the observations. He expounds this notion of knowledge by stating:

> What determines the value of the conceptual structures is their experiential adequacy, their goodness of fit with experience, their viability as means for the solving of problems, among which is, of course, the never-ending problem of consistent organization that we call understanding.
>
> (Ernst von Glasersfeld, 1987, p.5)

Two critiques of constructivism, as a theory of knowledge acquisition, are given by Jeremy Kilpatrick and Paul Ernest.

In his criticism of radical constructivism Jeremy Kilpatrick (1987) asserts first, that for the radical constructivist the act of knowing develops through adaptation of the mind to the world - and it never comes to know a reality outside itself. All it can learn about are the world's constraints on it: the things not allowed by what it has experienced as reality and what does not work. Second, that for the radical constructivist we are informationally closed systems and that "neither knowledge nor information flows in or out of us" (p.9). Consequently each of us constructs subjective meaning for the language we use as we build our experimental world of situations we have shared. Thus language and other forms of communication do not entail the interchange of ideas between us.

Furthermore, Jeremy Kilpatrick (1987) asserts that as a theory of knowledge acquisition, constructivism is not a theory of teaching or instruction. And, by means of examples, he illustrates that there is no necessary connection between the two theories. Thus, he suggests that

1. constructivists need to clarify and develop their ontological commitments because "cutting epistemology loose from metaphysics as a way of solving the epistemological dilemma does not provide a satisfactory resolution of our problems as educators" (p.19);
2. constructivists need to think through and spell out more clearly than they have done thus far the relationships between constructivism and both mathematics as a discipline and mathematics as a

school subject;

3. constructivism needs to address the claims of a new approach to the philosophy of mathematics - "quasi-empiricism", which studies the practice of mathematics in a sociohistorical context and which appears to be compatible with both realist and constructivist mathematics" (p.20); and,

4. constructivism needs to become more connected to reality - the reality of everyday scientific activity, mathematical investigation and classroom practice. Thus he says:

> If constructivism has something to say about what it means to come to know mathematics beyond the elementary school, about how indirect guidance of learning can be handled through the grades then it needs to find a language with which to speak to teachers on those matters.
>
> (Jeremy Kilpatrick, 1987, p.22)

Paul Ernest, on the other hand, argues that if we want to develop a theory of mathematics education which includes the learner, teacher, the classroom, the social and political context, and appropriate aims and values, it is necessary to put the "social" back into constructivism.

In his critique of constructivism Paul Ernest(1993) also finds it necessary - like Jeremy Kilpatrick (1987) - to distinguish different forms of constructivism by considering the basic principles as well as the underlying or assumed metaphors for the mind and world. Thus Paul Ernest distinguishes three forms of constructivism: Information Processing Constructivism (what Jeremy Kilpatrick calls trivial constructivism), Radical Constructivism and the newer form of constructivism, Social Constructivism.

A comparison of Paul Ernest's (1993) critique of trivial constructivism and radical constructivism with his exposition of social constructivism follows.

1. Trivial constructivism sees mind as an active but inanimate machine and radical constructivism sees mind as an evolving and adapting organism. Social constructivism regards individual minds and the realm of the social as integrated - thus there is no metaphor for the isolated individual mind. Instead, the underlying metaphor is that of conversation, comprising persons in meaningful linguistic and extra-linguistic interaction. Human subjects are thus formed through their interactions with each other (as well as by their individual processes) and mind is seen as part of a broader context "the social construction of meaning".

2. Trivial constructivism sees the world as absolute space with physical objects and radical constructivism sees the world as a subject's private domain of experience. On the other hand,

> ...the Social Constructivist model of the world is that of social reality a socially constructed world which creates (and is constrained by) the shared experience of the underlying physical and social worlds.
>
> (Paul Ernest, 1993, p.170)

3. Trivial constructivism brings with it an absolutist epistemology - the assumption that ultimate knowledge is possible - and locates the epistemological problematic exclusively in the mind of the learner. On the other hand, both radical constructivism and social constructivism have a fallibilist epistemology - the whole relationship between the knower and the known is problematized and they accept that no certain knowledge is obtainable by humans. But, Paul Ernest (1993) argues, where-as radical constructivism apparently has an exclusively individualistic focus, "social constructivism sees individuals as irreducibly implicated in and constituted by the social" (p.171).

Thus, according to Paul Ernest, although radical constructivism is a rich theory of learning, its individualistic emphasis - and the secondary role it accords to the social sphere - leads to the following significant weaknesses and dangers.

1. Its cognising subject seems to be isolated in a privately constructed world of its own and its construal of other persons is driven by whatever representations best fit the cognising subject's needs and purposes. Such a view, Paul Ernest argues, makes it hard to establish a social basis for interpersonal communication, for shared feelings and concerns, let alone for shared values - all of which society and its functions, and in particular education, depend on.
2. It can lead to overly child-centred teaching practices which sanction anything the child does as expressions of his/her individual creativity, and, which naively assume that the child can discover much of the conventional school knowledge on his/her own. Thus Paul Ernest maintains that

> Whilst there is a need to let learners construct their own meanings, the teacher (and peers) must interact with learners to negotiate a passage towards socially accepted knowledge.
>
> (Paul Ernest, 1993, p.172)

To me, constructivism seems to be a learning theory that makes all cognition a product of our own constructions, or mental acts. Thus we can have no direct or unmediated knowledge of any objective reality. Our subjective understanding of the world is mediated by personal and shared experiences, adapted and adjusted through the interaction of new experiences with our current knowledge, by meditative reflection and by social interaction through dialogue and the negotiation of meanings.

Thus, in short, if we understand mathematics to deal with the basic patterns that human beings find in (or imposed upon) their environments (Robert Davis, 1984; Alan Schoenfeld, 1990), then "learning mathematics" has to mean: constructing an understanding, through experience and by social interaction, of these basic patterns.

Constructivism, as a learning theory, has of necessity to be closely related to what might be called "a constructivist approach to teaching". Ernst von Glasersfeld (1987, p.12) noted that

> the primary goal of mathematics instruction has to be the student's conscious understanding of what he or she is doing and why it is being done.

He remarked further

dialogue, which ensues from the presentation, by the teacher, of the traditional proof of the famous Euler-Descartes formula for polyhedra (a formula, which after much trial and error, was a "conscious guess" by one of the students), Imre Lakatos demonstrates the development of mathematics. Thus, by means of this dialogue, the development of mathematical knowledge is portrayed as a process of coming to know collaboratively through negotiation. A process of coming to know via a "zig-zag" path between revising conclusions and examining and revising premises through the use of counterexamples or "refutations".

Also, in Proofs and refutations (1987, p.30), in the midst of an argument about guesswork and insight, the teacher announces:

> I abhor your pretentious 'insight'. I respect conscious guessing, because it comes from the best human qualities: courage and modesty.

Thus, through the voice of the teacher, Imre Lakatos seems to imply that, first, it takes courage to make conjectures (or what he calls "conscious guessing"); and, "making conjectures", an essential component of doing mathematics, should be encouraged. And, second, that if we accept no certain knowledge to be attained by humans, there should be a humility with regard to mathematics and mathematical knowledge.

Another view of doing mathematics is given by Alan Schoenfeld. In Reflections on Doing and Teaching Mathematics (1990, in press), he asserts:

> Our classrooms are the primary source of mathematical experiences (as they experience them) for our students, the experiential base from which they abstract their sense of "what mathematics is all about".
>
> (Alan Schoenfeld, 1990, p.1)

And, it is for this reason that he makes his problem solving courses for college students serve as "microcosms of selected aspects of mathematical practice and culture" (p.11). So that participating in that culture they come to develop as mathematical doers and thinkers.

Furthermore Alan Schoenfeld (1990, p.21) claims deflecting inappropriate teacher authority as one explicit goal of his problem solving courses, because:

> Mathematical authority resides in the mathematics, which - once we learn how to heed it - can speak through each of us, and give us personal access to mathematical truth. In that way mathematics is a fundamentally human (and for some, aesthetic and pleasurable) activity.

As a philosophical point of departure Alan Schoenfeld delineates mathematics as "the science of patterns". And then, from a liberal interpretation of the notion that mathematics is a pattern of science, by drawing an analogy between science and mathematics, he answers the question: What is mathematics?

Thus, from Alan Schoenfeld's point of view, as a descriptive end, and, very much like science, mathematics consists of observing and codifying - in general via abstract symbolic representation - regularities in the world of symbols and objects. Mathematics is all about a particular kind of sense-making of these "things" using well-established styles of reasoning for seeing how things fit together.

By reflecting on his own experiences and by means of illustrative examples, Alan Schoenfeld draws an analogy between "doing mathematics" and "doing science" in his effort to answer the question: What does it mean to <u>do</u> mathematics, or to act mathematically?

What follows is a summary of Alan Schoenfeld's views on what "doing mathematics" is all about.

1. Doing mathematics has to be a social rather than a mere individual or solitary act - there should be a mathematical community that shares and builds ideas.
2. Doing mathematics often evolves from a known or

suspected answer - knowing intuitively or by observations or otherwise that a certain result should be true. Proving this result true (or not true), mathematically, using the mathematician's tools: abstraction, symbolic representation and symbolic manipulation, may be straightforward without encountering much difficulty, or may follow one or more of the following paths:

2.1 Engage in argument and realizing after a while (sometimes months) that, that particular line of argument (or strategy) is not leading to the result. In doing mathematics choosing a wrong strategy is not unusual. So the mathematician admits that choosing that particular strategy was a mistake, abandons it, and starts afresh - using another strategy.

2.2 Engage in argument but at a certain point (or more) cannot get an intermediate result that seems necessary and decides to either

(a) search for (and find) resources to overcome the difficulty (difficulties), or

(b) share the problem encountered with local mathematicians and collaboratively find a solution to the problem.

2.3 Engage in argument but cannot find ways, as described in 2.2, to get the intermediate result so begins to think that the intuitive result might not be true. If not, there ought to be a counterexample to the intermediate result that seemed necessary. So the mathematician, in quest of finding - by trial and error - a counterexample that works, fails at all attempts to construct such one and sees that they all fail for the same reason - the reason that is the idea for the proof. So the intuitive answer might still be true.

3. In doing mathematics the <u>result</u> of mathematical thinking may be presented in "elegant clarity as a polished product" (p.6), yet the path that leads to that product is most often anything but a straightforward chain of logic from premises to conclusions.

4. Mathematicians in doing mathematics realize that "the products" of mathematics are provisional and that

> Truth in mathematics is that for which the majority of the community believes it has compelling arguments. In mathematics truth is socially negotiated, as in science.
>
> (Alan Schoenfeld, 1990, p.9)

Some studies done, e.g. Magdalene Lampert's (1990) case study of teaching and learning about knowing mathematics, explore whether it might be possible to make knowing mathematics in the classroom more like knowing mathematics in the "discipline". This case study, based on an activity of fifth graders (standard 3), illustrates several patterns of teacher and student interaction as well as student-student interaction. Problems are given to the students to do - how to get the answers is not explained, neither is the correctness of answers certified by the teacher. Finding the answers to problems posed by the teacher is not of importance. The real issue is developing mathematically legitimate strategies for finding the answers. To put it in Magdalene Lampert's own words:

> The content of the lesson is the arguments that support or reject solution strategies rather than finding the answers.
>
> (Magdalene Lampert, 1990, p.17).

Consequently Magdalene Lampert's students have to make their solution strategies public in the classroom for the correctness or not of these strategies to be debated by the class. Students' answers are put on the board - with a student's name often put next to his/her answer. The mathematical tools used by the students in these debates are: symbols and language (for communication and reasoning) and definitions and assumptions as they are negotiated. Also, in order to have these debates take place in a respectful (and humble) manner, the language she taught them to use if they disagree with an answer is, "I want to question so-and-so's hypothesis" (p.18).

Magdalene Lampert (1990) starts a lesson on the operation of exponentiation by challenging the class with the question: "How can we figure out the last digit to 5^4, 6^4 and 7^4 without multiplying?"

In trying to solve the problem, the students engage in suggesting various hypotheses about how to figure out the last digit. By comparing and debating answers the students test the legitimacy of these mathematical hypotheses. Next, in order to get the students to speculate whether their negotiated hypothesis would hold in a larger domain, Magdalene Lampert asks them to think about what the last digit of 7^5 would be. The discussion that follows this question is a zig-zag between proofs that the last digit should be 7 and refutations of alternative conjectures. Finally, reacting to a remark made by one of the students (after he had revised his thinking), she steers the students into extending the hypotheses they had been developing about how exponents work into a different domain: she elicits the response that 7^4 squared is the same as 7^8, thus $7^8 = 7^4 \times 7^4$, and gets the students to focus on more examples based on the general form: $7^{2n} = 7^n \times 7^n$.

Thus, based on the aforementioned theoretical arguments as well as the illustrative examples of ways of doing mathematics, a constructivist approach to teaching mathematics entails the following:

1. The classroom activities should be structured in a manner that will engage the students meaningfully and purposefully. To me this means that students should bring their own problems to class or be provided with situations from which they can develop their own problems. If the teacher provides the problem(s) care should be taken that the problems are such that the students can identify with them. In addition, problem situations should include the recognition of patterns.
2. As far as possible the learning process should not be allowed to take place individually and in isolation. Every student should be given the opportunity to digest, interpret and reflect on the "new" knowledge but at the same time be able to engage in discussion, reflection and interpretation with others (either as small groups or as a class). In this way meanings of concepts or solution strategies can be debated.
3. Teachers should refrain from certifying results. Through effective guiding (or mediation) controversial issues could be deflected back to the class (or group) for resolution or further investigation.
4. Teachers should pay careful attention to students' responses to try and determine "how" the responses were arrived at; the reasoning or the line of

argument is what is of importance in any learning experience.

5. Besides the usual mathematical tools (concepts, symbols, abstraction and so forth) students should be exposed to, and be made familiar with the other mathematical tools which might be useful in solving problems. Here I want specifically to refer to the use of counterexamples for refuting assertions.
6. For meaningful dialogue to take place all the participants, students as well as the teacher, should at all times be encouraged to respect each others' viewpoints.

At the beginning of this chapter I expounded on constructivism as a theory of knowledge acquisition. However, as was pointed out, since constructivism is not a theory of teaching, possible practical applications of what might be called a "constructivist approach to teaching" were also discussed. To what extent my teaching practice, which, as will be recalled, was clarified in the previous chapter, "fits" a constructivist approach to teaching, will be investigated in the next chapter.

CHAPTER 6

CONSTRUCTIVISM AND MY CLASSROOM PRACTICE

6.1 An Analysis of my Teaching Approach in terms of the Theoretical and Practical Underpinnings of Constructivism

The second and third examples of teaching episodes, described in Chapter 4, seem to illustrate that during the learning encounters the students were actively engaged in constructing their own mathematical knowledge and understanding. The problem arises when, after having recognised the existence of a pattern and upon reflection having organised their cognitive experiences, <u>language</u> has to play a crucial part in the generalizations - as is illustrated in Example 1. The first student when she says "But a minus and a plus is a <u>minus</u>", gives <u>her</u> rule or generalisation regarding the addition of integers of different signs. What is interesting is that she uses the same terminology when designating the sign of the product of integers with different signs. In my view, constructivism does not give the influence of <u>language</u> enough credit in the meaning making process. To me it is clear that if the individually constructed meanings are not shared and negotiated with other learners, wrong perceptions could manifest themselves easily - this view seems to support Paul Ernest's notion of social constructivism. Also, from this first example it is evident that the second student had not only constructed her own understanding of addition and multiplication of integers but was also able to analyze the reasoning of the first student. In addition she exhibited intellectual modesty when she said "I think that should be plus 3xy", though she very well realized that the assertion by the first student that it was minus 3xy was wrong.

My reaction, as the teacher, to the first student's admittance, in Example 1, that the minus signs confused her, was to make two suggestions: first, that we pay attention to the language we use when referring to multiplication and addition. And second, that we - the first student and I - explore her "thinking about" operations on negative and positive integers to find out whether her ideas about how they worked were congruent with mathematical conventions. This invitation I extended to the rest of the class for two reasons: first, more students might still have had similar problems (as was noted to be the case) and second, because I did not want to undermine the courage she exhibited by volunteering to do the problem on the board. The fear of being humiliated in public and the notion of being the only one not able to do mathematics were still very real and relevant issues. Furthermore, in the first example, the first student's reason for her answer also illustrates the greater importance of the reasoning that leads to an answer - rather than the answer itself.

In the second example the students were again required, not only to recognize the "patterns" which emerged from their geometrical constructions, but also to interpret their observations by extracting the mathematics involved. This lesson clearly exhibits that the students were learning about the nature of mathematical knowledge. First, the student who disagreed with the assertion "two angles and a side of one triangle constructed equal to two angles and a side of another triangle, will make the two triangles congruent" presented evidence, in the form of a counterexample, that the assertion could not be true. Second, when the third group admitted that "although we think the triangles should be congruent ... in number 3 we were able to construct two triangles ..." was a clear indication that they realized that truth in mathematics is established by gathering or producing evidence that will either support or disprove an assertion. In view of the counterexamples, and after reflection, the students were able, in both cases, to modify their assertions. Also, after negotiating the language to be used regarding the case of congruency involving two sides and an angle, the students decided that the word that communicated the "meaning" best was "tussen-in" hoek ("in-between" angle). Thus, in the process of meaning-making, the students were constructing their own socially accepted language to communicate the mathematics they had learned, again exhibiting mathematics as a product of human inventiveness and proving to themselves that they could also do mathematics.

The role that I took as teacher, in relation to the students' struggle to construct a triangle with the three sides as given, was to intervene and to give direction. Perhaps this is opposed to a constructivist view of teaching, and perhaps the students' would eventually have figured out the principles involved in constructing such a triangle. But, theory is one thing and practice another and, as teachers, we also have to accept the responsibility of making choices. I chose to mediate, first, because of the limited time we had at our disposal and second, because I believed that to leave them to struggle on their own, apparently unsuccessfully, would have directed their attention away from the mathematics they were doing to their inability to find a solution to the problem. Consequently, I believe, the confidence they had gained in their own mathematical abilities would have been undermined.

In Example 3 the students not only claimed ownership of the problems they constructed but also of the mathematics that evolved from these problems. This not only "humanized" the mathematics but also made it more meaningful. In solving the problems the students made use of knowledge gained through prior experience and which they brought along with them to the lesson. Their arguments were verbalized and then symbolised. By reflecting on the different operations which emerged, they were able to construct generalizations which appeared to be viable strategies for solving equations. Also, as noted earlier, this lesson transpired during the latter part of the year and the students had come a long way from expecting the teacher to say what was acceptable and what was not, to mostly revising their ideas amongst each other. This change in view of where the mathematical authority resides is illustrated, for example, by Gertrude's response "It will matter because we will have to times 3 by 9 also", to Welma's "Perhaps it does not matter". Furthermore, the mathematics involved was developed from the <u>students'</u> way of thinking and reasoning when solving similar problems "intuitively", rather than being presented as a fixed body of content.

Thus, judging from the above analysis, I unconsciously implemented some of the theories regarding the nature of mathematics, mathematics learning and teaching and what it means to do mathematics in my teaching practice. A social constructivist approach to teaching mathematics seems to underpin my teaching practice. But did this bring about a change in students' attitude towards mathematics? This is discussed in the next section.

6.2 Students' Attitudinal Change towards Mathematics

It has always been my practice to appraise my teaching strategies by means of a questionnaire administered to the students at the end of a course. This year (1993) eleven of my thirty students completed the questionnaire (Appendix 4). The analysis of their responses to the question: "Did your 'feeling' towards mathematics change somewhat?" yielded the following (the English translation of the responses is quoted):

6.2.1 No change in attitude.

Only one student claimed that her attitude towards mathematics had not changed. She ascribed her unchanged attitude to the fact that her last contact with formal mathematics learning was twenty years ago and, even then, she experienced problems. However, this student, through an answer to another question in the questionnaire, contradicted her emphatic denial of a change in attitude towards mathematics. She conceded that, regarding the lessons, she had enjoyed working in a group because "Where I worked amongst my peers I enjoyed it, at times I understood and at times I also voiced my opinion."

6.2.2 A change in attitude - with reservations

Two of the students, though acknowledging a change in their attitude towards mathematics, expressed reservations. The one student stated that "Although I do not feel myself 'mature' enough for the 'new mathematics', I should definitely fare better." The other student claimed: "I enjoyed the experiences I gained in class, but the fear I have of mathematics examinations is perhaps permanent. Perhaps I shall continue to be afraid of mathematics. I do not think I have developed enough self-confidence."

6.2.3 A change in attitude - without reservations

One of the two students falling in this category, explained her change in attitude towards mathematics as follows: "Interest in mathematics was stimulated. See subject from a better perspective and understand much better."

6.2.4 A change in attitude towards mathematics and the teaching of mathematics

Two of the students could be placed in this category. Their elaborations included the following: "...is now aware of the importance of establishing a thorough understanding of concepts as well as applying methods which will bridge the fear of mathematics" and "My interest in the subject intensified. Where at first I avoided teaching a mathematics lesson I shall be able to attempt one now."

6.2.5 Attributing attitudinal change to the attitude and teaching practice of the teacher

Three of the students, while conceding a change in their attitude towards mathematics, attributed this change to the image projected by the teacher. Their comments included: "... being able to communicate with the teacher"; "Your whole attitude and tolerance had a calming effect on me and this gave me a feeling of love for the subject" and "Your manner and methods contributed a great deal to my understanding of mathematics."

6.2.6 Claiming a definite change in attitude

One student was most emphatic about having experienced an attitudinal change towards mathematics. This student, while attributing her gain in self-confidence to the image projected by the teacher, ascribed the fulfilment of her learning needs to the fact that she could freely express her opinion.

The above analysis of the students' responses corroborate my observations during the course of the year: these students - like other students in the past - did experience a change in attitude towards mathematics. Also, judging from the responses, it is evident that a social constructivist approach to teaching mathematics contributes to allaying students mathematics anxiety.

As I went about trying to analyze my teaching practice I realised again how important it is for one not only to enjoy one's teaching - as I do, at least most of the time - but also to grow in one's teaching. This realization immediately evoked the question: How can "growth in teaching" be facilitated? Thoughts on this question gave rise to another question: How can one be a practising teacher and a researcher at the same time? These questions I shall try to address briefly in the next section.

6.3 The Teacher as Researcher

In order to "grow" as a teacher I have always, besides administering questionnaires at the beginning and at the end of the year, engaged in reflection after every teaching episode. The questions I asked myself included the following: Which aspects of my teaching could be improved on? Which aspects of the lesson did the students enjoy? Did the desired learning take place? This reflection in the past, however, always just focused on how I experienced the teaching and learning activity. It was a one-sided reflection on my teaching and on the students - not with them on my teaching and their learning.

Doing research for this mini-thesis has changed my perspectives. I now argue that collaborative critical reflection, brought about by ongoing consultation and dialogue, will not only empower me as the teacher educator but will also result in the empowerment of the students. The empowerment of the students resulting not only from their ability to do mathematics but also from being active in their own learning by providing possibilities for creating alternatives to teaching approaches which have a direct bearing on <u>their</u> learning of mathematics. Making a meaningful contribution towards matters affecting their understanding and doing of mathematics might also help reduce the feelings of "helplessness", "fear" and "anger" which manifest themselves at any encounter of mathematics. Thus, instead of a passive acceptance of whatever the teacher, as authority, thinks their needs are, the students can actively partake in the decision making regarding their own learning needs.

Finally, as a teacher educator, I believe that the chances are good that the students, by being active participants in "action research" (doing research in the classroom and reacting to the findings) in my classroom, will want to change those things that are unsatisfactory in their classrooms.

6.4 Conclusion

The purpose of this mini-thesis has been to analyse my approach to teaching mathematics, and especially to teaching mathematics to mature female practising teachers, to determine in which way my teaching attributed to a change in attitude towards mathematics and the learning of mathematics.

In my teaching I assumed that by simply telling the students that doing mathematics is not difficult and that they can all do mathematics, would not change their attitude towards mathematics or give them confidence in their mathematical abilities. I also assumed that any effort to change students' conceptions about mathematics and the learning of mathematics depends largely on creating learning situations which challenge their conceptions.

The examples of lesson episodes provide convincing evidence that my students were doing mathematics and were gaining confidence in their mathematical abilities - relying less and less on me as the teacher to evaluate their competence in mathematics. Their changing attitudes towards mathematics were also disclosed by their responses to the questionnaire (Appendix 4) discussed in section 6.2. Also, if their results in the final examination can be taken as a measure of the mathematical knowledge they acquired, then I must conclude that my students were indeed learning mathematics. All the students, except one student who withdrew from the course, scored more than 35%.

What has again become evident to me is the role the time factor plays in one's approach to mathematics teaching. With more time at my disposal I could have left the students to pursue the solution to a problem on their own for much longer before intervening; I could have held more interviews with individual students; and I could have generated more practise. In my view, all the above can be achieved by changing the curriculum, with teachers of mathematics actively involved in the change.

One of the questions that this study could not go into and that needs to be pursued by further research is the effect fear of examinations, especially mathematics examinations, has in the alienation of mature, female, practising teachers from mathematics. Another question that needs to be addressed in further research is whether the students who indicated that they intended continuing with mathematics - thus specializing in teaching mathematics - will carry some of their experiences of what mathematics is all about to their own mathematics classrooms.

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APPENDIX 1

DEPARTMENT OF INTERNAL AFFAIRS

CAPE TOWN 1982

DIPLOMA IN EDUCATION

SYLLABUS

MATHEMATICS
(ELEMENTARY)

AND

MATHEMATICS
(SENIOR PRIMARY)

AIMS

1. To give the student a thorough knowledge, insight and skill in the basic mathematical principles

2. To extend the student's background, to deepen his insight and to develop his appreciation of the logic and structure of Mathematics, in order to equip him to teach Mathematics with confidence and enthusiasm

3. To develop the student's ability to exercise independent thought in problem solving and to encourage students to ensure that this ability is transmitted to their pupils

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REMARKS

1. The arrangement of the content of the syllabus and sub-sections thereof are not necessarily an indication of the order in which it should be presented

   Wherever possible the various sub-sections should be integrated

2. The subject matter mentioned in this syllabus should be presented at such a level that students will not encounter any problems when teaching the corresponding subject matter in the syllabuses of the primary school

3. Where possible the subject matter and the method of presentation should be dealt with simultaneously

4. With the presentation of the course the students should be given ample opportunity for active participation and individual work

5. In this syllabus the basic requirements are set out

   Enrichment is left to the discretion of the lecturer

6. The introduction of a tutorial system is recommended

   One period per week should be set aside for the purpose of tutorials

   Tutorials must afford opportunity for the consolidation and application of mathematical concepts by means of assignments which students must do under supervision and guidance of the lecturer

   At all times the purpose of the tutorial must be to identify and solve the problems of individual students

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SYLLABUS

FIRST YEAR

MODULE 1

1. SETS

1.1 The concepts

Set; elements of a set; cardinal number; pairing; one-to-one mappings

1.2 Symbols and notation including simple cases of set builder notation

1.3 Special types of sets: finite and infinite sets; Universal set; empty set; subset; proper and improper subset; equal and equivalent sets; disjoint sets

1.4 Concepts of the following operations applied to sets with simple direct applications

Union, intersection and complement

1.5 Representation of the above operations using Venn diagrams (shading limited to two subsets of a Universal set)

1.6 Logic

Terms: all the; some of; none; contain; does not contain; or; and (intuitively only)

2. NUMBER CONCEPT

2.1 Number sets and their graphical representation on the number-line; natural numbers; counting numbers; integers (directed numbers); rational numbers; irrational numbers and real numbers

2.2 Geometric number patterns

Even and odd numbers; square numbers; triangular numbers; rectangular numbers; prime numbers and composite numbers (divisible numbers)

2.3 Subsets of counting numbers and their properties:

Even and odd numbers; factors; multiples; prime numbers; composite numbers; powers; indices

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2.4 Division tests

2.5 Concept and calculation of hcf and lcm using sets and factors (with simple applications of these two concepts to daily situations)

2.6 Knowledge of the Egyptian, Greek, Roman and Hindu-Arabic number systems

2.7 Number and place value principle in the Hindu-Arabic number system

3. BASIC OPERATIONS WITH COUNTING NUMBERS

3.1 Concept of the operations addition, subtraction, multiplication and division

3.2 Properties of the operations

Closure; the role of the commutative, associative and distributive properties in the algorithms of the basic operations must be clearly indicated; inverse element; identity element

4. COMMON AND DECIMAL FRACTIONS

4.1 Notation and symbols

4.2 Graphical representation

4.3 Equivalent fractions

4.4 Four basic operations

Addition; subtraction; multiplication and division (with combinations of these operations)

4.5 Recurring decimal fractions

4.6 Rounding off (approximations)

4.7 Percentages expressed as decimal fractions

4.8 Conversions of common fractions; decimal fractions and percentages

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4.9 Application to physical quantities (money, mass, length)

5. OPERATIONS WITH INTEGERS (DIRECTED NUMBERS)

5.1 Concept and notation

Distinction between positive and the plus operation and between negative and the minus operation

5.2 Four basic operations

Addition; subtraction; multiplication and division (with special reference to the use of the number line)

6. MONEY; MASS; MEASURE AND TIME

6.1 SI units as required for the primary school

6.2 Four operations applied to monetary units with specific references to commercial transactions

6.3 Different time units including the calendar; international time; modern notations; calculation of intervals between two dates

6.4 Four basic operations applied to mass and measurement units (simple calculations)

7. RATIO AND PROPORTION

7.1 Concept of ratio and proportion

7.2 Comparison of quantities

7.3 Division in a given ratio

7.4 Increase and decrease

7.5 Direct proportion

8. COMMERCIAL TRANSACTIONS

8.1 Profit and loss

8.2 Discount and commission

8.3 Simple interest

GEOMETRIC CONCEPTS

9.1 Description of: point; line; line segment; ray; plane; horizontal; vertical; perpendicular; angle; types of angles; degree; parallel lines

9.2 Triangle (types); polygons; quadrilaterals; rectangle; square; rhombus; kite; parallelogram

9.3 The following pairs of angles: adjacent; vertically opposite; alternate; corresponding; complementary; supplementary

9.4 Axioms and theorems (should be obtained (intuitively) using calculations and experimental approaches)

9.4.1 If two lines intersect then the sum of any pair of adjacent angles is equal to 180°, and the converse

9.4.2 If two lines intersect then the vertically opposite angles are equal

9.4.3 If two parallel lines are cut by a transversal then the corresponding angles are equal

alternate angles are equal

sum of the co-interior angles is equal to 180°

9.5 Simple applications of these geometric concepts and axioms (theorems)

10. CONSTRUCTION AND MEASUREMENT

10.1 The use and handling of geometric instruments

10.2 Line segments

Perpendicular; bisector; parallel lines

10.3 Angles

Bisecting and doubling of angles

10.4 Construction of unique triangles: (S;S;S); (S;∠;S); (S;S;Rt∠) (∠;∠;S)

10.5 Construction of non-unique triangles (S;S;∠) and (∠;∠;∠)

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10.6 Cases where construction of triangles is not possible

10.7 Construction of rectangles and squares

10.8 Simple scale-drawings of squares; rectangles and right-angled triangles

11. SHAPES

11.1 Concepts: perimeter, area and their units

11.2 Determination (practical) of the formulae for the calculation of the perimeter and area of the following discs: rectangle ; square ; rhombus; parallelogram; triangle and kite

11.3 Concepts: volume and content and the units

11.4 Determination (practical) of the formulae for the calculation of the volume of right prisms with square and rectangular bases

11.5 Only direct applications (with rational values) of the above formulae

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SECOND YEAR

MODULE I

1. SETS

1.1 Properties: Associative; Commutative; Distributive; De Morgans Law

Presentations of these properties using Venn Diagrams and shading

1.2 Solving problems using Venn diagrams (limited to 3 sets)

1.3 Sets of number pairs; ordering; Cartesian product; Cartesian system of axis; Cartesian plane;

Relations as subsets of the Cartesian product; range; domain; graphical representation of ordered number pairs

2. ALGEBRA

2.1 Interpretations of Algebra; Generalized Arithmetic; Written Geometry; and a Symbolic Language

2.2 Value; importance and historical background of Algebra

2.3 Important concepts: terms; expressions; factors; coefficients; expressions; degree of an expression; powers (ascending and descending); bases; indices

2.4 Operations: Addition on Subtraction of algebraic terms and expressions

2.5 Multiplication

2.5.1 Expressing to a power

2.5.2 Multiplication of a monomial with a polynomial

2.5.3 Multiplication of a binomial with a polynomial

2.5.4 Use and removal of brackets

2.6 Division

2.6.1 Division of a polynomial by a monomial

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2.6.2 Division of a polynomial by a binomial (long division)

2.7 Products and factors

2.7.1 Products of the following types (by inspection)

(i) $(ax \pm b)^2$

(ii) $(ax \pm b)(cx \pm d)$

(iii) $(ax + b)(ax - b)$

2.7.2 Factorization of polynomials of the following types:

(i) common factors

(ii) grouping: $ax \pm bx \pm ay \pm by$

(iii) difference between two squares: $a^2x^2 - b^2y^2$ where a and b are both monomials

2.8 Determination of Lcm of polynomials using factors (using the factors in 2.7.2)

2.9 Simplification of algebraic fractions (limited to two terms) using the factors 2.7.2(i) and 2.7.2(iii)

2.10 Substitution into formulas (using only rational values)

2.11 Changing the subject of a formula

2.12 Solution sets

2.12.1 Open and closed number sentences and phrases; concept of solution set; definition set

2.12.2 Solution of linear equations in one variable by inspection

2.12.3 Solution of linear equations in one variable with numerical coefficients with the aid of the Balance Algorithm

2.12.4 Solution of linear inequalities in one variable with numerical coefficients

2.12.5 Graphical illustrations of the above solution sets

3. ARITHMETIC

3.1 Compound interest;

calculation of interest and final amount (compounded yearly to a maximum of three years)

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3.2 Ratio and proportion

3.2.1 Direct and inverse proportions with graphical representation

3.2.2 Calculation of proportions: first; fourth; middle (mean) and third

3.3 Calculation of roots

3.3.1 Square roots and cube roots using factors

3.3.2 Square root calculation using the division method (answer correct to the first decimal point)

3.4 Number systems with other bases

3.4.1 Place value systems; notation and numeration of number systems with other bases; base two and base five

3.4.2 Conversions of decimal numbers to binary and quinary number systems and vice versa

3.4.3 Addition and subtraction of numbers in base two and base five

3.4.4 Application of the binary number system

4. GEOMETRY

4.1 Scale drawings which include the following

4.1.1 Clockwise bearings relative to the northerly direction

4.1.2 Compass bearings relative to the N-S axis

4.1.3 Angles of elevation and depression

4.2 Elementary and practical knowledge of geometric transformations: translations; rotation; reflections; symmetry (with reference to th square; rectangle and equilateral triangle)

4.3 Axioms and Theorems (without formal proof)

Practical and experimental approach should be followed

4.3.1 The sum of the interior angles of a triangle is equal to 180°

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4.3.2 The exterior angle of a triangle is equal to the sum of the two opposite interior angles

4.3.3 If two sides of a triangle are equal than the angles opposite these sides are equal

4.3.4 If two angles of a triangle are equal than the sides opposite these angles are equal

4.3.5 The opposite sides and angles of a parallelogram are equal

4.3.6 The diagonals of a parallelogram bisect each other

4.3.7 Simple applications of the above axioms and theorems

4.4 Triangles (practical and experimental approach)

4.4.1 Congruency of triangles (simple applications)

4.4.2 Theorem of Pythagoras (without formal proof)

Direct applications of the theorem

4.5 Introduction to Circles using the following terminology

centre; radius; diameter; chord; circumference; arc; secant; tangent; concentric circles

5. SHAPES (PRACTICAL AND EXPERIMENTAL APPROACH)

5.1 Circumference and area of circular discs

5.2 Calculation of the outer surfaces and volumes of right solid cylinders

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MODULE 2

SUBJECT DIDACTICS

The teaching of the subject content of the school syllabus for Mathematics Standards 2 to 4

1. The pupil

1.1 The background and milieu; the physical and psychological development; the level of thought, reasoning and experience of the normal primary school pupil and how these influence the presentation of Mathematics to pupils

1.2 Differentiated teaching in the primary school; gradation; individual learning rate

2. Aims of teaching Mathematics

2.1 Related to syllabus content: critical appraisal

2.2 The specific objectives in studying certain subsections of Mathematics in the various standards

2.3 Individual lesson aims and objectives

Variation from lesson to lesson, dependent on factors such as subject subsections, lesson content, lesson presentation

3. Lesson Structure

The following aspects may be considered as the basic elements according to which lessons may be structured

3.1 Lesson detail:

School
Standard
Composition of class
Subject
Subsection of subject
Lesson topic
Date
Time
Duration

3.2 Lesson notes

General Aims
Specific aims
Introduction and correlation with previous knowledge
Presentation of new subject matter
Assessment
Application

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3.3 Wherever applicable attention should be given to

Didactic form and principles
Method of development
Method of teaching
Learning and teaching methods
Remedial methods
Application
Material for exercises

The above is the general plan for a lesson when students are required to prepare carefully-planned lesson notes

4. Subject matter and syllabus (Standards 2 to 4)

4.1 Development and structuring of the syllabus

4.2 Role and function of the syllabus as official instruction

4.3 The manner in which the syllabus, as official instruction, is implemented

Use of aids such as textbooks, reference books and subject meetings

4.4 Study of the senior primary syllabus in Mathematics with a view to selection (choice of subject matter)

Division and arrangement of units of subject matter according to standard and groups of pupils

Progression and development of themes

5. Scheme of work - Standards 2 to 4

5.1 Necessity and purpose of a scheme of work

5.2 Planning the scheme of work (responsibility of the teacher, role of the syllabus, and the role of textbooks and reference works as aids in planning a scheme of work)

5.3 Planning and form of a scheme of work

Division of work, sequence and arrangement

Detailed planning, control, modification, completion of planned work

6. Record of completed work

6.1 Necessity and purpose; official prescriptions; control; reporting procedure

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7. Preparation

7.1 Form and control; written preparation (lessons, exercises, texts, revision)

7.2 Preparation with respect to subject matter and teaching methods

7.3 Preparation of teaching and learning aids

7.4 Preparation of pupils and by pupils for a particular lesson or lesson material

7.5 Homework: planning, control; keeping records

8. Teaching Aids and Writing Board Technique

8.1 Use of the writing board; setting out work; drawing and construction

8.2 The value and use of teaching aids in teaching Mathematics; technological aids

8.3 Construction of teaching aids

8.4 Discerning use of textbooks and appropriate reference works including periodicals

9. Evaluation

9.1 The necessity and purpose of evaluation

9.2 Testing (class tests)

9.2.1 Drafting a question paper; memorandum of marking, recording and records of marks

9.2.2 Diagnosis and remediation

9.3 Evaluation of homework

Necessity and various procedures

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STUDENT 1

APPENDIX 2

Bespreek u 'vrees' vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Dat ek die werk nie sal kan baasraak nie.
Dat ek skaam is vir vernedering as ek iets verkeerd gaan antwoord.
Dat mede studente 'n gek van my sal maak.

(b) Moontlike oorsake van die 'vrees' te gee. Het dit dalk iets met u geslag te doen?

Onsekerheid omtrent my vermoë m.b.t. my kennis.
Baie reëls het verander. Verskillende benamin[g]
Deesdae verskillende benadering tot werk dan voorheen toe ek op Kollege was.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf.**

As mens beskou ek myself as persoon wat nooit akademies in Wiskunde sal presteer nie. Gebrek aan selfvertrou[e]
As student is ek van mening dat ek altyd in wiskunde eksamen onder druk sal aflê omdat ek aan my vermoëns twyfel
As onderwyser was skaam vir vernedering om saam met ander leerkragte mee te ding.

STUDENT 2

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Onsekerheid.

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

As gevolg van die feit dat 'n termyn van 22 jaar laas met wiskunde te doen gehad het, was ek nie seker dat ek ooit die Wiskunde sal kan baasraak nie. Alle vorige Wiskunde onderwysers het vrees vir die vak by my ingeboesem. Ek kon nooit werklik sê wanneer ek iets nie verstaan nie.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf**.

Alle (sommige) studente het vertel hoe moeilik Wiskunde is. Ek kon ook nie altyd die Wiskunde terme verstaan nie. Toe ek die klas die eerste dag betree was ek onseker, senuweeagtig en bang vir die Vak. U aanbieding en verduideliking het my egter baie op my gemak gestel. My verhouding tot Wiskunde het heeltemal verander en ek vind dit nou aangenaam.

STUDENT 3

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

VREES VIR WISKUNDE KOM AL VANAF PRIMêRE SKOOL. JY AS PERSOON WAS BANG OMDAT DIE LEERKRAGTE STRENG WAS. JY WAS TE BANG OM N ANTWOORD TE GEE AS DIT VERKEERD WAS, WAS JY GESTRAF.

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

NEE, DIT HET NIKS MET DIE GESLAG TE DOEN NIE.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf**.

EK PERSOONLIK WAS "BANG" EK HET NOG NOOIT WISKUNDE KLASSE SO GENIET SOOS NOU NIE. EK HET GEDINK DAT DIE DOSENT DIE LESSE NET GAAN AANBIED OF JY VERSTAAN OF NIE. GELUKKIG IS DIT NIE SO NIE. SY VERDUIDELIK DIE WERK OP SO N MANIER DAT JY DIT MOET VERSTAAN. N GRAP WORD SOMS GEMAAK AS JY NIE VERSTAAN NIE. OP SKOOL WAS DIE GESIGSUITDRUKKING VAN DIE AANBIEDER, STUURS EN NORS. DIE MANIER VAN AANBIEDING VAN DIE DOSENT SPEEL N GROOT ROL

STUDENT 4 Patricia Cornelius

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Net gewonder of ek nog kan onthou wat ek op kollege en op skool aangaande Wiskunde geleer het.

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

Ek het 'n "vrees" vir Wiskunde ontwikkel toe ek op hoërskool kom. Die Wiskunde-onderwyser het gewoonlik net een keer 'n som verduidelik en dan het hy verwag dat jy die volgende dag die som moet verstaan.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf**.

Ek het net gevrees dat ek nie gou genoeg som snap om by te kan by die ander studente nie.

STUDENT 5

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Onsekerheid,
Bevrees dat as ek antwoord dit
verkeerd sal wees.

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

Wiskunde - onderwyser was baie
streng. Kon nooit sê as jy nie
verstaan met die gevolg het ek
ń afkeur in Wiskunde gehad in
St. 8.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf**.

U Rustige stemtoon en rustige
manier van les aanbieding het my
laat ontspan.
Geduld wat u aan die dag lê deur
elkeen individueel te verduidelik wat
nie mooi verstaan nie, het my laat
besef Wiskunde is glad nie so aaklig
nie, maar heel aangenaam.

Christa van Wyk.
STUDENT 6

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Bang — ek moes die klas bywoon en het daarna uitgeput gevoel. Ek het gevoel om alles te staak maar ek het besef dat wiskunde 'n noodsaaklikheid is.

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

1. Was 24jr. gelede nog in kontak met Wiskunde op die vlak.
2. Die gevolg was dat ek voel my handeling is uiters stadig — 'n verleentheid volg.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf**.

My gevoel — ek was baie onseker en voel kwaad as ek nie my doel kan bereik nie.
Daars nog vrees maar ek voel dat dit oorbrug kan word. Ek begin stadig van die vak te hou veral na 'n suksessie in die klas wat deur myself gedoen is.

STUDENT 7

Bespreek u `vrees´ vir Wiskunde deur ondere andere

(a) die vrees te beskryf

Onsekerheid

(b) Moontlike oorsake van die `vrees´ te gee. Het dit dalk iets met u geslag te doen?

As gevolg van die feit dat 'n termyn van dertien jaar verbygegaan het, sedert ek laas met Wiskunde op sekondère vlak kontak gehad het, was ek nie seker of ek moontlike vernuwingselemente sou kon baasraak nie.

(c) **gevolge van die vrees** - as mens, as student (bv. wat was u gemoedstoestand toe u hierdie wiskunde-klas die eerste dag betree?), as onderwyser, as wiskunde-onderwyser - **te beskryf.**

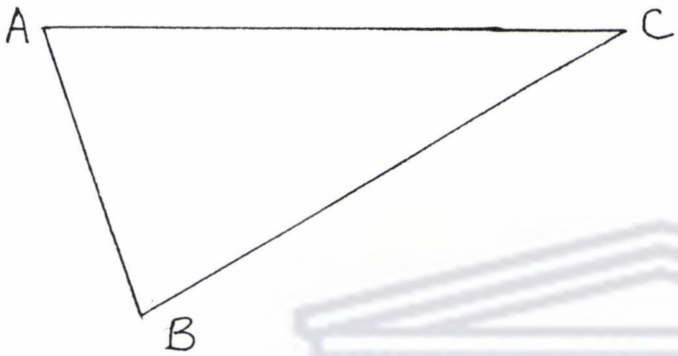
Ek het gehoor dat studente van terme praat waarvan ek nog nie gehoor het nie. Die eerste dag in die Wiskunde-klas het ek onseker en senuwee-agtig gevoel. My vrese is egter tot dusver uitgeskakel en die liefde vir die vak is weereens verstewig.

KONGRUENSIE VAN DRIEHOEKE

AKTIWITEIT 1

Gegee △ABC. Konstrueer nou △PQR met QR = AB, PQ = AC en PR = BC. Knip △PQR uit en pas dit op △ABC. Wat let u op?

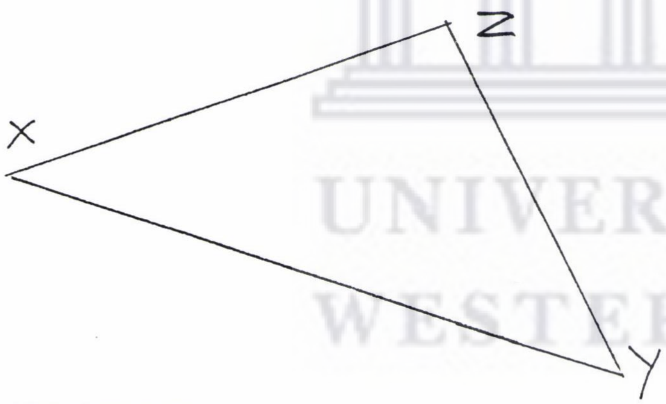
....



AKTIWITEIT 2

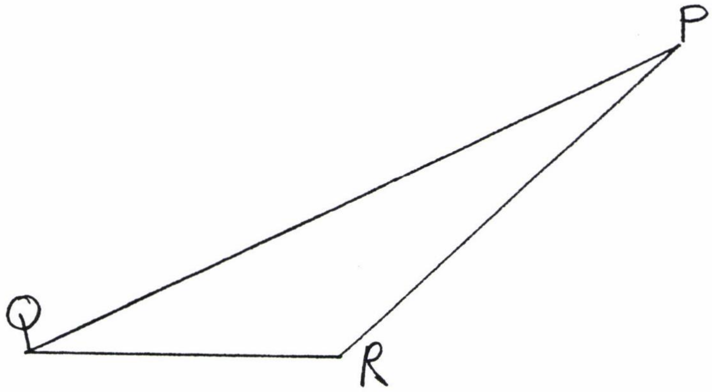
Gegee △XYZ. Konstrueer nou △MNR met MN = XY, $\hat{N} = \hat{Y}$ en NR = YZ. Knip △MNR uit en pas dit op △XYZ. Wat let u op?

....



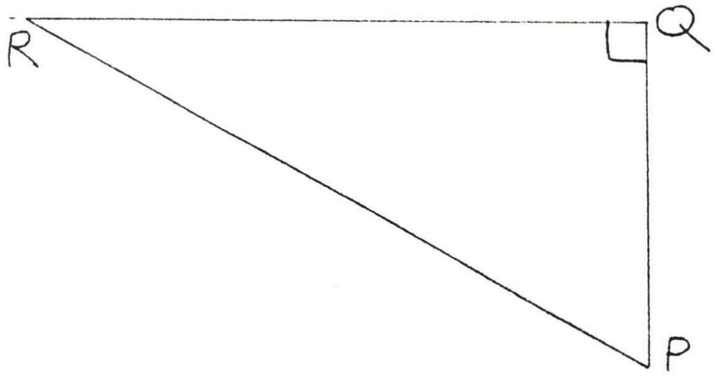
AKTIWITEIT 3

Gegee △PQR. Konstrueer nou △ABC met AB = PQ, $\hat{B} = \hat{Q}$ en AC = PR. Hoeveel driehoeke ABC kan u konstrueer? ....
Knip u driehoeke ABC uit. Pas hulle albei op △PQR? ....



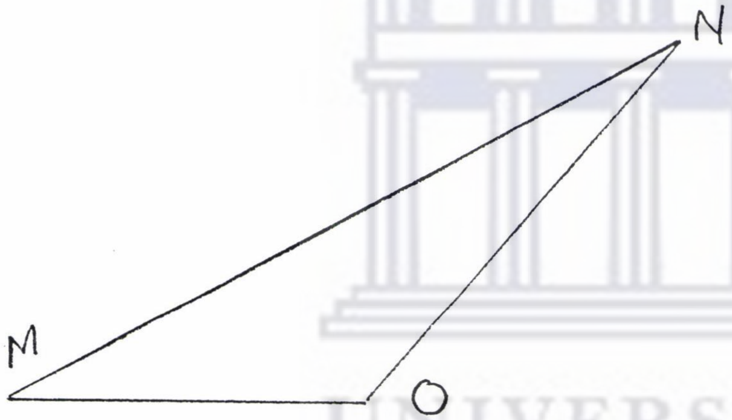
AKTIWITEIT 4

Gegee $\triangle PQR$ met $\hat{Q} = 90°$. Konstrueer nou $\triangle ABC$ met $\hat{B} = \hat{Q}$, AB = PQ en AC = PR. Knip u $\triangle ABC$ uit en pas dit op $\triangle PQR$. Wat let u op? ....



AKTIWITEIT 5

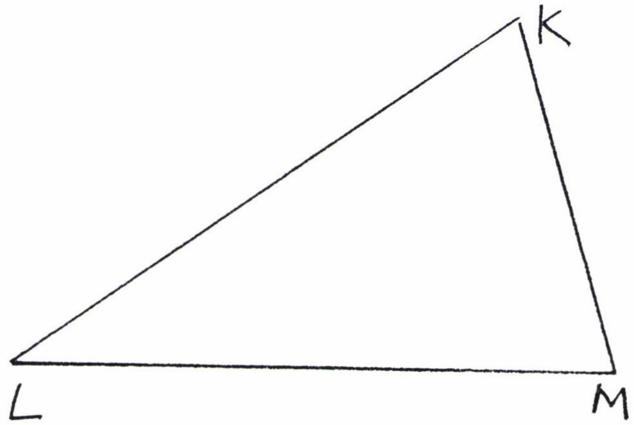
Gegee $\triangle MNO$. Konstrueer $\triangle SPT$ met SP = MN, $\hat{S} = \hat{N}$ en $\hat{P} = \hat{M}$. Knip $\triangle SPT$ uit en pas dit op $\triangle MNO$. Wat let u op?...........



AKTIWITEIT 6

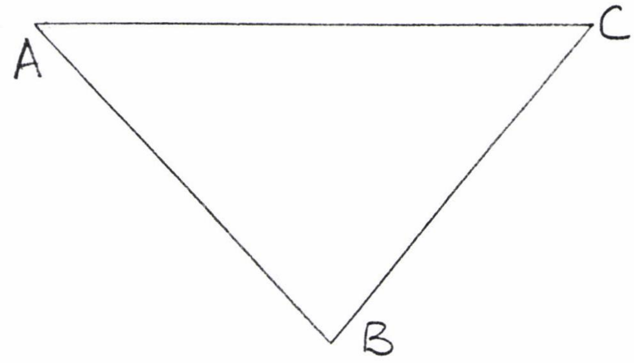
Gegee $\triangle KLM$. Konstrueer nou $\triangle DEF$ met EF = LM, $\hat{D} = \hat{L}$ en $\hat{F} = \hat{K}$. Knip u $\triangle DEF$ uit en pas dit op $\triangle KLM$. Wat let u op?

....



AKTIWITEIT 7

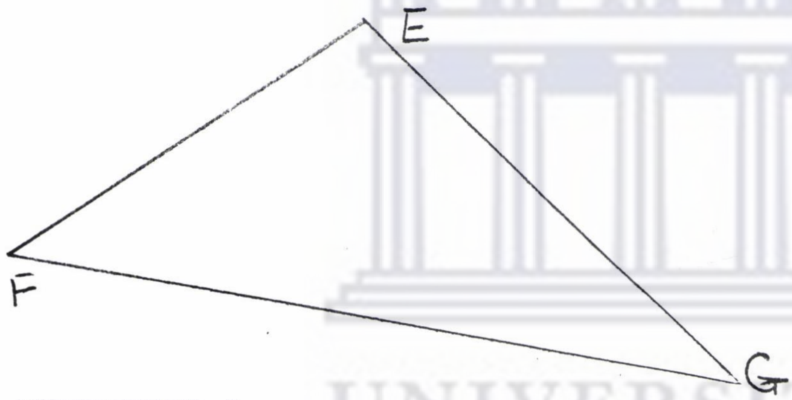
Gegee $\triangle ABC$. Konstrueer nou $\triangle PQR$ met $\hat{P} = \hat{A}$, $\hat{Q} = \hat{B}$ en $\hat{R} = \hat{C}$.
Wat let u op? Hoeveel driehoeke PQR kan u konstrueer?.......
Pas hulle almal op $\triangle$ ABC?.........



AKTIWITEIT 8

Gegee $\triangle EFG$. Konstrueer nou $\triangle HKL$ met HK = EG, $\hat{H} = \hat{E}$ en $\hat{L} = \hat{F}$.
Knip nou u $\triangle HKL$ en pas dit op $\triangle EFG$. Wat let u op?...........

....



AKTIWITEIT 9

1. Vir watter aktiwiteite het u driehoek PRESIES op die gegewe driehoek gepas? Nommer die aktiwiteite.

   ....

2. \`Veralgemeen´ nou die dimensies van die konstruksies in nommer 1 genoem.

   ....

   ....

   ....

   ....

3. Vir watter konstruksies was die driehoeke NIE kongruent nie? Gee moontlike redes.

   ....

   ....

1.

APPENDIX 4

VOORSPOED MET DIE EKSAMENS

Aan die begin van die jaar het van u ′n vraelys, wat gehandel het oor u `gevoel′ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei′ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ′n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: WELMA VISAGIE ......(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae′ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel′ teenoor Wiskunde enigsins verander?..Ja....

(b) Verduidelik

Ek het die ondervinding wat ek in die klas opgedoen het geniet, maar ek dink die vrees wat ek vir die wiskunde eksamen het is seker maar blywend. Ek sal seker maar bang bly vir Wiskunde. Ek glo nie ek het al genoeg selfvertroue opgebou nie.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Die besprekings, praktiese werk.
Omdat ek ′n beter begrip in die klas gekry het.

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Ek glo u aanbieding is goed. Ek het ′n veel beter begrip in u klas gekry as wat ek in ander wiskunde klasse ondervind het. Ek voel net die tyd vir al die werk wat ons moes dek was te min, omdat u baie vinnig, myns insiens, daardeur gegaan het.

2.

VOORSPOED MET DIE EKSAMENS

Aan die begin van die jaar het van u 'n vraelys, wat gehandel het oor u `gevoel' teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei' as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief 'n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

NAAM: ERILENE ADAMS ....(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae' is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u gevoel teenoor Wiskunde enigsins verander? JA.

(b) Verduidelik

Belangstelling in Wiskunde is gestimuleer. Sien vak uit 'n beter oogpunt en verstaan baie beter.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

(i) Besprekings

(ii) Praktiese werk

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Nie juis nie. Aanbieding was goed en maklik verstaanbaar.

3.

- VOORSPOED MET DIE EKSAMENS -

Aan die begin van die jaar het van u 'n vraelys, wat gehandel het oor u `gevoel' teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei' as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief 'n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM:........................(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae' is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel' teenoor Wiskunde enigsins verander?..Ja....

(b) Verduidelik

My belangstelling in die vak het verd op. Ek sal 'n wiskunde lesse kan aanpak waar ek eers weggeskram het daarvoor.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Besprekings, groepwerk: Ek kon hier uitvind waar my probleme was en groepwerk het my gehelp om dit uit te werk.

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Dat u nie te haastig moet wees met diegene wie nie mooi verstaan nie. Baie van die werk kan u ook asb. op meer eenvoudiger maniere stel. Ons sal verstaan.

4.

- VOORSPOED MET DIE EKSAMENS -

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: ...Norma Bacus........ (U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander? ...Ja...

(b) Verduidelik
Verfrissend om kennis op te knoop. Meer insig in die wiskunde op hoërskool en is nou bewus van die belangrikheid van die deeglike vaslegging van begrippe en aanwend van metodes om vrees vir wiskunde te oorbrug

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?
alles - interessant

....

....

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?
Voldoen aan vereistes.

....

....

- VOORSPOED MET DIE EKSAMENS -

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM:...Annie..............(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander?..Ja....

(b) Verduidelik

In die begin was ek vreeslik bekommerd omdat ek 26 jaar gelede Wiskunde gedoen het en bang was dat ek 'n mis oes van die vak sal maak...

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Besprekings en groepwerk

Deur die besprekings en samewerking met die ander het ek dit beter verstaan

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

..Hou....alles....so.......................................

....

....

6.

- VOORSPOED MET DIE EKSAMENS -

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: Ciréne Dänkers ...(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander? JA

(b) Verduidelik

Baie nuwe feite bygeleer. Alhoewel ek nog nie heeltemal ryp voel oor die nuwe Wiskunde nie, sal ek beslis beter vaar met so ´n knap dosent.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Besprekings en groepwerk. Tydens die besprekings en groepwerk kon ons deur andere se menings baie byleer.

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

U aanbieding is puik. Ek geniet alles wat u verduidelik. Knap!!

- VOORSPOED MET DIE EKSAMENS -

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: ....Claire............(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander?...Ja...

(b) Verduidelik

Dit was vir my ´n afskrikmiddel - ´n Vak wat ek weet, ek gaan difinitief druip. Juffrou se hele houding en verdraagsaamheid het ´n baie ~~kle~~ kalmerende houding/effek op my gehad en dit het my gevoel van ´n liefde vir die vak gegee

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

groepwerk en besprekings

....

....

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Hou alles net so

....

....

8.

VOORSPOED MET DIE EKSAMENS

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM:.........................(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander?..Ja.....

(b) Verduidelik

Daar was baie oefening ´n gedoen.

Ons kan kommunikeer met Leerkragte (doserte)

....

....

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

....

Besprekings

....

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

....

Geen.

....

9.

VOORSPOED MET DIE EKSAMENS

Aan die begin van die jaar het van u 'n vraelys, wat gehandel het oor u `gevoel' teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei' as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief 'n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: Ingrid Michaels ....(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae' is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel' teenoor Wiskunde enigsins verander? Ja

(b) Verduidelik

Die manier en metodes wat juffrou gebruik het, het baie bygedra dat ek die werk kan verstaan.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Groepwerk

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Aanbieding uitstekend - Miskien toetsing na elke nuwe werkstuk.

- VOORSPOED MET DIE EKSAMENS -

Ook aan u.
Mooi bly. Liefde
Christa

Aan die begin van die jaar het van u 'n vraelys, wat gehandel het oor u `gevoel' teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei' as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief 'n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

NAAM: Christa van Wyk (U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae' is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel' teenoor Wiskunde enigsins verander? Beslis.

(b) Verduidelik

U openbare beeld boesem selfvertroue. Die feit dat ek my sê mag sê ongeag hoe verbouereerd ek ook mag optree, ek nogsteeds die een was wat deur u bevredig was

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Besprekings en groepwerk. Jy voel nie uitgesonderd waar almal jou dophou. Groepwerk is uitstekend omdat jy deur andere gehelp mag word.

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

Juffrou, minder werk op 'n slag sou effektief wees. Ek weet daar was 'n tydsfaktor maar weens die agterstand voel 'n ek sou sleg omdat u u kant gebring het, ek u nie kon bevredig soos ek graag wou nie.

11.

VOORSPOED MET DIE EKSAMENS

Aan die begin van die jaar het van u ´n vraelys, wat gehandel het oor u `gevoel´ teenoor Wiskunde, vir my ingevul en ingehandig. U response het ek nie net vir my eie studies gebruik nie (dit gaan nou nog maar broekskeur om my verhandeling te voltooi), maar ek kon ook my hantering van ons wiskunde-lesse daarby aanpas. Ek is baie dank aan u verskuldig, en sê dus **baie dankie.**

Vir verdere ontwikkeling en afronding van my verhandeling - asook vir my eie `groei´ as wiskunde-onderwyser - benodig ek weer u samewerking. Knyp tog asseblief ´n paar minute af om die volgende vraelys te voltooi. U kan dit dan sommer Vrydag (29 Oktober) vir my gee of saam met u wiskunde skrifte inhandig.

....

NAAM: NAOMI VAN WYK ....(U kan besluit om anoniem te bly)

Voltooi asseblief die volgende vraelys so eerlik as moontlik. (Die `vrae´ is gebaseer op die response wat ek aan die begin van die jaar ontvang het.)

1(a) Het u `gevoel´ teenoor Wiskunde enigsins verander? Nee

(b) Verduidelik

Omdat ek twintig jaar gelede wiskunde gedoen het, en destyds al probleme ondervind het, is ek heelte -mal negatief ingestel vir die vak. Ek het darem 'n bietjie bygeleer, alhoewel dit soms vir my frustrerend was.

2. Watter aspekte van die wiskunde klasse het u geniet? (Besprekings, groepwerk, praktiese werk, ens.) Waarom?

Groepwerk, praktiese werk. Waar ek tussen ander mense gewerk het, het ek dit geniet, soms verstaan en soms my mening ook gelig.

3. Watter veranderinge sou u wou sien in my aanbieding van die vak?

U aanbieding is puik, en ek kan u net alle sukses toewens vir die toekoms. Dankie dat u so met my gesukkel het.